Single Pion Production Process in Pion-Nucleon Collision
and the Sakata Model

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Based on the Sakata model, we propose a model for the single pion production process in $\pi-N$ collision which is essentially an extension of the Lindenbaum-Sternheimer model. It is pointed out that the analyses of sub-Bev single pion production phenomena will give us useful information about the level scheme of the Sakata Model. As a first step of the investigation of this model an analysis of the single pion production process in $\pi^+p$ collision at 500 Mev is made. Our model predicts the contribution of the $I=2$ boson isobar as well as the $I=3/2$ fermion isobar (Lindenbaum-Sternheimer model) for this process. Such a prediction is consistent with the present experimental branching ratio $(\pi^+p\rightarrow\pi^+\pi^0+p)/(\pi^+p\rightarrow\pi^+\pi^+\pi^-n)=1.5$. We calculate the energy and angular distribution of the pion and nucleon which will be useful for obtaining information about the $I=2$ boson isobar and $I=3/2$ fermion isobar from this process.

§ 1. Introduction

In order to have a realistic understanding of the variety of the elementary particles and the success of the Nakano-Nishijima-Gell-Mann scheme for the strongly interacting particles, Sakata has proposed the idea that all the baryons and mesons are the compound particles composed of the three fundamental particles, proton, neutron and $A$-particle and their antiparticles. Much progress was made along this Sakata's idea on the elementary particle when Ogawa introduced the concept of full symmetry among the fundamental particles in the Sakata model. This full symmetry proposed by Ogawa has been further refined with collaboration of Ikeda and Ohnuki leading to the theory of unitary group $U(3)$ of degree three. In this theory a physical particle state (stable particle or unstable isobar state) is assumed to correspond to a basis vector of an irreducible representation space of $U(3)$.

Applying the semi-empirical mass formula originally proposed by Matumoto and reformulated so as to be adapted for the $U(3)$ theory, the present author and Yonezawa have studied the correspondence between the experimental information and the theoretical levels of the composite system derived from the $U(3)$ theory, and they have found that rather remarkable correspondence seems to exist between the theory and the experiment as far as the reliable experimental data are concerned, and they have stressed that all of the resonant states so far observed in the pion-nucleon and kaon-nucleon reaction can be interpreted as the particle levels

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of Sakata's composite model as well as the ordinary stable particle. Now we have some accumulation of the experimental information which seems to support the Sakata model and the $U(3)$ theory based on it, but the present experimental data are still insufficient to establish a hypothesis of composite model and the symmetry properties inherent in it. Especially we are in poverty of the experimental information about the boson isobar states, partly owing to the experimental difficulty of the direct meson-meson reaction.

It has been emphasized by many authors that the experimental study of the multiple meson production process will be prospectful for studying such boson resonant states, if they exist, and many theoretical investigations have been performed, through which the evidence of boson resonant state and their structure have been gradually clarified.

The end of this paper is to emphasize that the meson production process in $\pi-N$ collision will give us a useful information about these boson isobars whose existence is predicted in Sakata's composite particle model, and to propose a scheme that might be able to, in a certain sense, unify the various models that have been proposed to explain the multiple meson production phenomena. This discussion will be given in § 2.

Further, following the scheme given in § 2, we shall give an orientational information of the prediction of the Sakata model for the single pion production process at 500 Mev incident pion energy in $\pi^-p$ collision to serve for further experimental and theoretical analyses. Of course it is necessary to perform such an analysis covering all energy region and all channels. In order to obtain the clear evidence for the existence of the levels in our scheme, it is necessary to start the analysis with the phenomena which will involve only few parameters and allow us to confirm each level unambiguously. Then it will be appropriate at first to give the discussion of the phenomena near the threshold, where one has merely to concern with a few lower levels. As such one we choose the $\pi^-p$ collision at 500 Mev. In § 3 we shall discuss the theoretical energy spectra of the pion and nucleon and their angular correlations in this process.

§ 2. Some consideration based on the Sakata model

Boson resonant states or boson isobars are not new to the meson physics. For example, the strong pion-pion interaction has been proposed to explain the behavior of high energy pion-nucleon reactions, especially with respect to the $I=1/2$ resonant states below 1 Bev laboratory pion kinetic energy and the pion production process at this energy region. In this paper we shall also discuss the boson isobar states that will contribute to the reactions in this energy region. However, our point here is a little different from those previous analyses.

Now let us confine ourselves to the discussion of the single pion production process. There are two main, according to Pontecorvo, contradictory models
for the interpretation of this phenomenon, i.e. the strong pion-pion interaction model
and the \((3/2, 3/2)\) nucleon isobar model due to Sternheimer and Lindenbaum.\(^{11}\)

In our theory\(^*\) based on the Sakata model the second and the third resonances
of pion-nucleon reaction are considered due to the composite states which are
characterized by the three-body system, \(F_3^3(0, 1/2)\) and \(F_3^1(0, 1/2)\).\(^*\) \(F_3^3(0, 1/2)\)
and \(F_3^1(0, 1/2)\) are assigned to the \(I=1/2\) resonances at 600 Mev and 900 Mev
laboratory pion energies respectively. Since the major part of the features of the
total cross section of \(I=1/2\) state in the pion-nucleon reaction will be determin-
ed by the existence of these levels, we may not necessarily need to introduce either
the strong pion-pion interaction model or the nucleon isobar model for the inter-
pretation of the behavior of total cross section below 1 Bev.\(^**\) However, such
circumstance does not deny the existence of the strong pion-pion interaction nor
the success of the Lindenbaum-Sternheimer model. Rather the Sakata model will
allow us a consistent understanding of these contradictory models.

