$K^0 \leftrightarrow \bar{K}^0$ transition amplitude in the MIT bag model

Robert E. Shrock and S. B. Treiman

Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540
(Received 6 December 1978)

Estimates of the $K^0 \leftrightarrow \bar{K}^0$ transition amplitude and the resulting $K_1K_2$ mass difference are often based on an effective-Lagrangian treatment, where $I_{\text{eff}}$ is taken to be the four-quark operator extracted from a lowest-order calculation of the free-quark $s + d \rightarrow s + d$ scattering amplitude. For conventional SU(2) $\times$ U(1) models this effective Lagrangian has the form of a $(V - A)$ current-current interaction $O_{\mu
u}$, with coefficient $C$ parametrized by quark masses and mixing angles. An important question has to do with gluonic and other corrections to the coefficient, but, independent of this, it is also necessary to estimate the matrix element of $O_{\mu
u}$ between physical $K^0$ and $\bar{K}^0$ states. We focus on this aspect of the problem. Earlier treatments have relied on a "vacuum-insertion" approximation. For an alternative approach, we employ the static MIT bag model. With the standard values for the bag parameters the matrix element is smaller than that obtained with vacuum insertion by about a factor of 2. This result is reasonably stable for small variations of the bag parameters.

I. INTRODUCTION

An important problem which arises in the study of nonleptonic weak interactions is the calculation of matrix elements between hadron states of operator products of quark fields. An example of particularly great significance for the development of theories of weak interactions is the calculation of the $K^0 \leftrightarrow \bar{K}^0$ transition amplitude, which determines the $K_1K_2$ mass difference and associated $CP$ violation in the neutral-kaon mass matrix. The extremely small magnitude of the $K^0 \leftrightarrow \bar{K}^0$ amplitude, and thus the $K_1K_2$ mass difference $(m_L - m_S)/m_K \simeq 0.71 \times 10^{-14}$, can be understood as a consequence of the severe suppression of induced $\Delta S \neq 0$ neutral currents. Similarly, an appealing explanation of $CP$ violation in the neutral-kaon mass matrix is provided by the complex phases which can occur, if there are sufficiently many quarks, in the mixing matrix which specifies how the quark gauge-group eigenstates are composed of quark-mass eigenstates, or equivalently, in the charged-current quark couplings to the $W^\pm$ gauge bosons.

Historically, the absence of $\Delta S \neq 0$ neutral-current effects led to the incorporation of the Glashow-Iliopoulos-Maiani (GIM) mechanism in gauge theories of weak interactions. This mechanism guarantees that there is no nondiagonal part to the neutral current at the tree level in the Lagrangian. With the development of such renormalizable gauge theories it became feasible to undertake a one-loop calculation of the $K^0 \leftrightarrow \bar{K}^0$ transition amplitude in terms of quark fields. It was shown$^{2,3}$ that the GIM mechanism also works at the one-loop level to suppress this amplitude to the order $G_Q^2 \Delta m_q^2 \epsilon$, where $\Delta m_q^2$ refers to the difference of certain quark masses squared, and $\epsilon$ is a product of quark-mixing-matrix coefficients. In the treatment customarily adopted, one first computes the amplitude $A$ for the free-quark process $\bar{s}d \rightarrow s \bar{d}$ and then takes this as an effective Lagrangian, $L_{\text{eff}}$, relevant for the problem of the $K_1K_2$ mass difference. For the next step it becomes necessary to estimate the matrix element $\langle \bar{K}^0 | \mathcal{O}_{\text{eff}} | K^0 \rangle$ where $\mathcal{O}_{\text{eff}} = -L_{\text{eff}}$. The operation of the GIM mechanism is incorporated in the first step of the calculation, which yields $\mathcal{L}_{\text{eff}}(\bar{s}d \rightarrow s \bar{d})$. However, the second step is no less important for the overall problem of computing $\Delta m_{K'}$. Unfortunately, this step has the standard difficulties of strong-interaction physics, since the ability to calculate such a matrix element exactly would require a complete theory of the structure and interactions of hadrons. In the absence of such a theory one was led to rely$^{2,3}$ upon an approximate technique of inserting a sum over a complete set of states between the currents in $\langle \bar{K}^0 | \mathcal{O}_{\text{eff}}(\bar{s}d - s \bar{d}) | K^0 \rangle$, and then truncating this sum with the contribution of the vacuum state.

In this paper we shall reexamine the problem of calculating the matrix element $\langle \bar{K}^0 | \mathcal{O}_{\text{eff}}(\bar{s}d - s \bar{d}) | K^0 \rangle$. The general importance of improving the accuracy of the computation of $A(K^0 \leftrightarrow \bar{K}^0)$ is obvious since this amplitude determines the $K_1K_2$ mass difference and $CP$ violation in the neutral-kaon mass matrix. Moreover, there is a particular application which lends some urgency to this problem at the present time, namely the use of these two quantities to set bounds on certain quark mixing angles.

We shall restrict our attention here to theories in which the weak neutral current is diagonal in quark-mass eigenstates at the Lagrangian level. We shall use as our model of weak and electromagnetic interactions the sequential Weinberg-Salam$^4$...
(WS) SU(2) × U(1) theory with three (left-handed) doublets of quarks and leptons, although, as will be clear, much of our analysis is independent of this assumption. This model, first discussed by Kobayashi and Maskawa (KM),\(^5\) incorporates all of the known fermions and seems to be compatible with a wide variety of charged- and neutral-current weak-interaction data. It is not out of the question that additional quarks and leptons will be found; based on the success of the KM version of the WS model, it is reasonable to assume that these new fermions will transform under weak SU(2)\(_L\) × U(1) in the same way as the known ones.

