SHORT NOTES.

On the Lifetime of the Pseudoscalar Meson.

By Shoichi Sakata.

It is well known that there is a serious discrepancy between the observed and the theoretical value of the ratio between the lifetimes of the free meson and of light beta-radioactive nuclei. In the previous paper, I have pointed out that such a difficulty disappears if one adopts the pseudoscalar theory of the meson. This conclusion was obtained in the following way. The reciprocal proper lifetime of the meson calculated from the pseudoscalar theory is given by

$$\frac{1}{\tau_0} = \frac{m_e c^2}{2h} \left( \frac{f_1'}{\sqrt{\hbar c}} \right) + \left( \frac{f_2'}{\sqrt{\hbar c}} \right) \left( \frac{m}{m_e} \right)^{1/2}, \quad (1)$$

where it is noticeable that the term involving $f_2'$ gives a very small contribution to the meson decay. On the other hand, the interaction energy which gives rise to the nuclear beta-decay has the following form in this theory:

$$H_\beta = \int G(r) F(r) dr,$$

and all other notations are the same as in I. Operating the differentiation upon $F(r)$, (2) reduces to

$$H_\beta = \int \left[ f_2 f_2' \left( \frac{\langle \tilde{M}(r_0), M'(r_0) \rangle}{r} \right) + \left( \frac{f_1'}{\sqrt{\hbar c}} \right) \left( \frac{m}{m_e} \right)^{1/2} \right] F(r). \quad (3)$$

As the wave lengths $\lambda$ of the light particles emitted in ordinary beta-disintegration are always large compared with $1/\kappa$, the integration with respect to $r_0$ can be carried out by ignoring the space dependence of the light particle wave functions. Thus (4) becomes

$$H_\beta = \frac{4\pi f_2 f_2'}{3\hbar c^2} \int \langle \tilde{M}(r_0), M'(r_0) \rangle dr_0, \quad (5)$$

which is identical with the Gamow-Teller interaction in Fermi's theory of beta-disintegration; the only difference being that the Fermi constant is here replaced by $4\pi f_2 f_2'/3\hbar c^2$. From (1) and (5), it is easily seen that we can determine the value of $f_1'$ from the meson decay and that of $f_2'$ from the beta-decay separately and hence we may conclude that the above mentioned discrepancy, between the theory and the experiment, disappears in the pseudoscalar meson theory.

Recently it has however occurred to me that this conclusion deduced from the current meson theory is merely apparent, because the expression (5) originates precisely from the singular term in (2) which would be dropped out if the consequent field theory

were established. This can be shown by using Gauss' theorem in the following way.

By writing (2) as

$$H_\beta = \frac{-f_1'}{\kappa} \int d\tau \rho \{ M' (\tau), \text{grad} \} \cdot \nabla (M (\tau), \text{grad} \cdot \nabla) - f_1' \int d\tau \rho \{ F (\tau), \text{grad} \cdot \nabla \}
$$

and by performing partial integrations, we obtain

$$H_\beta = \frac{-4\pi f_1'}{\kappa} \int \{ f_2 M' + f_2' M_0' \} \{ f_2 M + f_2' M_0 \} d\tau + \int \{ f_1 W + f_1' W' \} \{ f_1 W + f_1' W' \} d\tau$$

which is identical with (5), if we neglect the terms involving \( \text{grad} \cdot \nabla \) which are of the order of \( 1/\kappa^2 \). The appearance of the first term in (6) depends obviously upon the fact that \( d\tau \rho \cdot \nabla \) has a singularity of the order of \( 1/r^2 \) at the origin. As is well known, it was just this singularity which has led to the nuclear interaction involving the term \( 1/r^3 \), and as such an interaction does not give any finite binding energy for the deuteron, it is reasonable to cut off the theory in some way, even if the convergent result is obtained. If we omit the first term completely from (6), the numerical values of the interaction constants \( f_1' \) and \( f_2' \) must be chosen much larger than before, in order to account for the lifetime of the nuclear beta-decay. So that the resulting lifetime of the meson becomes too short to coincide with experiment. Moreover the energy distribution of the beta-rays differs then entirely from Fermi's type, because the remaining interactions in (6) contain the derivatives of the light particle wave functions. Consequently, if such a cutting off prescription is correct, the beta-disintegration phenomena cannot be accounted for on the basis of the pseudoscalar meson theory. In order to get rid of this difficulty, it is necessary either to adopt Fermi's theory of beta-disintegration(2), or to assume the existence of two kinds of mesons(3).

It is however noticeable that in each of the four types of the meson theories there is an ambiguity with respect to the existence of the direct interactions of the \( \delta \)-function type between heavy and light particles. Though it is usual to choose the formalism which does not involve the static interaction of this type, there is no a priori reason for this choice. In fact we may add the invariant quantity

$$-\frac{4\pi}{\kappa^2} \int \left( f_2 \bar{M} + f_2' \bar{M}' \right) \left( f_2 M + f_2' M' \right) d\tau$$

and by the total Hamiltonian (1) in I of the pseudoscalar meson theory(4). In this case, we cannot deduce any definite conclusion, because it is difficult to decide whether the interaction of the \( \delta \)-function type vanishes or spatially extends when the consistent field theory is established.

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(3) S. Sakata and T. Inoue, On the Relation between the Meson and the Yukawa-Particle (Japanese), Nippon Sūkan-Butsurigakkaisi, 16 (1942), 232.