From the pion energy spectrum of the single pion production process in the
pion-nucleon collision around 1 Bev incident pion energy, we know that the \((3/2, 3/2)\)
nucleon isobar, or \(F_3^3(0, 3/2)\) in our notation, is also produced as well as the
stable particles such as nucleons and pions.\(^{10,11}\) In Sakata's composite particle
model this seems to be very natural occurrence, since in this theory all the stable
particles and the unstable resonant states bear the same physical significance, and the
only matter that makes them appear so different from each other is their life-time.

Accordingly, we can expect that other unstable composite states of the Sakata
model will behave similarly as the \((3/2, 3/2)\) isobar state does if the appropriate
condition is fulfilled, although now we do not know the condition exactly.

We will shortly see what circumstances are expected when we assume that
such a viewpoint holds generally and the single pion production process can be
reduced to the two-body cascade decay of the intermediate state such as
\[
\begin{align*}
\pi + N &\rightarrow B + N \\
&\rightarrow \pi + \pi, \\
\pi + N &\rightarrow F + \pi \\
&\rightarrow \pi + N,
\end{align*}
\]
where \(\pi\) and \(N\) indicate the pion and the nucleon respectively and \(B\) and \(F\) are
the unstable composite states of the Sakata model with the indicated decay modes.\(^***\)

\(^*\) For fermion state we use the notation \(F_j^i(S, I)\) where subscript \(j\) expresses the number
of particles plus antiparticles of which the fermion is composed, and superscript \(i\) the class to which this
state belongs (each class corresponds to each irreducible constituent). \(S\) and \(I\) in brackets are its
strangeness and isotopic spin respectively. For boson state we write similarly as \(B_j^i(S, I)\). See
reference 5).

\(^**\) Our present theoretical information of these composite states is limited to their masses, and
we do not know how these composite states will appear with definite spin and parity. Its clarification
will be an important task for the composite model and will make clear the role of \(\pi-\pi\) interaction, etc.,
if they have some contribution in the actually observed resonance phenomena.

\(^***\) The interference between (1a) and (1b) does not occur, since in our present model the first
step of each reaction is assumed to be a real process.
According to the scheme (1) we give the production threshold energy of the states that may contribute to the single pion production process in Fig. 1. The masses of the levels given in Fig. 1 are those of the experimental data for the fermion states and those of mass formula (3) of the previous article for the boson states. Unfortunately we have not information about the spin and parity of these states at present, but the lack of such information does not prevent one from obtaining the information on mass levels from such a kind of analysis as that in § 3.

![Threshold Energy Graph](http://ptp.oxfordjournals.org/)

**Fig. 1.** Threshold energy of the unstable particles according to the scheme (1).

**Table I**

<table>
<thead>
<tr>
<th>process</th>
<th>( \pi^+ + p \rightarrow )</th>
<th>( \pi^- + p \rightarrow )</th>
<th>( \pi^- + p \rightarrow )</th>
<th>( \pi^0 + \pi^0 + n )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \pi^+ + \pi^0 + p )</td>
<td>( \pi^- + \pi^0 + n )</td>
<td>( \pi^0 + \pi^0 + n )</td>
<td>( \pi^- + \pi^- + n )</td>
</tr>
<tr>
<td>( B(0, 2) )</td>
<td>( \frac{1}{5} \sigma_3 )</td>
<td>( \frac{4}{5} \sigma_3 )</td>
<td>( \frac{9}{45} \sigma_3 )</td>
<td>( \frac{2}{45} \sigma_3 )</td>
</tr>
<tr>
<td>( B(0, 1) )</td>
<td>( \sigma_3 )</td>
<td>0</td>
<td>( \frac{1}{9} (4\sigma_1 + \sigma_3 + 4\alpha) )</td>
<td>( \frac{1}{9} (2\sigma_1 + 2\sigma_3 - 4\alpha) )</td>
</tr>
<tr>
<td>( B(0, 0) )</td>
<td>0</td>
<td>0</td>
<td>( \frac{4}{9} \sigma_1 )</td>
<td>( \frac{2}{9} \sigma_1 )</td>
</tr>
<tr>
<td>( F(0, 3/2) )</td>
<td>( \frac{4}{15} \sigma_3 (\text{extra } \pi^+ \text{)} )</td>
<td>( \frac{2}{15} \sigma_3 )</td>
<td>( \frac{1}{135} (10\sigma_1 + 16\sigma_3 - 40b) ) (extra ( \pi^- ))</td>
<td>( \frac{1}{135} (45\sigma_1 + 18\sigma_3 + 90b) ) (extra ( \pi^+ ))</td>
</tr>
<tr>
<td></td>
<td>( \frac{9}{10} \sigma_3 (\text{extra } \pi^0 \text{)} )</td>
<td>( \frac{1}{135} (10\sigma_1 + \sigma_3 - 10b) ) (extra ( \pi^0 ))</td>
<td>( \frac{1}{135} (5\sigma_1 + 8\sigma_3 - 20b) ) (extra ( \pi^- ))</td>
<td>( \frac{1}{135} (20\sigma_1 + 2\sigma_3 - 20b) )</td>
</tr>
<tr>
<td>( F(0, 1/2) )</td>
<td>( \frac{1}{3} \sigma_3 (\text{extra } \pi^+ \text{)} )</td>
<td>( \frac{2}{3} \sigma_3 )</td>
<td>( \frac{1}{27} (4\sigma_1 + \sigma_3 + 4\alpha) ) (extra ( \pi^- ))</td>
<td>( \frac{1}{27} (8\sigma_1 + 2\sigma_3 + 8\alpha) ) (extra ( \pi^- ))</td>
</tr>
<tr>
<td></td>
<td>( \frac{1}{3} )</td>
<td>( \frac{2}{3} \sigma_3 )</td>
<td>( \frac{1}{27} (4\sigma_1 + 4\sigma_3 - 8\alpha) ) (extra ( \pi^0 ))</td>
<td>( \frac{1}{27} (2\sigma_1 + 2\sigma_3 - 4\alpha) )</td>
</tr>
</tbody>
</table>
From Table I we can easily obtain the charge ratio in each channel. In this Table $\sigma_i$ and $\sigma_3$ mean the cross section of $I=1/2$ and $I=3/2$ states of each channel where the notation specifying the channel has been omitted, and $a=\sqrt{\sigma_1\sigma_3}\cos\varphi$ and $b=\sqrt{2/5}a$ where $\varphi$ is the relative phase between $I=1/2$ and $3/2$ states. The term denoted by (extra $\pi$) is the contribution from the process in which the designated $\pi$ is emitted associated with $F$ (cf. reference 11).