In order to fix notation we exhibit the quark doublets in the WS-KM model:

\[
\begin{pmatrix}

u \\
(d')_L
\end{pmatrix}
\begin{pmatrix}

(c) \\
(s')_L
\end{pmatrix}
\begin{pmatrix}

l \\
(b')_L
\end{pmatrix},
\]

(1.1)

where the gauge group eigenstates \(d'_L\), \(s'_L\), and \(b'_L\) are linear combinations of the left-handed components of the corresponding mass eigenstates \(d_L\), \(s_L\), and \(b_L\), as described by the 3×3 unitary quark mixing matrix \(V\):

\[
\begin{pmatrix}

d' \\
s' \\
b'
\end{pmatrix}_L = V
\begin{pmatrix}

d \\
s \\
b
\end{pmatrix}_L.
\]

(1.2)

(Needless to say, we are presuming that the \(t\) quark exists and will eventually be found in the form of \(tq\), \(tgq\), and \(tT\) hadrons.) The generalization to a world with \(n\) quark doublets is immediate. It is easy to show in the general \(n×n\) case that unitarity of \(V\), together with the usual freedom in redefining the phases of quark fields, implies that \(V\) depends on \((n-1)^2\) (real) parameters, of which \(n(n-1)/2\) are CP-conserving rotation angles and \((n-1)(n-2)/2\) are CP-violating phases. The explicit form of \(V\) for the WS-KM model is given in Eq. (2.7) of Sec. II.

In the free-quark approximation it is a straightforward matter to compute the amplitude for \(\bar{s}d\rightarrow\bar{s}d\). This has the form of a current-current interaction, with coefficient parametrized by quark masses and by the mixing angles that characterize the matrix \(V\). We shall offer some passing comments about the validity of treating this amplitude as an effective Lagrangian; however, the bulk of the paper—accepting that this can be done—is concerned with estimates of the matrix element of \(\mathcal{K}_{\text{eff}}\) between the physical \(K^0\) and \(\bar{K}^0\) states.

The paper is organized as follows. In Sec. II we discuss the "conventional" vacuum-insertion calculation of the \(K^0\rightarrow\bar{K}^0\) amplitude and associated \(K, K_S\) mass difference and CP violation. In Sec. III the contribution of the one-pion intermediate state to the matrix element \(\langle K^0|\mathcal{K}_{\text{eff}}(s\bar{d}→s\bar{d})|K^0\rangle\) is estimated and found to be of the same order of magnitude as that of the vacuum state, thereby raising serious questions of principle about the reliability of the vacuum insertion method. Section IV contains the main result of our work, namely a calculation of the \(K^0\rightarrow\bar{K}^0\) amplitude in the static, spherical approximation to the MIT bag model. We find that the MIT bag model result, as compared with that computed via the vacuum-insertion approximation, is of the same sign but about 60\% smaller in magnitude.

II. THE CONVENTIONAL FREE-QUARK CALCULATION OF THE \(K_L K_S\) MASS DIFFERENCE

The conventional calculation\(^2\) of the amplitude \(A(K^0\rightarrow\bar{K}^0)\) and the associated \(K_L K_S\) mass difference begins with the determination of the free-quark transition amplitude \(A(s\bar{d}→s\bar{d})\), in the limit where the external quark momenta are assumed to be negligible compared to the loop momenta and virtual quark masses which control the contribution of the Feynman diagrams for this process. In the sequential WS model which we are using here there are two classes of diagrams which contribute in lowest (one-loop) order to the \(s\bar{d}→s\bar{d}\) transition; the one shown in Fig. 1 and its crossed version. These give the same contribution. A standard calculation yields the free-quark amplitude \(A(s\bar{d}→s\bar{d})\), or equivalently, the effective current-current Lagrangian

\[
\mathcal{L}_{\text{eff}} = C\Theta_{\text{eff}} + \text{H.c.},
\]

(2.1)

where

\[
C = \frac{G_F}{\sqrt{2}} \frac{\alpha}{16\pi} \csc^2 \theta_W \int_0^\infty dx \left[ \sum_{i=1}^n \frac{\lambda_i \xi_i}{X + \epsilon_i} \right]^2
\]

(2.2)

and

\[
\Theta_{\text{eff}} = \left[ \bar{\psi} \gamma_\mu (1 - \gamma_5) \psi \right] \left[ \bar{\psi} \gamma^\nu (1 - \gamma_5) \psi \right].
\]

(2.3)

In Eq. (2.2) \(G_F\) is the Fermi coupling constant, \(\theta_W\) the Weinberg angle,

\[
\lambda_i = V_{i\alpha}^\dagger V_{\alpha d}
\]

(2.4)

and

\[
\begin{array}{c}
\bar{s}_d \quad q(2/3) \\
\end{array}
\]

\[
\begin{array}{c}
\bar{q}_d \\
r(2/3) \quad \bar{s}_d
\end{array}
\]

FIG. 1. Feynman diagram for the "scattering" term in the \(s\bar{d}→s\bar{d}\) transition amplitude. The "annihilation" term arises from the crossed graph.
\[ \epsilon_i = \frac{m_i^2}{m_w^2}, \]  

where \( m_i \) is the mass of the \( i \)-th charge \( \frac{2}{3} \) quark in the mass eigenbasis defined by the \( n \)-dimensional generalization of Eq. (1.2) (so that \( i = 1, 2, \) and \( 3 \) correspond to \( u, c, \) and \( t \), respectively). In Eq. (2.3) and elsewhere in this paper our metric and \( \gamma \)-matrix conventions are such that \( g_{\mu\nu} = \text{diag}(1, -1, -1, -1) \) and \( \gamma^\mu = (1 - \gamma^4) \). The free-quark result given in Eq. (2.2) reflects the GIM relation

\[ \sum_{i=1}^{n} \lambda_i = 0, \]

and incorporates the reasonable approximations of neglecting external quark momenta, setting \( m_w = 0 \), and dropping terms of order \( (m_q^2/m_w^2)^2 \) relative to unity.

For our present analysis it is the operator \( \Theta_{ij} \) rather than the constant \( C \) which is of primary interest. However, it is useful to include several remarks here for completeness. The GIM mechanism guarantees that for \( m_q^2 \ll m_w^2 \) the integral in Eq. (2.2) is of order \( m_q^2/m_w^2 \), and, moreover, that \( C \) would vanish in the hypothetical case where the masses of the virtual \( Q = \frac{2}{3} \) quarks were all equal. In the simplest case, namely the WS-GIM four-quark model, one finds that

\[ C = \frac{G_F}{\sqrt{2}} \frac{2\alpha}{16\pi} \left( \frac{m_q}{37.3 \text{ GeV}} \right)^2 \sin^2 \theta_C \cos^2 \theta_C, \]

where \( \theta_C \) is the Cabibbo angle. It is interesting to note that for \( n > 1 \) (where \( n \) is the number of quark doublets) \( C \) can have an imaginary (as well as real) part, which might account for CP violation in \( K \) decays. Indeed, it was this feature of the six-quark version of the WS model which served as the focus of the first study of the model by Kobayashi and Maskawa. In order to give the real and imaginary part of \( C \) in the WS-KM, it is useful to list the explicit form for the quark mixing matrix \( V \). In the convenient KM parametrization it is

