Beta-Decay and Mesotron Lifetime

Yukawa, in his original discussion of mesotrons and nuclear forces, suggested that the mesotron has a finite lifetime, disintegrating into an electron and neutrino, and that the $\beta$-decay should be described as taking place through a mesotron as intermediary, rather than through direct disintegration of the proton and neutron. For the mesotron of spin zero considered by Yukawa, this point of view led to a $\beta$-decay theory fully equivalent to Fermi's, and to a definite prediction of the lifetime of the mesotron in terms of $\beta$-lifetimes.

More recently a number of investigators have come to the conclusion that the sign and spin dependence of nuclear forces can best be understood on the supposition that the mesotron has spin one, and obeys the Proca equations. But with this theory serious objections to Yukawa's description of $\beta$-decay arise. Treating the heavy particles non-relativistically, the coupling energy between heavy particles and mesotrons takes the form

$$\psi^\dagger g \cdot \partial U + \kappa g \cdot \partial \psi,$$

with $U = (U_p, U)$ the four-vector mesotron wave function, $\kappa^{-1} = \hbar/\mu_c$. The coupling energy between light particles and mesotrons is

$$\psi^\dagger \gamma^\nu_\mu E \psi + \kappa g \cdot \partial \psi = [\partial U_p/\partial \chi - \partial U_p/\partial \chi^\dagger] \psi.$$

Both terms in (1) are proportional to the momentum of the mesotron; the second term because it involves derivatives of $U$, the first because of the relation $C^{-1} U^3 = -\text{div} \, U$, which for a mesotron of momentum $p$ and energy $E$ becomes $U^3 = U_p - p_c/E$. Since the mesotron momentum is equal to the sum of the momenta of electron and neutrino, the heavy particle matrix element will thus contribute two powers of the light particle momenta in the expression for the $\beta$-lifetime; the lifetime will be connected to the upper limit of the $\beta$-spectrum by a seventh power law, rather than the fifth power law required by experimental evidence. The only escape is to deny the mesotron any role in $\beta$-decay, and return to direct emission of the light particles by the heavy ones, for example by an interaction

$$f_1(\psi^\dagger \gamma^\nu_\mu \psi^\dagger \gamma^\nu_\mu \psi + f_2(\psi^\dagger \gamma^\nu_\mu \psi^\dagger \gamma^\nu_\mu \psi).$$

One is thus forced to give up any theoretical connection between $\beta$-decay and mesotron decay; both can take place, but they must be supposed completely independent processes.

This conclusion is not in contradiction with the results of Yukawa and his collaborators for mesotrons obeying the Proca equation, although their calculations seem to give both a satisfactory $\beta$-theory and a relation between $\beta$-lifetimes and mesotron lifetime. Yukawa does in fact include, in his Lagrangian for the field, terms of type (3). An apparent connection between the lifetimes arises only because he assumes a definite ratio between these terms and terms (1) and (2) $f_1 = \kappa^2 g \cdot \psi^\dagger \psi^\dagger$, $f_2 = \kappa g \cdot \psi^\dagger \psi$. With this choice the $\beta$-decay is determined entirely by (3), the contribution of (1) and (2) being smaller by a factor $(E_{\text{max}}/m_{\mu c})^3$. Since Yukawa's choice seems inadequately motivated, the discrepancy noted by Nordheim between the calculated and observed mesotron lifetimes is purely fictitious.

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The Fission of Protactinium

In connection with the recent observation by v. Grosse, Booth and Dunning, that it is possible to produce fission in protactinium by neutrons of less than 2 Mev energy but not by thermal neutrons, we should like to point out that this important discovery would seem to fit very well with the theoretical considerations about the fission mechanism developed by us in a recent paper. This theory rests upon the idea that fission, like other nuclear transmutations initiated by collisions or radiation, takes place in two stages. Of these the first is the formation of a compound nucleus in which the energy is temporarily stored among the different degrees of freedom in a way resembling thermal agitation; the second stage is the transformation of a sufficient portion of this energy into potential energy of deformation of the compound nucleus to lead to its division. The possibility of fission by impact of neutrons of given energy depends, therefore, on the difference between the critical energy $E_f$ of such an unstable deformation and the excitation energy of the compound nucleus, which is determined by the binding energy $W_a$ of the added neutron. The considerations in our paper lead to the estimates for these quantities given in Table I.

According to this table, and in agreement with the observations of v. Grosse, Booth and Dunning, we shall just expect that fission is produced in protactinium more easily than in thorium but less easily than in the isotope $^{238}_{\text{U}}$ which, according to the theory, is responsible for the large fission yield of thermal neutrons in uranium. While the accuracy of the estimates of $E_f - W_a$ should be amply sufficient for such qualitative conclusions, it hardly permits talking.

Table 1. Estimates of the differences in Mev between the critical energy $E_f$ of unstable deformations and the binding energy $W_a$ of the added neutron.

<table>
<thead>
<tr>
<th>COMPOUND</th>
<th>$E_f$</th>
<th>$W_a$</th>
<th>$E_f - W_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{235}_{\text{U}}$</td>
<td>5.0</td>
<td>5.4</td>
<td>-0.4</td>
</tr>
<tr>
<td>$^{233}_{\text{Th}}$</td>
<td>5.3</td>
<td>6.4</td>
<td>-1.1</td>
</tr>
<tr>
<td>$^{236}_{\text{Th}}$</td>
<td>6.5</td>
<td>5.4</td>
<td>+0.1</td>
</tr>
<tr>
<td>$^{236}_{\text{U}}$</td>
<td>5.9</td>
<td>5.2</td>
<td>+0.7</td>
</tr>
<tr>
<td>$^{235}_{\text{Th}}$</td>
<td>6.9</td>
<td>5.2</td>
<td>+1.7</td>
</tr>
<tr>
<td>$^{235}_{\text{U}}$</td>
<td>6.5</td>
<td>5.3</td>
<td>+1.2</td>
</tr>
</tbody>
</table>

* By an unfortunate error this quantity was given as 6.4 Mev in Table II of reference 2. It is clear, however, that the case of $^{235}_{\text{U}}$ is comparable not to that of $^{238}_{\text{U}}$, but to that of $^{235}_{\text{Th}}$, in which the removal of a neutron leads from an isotope of odd neutron number to one of even neutron number.