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On the Correlations between Mesons and Yukawa Particles*.

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Introduction

1. At the present stage of its development, meson theory, which generated from the original idea of Yukawa, is confronted with several grave difficulties. Perhaps some of these difficulties, as has on many occasions been pointed out by Heisenberg(1), have close connections with the existence of an "universal length" that limits the validity of the present relativistic quantum mechanics. But on the other hand, it seems very likely too, that some of them are not of such essential nature, but of complicated character. In fact, it was recently shown that certain problems are of purely mathematical nature which results from an inappropriate application of perturbation theory as had been insisted on by Bhabha(2) from earlier times, and if the reaction of field is properly considered, can be removed(3). The object of the present paper is to show that a sequence of difficulties can be removed by the modification of the fundamental nature of elementary particles from a new stand-point.

Initially, the Yukawa theory was proposed in order to explain the problems of nuclear forces and beta-decay phenomena in unification, and the identification of the particle introduced in this theory (Yukawa particle) with a new particle discovered in the hard component of cosmic rays (meson), at once led to theoretical stand-point from which very clear and reasonable accounts could be given for many problems on cosmic ray phenomena. For example, the theoretically anticipated instability of the Yukawa particle gave reasonable accounts for various effects on the variation of intensity of the hard component in cosmic rays (temperature effect, density effect etc.). But the more precise and quantitative the comparison between

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theory and experiment became, discrepancies manifested themselves so much the more markedly. For the lifetime of mesons, experimental results exceed the theoretical by $10^4$ in magnitude\(^{(6)}\). On the other hand, for the cross section of slow mesons with energy $2-20 \times 10^6$ eV, maximum estimation of experimental results\(^{(6)}\) gave a cross section smaller than the calculated one by $10^{-2}$ in magnitude. In order to remove these difficulties, many trials\(^{(9)}\) have been made by several authors, but they still seem unsatisfactory. In this paper, with the main aim to solve the above-mentioned two difficulties simultaneously, we propose a new theory of mesons developed from the stand-point based on the following assumption: the meson is an elementary particle which has close correlations to the Yukawa particle, but it should be considered as an elementary particle of a different sort.

For a meson theory from such a stand-point, two alternatives are possible: Bose meson and Fermi meson theory. In the following, we adopt the latter alternative\(^{(8)}\).

**Interaction between Meson and Yukawa Particle.**

2. According to the Yukawa theory, heavy particles (nucleon) and light particles interact with Yukawa particles by the following process (schematically represented) respectively:

\[
\begin{align*}
P & \leftrightarrow N + Y^+, \\
N & \leftrightarrow P + Y^-, \\
e^- & \leftrightarrow \nu + Y^-, \\
\nu & \leftrightarrow e^- + Y^+. 
\end{align*}
\]

\(P\): proton, \(N\): neutron, \(e^-\): electron, \(\nu\): neutrino, \(Y^\pm\): positively and negatively charged Yukawa particles.

In our theory, it is assumed that the meson is a Fermi particle with spin $\frac{1}{2}$, and furthermore corresponding to (I) and (II) the following interactions are introduced:

\[
\begin{align*}
m^\pm & \leftrightarrow n + Y^\pm, \\
n & \leftrightarrow m^\pm + Y^\pm. 
\end{align*}
\]

\(m^\pm\): positively and negatively charged meson, \(n\): neutral meson which is assumed in the following discussions to have a negligible mass, and consequently may be regarded as equivalent with the neutrino.