\[ V = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 s_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & -c_1 c_2 s_3 - s_2 s_3 e^{i\delta} & c_1 s_2 c_3 - c_2 s_3 e^{i\delta} \end{pmatrix}, \]

where \( c_1 = \cos \theta_1, s_1 = \sin \theta_1, \) etc. Then

\[
\text{Re}(C) = \frac{G_F}{\sqrt{2}} \frac{2 \alpha}{16\pi} \left[ \lambda_R m_q^2 + \lambda_I R m_t^2 + \frac{2 \lambda_R \lambda_I R m_q^2}{1 - m_e^2/m_t^2} \ln \left( \frac{m_q^2}{m_e^2} \right) \right] - \lambda_I \frac{1}{2} \left( m_q^2 + m_t^2 - \frac{2 m_e^2}{1 - m_e^2/m_t^2} \ln \left( \frac{m_q^2}{m_e^2} \right) \right),
\]

and

\[
\text{Im}(C) = \frac{G_F}{\sqrt{2}} \lambda_I \left[ \lambda_R m_q^2 - \lambda_I R m_t^2 + \frac{2 \lambda_R \lambda_I R m_q^2}{1 - m_e^2/m_t^2} \ln \left( \frac{m_q^2}{m_e^2} \right) \right],
\]

where

\[ \lambda_{jk} = \text{Re}(\lambda_{ij}), \]

and

\[ \lambda_{ij} = \text{Im}(\lambda_{ij}), \]

with \( j = c \) or \( t \). Note that \( \lambda_{ji} = -\lambda_{ij} \).

We digress briefly to comment on the use of the four-quark operator effective Lagrangian. For this operator the relevant scale of Euclidean momentum is usually set by the heavy-quark masses \( m_q \) (by \( m_c \), in the four-quark model). If these are large enough, so that the effective strong-interaction coupling parameter \( \alpha(m_q^2) \) is small, it may not be unreasonable to ignore gluon corrections and to ignore also operators of higher dimension. A more accurate treatment would of course have to allow for these effects, as also for "traditional" contributions from low-mass intermediate states \( X \) in \( K^0 \to X \to K^0 \). We cannot address these difficult issues here. We simply accept the view that the effective Lagrangian is dominated by the four-quark operator \( \Theta_{ij} \) and leave to ongoing discussion whether the coefficient \( C \) receives important strong-interaction corrections. Our task is to estimate the matrix element of \( \Theta_{ij} \) between physical kaon states.

In order to compute the physical \( K^0 \to K^0 \) transition amplitude, one must estimate the matrix element \( \langle K^0 | \Theta_{ij} | K^0 \rangle \). In the conventional approach this is done by formally inserting a complete set of intermediate states in all possible ways, but then saturating this set with the contribution only of the vacuum state. In this approximation one
vector current contributes to the \( K^0 \)-vacuum matrix element, so only the vector current contributes to the \( K^0 \rightarrow \bar{K}^0 \) matrix element. Because of the opposite charge-conjugation property of vector and axial-vector currents, the analog for \( \mathcal{M}_{\text{vac}} \) of the relation
\[
\langle \bar{K}^0 | \bar{\psi}_d \gamma_\mu \gamma_5 \psi_\pi | 0 \rangle = \langle 0 | \bar{\psi}_d \gamma_\mu \gamma_5 \psi_\pi | K^0 \rangle^*, \quad (3.3a)
\]
which was utilized in the evaluation of \( \mathcal{M}_{\text{vac}} \), is
\[
\langle \bar{K}^0 | \bar{\psi}_d \gamma_\mu \gamma_5 \psi_d | \pi^0 \rangle = -\langle \pi^0 | \bar{\psi}_d \gamma_\mu \psi_d | K^0 \rangle^*, \quad (3.3b)
\]
It is possible to estimate \( \mathcal{M}_{\text{vac}} \) since \( \langle \pi^0 | \bar{\psi}_d \gamma_\mu \psi_d | K^0 \rangle \) is related by an SU(3) transformation to the matrix elements \( \langle \pi^0 | \bar{\psi}_d \gamma_\mu \psi_d | K^0 \rangle \) and \( \langle \pi^0 | \bar{\psi}_d \gamma_\mu \psi_d | K^0 \rangle \), which are experimentally measured via the decays \( K^+ \rightarrow \pi^+ \nu \nu \) and \( K^+ \rightarrow \pi^+ \nu \nu \), where \( I = e \) or \( \mu \):
\[
\langle \pi^0 | \bar{\psi}_d \gamma_\mu \psi_d | K^0 \rangle = \langle \pi^0 | \bar{\psi}_d \gamma_\mu \psi_d | K^0 \rangle^* = -\frac{1}{\sqrt{2}} \langle \pi^0 | \bar{\psi}_d \gamma_\mu \psi_d | K^0 \rangle. \quad (3.4)
\]
The Lorentz structure of all these matrix elements is given by
\[
\langle \pi(|p_+)| J_\mu | K(|p_\pi)| = f_+(q^2)(p_\pi + p_+)^\mu + f_-(q^2) q_\mu , \quad (3.5)
\]
where
\[
q = p_\pi - p_+, \quad (3.6)
\]
and \( J_\mu \) represents either the \( \bar{\psi}_d \gamma_\mu \psi_d \) or the \( \bar{\psi}_d \gamma_\mu \psi_d \) current. For small \( q^2 \), a standard parametrization of the \( K_{\text{vac}} \) form factors is specified by the linear formula
\[
f_+(q^2) = f_+(0) \left( 1 + \frac{q^2}{m_\pi^2} \right), \quad (3.7)
\]
and together with
\[
f_-(q^2) = \xi(0) f_-(q^2), \quad (3.8)
\]
where \( \lambda_+ \) is the slope parameter. These are low-\( q^2 \) approximations to what one expects are form factors better represented by a sum of one or more first-order poles. However, since the order of the pole cannot be determined from \( K_{\text{vac}} \) decay data, and since a single first-order pole decreases quite slowly for large \( q^2 \), we shall consider, for purposes of comparison, a parametrization based alternatively on single first- or second-order poles. We thus write
\[
f_+(q^2) = \frac{f_+(0)}{(1 - q^2/m(n)^2)^n}, \quad (3.9)
\]
where \( n = 1 \) or \( 2 \), and \( m(n) \) is fixed by the constraint
\[
m(n)^2 = n m_\pi^2 / \lambda_+. \quad (3.10)
\]
The intercepts are predicted by SU(3) to satisfy

