On a Gauge Theory of Elementary Interactions.

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Summary. — A theory of strong as well as weak interactions is proposed using the idea of having only such interactions which arise from generalized gauge transformations.

1. — Introduction.

One of the problems engaging current interest in field theory is the problem of determining which fields are « elementary » in some fundamental sense, and which are not. An equally, if not more, important, problem is that of finding a guiding principle for writing fundamental interactions of fields. The only such principle which exists at the present time seems to be the gauge-principle. Whenever a symmetry property exists, the associated gauge transformation leads in a definite manner to the postulation of an interaction through the mediation of a number of intermediate particles. There exist, at present, numerous attempts to understand all known elementary interactions in this manner. In an earlier paper (1), the authors considered a gauge-transformation in [3] « charge-space » to generate weak and electro-magnetic interactions. Recently J. J. Sakurai (2) has used similar ideas to postulate five intermediate vector mesons which may be responsible for mediating strong interactions.

All these attempts suffer from certain weaknesses. Our earlier attempt (1) to understand weak and electro-magnetic interactions produced only parity-

conserving interactions in a natural manner. Besides, the weak interactions
did not obey the $\Delta I = \frac{1}{2}$ rule. Sakurai’s strong Lagrangian suffers from the de-
fect that it contains no Yukawa-like terms permitting single emission of pions
or K-mesons by baryons.

In this note we wish to reconsider the problem. Our basic postulate is
that it should be possible to generate strong, weak and electro-magnetic inter-
action terms (with all their correct symmetry properties and also with clues
regarding their relative strengths), by making local gauge transformations on
the kinetic-energy terms in the free Lagrangian for all particles. This is the
statement of an ideal which, in this paper at least, is only very partially realized.

It may however be of interest to set down the procedure which has been
followed.

2. – A simple model.

Consider the $\pi$-nucleon system. Following SCHWINGER we assume the ex-
istence of a scalar iso-scalar particle $\sigma$. Write the free Lagrangian kinetic
energy terms in the form

\begin{equation}
N_L^+ \gamma_\mu \partial_\mu N_L + N_R^+ \gamma_\mu \partial_\mu N_R + \frac{1}{2} (\partial_\mu \pi) \cdot (\partial_\mu \pi) + \frac{1}{2} (\partial_\mu \sigma)^2,
\end{equation}

where

\[ N_L = \frac{1}{2} (1 + \gamma_5) N, \]
\[ N_R = \frac{1}{2} (1 - \gamma_5) N. \]

Following a suggestion made by SCHWINGER (3), GELL-MANN and LEVY (4)
one may consider $\left( \begin{array}{c} N_L \\ N_R \end{array} \right)$ as forming a spinor and $\left( \begin{array}{c} \sigma \\ \pi \end{array} \right)$ a 4-vector in a [4] Eucli-
dean space. Since there are 6 rotations in such a space, the gauge-principle
will give rise to six fields $X$, $Y$ with the interactions

\begin{equation}
L_{\text{int}} = \frac{1}{2} \left( \sigma \frac{\partial \pi}{\partial x_\mu} - \frac{\partial \sigma}{\partial x_\mu} \pi + \pi \left( \frac{\partial \pi}{\partial x_\mu} - \frac{\partial \pi}{\partial x_\mu} \right) \right) \cdot X_\mu +
\end{equation}

\[ + i N_L^+ \gamma_4 \gamma_\mu \pi \cdot X_\mu N_L + \frac{1}{8} (X \cdot \pi)^2 + \frac{1}{8} (\pi \wedge X)^2 + \frac{1}{8} X^2 \sigma^2 + \]

\[ + \frac{1}{2} \left[ \left( \sigma \frac{\partial \pi}{\partial x_\mu} - \frac{\partial \sigma}{\partial x_\mu} \right) \pi \right] \cdot Y_\mu + \]

\[ + i N_R^+ \gamma_4 \gamma_\mu \pi \cdot Y_\mu N_R + \frac{1}{8} (Y \cdot \pi)^2 + \frac{1}{8} (\pi \wedge Y)^2 + \frac{1}{8} Y^2 \sigma^2. \]

The free Lagrangians for the $X$ field is as follows
\[ \left[ \left( \frac{\partial}{\partial x_{\mu}} X_{\nu} - X_{\mu} \right) \right. \left. \left( \frac{\partial}{\partial x_{\nu}} X_{\mu} - X_{\nu} \right) \right]^{*}, \]
with a similar expression for the $Y$ field.