We cannot answer the question what the magnitude of cross section of each process is and how much of the phenomena can be explained by the scheme (1), since we do not yet have the dynamical theory of the composite model. In a rough approximation, Fermi's statistical theory will give us good estimation.

At the low energy near the threshold of the single pion production process we have $F_2^1(0, 3/2)$ as $F$ of the scheme (1). As to $B$, we have to consider the contribution of $B_1^2(0, 2)$ and $B_3^2(0, 0)$ states. The estimation of the mass by the mass formula (3) of reference 5 gives $2m_\pi$ for $B_1^2(0, 2)$ where $m_\pi$ is the pion mass. Since no reliable evidence of charge particles with the mass $\sim 2m_\pi$ has been reported, it will be natural to consider that $B_1^2(0, 2)$ may have the mass greater than $2m_\pi$ and decay rapidly into two pions. With respect to $B_3^2(0, 0)$ there are two alternatives, (a) $B_3^2(0, 0)$ is the excited state of the vacuum, and not a new particle with the mass $\sim 2.5m_\pi$, or (b) $B_3^2(0, 0)$ will be new particle and not the excited state of the vacuum. We do not know which alternative corresponds to the reality. In the case (b) there will be two possibilities for the decay. If $B_3^2(0, 0)$ is $0^+$ or $2^+$, it will decay into two pions very rapidly and contribute to the single pion production process. On the other hand, if it is $0^-$ or $1^+$ and has the mass $<3m_\pi$, then its dominant decay mode will be $\rightarrow 2\gamma$ or $\rightarrow \pi^0 + \gamma$ and will be observed as metastable particle.

As to the experimental data of $\pi^-p$ reaction near the threshold, the analyses have been performed based on the strong pion-pion interaction model and the Lindenbaum-Sternheimer model respectively. It has been reported that some features of the energy spectrum and angular distribution of pions are consistent with the theoretical prediction of the nucleon isobar model and, on the other hand, the large experimental cross section of the single pion production process can be explained by the strong pion-pion interaction. Although the experimental and theoretical investigations are at the very preliminary stage and it is not the purpose of the present article to criticize the analyses so far made, such a situation seems to have some connection with our present scheme that $B_1^1(0, 2)$ and $F_2^1(0, 3/2)$ (and possibly $B_3^2(0, 0)$) will contribute to the single pion production process near the threshold. If the mass of $B_1^1(0, 2)$ is $<3m_\pi$, $B_1^1(0, 2)$ will give larger contribution than $F_2^1(0, 3/2)$ does at the low energy near the threshold. As to the problem of a large cross section compared with the static pion theory, we make no comment, since we have no appropriate way to estimate the cross section. However, a large cross section of the single pion production process will not be unreasonable if we take into account of the phase volume available for
the two-particle process and three-particle process at these energies and there are two (or possibly three) channels opened for the single pion production process.

As the incident pion energy increases, we will see from Fig. 1 that the next higher level \( B/\text{0,} 0 \) or \( n\text{°} 1 \) will have influence above 600 Mev incident pion energy. Then, if our scheme is right, we can understand the phenomena of single pion production under 600 Mev incident pion energy in terms of \( B/\text{0,} 2 \) and \( F_3\text{3/2} (0, 3/2) \) (and \( B/\text{2,} 0 \)). In this connection there are some interesting experimental data. They are the data on \( \pi^+ + p \) reaction at 500 Mev pion energy.\(^\text{14}\) It will be concerned only with \( B/\text{0,} 2 \) and \( F_3\text{3/2} (0, 3/2) \) isobars in our scheme. The data give the branching ratio \( \frac{\pi^+ + p \rightarrow \pi^+ + \pi^+ + n}{\pi^+ + p \rightarrow \pi^+ + \pi^+ + n} \approx 1.5 \pm 0.5 \) for the single pion production process.

Although the statistics is still very poor, the interesting point is that this value is far from 6.5 which is the prediction of the Sternheimer-Lindenbaum model. Now, besides \( F_3\text{3/2} (0, 3/2) \), we have \( B/\text{0,} 2 \) which gives 1/4 for this branching ratio and we will have the value which is much smaller than 6.5 of the nucleon isobar model. (It is noted that the contribution of the boson isobar with \( I=1 \) mal-\text{c} to increase the ratio from 6.5 and \( I=0 \) isobar does not contribute entirely.) We will discuss the energy spectrum of pions, etc., of this reaction in the next section in view of its interesting features.