III. ONE-PION CONTRIBUTION TO THE \( K^0 \rightarrow \bar{K}^0 \) AMPLITUDE

In order to test the reliability of the vacuum insertion technique for determining the matrix element \( \langle \bar{K}^0 | \mathcal{M}_{\text{eff}} | \pi \rangle \), we shall compute the contribution of the one-pion intermediate state to this matrix element. We define the ratio of the vacuum and one-pion contributions as
\[
R_{\text{1p/vac}} = \frac{\mathcal{M}_{\text{1p}}}{\mathcal{M}_{\text{vac}}}, \quad (3.1)
\]
where
\[
\mathcal{M}_{\text{vac}} = \langle \bar{K}^0 | \bar{\psi}_d \gamma_\mu (1 - \gamma_5) \psi_\pi | 0 \rangle \times \langle 0 | \bar{\psi}_d \gamma_\mu (1 - \gamma_5) \psi_\pi | K^0 \rangle, \quad (3.2a)
\]
and
\[
\mathcal{M}_{\text{1p}} = \int d^4p_\pi \delta_+(p_\pi^2 - m_\pi^2)(2\pi)^{-3}
\times \langle \bar{K}^0 | \bar{\psi}_d \gamma_\mu (1 - \gamma_5) \psi_\pi | \pi^0 \rangle
\times \langle \pi^0 | \bar{\psi}_d \gamma_\mu (1 - \gamma_5) \psi_\pi | K^0 \rangle. \quad (3.2b)
\]
Note the obvious facts that, just as only the axial-
\[
-f_+(0)_{K^0\to\pi^0} = f_+(0)_{K^+\to\pi^0} = \frac{1}{\sqrt{2}} f_+(0)_{K^0\to\pi^-} = \frac{1}{\sqrt{2}}.
\]

(3.11)

As expected, the \(q^2\) dependences of the form factors for \(K_\mu^0\) and \(K_\mu^+\) decays are similar; for our purposes it is sufficient to take the rough values \(\lambda = 0.03\) and \(\lambda(0) = -0.2\). It should be noted that the \(q^2\) dependence of \(f_+(q^2)\) could be somewhat different from that of \(f_-(q^2)\); the simplifying assumption of Eq. (3.8) that they are the same is made because the accuracy of the experiments is not great enough to determine the \(q^2\) dependence of \(f_-(q^2)\). However, this possible difference is of negligible importance for our calculations.

The amplitude \(\mathfrak{M}_{\pi}\) is then given by the integral

\[
\mathfrak{M}_{\pi} = \left( \frac{1}{16\pi m_K^3} \right) \int_{q_{\max}^2}^{q_f^2} d^2 q' \lambda^{1/2} (q^2, m_x^2, m_y^2) \times \left\{ |f_+(q^2)|^2 (2m_X^2 + 2m_Y^2 - q^2) + |f_-(q^2)|^2 q^2 + 2 \text{Re} [f_+(q^2) f_-(q^2)] (m_Y^2 - m_X^2) \right\},
\]

(3.12)

where

\[
q_{\max}^2 = (m_K - m_\pi)^2,
\]

\[
\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + yz + xz),
\]

and the \(m_x^2\) terms are included for completeness only; they are numerically insignificant. The cut-off \(q_f^2 < 0\) is placed on the \(q^2\) integration because the pole-dominated form factors cannot be expected to provide an accurate dynamical model for the \(K^0\to\pi^0\) matrix element for \(|q^2| \gg m(n)^2\). As illustrative results we list the following:

\[
R_{1\pi/\pi^0}(n = 1) = \begin{cases} -1.1, & \text{for } (-q_c^2)^{1/2} = 1.5 \text{ GeV}, \\ -2.1, & \text{for } (-q_c^2)^{1/2} = 2.0 \text{ GeV}, \end{cases}
\]

(3.15)

and

\[
R_{1\pi/\pi^0}(n = 2) = \begin{cases} -0.65, & \text{for } (-q_c^2)^{1/2} = 1.5 \text{ GeV}, \\ -0.91, & \text{for } (-q_c^2)^{1/2} = 2.0 \text{ GeV}. \end{cases}
\]

(3.16)

The ratio \(R_{1\pi/\pi^0}\) evidently depends sensitively upon the assumed cutoff \(q_c^2\) and type of form factor; accordingly, no particular one among the numerical values in Eqs. (3.15)-(3.16) should be taken too seriously. However, the fact that the sign of \(\mathfrak{M}_{\pi}\) is opposite to that of \(\mathfrak{M}_{\pi}\) is a general consequence of the opposite transformation properties of vector and axial-vector currents under charge conjugation, combined with the positivity of the integrand in Eq. (3.12) for the values of \(f_+(q^2)\) determined from \(K_{1\pi}\) experiments. These results indicate that the one-pion contribution to the \(K^0\to\bar{K}^0\) amplitude is roughly comparable to that of the vacuum state in magnitude and opposite in sign, and hence there is no reason to assume that the vacuum insertion method will yield an accurate estimate of this amplitude.

One might consider going further, enumerating various other allowed intermediate states and estimating their contributions. However, there is little point in this, for several reasons. First, since there are infinitely many such states, the inclusion of several more does not enable one to approach closer to the complete answer. Secondly, the one-pion contribution is the only case for which one has much knowledge of the required matrix element. As a function of increasing mass, the next important state is the two-pion one. Here again one could, in principle, relate the required matrix element to one which is measured in \(K_{3\pi}\) decay. However, because the matrix element is considerably more complicated than in the case of \(K_{13}\) decay, while at the same time much less is known about it, we do not consider it possible to estimate the \(2\pi\) contribution in a reliable enough way for it to yield any useful information regarding the amplitude under study. But it is also not necessary; the fact that the one-pion state gives a contribution which is comparable in magnitude to that of the vacuum and opposite in sign is already quite sufficient to raise serious doubts about the reliability of the vacuum-insertion method.