Furthermore, if we introduce a neutral Yukawa particle \((Y^0)\) in order to explain the proton-proton and neutron-neutron forces and set up the
following interactions:

\[ P \xrightarrow{\text{\( (I') \)}} P + Y^o, \quad N \xrightarrow{\text{\( (I') \)}} N + Y^o, \]

it would be natural in our theory to introduce the following interactions

\[ m^\pm \xrightarrow{\text{\( (III') \)}} m^\pm + Y^o, \quad n \xrightarrow{\text{\( (III') \)}} n + Y^o. \]

In the following, results obtained by the introduction of new interactions (III) and (III') will be discussed. Consequently, it is concluded that if \( g, g' \) and \( r \) are the natural constants which represent the strength of interactions (I), (II) and (III) or (III') respectively and we adopt as their value \( g^2/\hbar c \sim 10^{-1}, g'^2/\hbar c \sim 10^{-10} \) and \( r^2/\hbar c \sim 10^{-8} \), it is possible to account for the phenomena in atomic nuclei and cosmic rays consistently, without aiming to touch the inherent difficulties of field theory.

**Mass and Lifetime of Yukawa Particle.**

3. For phenomena in atomic nuclei (nuclear forces, beta-decay etc.), Yukawa theory is conserved in its original form. But it is to be noted that the particle with mass determined from the range of nuclear forces, is the Yukawa particle and not the meson found in cosmic rays. This point is advantageous to explain the experimental results\(^6\) about nuclear force range (\( \frac{\hbar}{m \mu c} \)) which gives half the value (\( \frac{\hbar}{\mu c} \)) obtained from the meson mass data (\( m_u \): mass of Yukawa particle, \( \mu \): mass of meson). In order to account for these results, \( m_u = 2\mu \) is a reasonable assumption. More generally speaking, it is allowable to assume \( m_u > \mu \).

4. As the consequence of the above assumption \( m_u > \mu \), it becomes possible that a Yukawa particle transforms into a meson in vacuum by the following process (spontaneous disintegration of Yukawa particle):

\[ Y^\pm \rightarrow m^\pm + n \quad (m_u > \mu) \quad (IV) \]

The reciprocal of the proper lifetime of Yukawa particles (\( \tau_0 \)) calculated in vector theory is given by the following expression

\[ \frac{1}{\tau_0} = \frac{m_u^2}{2\hbar} \left( \frac{2}{3} \frac{r_1^2}{\hbar c} + \frac{1}{3} \frac{r_2^2}{\hbar c} \right) \left( 1 - \left( \frac{\mu}{m_u} \right)^2 \right) \quad (1) \]

where \( r_1 \) and \( r_2 \) are the constants of vector and tensor interactions respectively. If we assume \( r_1^2 \sim 10^{-6} \frac{\hbar}{c}, r_2^2 \sim 10^{-4} \frac{\hbar}{c} \) (as later shown, these figures
are compatible with the considerations of meson decay), we obtain $t_0 \sim 10^{-24}$ sec. for $m_u/\mu \sim 2$. (Mean free path of Yukawa particle with the energy $10^{10}$ eV. is about $10^{-8}$ cm). Naturally these values depend on the ratio $m_u/\mu$. As $m_u$ approaches $\mu$, the lifetime of Yukawa particles is prolonged by the last factor in (1); and the limit $m_u = \mu$, it becomes infinite. But in this case the Yukawa particle decays by the original process $Y \rightarrow e + \nu$. Consequently, the lifetime of the Yukawa particle does not become greater than $10^{-8}$ sec.

5. Process (IV) represents the creation of the hard component in cosmic rays. For the analysis of cosmic ray phenomena, we shall take up proton primary hypothesis which has been proposed by Schein et al. Then primary incidental protons at first create Yukawa particles by collisions with nuclei of $N$ or $O$ atoms existing in the atmosphere. These Yukawa particles transform into mesons instantaneously by the above process. The interaction of the latter with matter is smaller than that of the former $(\tau < \gamma)$. Thus we observe these mesons as hard components of cosmic-rays. Previously, Nordheim in his analysis on cosmic-ray suggested that the absorption process of hard component must have cross sections smaller by a factor of order 10 compared to the creation processes. This difficulty, which indicates an asymmetry inconsistent with the original Yukawa theory, is overwhelmed by the insertion of process (IV).

Decay and Scattering of Meson.