Above 600 Mev incident pion energy, the states \( B_4\text{4,} 0 \), \( B/\text{4,} 0 \) and \( B/\text{4,} 1 \) will newly appear and contribute to the single pion production, if they decay rapidly into two pions. \( B/\text{4,} 0 \) is the antiparticle of \( B/\text{4,} 0 \), so that they have the same mass, spin and parity. The estimation of their masses based on the two-body approximation (mass formula (3) of reference 5\)) gives the value \( \sim 750 \) Mev. Then they will become influential only above 800 Mev incident pion energy when we assume this value of mass. It is, however, necessary to take into account the theoretical error of order 200 Mev for the estimation by the mass formula, when we apply the formula to the four-body system, and we must consider the possibility that these levels appear at lower energies. The existence of the isobar states having \( I=1 \) and the corresponding energy have been discussed often\(^\text{6,7,16}\).

In respect of the opening of the channels of \( N+B/\text{4,} 0 \), \( N+B/\text{4,} 1 \), \( \pi+F_3\text{3/2} (0, 1/2) \) and \( B/\text{4,} 1 \) \( + F_3\text{3/2} (0, 3/2) \), that may happen to concentrate in the energy region \( 700 \sim 900 \) Mev incident pion energy, it will be interesting to note that the shoulder behavior of the \( \pi^+ + p \) total cross section around 800 Mev pion energy\(^\text{10}\)\* may have some connection with it. Further information of the masses and spin-parity of \( B/\text{4,} 0 \), \( B/\text{4,} 1 \) and \( B/\text{4,} 2 \) may bring some clarification on this point. Since for the phenomena above 600 Mev we must newly take into account \( B/\text{4,} 0 \), \( B/\text{4,} 1 \), \( B/\text{4,} 1 \), \( F_3\text{1,} 1/2 \) and \( F_3\text{1,} 1/2 \) besides \( B/\text{4,} 2 \) and \( F_3\text{3/2} (0, 3/2) \) (and \( B/\text{2,} 0 \)) successively as the incident energy

\* Another possible interpretation from the Sakata model is that it is due to the resonance states \( F_3\text{3/2} (0, 3/2) \), and this interpretation will be allowed if the fourth broad resonance around 1.3 Bev consists of two resonances, and not three resonances.
increases, the circumstance becomes complicated. But still it will be possible to obtain the information on these levels if we have sufficient accumulation of the experimental data about the branching ratio, energy spectrum of pions and nucleons, angular correlations, etc., in addition to the information obtained from the analyses of the reaction at lower energies. It is evident from the scheme (1) the fermion isobars $F$’s manifest themselves most clearly in the pion energy spectrum, and the boson isobars $B$’s in the nucleon energy spectrum. In regard to this, it will be useful to point out that the pion energy spectrum due to the decay of $B$ is a monotonous function of energy so that the contribution from this channel does not generally wash away the characteristic features of the pion spectrum of the nucleon isobars. For this reason and from the fact that $F_3^1(0, 3/2)$ channel will be dominant in $\pi^-p$ reaction even at 1 Bev, the qualitative success of the Sternheimer-Lindenbaum model at this energy, which has been shown mainly about the pion energy distribution, will not be unexpectable.

§ 3. Pion production in $\pi^+p$ collision at 500 Mev

In the preceding section we have given the prediction that would be expected in the single pion production process initiated by the pion-nucleon collision if we consider the phenomena based on the level scheme of the Sakata model. Our interest in the single pion production phenomena is, first of all, that it will give the information which may serve for establishing the level scheme of composite particle. For this purpose, we need to obtain information about each level. It will be clever to start with the analysis of the events that concern with parameters as few as possible, and then to extend the investigation to the more complicated cases.

As such a simple case, we choose the $\pi^+p$ reaction at 500 Mev. As shown in § 2, we will be concerned only with the total isospin $I=3/2$ state and $B_1^1(0, 2)$ and $F_2^1(0, 3/2)$ as the unstable isobars in this case. The energy 500 Mev will be high enough to produce these isobars and low enough to avoid the contributions of higher levels, $B_1^4(0, 1)$, etc. We have now no correct knowledge about higher levels, $B_1^1(0, 1)$, $B_2^1(0, 1)$ and $B_2^6(0, 1)$. The estimation of two parameter mass formula gives 750 Mev for $B_1^4(0, 1)$ and $B_2^6(0, 1)$ and 850 Mev for $B_1^1(0, 1)$. One may of course expect an error in this estimation. But the maximum mass value of the boson isobar $B$ that can be created at 500 Mev incident pion energy is $\sim510$ Mev, so we can reasonably expect that they will not give a large contribution to the process even if they have relatively small mass. The fact that only $B_1^1(0, 2)$ and $F_2^1(0, 3/2)$ will give large contribution to the process in question is a salient feature of the present model.

Now the cross section of each process is given from Table 1 by the expression

$$\sigma(\pi^+ + p \rightarrow \pi^- + \pi^0 + p) = \frac{1}{5} \sigma_3(B_1^1(0, 2)) + \frac{13}{15} \sigma_3(F_2^1(0, 3/2)),$$
\[ \sigma (\pi^+ + p \rightarrow \pi^+ + \pi^+ + n) = \frac{4}{5} \sigma_3 (B_1^1 (0, 2)) + \frac{2}{15} \sigma_3 (F_3^1 (0, 3/2)) , \tag{3} \]

where \( \sigma_3 (B_1^1 (0, 2)) \) is the total cross section of the first step of the reaction \((1a)\) in which \(B_1^1 (0, 2)\) particle is emitted as \(B\) and \(\sigma_3 (F_3^1 (0, 3/2))\) is that of the reaction \((1b)\). From this we can naturally expect that the branching ratio is in the range

\[ 0.25 \leq \frac{\sigma (\pi^+ + p \rightarrow \pi^+ + \pi^+ + n)}{\sigma (\pi^+ + p \rightarrow \pi^+ + \pi^+ + n)} \leq 6.5. \]

The value 6.5 corresponds to the case when only \(F_3^1 (0, 3/2)\) channel exists.