IV. CALCULATION BASED ON THE MIT BAG MODEL

In view of the above doubts about the reliability of the vacuum-insertion method of evaluating the matrix element

\[
\mathfrak{M} = \left\langle \bar{K}^0 | \mathcal{O}^{\mu\nu} | K^0 \right\rangle
\]

(4.1)

and because of the impossibility of carrying out any reasonably complete sum over other exclusive intermediate states, it is clearly desirable to use a different, inclusive method for computing this matrix element. The MIT bag model of hadrons provides such a method. This model incorporates quark and gluon confinement as an assumed property and has achieved a number of successes in describing the static properties of low-lying hadrons, such as masses, magnetic moments, charge radii, and axial-vector coupling constants.

Before embarking on the computation one might inquire as to how accurate a result could be expected from the model. Our application to the \(K^0\to\bar{K}^0\) transition amplitude has the feature of involving a transition between states related to each other by charge conjugation. From the point of
view of mass and radius it is therefore effectively a static, diagonal matrix element. For this reason one could expect that the accuracy of the bag-model prediction for the amplitude might be comparable to that of the predictions for static hadron parameters. A recent application of the MIT bag model to the study of nonleptonic hyperon and $K$ decays\(^\text{12}\) may possibly give some indication of the accuracy to be expected from the model in the evaluation of matrix elements of current-current operators. However, this application differs from the present one precisely because it concerns decays, as opposed to the special charge-conjugation transition $K^0\rightarrow\bar{K}^0$. Unfortunately, the standard (static, spherically symmetric) MIT bag model is not well adapted to deal with such decay processes. In order to mitigate this problem, the authors of Ref. 12 used partial conservation of the axial-vector current (PCAC) to reduce a nonleptonic decay amplitude of the form \(\langle h'\mid \mathbf{g}_{\alpha\beta} \mid h \rangle\) to \(f_x^{-1}\langle h'\mid \mathbf{g}_{\alpha\beta} \mid h \rangle\), where \(h\) and \(h'\) are initial and final hadrons in the decay (baryons, in the case of hyperon decay, and \(K\) and \(\pi\), respectively, in the case of \(K\rightarrow 2\pi\) decay), and \(f_x\) is the pion decay constant. The matrix element thereby obtained is still not diagonal in mass, so the approximation is adopted of setting \(m_h = m_{h'}\) and \(R_h = R_{h'}\). The resulting predictions for nonleptonic decay amplitudes based on the same set of MIT bag model parameters as the one used in our present work (and including estimates of short-distance enhancement effects) are found to be smaller than the experimental values by factors of 2–3. It would be difficult and inappropriate to try to ascertain the causes of this disagreement here.\(^\text{12}\)

We may infer, however, that the bag model applied to the problem under present consideration should be at least reasonably accurate, say to within a factor of 2–3.

From another direction one might raise the following question: If, indeed, the vacuum state did saturate well the sum over intermediate states (contrary to the indications from the previous section) would it be natural for the MIT bag model to reproduce the vacuum state contribution? At first glance one might think not since, as is well known, the standard MIT bag model does not incorporate broken chiral symmetry in a natural way, and the vacuum contribution involves \(f_{\text{vac}}\), a parameter characterizing the spontaneous symmetry breaking of SU(3)×SU(3). However, this may not be so serious a problem as it would first appear. It is true, for example, that the model predicts a pion mass which is roughly twice the actual value; however, its prediction for the kaon mass, which is the one relevant for our work, differs by less than \(-1\%\) from the experimental value.\(^\text{13}\)

We proceed to describe our calculation of the matrix element \(\mathcal{M}\) based on the static, spherically symmetric approximation to the MIT bag model. Since this model has been discussed in detail in a number of references,\(^\text{11}\) we shall be quite brief in our review of it. In the bag theory quarks (and gluons) are supposed to be confined by a bag pressure \(B\) to the interior of a certain region of space constituting a hadron. To zeroth order in \(\alpha_s = g_s^2/(4\pi)\), the strong coupling parameter, each quark or antiquark obeys a free-particle Dirac equation

\[
(i\gamma\cdot\mathbf{p} - m_\alpha)\psi_\alpha = 0
\]

inside the boundary of the hadron, where \(\alpha\) labels the fermion. In the static, spherically symmetric treatment adopted here, Eq. (4.2) holds for \(r < R\), where \(R\) is the radius of the bag. There are two boundary conditions which apply on the surface of the sphere. The first is a linear one

\[
-i\gamma\cdot\mathbf{v}\psi = \psi,
\]

where \(n\) is the inward normal to the surface, which in the rest frame of the hadron has the form \(n^a = (0, -\mathbf{p})\). This condition guarantees that from the Lagrangian

\[
\mathcal{L} = \sum_\alpha \bar{\psi}_\alpha (i\not\!{\partial} - m_\alpha) \psi_\alpha - B\delta(R - r),
\]

the partial integration necessary to derive the field equation (4.2) yields a vanishing surface term. The second boundary condition, which is nonlinear, reads

\[
-\sum_\alpha \frac{\partial}{\partial r} (\bar{\psi}_\alpha \psi_\alpha) = 2B \quad \text{at} \quad r = R.
\]

This condition implies that only quark modes with total angular momentum \(j = \frac{1}{2}\) can exist in the bag. Two such solutions, with opposite parities, are

\[
\psi^{(\text{cl})}_{\alpha(\kappa=1)}(\mathbf{r}) = \frac{N_{\alpha(\kappa=1)}}{\sqrt{4\pi}} \left( -i\epsilon_\alpha j_1(\mathbf{p}_\alpha r) \mathbf{\hat{r}} \cdot \mathbf{\hat{p}} \chi \right),
\]

and

\[
\psi^{(\text{cl})}_{\alpha(\kappa=1)}(\mathbf{r}) = \frac{N_{\alpha(\kappa=1)}}{\sqrt{4\pi}} \left( i\epsilon_\alpha j_1(\mathbf{p}_\alpha r) \mathbf{\hat{r}} \cdot \mathbf{\hat{p}} \chi \right),
\]

where \((\text{cl})\) means "classical," and \(\alpha\) labels the parameters of the quark mode, which include flavor \(f = s\) or \(d\), energy \(E\), which is quantized as a function of \(\kappa\), the parity index, and a number \(n\) which plays the role of a principal quantum number and labels each set of \(\kappa = \pm 1\) mode energies in order of increasing values. In Eqs. (4.6)–(4.7) \(j_{\pm 1}(z)\) and \(j_1(z)\) are spherical Bessel functions,

\[
\epsilon_\alpha = \left( \frac{E_\alpha - m_\alpha}{E_\alpha + m_\alpha} \right)^{1/2},
\]
and the normalization constants are defined such that
\[ \int_{bag} d^3x \phi_{n_\kappa}^{(c_1)} \phi_{n_\kappa}^{(c_2)} = 1. \] (4.9)

