The Self-Energy of the Electron and the Mass Difference of Nucleons

It has been made clear\(^{(1)}\) that Bopp's new electromagnetic theory\(^{(2)}\) turns out to be, on quantization, a mixed theory of Maxwells electromagnetic field and the neutral vector Yukawa field. As a sequel, however, the following two new difficulties present themselves. Firstly, the energy of the meson becomes negative, and secondly, the self-energy diverges due to the fluctuation of the field. Inoue and Takagi advocate the adoption of a scalar field as the Yukawa field, in order to remove the first difficulty\(^{(3)}\). We desire to point out that the second difficulty, too, can be obviated by this combination of fields.

The self-energy of an electron at rest due to an electromagnetic field becomes, on calculating positron-theoretically in accordance with Weiskopf\(^{(4)}\),

\[
W_E = \frac{e^2}{\pi} \frac{m_e^2}{\kappa} \left( \frac{3}{2} \lim_{\kappa \to \infty} \log \kappa \right) + \frac{3}{2} \log 2 - \frac{1}{2}
\]

where \(m\) denotes the mass of the electron, \(e\) its charge, \(2\pi \hbar\) Planck's constant, \(c\) the light velocity. This is the sum of the electrostatic self-energy and the electrodynamical one due to the fluctuation of the field, and differs from the result of the ordinary one-body problem calculation in that it diverges logarithmically. Carrying out exactly similar calculations for the self-energy due to the meson field, we obtain

\[
W_M = -\frac{f^2}{\pi} \frac{m_n^2}{2\pi} \left( \frac{3}{2} \lim_{\kappa \to \infty} \log \kappa \right) - \frac{3}{2} \log m_n + \frac{1}{4} - \frac{1}{2} \log 2
\]

Here we have considered, just as Inoue and Takagi, only the scalar type of interaction between the electron and the meson field, and denoted its constant by \(f\). Further, \(m_n\) represents the mass of the meson.

The expression (2) diverges, just as (1), logarithmically, and if we postulate the relation

\[
\alpha^2 = \frac{f^2}{2}
\]

between the constants of the interactions, the result becomes finite, giving as the total self-energy of the electron

\[
W_e = W_E + W_M = \frac{e^2}{\pi} \frac{m_e^2}{\kappa} \left( \frac{3}{2} \log \frac{2m_n}{m} \right) + \frac{1}{2} \log 2 - \frac{3}{4}
\]

If we take \(m_n \sim 200\) m, this value is so small compared with the electron mass, that the additional mass due to the self-energy may, in general, be neglected.

As to the theories obviating the divergence of self-energy, there is, besides those following the line of Born and Bopp, which we have examined, another originating in Wentzel\(^{(5)}\) and developed by Dirac and Pauli.\(^{(6)}\) This theory employs manipulation called the \(\lambda\)-limiting process, together with a new method of quantization involving the introduction of negative energy photons, and is claimed by Pauli to be capable of removing all the divergences in the quantum theory of fields. In particular, as concerns self-energy, the \(\lambda\)-limiting process is instrumental in eliminating the electrostatic part, while the new method of quantization is effective in annulling that part due to the fluctuation of the field. But this is the result of a one body-problem type of computation, and if the correct, positron-theoretical method of calculation is followed, the self-energy diverges once more and neither the \(\lambda\)-limiting process nor the new quantization is of any avail here. As Pauli says, although this theory is extremely unsatisfactory in that it simultaneously employs two altogether independent methods, and further requires a reinterpretation of the results, there is something in it
that reminds us of the eve of the advent of the quantum mechanics. But, it must be admitted to be a fatal defect of their theory that it loses sight of the solution of the self-energy problem, which was its original subject matter.

Our theory can be directly extended to protons as well, and gives, as its self-energy, the expression

\[ W_p = -\frac{e^2}{\hbar c} \frac{M^2}{\pi} \left( \frac{1}{4} + \frac{m_u}{m_n} \tan^{-1} \frac{M}{m_u} \right) \]

where \( M \) denotes the intrinsic mass of the proton, and it is assumed that \( M > m_u > m_n \).

\( \Delta M = W_p/\hbar^2 \) may be regarded as giving the mass difference between neutrons and protons, and, if we take \( m_u \approx 2\theta m \), we obtain \( \Delta M \approx 2m \), which is approximately of the same order as the observed value. Our theory may be claimed noteworthy as there seems to be, at present, no other capable of explaining this \( \Delta M \).

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References

(3) T. Inoue & S. Takagi, Kagaku. 16 (1946), 205.
In this theory, it is assumed \( \phi^a = f_a \).
(6) Pauli, Rev. Mod. Phys. 15 (1943), 175.