The preliminary experiment at this energy was performed by Willis, which gives \(1.5_{-1.2}^{+1.5}\) for this ratio.\(^{14}\) This means \(\sigma_3 (B_1^1 (0, 2)) / \sigma_3 (F_3^1 (0, 3/2)) \approx 2/3\). Although this experiment is a preliminary one and we have not an appropriate way to calculate the magnitude of the cross section, this value may not be unreasonable.\(^{**}\)

Unfortunately we have not a reliable experimental information about pion energy spectrum, etc., at this incident energy. However, it is expected that the information about these quantities will increase rapidly so that it will be useful to make clear some features of the spectra of our model.

3-1. **Energy spectrum of pion**

The characteristics of the energy spectrum and angular correlation of the pion and nucleon are essentially determined by kinematical relations in the present model. So if we have full understanding about kinematical relations among the final particles it is rather easy to deduce what the spectra are. Hereafter, discussion is given in the c.m.s. of the total system.

The energy spectrum of the pion consists of three parts. The first is coming from the decay of \(F_3^1 (0, 3/2), I_1 (E_\pi)\), the second is the extra pion which is emitted in company with \(F_3^1 (0, 3/2), I_2 (E_\pi)\), and the third is the pion resulted from the decay of \(B_1^1 (0, 2), I_3 (E_\pi)\).

Assuming the independency of the matrix elements of the isobar momentum and spin state, the energy spectrum in each case can be expressed in c.m.s. as:

\[ I_1 (E_\pi) dE_\pi = N_1 \int_{m_{F_{\pi}^{\text{min}}}^{}}^{m_{F_{\pi}^{\text{max}}}^{}} dm_{\pi} \rho (m_{\pi}) F_1 (E_\pi) dE_\pi , \tag{5} \]

where

\[ m_{F_{\pi}^{\text{max}}} = \left( b_2 \pm \sqrt{b_2^2 - a_1 c_1} \right)^{1/2} : \text{the available maximum (minimum) mass of } F_3^1 (0, 3/2) \text{ for a fixed energy } E_\pi \text{ of decay pion,} \]

\(^*\) The experiment at 300~750 Mev gives 1.0 for this ratio.\(^{14}\)

\(^{**}\) From the argument of the phase volume and weight of spin state, we have \(\sigma_3 (B_1^1 (0, 2)) / \sigma_3 (F_3^1 (0, 3/2)) \approx 0.77\) for spinless \(B_1^1 (0, 2)\) with mass 350 Mev and \(F_3^1 (0, 3/2)\) with mass 1230 Mev.
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\[ a_t = W^2 + 2m_\pi^2 - 2W E_\pi, \]
\[ b_t = m_N^2 W^2 + m_\pi^2 + (W^2 - m_N^2) W E_\pi - 2W^2 E_\pi^2, \]
\[ c_t = (m_N^4 - 2m_N^2 m_\pi^2 - m_\pi^4 + m_\pi^2 W^2) W^2 + m_\pi^6 + 2(m_N^2 - m_\pi^2)(W^2 - m_\pi^2) W E_\pi, \]
\[ G_{\pi1} = \frac{m_F}{2p_F p_{\pi F}}, \]
\[ F_1 = E_{\pi} p_{\pi F}, \]
\[ E_{\pi F} = \frac{W^2 + m_F^2 - m_\pi^2}{2W} : \text{the energy of } F_1^i(0, 3/2) \text{ in c.m.s.,} \]
\[ p_{\pi F} : \text{the momentum of } F_1^i(0, 3/2) \text{ in c.m.s.,} \]
\[ E_{\pi F} = \frac{m_F^2 + m_\pi^2 - m_N^2}{2m_F} : \text{the energy of decay pion in the rest system of } F_3^i(0, 3/2), \]
\[ p_{\pi F} : \text{the momentum of decay pion in the rest system of } F_3^i(0, 3/2), \]
\[ W : \text{the total energy of the system in c.m.s.} \]
\[ I_2(E_\pi) dE_\pi = N_3 \rho(m_F) F_1 G_{\pi2} dE_\pi, \tag{6} \]
where
\[ G_{\pi2} = \frac{W}{m_F}. \]
\[ I_3(E_\pi) dE_\pi = N_3 \int_{m_{B_{\pi_{\min}}}}^{m_{B_{\pi_{\max}}}} dm_B \rho(m_B) F_2 G_{\pi3} dE_\pi, \tag{7} \]
where
\[ m_{B_{\pi_{\min}}} = \left(b_3 \pm \sqrt{b_3^2 - 2a_3 c_3}\right)^{1/2} : \text{the available maximum (minimum) mass of } B_4^i(0, 2) \text{ for a fixed energy } E_\pi \text{ of decay pion,} \]
\[ a_3 = W^2 + m_\pi^2 - 2W E_\pi, \]
\[ b_3 = m_\pi^2 (W^2 + m_N^2) + (W^2 - m_N^2) W E_\pi - 2W^2 E_\pi^2, \]
\[ c_3 = m_\pi^2 (W^2 - m_N^2)^2, \]
\[ G_{\pi5} = \frac{m_B}{2p_B p_{\pi B}}, \]
\[ F_3 = E_N p_N, \]
\[ E_N = \frac{W^2 + m_N^2 - m_\pi^2}{2W} : \text{the energy of nucleon in c.m.s.,} \]
\[ p_N : \text{the momentum of nucleon in c.m.s.,} \]
\[ E_B = W - E_N : \text{the energy of } B_4^i(0, 2) \text{ in c.m.s.,} \]
$p_B$ : the momentum of $B_1^1(0, 2)$ in c.m.s.,

$E_{\pi n} = \frac{m_B}{2}$ : the energy of decay pion in the rest system of $B_1^1(0, 2)$,

$\rho_{\pi n}$ : the momentum of decay pion in the rest system of $B_1^1(0, 2)$.