6. According to our theory, the decay of mesons occurs by following process:

$$m^\pm \rightarrow e^\pm + n$$

(V)

The reciprocal of the proper lifetime of mesons ($\tau_0$) in vector and pseudoscalar theory are given as follows, where $f^\prime$, $\bar{f}$ are interaction constants in pseudoscalar theory corresponding to $g^\prime$, $\gamma$ in vector theory.

$$\frac{1}{\tau_0} = \frac{\mu c^2}{\pi \hbar} \left\{ \left( \frac{g_1^\prime \bar{f}_1}{\hbar c} \right) I_1^{(\nu)} + \left( \frac{g_2^\prime \bar{f}_2}{\hbar c} \right) I_2^{(\nu)} \right\}, \quad (2)$$

(vector theory)

$$\frac{1}{\tau_0} = \frac{\mu c^2}{\pi \hbar} \left\{ \left( f_1^\prime \bar{f}_1 \right) I_1^{(\nu)} + \left( f_2^\prime \bar{f}_2 \right) I_2^{(\nu)} \right\}, \quad (2')$$

(pseudoscalar theory)
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\[ I_1^{(a)} = \frac{1}{(1+A)^2} \left\{ \frac{300 A^4 + 870 A^3 + 670 A + 603}{1440} - \frac{A(A+1)}{24} \left( \frac{5A^2 + 12A + 6}{A+1} \right) \times \log \frac{A+1}{A} \right\}, \]

\[ I_2^{(a)} = \frac{1}{(1+A)^2} \left\{ \frac{30 A^3 + 285 A^2 + 310 A + 61}{1446} - \frac{A(A+1)(A^2 + 9A + 6)}{48} \times \log \frac{A+1}{A} \right\}, \]

\[ I_3^{(a)} = \frac{6 A + 1}{16} - \frac{A(3A+2)}{8} \log \frac{A+1}{A} \quad I_4^{(a)} = \frac{1}{(1+A)^2} \frac{1}{480}, \]

\[ A = \left( \frac{m_n}{m} \right)^2 - 1. \]

The value of \( \tau_0 \) determined from experiments is about \( 10^{-8} \) sec. Using this value and characteristic constants of beta-radioactive nuclei, which depend on constants \( g' \) or \( f' \), we can determine the value of interaction constants \( r \) or \( \bar{r} \). Using only the first term in (2), we can determine \( r_1 \sim 0.025 \) fi for \( m_n/\mu \sim 2 \). Determining \( r_2 \) from the second term only, we obtain a magnitude of the same order.

7. Introducing the neutral Yukawa particle and taking interaction (III'), the scattering of mesons occurs by the following process:

\[ m^+ + N \rightarrow m^+ + Y^0 + N \rightarrow m^+ + N' \]  

(VI)

The cross section of these processes is determined by the interaction constant of (III'). Taking the same constant \( r \) of (III) for this (analogous to Kemmer's symmetrical theory of nuclear forces), we have to examine whether the above-determined \( r \) leads to a cross section consistent with scattering experiments or not. Furthermore, we must take into account the following process which has a probability of the same order of magnitude as that of the scattering process.

\[ m^+ + N \rightarrow m^+ + Y^- + P \rightarrow n + P \]  

(VI')

If the cross section of the latter process becomes appreciably large, it is provable that an attempt to account for the creation of neutral mesons by this process may lead to a contradiction with experimental results. From these considerations, it is desirable to take as small a value for \( r \) as
possible\(^{(9)}\)(\(*\)) In fact, taking into account that expressions (2) and (2') consist of two terms which involve \(g'_1, g'_2\), (or \(f'_1, f'_2\)) separately, and adjusting their values properly, we can make the values of \(\gamma\)'s sufficiently small. For example, in the vector theory if we assume \(g_1 = 0\), then \(g'_1\) becomes irrelevant to the \(\beta\)-decay process and we can choose a sufficiently small value for \(\gamma_1\). Furthermore, if it is assumed that meson decay occurs by \(\gamma_1\)-interaction only, the choice of \(\gamma_2\) can be determined from the scattering cross section freely. In pseudo-scalar theory, if we take into consideration that \(f_1, f'_1\) are not responsible for nuclear force and \(\beta\)-decay process respectively, we can make \(\gamma_1, \gamma_2\) sufficiently small simultaneously in an analogous way.