One can rewrite the above interaction slightly differently, introducing fields
\[ \frac{1}{2}(X + Y) = u, \]
\[ \frac{1}{2}(X - Y) = v. \]

Thus
\begin{equation}
L_{\text{int}} = \left[ \pi \left( \frac{\partial \pi}{\partial x_{\mu}} - \frac{\partial \pi}{\partial x_{\mu}} \pi \right) + N^\dagger \gamma^\mu \gamma^5 \tau \Sigma \right] \cdot u + \nonumber \end{equation}
\[ + \left[ \frac{\partial \sigma}{\partial x_{\mu}} - \frac{\partial \sigma}{\partial x_{\mu}} \pi + N^\dagger \gamma^\mu \gamma^5 \tau \Sigma \right] \cdot v + \text{terms quadratic in } u \text{ and } v. \]

We wish to identify the $\sigma$-containing part of the above Lagrangian as representing strong interactions and the remaining Lagrangian as representing weak interactions. The leptons ($e^+, \nu_e, e^-$) or ($\mu^+, \nu_\mu, \mu^-$) form a 3-vector in the space we are considering and a gauge-transformation will only link them with the $u$-field. The strength of the strong coupling comes about if we assume that the vacuum expectation value of $\sigma$ ($\langle \sigma \rangle_0$) does not equal zero but equals\[ (g_s/2M_\pi)(1/g_o) \]
\[ (\dagger) \]
Thus terms in the Lagrangian with $\sigma(\partial \pi/\partial x_{\mu}) \cdot v$ and $M^+ \gamma^\mu \gamma^5 \tau \cdot v N$ together give the conventional pseudo-vector strong Yukawa interaction with pions emitted singly. This is not to say that we are considering $\sigma$ as an alternative expression for the strong coupling constant. There is every possibility that $\sigma$-particles are emitted (and absorbed) as physical particles.

Notice that the weak interactions in this model conserve parity. This unfortunate situation seems to persist in subsequent work also (\dagger\dagger).

3. – Extension to strange particles.

In the above model $\sigma, \pi$ form a 4-vector in a [4]-Euclidean manifold while $N_L$ and $N_R$ form a 4-spinor. A direct extension of this to include K-mesons is possible, provided we consider $\sigma, \pi$ and K-particles to form a vector in an [8]-space while the sixteen baryons (eight-baryons each decomposed into their left and right components) form a 16-component spinor in such a space.

\[(\dagger) \text{ Notice the terms } \sigma^2(X^2 + Y^2) \text{ in (2) could give the mass-terms for } u \text{ and } v \text{ particles.} \]

\[(\dagger\dagger) \text{ The theory of weak interactions recently proposed by Gell-Mann and \text{Lévy} (\dagger) effectively proceeds by identifying the terms containing } \bar{X}_{\mu} \text{ only in (2) with the strangeness-conserving weak Lagrangian. The gauge-transformations giving rise to } \bar{Y}_{\mu} \text{ fields are not considered.} \]
The formalism we use was essentially developed by Tiomno (5). Let us first recapitulate this. Write
\[
\frac{1}{\sqrt{2}} (\Lambda^0 - \tau \cdot \Sigma) = \begin{pmatrix} Z^0 & \Sigma^+ \\ \Sigma^- & Y^0 \end{pmatrix} = (\Sigma_2, \Sigma_1),
\]
\[K^+ = K_1 - iK_2, \quad K^0 = K_3 - iK_4.\]

If all \(K\)-coupling constants are equal, the conventional p.s. (or p.v.) strong \(K\)-Lagrangian can be written as
\[
L_K = (N^+ \Sigma^+) (i\gamma_4 \gamma_5) \begin{pmatrix} K^0 & K^+ \\ K^- & -K^0 \end{pmatrix} \begin{pmatrix} \Sigma_1 \\ \Sigma_2 \end{pmatrix} + \text{h.c.}.
\]