In Eqs. (5) ~ (7) $N_1$, $N_2$ and $N_3$ are the normalization constants. $\rho(m_F)$ is the mass spectrum of $F_1^3(0, \frac{3}{2})$ particle and $\rho(m_B)$ is the one of $B_1^1(0, 2)$ particle. $F_1$ and $F_3$ are the two-body phase space factors of the first steps of the reactions (1b) and (1a) divided by the boson (pion or $B_1^1(0, 2)$) energy. $G_*$'s are the factors which give the energy spectrum of the pion from the decay of the isobars of masses $m_F$ and $m_B$. Our discussion is essentially the same with the L-S model except for a few minor simplifications.

We have not correct information on the mass spectrum of each level, and it is very dependent on the unknown properties of the system (spin, parity, formation interaction, decay interaction, etc.). We assume the form

$$\rho(m_F) \sim (m_F - m_N - m_\pi) \times \exp \left[ -\frac{(m_F - m_F^*)^2}{F_F^*} \right].$$

However, this assumption will not be so essential for our present purpose. The parameters of $\rho(m_F)$ are chosen so as to approximately give the energy dependence of the total cross section of $\pi^+ - p$ reaction at low energy ($m_F^* = 1230$ Mev, $F_F^* = 66$ Mev).

Fig. 2. Calculated momentum spectrum of pion in the reaction (1b).

Fig. 3. Calculated momentum spectrum of pion in the reaction (1a).
For the $B_1^1(0, 2)$ particle the parameters are more arbitrary. The theoretical estimation of mass of $B_1^1(0, 2)$ gives $2m_e$, but owing to the situation that no evidence of charged boson with mass $\sim 2m_e$ and $B_1^1(0, 2)$ will decay rapidly. Even in this case $\rho(m_B)$ is still too arbitrary. Similar to the case of $\rho(m_F)$, we choose

$$\rho(m_B) \sim (m_B - 2m_e) \exp[-(m_B - m_B^0)^2/\Gamma_B^2],$$

with $m_B^0 = 300, 350, 400$ Mev and $\Gamma_B = 30$ Mev. The feature of the spectrum with different parameters can be guessed from these data to some extent. The momentum spectra $I_1(p_e)$ are given from the corresponding energy spectrum $I_1(E_e)$ by the relation $I_1(p_e) = I_1(E_e) dE_e/dp_e$.

In Fig. 2 we give the momentum distribution of the pion calculated from $I_1(E_e)$ and $I_2(E_e)$. In Fig. 3 we give the distribution due to $I_3(E_e)$.

The energy spectrum in each process can be expressed in terms of $I_1(E_e)$, $I_2(E_e)$ and $I_3(E_e)$ as

$$\pi^+ p \rightarrow \pi^+ + p$$

$$I(E_e) dE_e = 15 \sigma_s(F_3^1(0, 3/2)) \{9 I_1(E_e) + 4 I_2(E_e)\}$$

$$+ \frac{1}{5} \sigma_s(B_1^1(0, 2)) I_3(E_e) \} dE_e, \quad (10)$$

$$\pi^0 p \rightarrow \pi^- + n$$

$$I(E_e) dE_e = 15 \sigma_s(F_3^1(0, 3/2)) \{4 I_1(E_e) + 4 I_2(E_e)\}$$

$$+ \frac{1}{5} \sigma_s(B_1^1(0, 2)) I_3(E_e) \} dE_e, \quad (11)$$

$$\pi^+ p \rightarrow \pi^- + n$$

$$I(E_e) dE_e = 15 \sigma_s(F_3^1(0, 3/2)) \{I_1(E_e) + I_2(E_e)\}$$

$$+ \frac{4}{5} \sigma_s(B_1^1(0, 2)) I_3(E_e) \} dE_e. \quad (12)$$

For the interest of the reader, the theoretical momentum spectrum of the pion in each process is given in Figs. 4 and 5 for the case of $m_B^0 = 350$ Mev and $\sigma_s(B_1^1(0, 2))/\sigma_s(F_3^1(0, 3/2)) = 2/3$. The experimental data of Willis are also given in Figs. 4 and 5.

Although it will be too early to discuss these experimental data, Willis has noted that in the $\pi^- p \rightarrow \pi^- + \pi^- + n$ process the pions are emitted with very high or low energies. A similar observation has also been made for the experimental data of Blevins et al.\(^4\) Apparently the characteristic feature of the spectrum of $F_3^1(0, 3/2)$ channel does not show such a tendency. The existence of $B_1^1(0, 2)$ will reduce such a discrepancy, if not satisfactory.
Fig. 4. Calculated and experimental momentum spectra of $\pi^+$ in $\pi^+ + p \rightarrow \pi^+ + \pi^0 + p$ (solid curve) and $\pi^0$ in $\pi^+ + p \rightarrow \pi^+ + \pi^0 + p$ (dashed curve).

Fig. 5. Calculated and experimental momentum spectrum of $\pi^+$ in $\pi^+ + p \rightarrow \pi^+ + \pi^+ + n$.

3.2. Energy spectrum of nucleon

The energy spectrum of nucleon consists of two parts. One is the decay product of $F_1^1(0, 3/2)$, $J_1(E_N)$, and the other is the extra nucleon which is emitted in company of $B_4^1(0, 2)$, $J_2(E_N)$.