In the above consideration we took up the symmetrical formalism. But even if we assume that for the interaction with heavy particles the interaction constants of \(Y^o\) are greater than that of \(Y^*\) as in Heitler's theory or Bethe's neutral theory, it is still possible to make \(\gamma_1, \gamma_2\) sufficiently small.

Some Remarks on the Results.

8. As shown above, there is certainly a possibility that the contradiction between Yukawa theory and experimental results be removed. In the following, some related problems which are characteristic to our theory will be discussed.

(i) One of characteristics of our theory is that the meson has spin \(\frac{3}{2}\). This point is interesting in relation with the results of analysis on radiative effects of the meson which are sensitively affected by the spin value. According to the calculation by Christy and Kusaka\(^{(9)}\) of the sizes of bursts produced by mesons, our choice of spin \(\frac{3}{2}\) for the meson is supported by experiment, in contrast to the usual theory with spin \(\frac{1}{2}\).

(ii) The role of neutral particles in the original Yukawa theory has been frequently discussed. If we take the symmetrical theory on nuclear force and furthermore assume \(m_\sigma > 2\mu\), the neutral Yukawa particle decays

\(^{(9)}\)(\(*\)) Naturally, we cannot take too small a value for \(\gamma\), in order that it be consistent with the decay probability of Yukawa particles. Eventually, it seems appropriate to take \(\gamma \sim 10^{-4} \text{sec}\) (or a slightly smaller value).

\(^{(9)}\)(\(*\)) If we take the formalism which excludes the neutral Yukawa particle, the scattering of mesons takes place only by a process of the fourth order, which results in a much smaller cross section than that of (VI). But, in this case, process (VI') is not excluded.
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by the following process: \( (\text{spontaneous disintegration of neutral Yukawa particle}) \)

\[
Y^0 \rightarrow m^+ + m^- \quad (V')
\]

and its proper lifetime is of the same order as that of charged Yukawa particles calculated above \( (\sim 10^{-21} \text{ sec}) \). This value is smaller than that obtained from the previously discussed process\(^{(18)}\), in which the neutral Yukawa particle (neutretto) transforms into photons. As a result, the Yukawa particle transforms into mesons (penetrating component) with a greater probability than into photons (shower producing component). This is very favourable in interpreting the results of experiment by Schein et al\(^{(10)}\) which denied the existence of shower producing components in upper atmosphere and suggested, as the possible alternatives which was at first suggested by Taketani\(^{(14)}\), the introduction of some process in which the neutral Yukawa particle transforms into the penetrating component with a greater probability than into shower-producing components, or otherwise the elimination of neutral Yukawa particle from the formalism.

(iii) Also, it is to be noted that as the decay product of one meson, three particles are generated and two of them are neutral particles which have smaller interaction with matter. This consequence seems to be in agreement with the fact\(^{(15)}\) that when mesons are stopped in matter, decayed electrons have rarely been observed, or that when mesons decay in a Wilson chamber, only slow electrons have been observed. Furthermore, the fraction of the total mesonic energy given to the electron by decay in this theory is smaller than that in the usual theory. This point is, too, supported by Nordheim's analysis\(^{(16)}\) on the intensity of cascade showers. But eventually, the validity of introduction of such processes should be fully discussed in future, based on further experimental and theoretical investigations on cosmic rays.

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(8) Another alternative which involves Bose-meson distinguished from Yukawa particle are adopted by Y. Tanikawa.


(10) Schein, Wollan and Jesse: Phys. Rev. 59 (1941), 615.


(14) Taketani: Kagaku 1 (1941), 523.