Write \(\psi = \begin{pmatrix} N \\ \Sigma \end{pmatrix}\); then \(L_K\) equals
\[
\frac{4}{\sqrt{2}} \sum_{\alpha=1}^{4} \psi^+ i\gamma_4 \gamma_5 \Gamma_{\alpha} K_{\alpha} \psi, \]

where
\[
\Gamma = \begin{pmatrix} \tau \times 1 \\ \tau \times 1 \end{pmatrix}, \quad \Gamma_4 = \begin{pmatrix} i \times 1 \\ i \times 1 \end{pmatrix}.
\]

These \(\Gamma\) matrices are \(8 \times 8\) matrices pertaining to a (six or) seven dimensional manifold. The spinor \(\psi\) is an \(8 \times 1\) column.

It is possible to write the conventional p.s. (or p.v.) \(\pi\)-interactions in term of \(\psi\).

Define three additional matrices,
\[
\Gamma_{5,6,7} = \begin{pmatrix} 1 \times \tau \\ 1 \times \tau \\ -1 \times \tau \\ -1 \times \tau \end{pmatrix} = \tau_3 \times 1 \times \tau.
\]

Then the matrices \(\Gamma_1, \Gamma_2, ..., \Gamma_7\) anti-commute. If all \(\pi\)-couplings are equal (and in particular if \(g_{\mu,\nu} = -g_{\pi,\gamma}\)), one can write the \(\pi\)-Lagrangian
\[
L_\pi = \sum_{\alpha=5,6,7} \psi^+ i\gamma_4 \gamma_5 (\Gamma_{\alpha} \pi_{\alpha}) \psi.
\]

It is clear that \(\pi\)'s and \(K\)'s form a vector in a \([7]\)-space.

So much for Tiomno's formalism. We can now follow a procedure analogous to Section 2 and obtain an expression which would contain terms like
\[
\frac{\sigma}{C} \frac{\partial K_\tau}{\partial x_{\mu}} \mathcal{V}_\mu, \quad \frac{\sigma}{C} \frac{\partial Z_\tau}{\partial x_{\mu}} \mathcal{Z}_\mu, \quad \sum_{\alpha=1}^{4} \psi^+ (i\gamma_4 \gamma_5 \gamma_3) \Gamma_{\alpha} Z_{\alpha} \psi,
\]

to give an effective strong p.v. Lagrangian of the Tiomno type. Write
\[
L_I = \psi_L^+ \gamma_\mu \gamma_\nu \gamma_\rho \psi_L + \psi_R^+ \gamma_\mu \gamma_\nu \gamma_\rho \psi_R + \frac{1}{2} [\partial_\mu \sigma + (\partial_\mu \pi)^2 + (\partial_\mu K_\alpha)^2].
\]
(9)

The sixteen-component entity \( \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \) forms a spinor in [8]-space. One anti-commuting Dirac set for such a space is
\[
\begin{align*}
\Gamma_{1,2,3}^{(8)} &= \tau_1 \times (\tau_1 \times \tau) \times 1, \\
\Gamma_4^{(8)} &= \tau_1 \times (\tau_2 \times 1) \times 1, \\
\Gamma_{5,6,7}^{(8)} &= \tau_1 \times (\tau_3 \times 1) \times \tau, \\
\Gamma_8^{(8)} &= \tau_2 \times 1 \times 1 \times 1.
\end{align*}
\]
(10)

In [8] space there are 28 rotations. Seven of these rotations \((\sigma \rightarrow \sigma + \mathbf{e} \cdot \pi + \varepsilon_\lambda K_\lambda, \pi \rightarrow \pi - \varepsilon_\sigma, \text{etc.})\) with the corresponding spinor rotation matrices
\[
\frac{1}{2i} (\Gamma_8^{(8)} \Gamma_5^{(8)} - \Gamma_7^{(8)} \Gamma_6^{(8)}),
\]

give
\[
L_{int} = \frac{1}{2} \left( \partial \pi \cdot r_\mu + \partial \sigma Z_\mu - \frac{\partial \sigma}{\partial x_\mu} \mathbf{v} - \frac{\partial \sigma}{\partial x_\mu} K_\lambda Z_\lambda \right)
\]