$J_1(E_N)$ and $J_2(E_N)$ are expressed as:

$$J_1(E_N) \, dE_N = N_1' \int_{m_{F_{\text{min}}}}^{m_{F_{\text{max}}}} dm_F \rho(m_F) F_1 G_{N_1} dE_N,$$  \hspace{1cm} (13)

where

$$m_{F_{\text{min}}} = \left( \frac{a + \sqrt{b^2 - ac}}{a} \right)^{1/2}$$ : the available maximum (minimum) mass of $F_3^1(0, 3/2)$ for a fixed energy $E_N$ of decay nucleon,
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\[ a = W^2 + m_\pi^2 - 2WE_N, \]
\[ b = m_\pi^2(W^2 + m_N^2) + (m_N^2 - 2m_\pi^2)WE_N - 2W^2E_N^2, \]
\[ c = m_N^2(W^3 - m_\pi^2)^2 + (m_N^2 - m_\pi^2)^2W^2 - 2(m_N^2 - m_\pi^2)(W^2 - m_\pi^2)WE_N, \]

\[ G_N = \frac{m_F}{2p_Fp_{NF}} = G_{s1}. \]

\[ J_1(E_N)\,dE_N = N'_2\rho\,(m_N)\,F_3G_{N2}\,dE_N, \]

where

\[ G_{N2} = \frac{W}{m_N}. \]

The notation \( N'_1, N'_2, G_{s1}, \) and \( G_{N2} \) should be understood analogously as in the case of the pion energy spectrum. In Fig. 6 we give the momentum spectrum of nucleon calculated from \( J_1(E_N) \) and \( J_2(E_N) \).

![Fig. 6. Calculated momentum spectrum of nucleon.](image)

The energy spectrum of nucleon in each process can be expressed in terms of \( J_1(E_N) \) and \( J_2(E_N) \) as

\[ p \text{ spectrum in } \pi^+ + p \rightarrow \pi^+ + \pi^0 + p \]

\[ J(E_N)\,dE_N = \left\{ \frac{13}{15} \sigma_s(F_3^+(0, 3/2))\,J_1(E_N) + \frac{1}{5} \sigma_s(B_1^+(0, 2))\,J_2(E_N) \right\}\,dE_N, \]

(15)
Fixing the $\frac{\sigma_3(B_1^+(0, 2))}{\sigma_3(F_3^+(0, 3/2))}$ ratio by the experimental data of Willis, and assuming $m_B = 350$ and 400 Mev, we obtain the spectra in Figs. 7 and 8.

\begin{equation}
J(E_N)dE_N = \left\{ \frac{2}{15} \sigma_3(F_3^+(0, 3/2))J_1(E_N) + \frac{4}{5} \sigma_3(B_1^+(0, 2))J_2(E_N) \right\}dE_N.
\end{equation}

Fig. 7. Calculated and experimental momentum spectrum of $p$ in $\pi^+ + p \rightarrow \pi^+ + \pi^0 + p$.

Fig. 8. Calculated and experimental momentum spectrum of $n$ in $\pi^+ + p \rightarrow \pi^+ + \pi^0 + n$.

3-3. Angular correlations

In the preceding discussions we have given information on the momentum spectrum of the pion and nucleon. The obtained spectra have characteristic features in each
case, so that even with the momentum spectrum one can make a certain detailed
discussion for our present scheme, if sufficient experimental data are supplied.

In this section we will give some crude information about the angular cor-
relation between final products. It will also be useful in comparing the present
scheme with experiment.

Different from the case of momentum spectrum, the feature of the angular
correlation is rather strongly dependent on the mass of unstable isobars.

So, for the angular correlation, we give the theoretical spectrum when
\[ \rho(m_F) \sim \delta(m_F - m_f^0) \] and \[ \rho(m_B) \sim \delta(m_B - m_b^0). \] Such a discussion may be helpful
to understand the mutual correlation of the mass spectrum and the angular
correlation.

\( a) \) Angular correlation between pions

The angular correlation between pions consists of two parts. One is the cor-
relation between the extra pion and the decay pion in the reaction (1b), \( K_1(\theta_{\pi\pi}) \),
and the other is the correlation between the pions which are the decay products
of the reaction (1a), \( K_2(\theta_{\pi\pi}) \). They can be written as

\[
K_1(\theta_{\pi\pi}) \cos \theta_{\pi\pi} = \int dm_F \rho(m_F) \frac{\gamma_F^2(x_{\pi} - \delta_{\pi})^3}{2(1 - \gamma_F^2 x_{\pi}) \cos^2 \theta_{\pi\pi}} d(\cos \theta_{\pi\pi}),
\]

(17)

where \( x_{\pi} \) is given by the relation \( \frac{\sqrt{1 - x_{\pi}^2}}{\gamma_F(x_{\pi} - \delta_{\pi})} = \tan \theta_{\pi\pi} \), and \( \gamma_F = \frac{E_F}{m_F} \) and
\( \delta_{\pi} = p_F E_{\pi F}/E_F p_{\pi F} \).

\[
K_2(\theta_{\pi\pi}) \cos \theta_{\pi\pi} = \int dm_B \rho(m_B) \frac{(\gamma_B^2 - 1) y^3 + (1 - \gamma_B^2 \delta_B^2) |y|}{4 \gamma_B^2 \delta_B^2 y^2 (1 - \gamma^2) + \gamma_B^2 (1 - \delta_B^2)} \cos^2 \theta_{\pi\pi} d(\cos \theta_{\pi\pi}),
\]

(18)

![Graph of K_1 and K_2](http://ptp.oxfordjournals.org/)

Fig. 9. The graphs of \( K_1 \) and \( K_2 \).
where $y$ is given by the relation 
\[ \frac{2\gamma_B \delta_B \sqrt{1 - y^2}}{(1 - \gamma_B^2)} = \tan \theta \pi, \] 
and $\gamma_B = \frac{E_B}{m_B}$
and $\delta_B = \frac{p_B E_B}{E_B p_B}$. These $K_1(\theta \pi)$ and $K_2(\theta \pi)$ are given in Fig. 9 with $m_B^0 = 1180, 1230, 1280$ Mev and $m_B^0 = 300, 350, 400$ Mev.