The quantity \( \xi = pR \) is determined, as a function of \( mR \), by the eigenvalue condition which results from the linear boundary condition (4.3):
\[ \tan \xi = \frac{k \xi}{\kappa - \kappa m \sigma R + E_\alpha R}, \] (4.10)

where
\[ E_{\alpha(n=0)}R = [\xi^2 + (m \sigma R)^2]^{1/2}. \]

Since the usual SU(3) 56 baryons and 35 mesons, in particular the kaon, are ground-state hadrons, i.e., they have orbital angular momentum \( L = 0 \), each of the quark modes for these hadrons will have an energy given by the lowest positive solution to Eq. (4.10). For \( mR = 0 \) this solution is
\[ \xi_{n=1, \kappa = -(mR = 0)} = 2.043. \] (4.11)

As \( mR \) increases from zero to infinity, this lowest solution increases smoothly from the above value to \( \pi \). As indicated in Eq. (4.10), the (positive-energy) quark modes thus have energies given by
\[ E_\alpha = (p_\alpha^2 + m_\alpha^2)^{1/2}, \]

where \( p_\alpha = \xi_\alpha / R \).

Up to this point the bag model is a classical field theory, although we have used terminology which anticipates the second quantization to follow. To quantize the theory one introduces creation operators \( b^{\dagger (i)}_\alpha \) and \( d^{\dagger (i)}_\alpha \), respectively, for quarks and antiquarks with color \( i \) and other properties (such as flavor, momentum, and spin) labeled by \( \alpha \). These satisfy the usual anticommutation relations
\[ \{b^{\dagger (i)}_\alpha, b^{(j)}_{\alpha'}\} = \delta_{ij} \delta_{\alpha \alpha'} \delta^3(p - p'), \] (4.12)

and
\[ \{d^{\dagger (i)}_\alpha, d^{(j)}_{\alpha'}\} = \delta_{ij} \delta_{\alpha \alpha'} \delta^3(p - p'). \] (4.13)

The quantum field operator \( \psi^{(i)}_\alpha \) is defined in terms of these by the equation
\[ \psi^{(i)}_\alpha = b^{(i)}_\alpha \psi_\alpha + d^{(i)}_\alpha \psi_\alpha, \] (4.14)

where
\[ \psi_{\alpha(n=1)} \equiv \psi^{(d)}_\alpha, \] (4.15)

and
\[ \psi_{\alpha(n=1)} \equiv \psi^{(u)}_\alpha. \] (4.16)

The above assignment for the antiquark spinor follows from the standard hole interpretation of an antifermion together with the symmetry relations
\[ \xi_{n, \kappa} = -\xi_{w, -\kappa}, \] (4.17a)

and
\[ E_{n, \kappa} = -E_{w, -\kappa}. \] (4.17b)

In order to use the bag model to calculate the mass of a hadron one includes, first, the volume energy, which stabilizes the bag against indefinite expansion:
\[ E_v = \frac{4}{3} \pi R^3 B \] (4.18)

and the (anti)quark energy
\[ E_\alpha = \sum_\alpha E_\alpha. \] (4.19)

In addition, there is a contribution to the mass arising from the zero-point fluctuations of the quark fields. This is represented phenomenologically by the term
\[ E_0 = -\frac{Z_0}{R}, \] (4.20)

where \( Z_0 \) is an empirically determined constant. Finally, one may include the lowest-order quark-quark interactions via the terms \( E_B \) and \( E_H \) for the color electric and magnetic interactions, respectively. These are proportional to \( \alpha_s / \lambda \), where \( \alpha_s \) is assumed to be a constant, again determined empirically.

The mass of a hadron is then given by
\[ m = E_v + E_\alpha + E_B + E_H. \] (4.21)

As defined, the model depends on the parameters \( B, Z_0, \alpha_s, \lambda \), and the quark masses \( \alpha \).

These parameters can be determined by a fit to various hadron properties. The MIT group\(^{11}\) carried out two such fits, which used the masses of the 0 and 1 mesons, and \( \frac{3}{2} \) and \( \frac{1}{2} \) baryons as the properties to be predicted. In the first of these fits, labeled "case A" by the authors of Ref. 11, the constraint \( m_u = m_d = 0 \) was imposed. The resulting set of parameters was found to be \( B_{fit} = 0.145 \) GeV, \( Z_0 = 1.84, \alpha_s = 0.55 \), and \( m_s = 0.279 \) GeV. These are universal parameters, while the radius \( \lambda \) differs for each hadron. In the case of interest, the kaon, \( R_K = 3.26 \) GeV\(^{-1}\); furthermore, the kaon mass is predicted to be \( m_K^{\text{fit}} = 0.497 \) GeV, in very good agreement with experiment. It is this fit, case A, which we shall use for our calculations. The MIT group also performed another fit of the bag model to the same set of hadron masses, in which the \( u \) and \( d \) quark masses were subject to the constraint \( m_u = m_d = m_s \), but \( m_s \) was not forced to vanish. We have not used the bag parameters derived from this fit since the kaon mass is not predicted well; \( m_K^{\text{fit}} \) (case B) = 0.371 GeV. It might be noted that the kaon bag radius for the case B fit is substantially different than for the case A fit; \( R_K \) (case B) = 0.73 GeV\(^{-1}\). One may alternatively choose to determine the set of fundamental bag parameters by a fit to quantities
other than just masses, such as magnetic moments and axial-vector coupling constants. Such an approach has been followed by Golowich et al.,\textsuperscript{12} within the framework of the simplified MIT bag model without the zero-point or gluon-quark interaction energy terms in Eq. (4.21). The results of this different fit, which applied to baryons only, were of comparable quality to those of the A or B MIT fits. For further details on the structure of the MIT bag model, the reader should consult Refs. 11 and 12.

