Measuring CP asymmetries in B decays

M. V. Purohit
Princeton University

Abstract

CP violation asymmetries will be measured using rate differences between $B^0$ and $\bar{B}^0$ decays into CP eigenstates. The final result will undoubtedly utilize both the asymmetry in the total rate as well as the non-exponential nature of the decay times. Here the two effects are analyzed separately and compared. A maximum likelihood analysis of the time dependence of decays is carried out and analytic expressions are obtained. We display the results which show that, contrary to what one might expect, the error from fitting the time dependence is comparable to that from the total rate for the $B_d^0$ and much smaller than the error from the total rate in the case of the $B_s^0$.

Introduction

Observation of CP violation with B mesons is best carried out using their decays into CP eigenstates. The decay of a system that starts out as a $B^0$ at time $t=0$ into a CP eigenstate $f$ at time $t$ is given by

$$ P(t) = e^{-t}[1 - a \sin(xt)] $$

(1)

Similarly, if we start with a $\bar{B}^0$ we get

$$ \bar{P}(t) = e^{-t}[1 + a \sin(xt)] $$

(2)

where time is measured in units of $B$ lifetime, $a$ is the CP asymmetry and $x$ is the mixing parameter ($\Delta M/\Gamma$).

At an $e^+e^-$ B-factory the origin of time is when either the tagging B or the B decaying into a CP eigenstate decays, whichever comes first. This is due to the coherence of the $B\bar{B}$ pair until they decay. In any case, it is critical to measure the sign of $t$, which may be positive or negative depending on which of the two Bs decays first. At a hadron collider the origin of time is easily taken to be the primary collision, but one expects to make a vertex separation cut which is equivalent to a cut to exclude decays prior to some minimum time, say $t_1$. 

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The time integrated asymmetry $A$ is defined by

\[ A = \frac{N^0 - \bar{N}^0}{N^0 + \bar{N}^0} \]  

(3)

where $N^0$ and $\bar{N}^0$ are the total number of $B^0$ and $\bar{B}^0$ particles observed to decay into the final state $f$. It is easy to show that the time independent asymmetry is given by

\[ A = \frac{-ax}{1 + x^2} \]  

(4)

if we consider all times and equal to

\[ A = \frac{-a}{1 + x^2}(x \cos(xt_1) + \sin(xt_1)) \]  

(5)

if we start our clock at time $t_1$. This is eloquently stated as a dilution of the CP violation asymmetry $a$ where the dilution factor, $D$ is defined by

\[ A = Da \]  

(6)

and can be written as

\[ D = \frac{1}{1 + x^2}(x \cos(xt_1) + \sin(xt_1)) \]  

(7)

If there are (approximately) $N$ events from $B^0$ mesons and $N$ from $\bar{B}^0$ mesons, one expects then that the error on $a$ is given by

\[ \Delta a = \frac{1}{D\sqrt{2N}} \]  

(8)

Of course, any detector which can separate vertices can measure time, even those that aim to detect no more than the sign of $t$. (The issue of resolution is considered below). One may then determine $a$ from the deviation of the lifetime distribution from a pure exponential. Such a determination provides dramatic confirmation of CP violation. Again, $a$ will be determined by maximizing the likelihood of the data where the likelihood of the $i^{th}$ data point is given by

\[ l_i = e^{-(t-t_i)} \frac{(1 - a \sin(xt_i))}{(1 - aD)} \]  

(9)
Defining the error in the standard way by

\[ \Delta a = \left[ \frac{1}{2} \frac{\partial^2}{\partial a^2} (-2 \ln L) \right]^{-1/2} \]  \hspace{1cm} (10)

one can show that

\[ \Delta a = \frac{1}{\sqrt{2N \ Var(g)}} \]  \hspace{1cm} (11)

where \( Var(g) \) is the variance of

\[ g = \frac{\sin(xt)}{1 - a \, \sin(xt)} \]  \hspace{1cm} (12)

and \( 2N \) is the total number of decays into \( f \) (from both \( B^0 \) and \( \bar{B}^0 \)). It is easy to show further that

\[ Var(g) = D'^2 - D^2 \]  \hspace{1cm} (13)

where \( D \) is defined as above and

\[ D'^2 = \frac{1}{2} \left( 1 + \frac{2x \sin(2xt_1) - \cos(2xt_1)}{1 + 4x^2} \right) \]  \hspace{1cm} (14)

under the assumption of a small asymmetry \( a \).

In the table below we present results for relevant values of \( x \) and \( t_1 \), including the obvious cases of \( t_1 = 0 \) and \( x = x_d = 0.7 \) and also \( x = x_s = 7 \). It is seen that for the \( B^0 \) the time dependent measurement is by far the superior one and even in the case of the \( B^0 \) it measures \( a \) almost as well as the time-independent case.
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<tr>
<th>( t_1 )</th>
<th>( x )</th>
<th>( \sqrt{2N} \Delta a ) (Time-dep.)</th>
<th>( \sqrt{2N} \Delta a ) (Time-ind.)</th>
<th>Ratio</th>
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Table 1. Columns 3 and 4 are the numerators \( k \) in the expression \( \Delta a = k/\sqrt{2N} \) for the time dependent and time independent cases using equations 8 and 11 in the text. \( 2N \) is the total number of decays observed from both the \( B^0 \) and \( \overline{B^0} \) mesons.

It has been pointed out\(^2\) that folding in a resolution function merely dilutes the asymmetry further. The distribution specified by (1) above becomes, in the measured variable \( t' \) (\( t' \) is \( t \) smeared by a Gaussian of width \( \sigma_t \))

\[
P(t') = \int_{-\infty}^{+\infty} dt \ e^{-(1-a \sin(\pi t))} \frac{e^{-\frac{(t-t')^2}{2\sigma_t^2}}}{\sigma_t \sqrt{2\pi}}
\]  
\begin{equation}
(15)
\end{equation}
i.e.,

\[
P(t') = e^{-\frac{(t'-\sigma_t^2)^2}{2}} [1 - e^{-\frac{x^2}{2\sigma_t^2}} a \sin x(t' - \sigma_t^2)]
\]  
\begin{equation}
(16)
\end{equation}

Effectively, the asymmetry has been further diluted by a factor \( e^{-\frac{x^2}{2\sigma_t^2}} \).
REFERENCES


2 K. McDonald, private communication.