**b) Angular correlation between pion and nucleon**

The angular correlation between nucleon and pion consists of three parts, the correlation between the extra pion and the nucleon resulted from the decay of isobar $F_1^0(0, 3/2)$, $L_1(\theta \pi N)$, the correlation between the pion and the nucleon which are the decay products of $F_1^0(0, 3/2)$, $L_2(\theta \pi N)$ and the correlation between the extra nucleon and the pion from the decay of $B_2^1(0, 2)$, $L_3(\theta \pi N)$. They are expressed respectively as

\[ L_1(\theta \pi N) d(cos \theta \pi N) = \int dm_P \rho(m_P) \frac{\gamma_F^2 (x_N - \delta_N)^2}{2(1 - \delta_N x_N) \cos^2 \theta \pi N} d(cos \theta \pi N), \]

where $x_N$ is given by the relation 
\[ \frac{\sqrt{1 - x_N^2}}{(1 - x_N - \delta_N) \cos \theta \pi N} = \tan \theta \pi N, \] 
and $\delta_N = \frac{pE_{P \pi F}}{E_{P \pi F}}$;

\[ L_2(\theta \pi N) d(cos \theta \pi N) = \int dm_P \rho(m_P) \]

\[ \times \frac{[(\gamma_F^2 - 1) z^2 + \gamma_F^2 (\delta_N - \delta_N) z + (1 - \gamma_F^2 \delta_N \delta_N) z^2]}{2\gamma_F^2 (\delta_N + \delta_N)^2 \left[ - (\gamma_F^2 - 1) z^2 + (2\gamma_F^2 - \gamma_F^2 \delta_N \delta_N - 1) z + \gamma_F^2 (\delta_N - \delta_N) \right] \cos^2 \theta \pi N}, \]

where $z$ is given by the relation 
\[ \frac{\gamma_F (\delta_N - \delta_N) \sqrt{1 - z^2}}{\gamma_F^2 (z - \delta_N) (z + \delta_N) + (1 - z^2)} = \tan \theta \pi N; \]

\[ L_3(\theta \pi N) d(cos \theta \pi N) = \int dm_P \rho(m_P) \frac{\gamma_B^2 (x_N - \delta_B)^2}{2(1 - \delta_B x_N) \cos^2 \theta \pi N} d(cos \theta \pi N), \]

Fig. 10. The graphs of $L_1$ and $L_2$. 

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where \( x_N \) is given by the relation \( \sqrt{1-x_N^2} = \tan \theta_{x_N} \), \( L_1(\theta_{x_N}) \), \( L_2(\theta_{x_N}) \) and \( L_3(\theta_{x_N}) \) are given in Figs. 10 and 11.

In our opinion, the investigation of the pion-pion angular correlation seems to be more prospective than the pion-nucleon correlation to make clear the mutual correspondence between our model and the experiment.

Besides the investigation of the angular correlations between final products, the analysis of the correlations between the incident particle and the secondary particles is also important. For example, the Adair type analysis\(^{17}\) will be interesting for the determination of the spin and parity of the isobar.

§ 4. Some remarks

In the preceding sections we have given an analysis of the single pion production process in \( \pi-N \) collision. Our analysis is essentially an extension of the Lindenbaum-Sternheimer model to the case where the boson isobar exists besides the fermion isobar. Our aim in this paper is to give rough information in order to help further theoretical and experimental investigation.

For the present stage of the composite theory, it is one of the important problems to find out what a correspondence exists between the theoretical levels (such as \( U(3) \) theory) and the experimental levels. Although we believe the composite levels manifest themselves as stable particles or as resonant states in nature, it is rather difficult at present to show theoretically how these unstable composite levels show themselves in the interaction. But this will not give an objection to the necessity of such a kind of phenomenological analysis so far made. Our feeling for the necessary step to the future advance is to confirm a corre-
spondence between the theoretical levels and the experimental levels and to study the various processes such as the transitions between levels of composite system as far as possible, even if it were based on a rather naive picture. For such a purpose, we have proposed a systematic investigation of the pion production process, starting from the low energy phenomena which may involve few unknown parameters.

The theoretical analysis of the pion production process has so far been performed along two main different approaches (the L-S model and the strong pion-pion interaction model) rather independently, and the analyses in each model have shown some consistency with the experimental results. Our standpoint here proposed may give a logical basis for these models and show that both, if we use old terminology, the L-S model and the strong pion-pion interaction model are necessary to explain the phenomena.

Finally, it is also noted that the unstable composite states may appear as the reaction products and play important roles, not only in the multiple meson production process initiated by the pion collision, but also in other processes such as the nucleon-nucleon collision and nucleon-antinucleon annihilation. For example, we can increase the multiplicity of pion without taking a large interaction volume, if we take account of the boson isobars predicted in the Sakata model, although it will require further extensive work to find out what is really going on in these phenomena.

We hope that experimental and theoretical analyses will be performed so as to make clear the scheme proposed in this paper.

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