\[
\mathbb{M} = 2(\vec{K}^0) \{ \sum_{i} [ - \mathcal{B}^{(t)B} (d_{s}^{(t)B} d_{s}^{(t)B})] [\bar{u}_{s} \gamma^{\mu} (1 - \gamma_5) u_{s}] [\bar{u}_{s} \gamma^{\mu} (1 - \gamma_5) u_{s}] \\
+ (d_{s}^{(t)B} d_{s}^{(t)B})] [\bar{u}_{s} \gamma^{\mu} (1 - \gamma_5) u_{s}] [\bar{u}_{s} \gamma^{\mu} (1 - \gamma_5) u_{s}] \} |K^0^0\rangle, \tag{4.22}
\]

where, as indicated before, \( b_{fn}^{(t)} \) is the creation operator for a quark of color \( t \), flavor \( f = s \) or \( d \), and spin \( s_{f} \), as labeled in Fig. 1, and similarly with the other operators. The contractions of quark color indices, combined with the color normalization factors in the \( K^0 \) and \( K^0 \) wave functions, yield a weight factor of 3 for the scattering term and 3 for the annihilation term in Eq. (4.22). Next, one can use a Fierz identity to replace the spinor expression in the annihilation term by minus the corresponding expression for the scattering term. Carrying out the indicated operations, one is then left with spin matrix elements to be evaluated and an integral over \( r \) to be performed. We find that

\[
\mathbb{M}_{\text{bag}} = 8N_{s}^{2} N_{c}^{2} (4\pi)^{-1} \int_{0}^{R} d r r I(r), \tag{4.23}
\]

where

\[
I(r) = 2 \left[ - (f_{os} f_{od})^{2} - (f_{1s} f_{1d})^{2} + (f_{os} f_{id})^{2} \\
+ (f_{1s} f_{1d})^{2} + 4 (f_{os} f_{1d} f_{1s} f_{1d}) \right]. \tag{4.24}
\]

In Eq. (4.24) we have used the abbreviations \( f_{os} = f_{s}(p_{o} r) \) and \( f_{1s} = f_{s}(p_{o} r) \), with \( f = s \) or \( d \).

Performing the integration over \( r \), we finally obtain the result

\[
\mathbb{M}_{\text{bag}} = 0.72 \times 10^{-2} \text{ GeV}^{3}, \tag{4.25}
\]

from which it follows that

\[
R_{0J} = \frac{\mathbb{M}_{\text{bag}}}{\mathbb{M}_{\text{vac}}} = 0.42. \tag{4.26}
\]

From the definition of \( R_{0J} \), one has

\[
\left( \frac{m_{s} - m_{s}}{m_{b}} \right)_{\text{bag}} = R_{0J} \left( \frac{m_{s} - m_{s}}{m_{b}} \right)_{\text{vac}}. \tag{4.27}
\]

Thus the MIT bag model yields a matrix element \( \mathbb{M}_{\text{bag}} \), which, as compared with the vacuum-insertion result \( \mathbb{M}_{\text{vac}} \), is of the same sign and comparable in magnitude, although smaller by \( \sim 60\% \).

The above result for \( \mathbb{M}_{\text{bag}} \) is based on the best available set of MIT bag-model parameters, namely those of the MIT case A fit. In particular, with this set of parameters, \( m_{s} = 0 \), \( m_{s} = 0.279 \text{ GeV} \), and \( R_{0} = 3.26 \text{ GeV}^{-1} \).

In order to see how stable the relation in Eq. (4.25) is, we have examined the functional dependence of \( \mathbb{M}_{\text{bag}} \) on these three parameters. First, one may observe that in the hypothetical situation in which \( m_{s} = 0 \) for \( q = s \) as well as \( q = d \), on dimensional grounds alone there would follow the exact scaling relation

\[
\mathbb{M}_{\text{bag}} (m_{s} = 0) \propto R^{-3}. \tag{4.28}
\]

Most generally, one can write

\[
\mathbb{M}_{\text{bag}} = R^{-3} f(m_{s} R, m_{d} R), \tag{4.29}
\]

where \( f \) is a dimensionless function which can be extracted in a straightforward manner from Eqs.

\[\text{FIG. 2. Plot of the function } f(m_{s} R, m_{d} R) = R^{-3} \mathbb{M}_{\text{bag}}. \text{ for a range of } m_{s} R \text{ around the central point } 0.910. \text{ The three curves are for } m_{s} R = (a) 0, \text{ (b) } 8.15 \times 10^{-2}, \text{ and (c) } 1.63 \times 10^{-1} \text{ corresponding to } R = 3.26 \text{ GeV}^{-1} \text{ and } m_{s} = (a) 0, \text{ (b) } 0.029 \text{ GeV}, \text{ and (c) } 0.050 \text{ GeV}. \text{ The standard value of } f \text{ is } f(0.910, 0) = 0.249 \text{ as marked with a dot on curve (a).}\]
As is evident from Eq. (4.23), the terms which contribute to the integrand for \( \langle K^0 - \bar{K}^0 \rangle \) enter, in general, with different signs, and the resulting matrix element is therefore not necessarily positive definite. Indeed, as our numerical studies indicate, there is substantial cancellation which occurs among individual terms. In Fig. 2 we plot \( f(m_2 R, m_3 R) \) for a range of \( m_2 R \) centered at the point \( m_2 R = 0.910 \), corresponding to the MIT case A fit, and for three values of \( m_3 R \), corresponding to \( m_3 R = (a) \) 0, (b) 0.025 GeV, and (c) 0.050 GeV, with \( R = 3.26 \text{ GeV}^{-1} \). In computing \( f \) we have, of course, solved the eigenvalue equation, Eq. (4.10) to obtain the correct \( \varepsilon_2 \) and \( \varepsilon_3 \) for each pair of \( m_2 R \) and \( m_3 R \) values. The central value of \( f \) is \( f(0.910, 0) = 0.249 \), as marked with a dot on curve (a). Combined with the value \( R = 3.26 \text{ GeV}^{-1} \) this yields the result for \( \Upsilon_{\text{vac}} \) quoted in Eq. (4.25). As one can see from the graph, \( f \) is an approximately linear, decreasing function of both its arguments and, although positive for a reasonably large range of \( m_2 R \), including the central value, it actually vanishes at \( m_2 R = 1.21 \) (for \( m_2 R = 0 \)) and is negative for larger values of \( m_2 R \). Thus, because of the strong cancellations which occur, \( \Upsilon_{\text{vac}} \) does not scale like \( R^3 \). The resulting dependence of \( \Upsilon_{\text{bag}} \) and hence the ratio \( \Upsilon_{\text{bag}} \) on \( m_3 R \), \( m_3 R \), and \( R \) can be inferred from Eq. (4.29). It should be stressed that this analysis is intended solely to illustrate the functional dependence of \( \Upsilon_{\text{bag}} \) on \( m_3 R \), \( m_3 R \), and \( R \); realistically one is not at liberty to vary these parameters independently, but must use a consistent set, such as that derived from the MIT case A fit. As Fig. 2 indicates, for a reasonable range of \( m_3 R \), \( m_3 R \), and \( R \), \( \Upsilon_{\text{bag}} \) is positive, and near to the central value quoted in Eq. (4.25).