(11)
i.e. Tiomno Lagrangian with \( (i \gamma_4 \gamma_5 \tau \cdot \pi) \) replaced by \( (i \gamma_4 \gamma_5 \gamma_6 \pi \cdot \mathbf{v}) \) and \( (i \gamma_4 \gamma_5 \Gamma_\lambda \pi) \) replaced by \( i \gamma_4 \gamma_5 \gamma_6 \Gamma_\lambda Z_\lambda \mu \). The fields \( V_\mu \) and \( Z_\mu \) behave, so far as isotopic spin, etc., is concerned just like \( \pi \) and \( K \) mesons.

In so far as these seven rotation matrices do not form a Lie-Algebra, the interaction Lagrangian must contain 21 other fields corresponding to the remaining 21 rotations. From the point of view adopted in Section 2, these give weak interactions only. A general analysis of these terms has been given by Gürsey (6) in a recent paper which also adopts the Tiomno formalism to give an analogue of the Gell-Mann–Lévy theory of weak interactions.

It may be more profitable from our point of view to consider two fields \( \sigma \) and \( \sigma' \) in such a way that \( (\sigma, \pi) \) form a 4-vector and \( (\sigma', K_\lambda) \) a 5-vector. The resulting strong Lagrangian would then contain two coupling parameters \( \langle \sigma \rangle_0 \) and \( \langle \sigma' \rangle_0 \). \( (\sigma, \pi) \) and \( (\sigma', K_\lambda) \) spaces in a sense form two (disconnected) pieces of a [9]-space. Even for the Tiomno Lagrangian it was possible to consider \( \pi \) and \( K \)-mesons as particles corresponding to disjunct pieces of the 7-dimen-

ional space. Thus if we replace \( \Gamma_\alpha \pi_\chi \) in eq. (8) by \( \Gamma_\alpha' \pi_\chi \) where \( \Gamma_\alpha' = 1 \times 1 \times \tau \) (i.e. \( g_{\pi N} = + g_{\pi N} \)) we see that \( \pi \) and \( K \) no longer form a 7-vector. The per-

mitted rotations have to be limited in such a way that $\pi$-mesons are not transformed into $K$-mesons.

Returning to the [9] space, if $(\sigma, \pi)$ form a [4] subspace and $(\sigma', K)$ a [5] sub-space, it is clear that the total number of intermediate bosons will be $6 + 10 = 16$. Of these seven will mediate strong (p.v.) interaction, and 9 will mediate weak (v.) interactions. All these interactions conserve parity and isotopic-spin.

4. $- \Delta |I| = \frac{1}{2}$ rule.

One simple way to introduce strangeness violation consistent with the $\Delta |I| = \frac{1}{2}$ rule is to remark that the other field besides $\sigma$ (or $\sigma'$) which can have a non-zero expectation-value is the field corresponding to the $6^0$ particle ($CP = + I$). In the notation above $\langle K \rangle \neq 0$ and would in fact be proportional to $g_\pi$ even in the conventional theory. Thus all strangeness conserving terms like $\bar{\Lambda}N\tau \cdot \pi K$ describe also the matrix-elements for (parity-conserving) weak decay of $\Lambda \rightarrow N^0 + \pi$ consistent with $\Delta T = \frac{1}{2}$ provided we take the vacuum expectation value of the K-meson.

The non-zero expectation value of $6^0$ is the perfect realization of the spurious idea of Wentzel so that it may not be necessary to introduce any additional fields to violate strangeness.

From what has been said above, it is clear that seven fields (three with transformation character of $\pi$-mesons and four with that of $K$-mesons) are necessary to mediate strong-interactions. The number of those necessary for weak interactions depends on the model used. However in all this work parity-violation for weak interactions remains a complete mystery.

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RIASSUNTO (*)

Si propone una teoria delle interazioni sia forti che deboli nell'ipotesi di avere solo interazioni dovute a trasformazioni di gauge generalizzate.

(*) Traduzione a cura della Redazione.