It is worth remarking that although the actual sign of the coefficient \( \text{Re}(C) \) in \( L_{\text{eff}} \) and hence of \( \text{Re}(\langle K^0 - \bar{K}^0 \rangle) \) depends on the quark masses (in particular the \( t \)-quark mass, in the KM model) and mixing angles, some of which are not known precisely, for a wide range of realistic values of these parameters, \( \text{Re}(C) > 0 \). Hence, at least both the vacuum insertion method and the MIT bag model predict the correct sign for the \( K^0 \bar{K}^0 \) mass difference, \( m_{-} - m_{+} > 0 \).

In conclusion, given the importance of having an estimate of the amplitude \( \langle K^0 - \bar{K}^0 \rangle \) as possible, we have carried out an improved calculation of the matrix element,

\[
\Upsilon = \langle K^0 | \left[ \bar{\psi}_g(1 - \gamma_5) \psi_d \right] \left[ \bar{\psi}_d^\gamma(1 - \gamma_5) \psi_d \right] | \bar{K}^0 \rangle,
\]

using the MIT bag model. It has always been recognized that the vacuum-insertion technique for evaluating \( \Upsilon \) has little theoretical justification. We have indeed found from an illustrative calculation that the contribution to \( \Upsilon \) from the one-pion intermediate state is comparable in magnitude to that of the vacuum state and opposite in sign. In contrast, the MIT bag model should provide a reasonably accurate means of computing \( \Upsilon \). We have studied the dependence of \( \Upsilon_{\text{vac}} \) on the input parameters \( m_3 R \), \( m_3 R \), and \( R \) and find that, for the small variations allowed phenomenologically, our result is reasonably stable. For standard values of these parameters we find that \( \Upsilon_{\text{bag}} / \Upsilon_{\text{vac}} \approx 0.42 \).

**ACKNOWLEDGMENTS**

We would like to thank J. F. Donoghue and R. L. Jaffe for valuable discussions, and F. Wilczek for comments. We also record here the debt which this paper owes to the pioneering work of B. W. Lee on this subject. This research was supported in part by the Department of Energy and the National Science Foundation under Contract Nos. EY-C02-3072 and PHY-78-01221, respectively.

---

2. B. W. Lee, J. R. Primack, and S. B. Treiman, Phys. Rev. D 2, 510 (1973). For a different approach to estimating the matrix element \( \langle K^0 | \left[ \bar{\psi}_g(1 - \gamma_5) \psi_d \right] \left[ \bar{\psi}_d^\gamma(1 - \gamma_5) \psi_d \right] | \bar{K}^0 \rangle \), see T. Appelquist, J. D Bjorken, and M. Chanowitz, Phys. Rev. D 1, 225 (1973).
7. This value of \( f_K \) is obtained from \( \Gamma(K - \mu) \) using the value of \( |V_{12}|^2 = 0.219 \) derived from the new generalized Cabibbo fit by R. E. Shrock and L.-L. Wang, Phys. Rev. D 41, 1602 (1979).
8. For a previous estimate of \( \Upsilon_{\text{vac}} \), see K. Kang and J. E. Kim, Phys. Rev. D 14, 1903 (1977); and erratum (to be published).
9. For a discussion of the parametrization of \( K_{13} \) form factors and present experimental values of \( \lambda^2 \) and \( (0^P \) see Particle Data Group, Phys. Lett. 72B, 1 (1978).
For an analysis of the structure of the $K_{11}$ matrix element, see e.g., A. Pais and S. B. Treiman, Phys. Rev. 168, 1858 (1968). Some recent work which attempts to estimate the 2π contribution to the $K^0 \to \bar{K}^0$ amplitude has been performed by P. D. Burnett and D. G. Sutherland, Glasgow report (unpublished).

The MIT bag model was proposed by A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf, Phys. Rev. D 9, 3471 (1974), and developed by A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn, ibid. 10, 2599 (1974). In T. DeGrand, R. L. Jaffe, K. Johnson, and J. Kiskis, ibid. 12, 2060 (1975); a detailed fit of the full bag model, including nonzero quark masses, $E_0$, $E_b$, and $E_m$, to hadron masses is carried out and used to derive the case "A" and "B" sets of parameters. There are a number of later papers by the MIT group and others on various topics related to the bag model. A useful review with references to the original literature has been written by P. Hasenfratz and J. Kuti, Phys. Rev. Reps. 40C, 75 (1978). See also Refs. 12 and 13.

J. Donoghue, E. Golowich, and B. Holstein, Phys. Rev. D 12, 2875 (1975); J. Donoghue and E. Golowich, ibid. 14, 1386 (1976). In the first paper a set of parameters for the simplified bag model (with $m_3 = 0$ but without the $E_0$, $E_b$, or $E_m$ terms) is derived by a fit to the degenerate $N-\Delta$ mass, $g_A$, and the $\sigma$ term. In the second paper the more sophisticated MIT bag model and the MIT group's case A set of parameters are adopted. It is the nonleptonic decay amplitudes computed in this second paper which are a factor of 2-3 times smaller than experiment. Gluon and sea-quark corrections to the lowest-order, valence-quark amplitudes for nonleptonic decays have been studied by J. Donoghue and E. Golowich, Phys. Rev. D 15, 3421 (1977), Phys. Lett. 69B, 437 (1977); J. Donoghue, E. Golowich, and B. Holstein, Phys. Rev. D 15, 1341 (1977).

For an attempt to compute the decay constants $f$, and $f_L$ within the MIT bag model, see P. Hays and M. Uehlka, Phys. Rev. D 15, 1339 (1977); 15, 903(E) (1977).