Some decays of the 36 $\bar{q}q$ mesons are calculated within the static quark model. Introducing as new parameter the radius of the wave function we calculate the $V\gamma$ vertex, the $P\gamma$, the $P\mu\nu$ and the $\eta\rightarrow2\pi\gamma$ decays.

The static quark model is surprisingly successful in explaining the partial width for decays like

$$B^* \rightarrow B + P', \quad V \rightarrow P + P',$$

or

$$V \rightarrow P + \gamma, \quad P \rightarrow V + \gamma. \quad [1]$$

In view of this success, it seems desirable to check on whether it is pure coincidence or real support for the model.

For reference let us review the decay $X^0 \rightarrow \rho^0 + \gamma$ [2]. Taking into account the phenomenological $X^0$-$\eta$ mixing of about 100, one predicts a partial width of less than 10 keV. Non-resonating background can be estimated to be less than 10 keV. Since empirically only $\Gamma(X \rightarrow \rho + \gamma) = 1/2$ is known [3], one just gets the prediction $\Gamma X \approx 0.5$ MeV.

Further results of the quark model require another parameter, namely the spatial extension of the quark-antiquark wave function $\psi(x)$. For one cannot make a precise prediction, but putting $[\psi(0)]^2 = r^{-3}$ one can check whether we get a reasonable value for $r$. This is a compromise between a (long-ranged) hydrogen-type wave function and a sharply cut-off sphere.

Considering first the vector meson-$\gamma$-vertex one defines a coupling constant $f_{V\gamma}$ by

$$\langle V | j_\mu(x) | 0 \rangle = (2m_V\Omega)^{-1} \epsilon_V \epsilon_\mu e^{ipx} \langle 1 \rangle$$

where $\epsilon_\mu$ is the polarization vector and $\Omega$ the normalization volume. The electric current $j_\mu(x)$ is proportional to

$$j_\mu \sim \frac{1}{2} \mu \gamma_\mu (\not\rho + \not\eta) \gamma_\mu - \frac{1}{2} \lambda \gamma_\mu$$

where $(p, n, \lambda)$ for the quark triplet [4]. $\Gamma_\mu$ is given by

$$\Gamma_\mu = e_\gamma \not\mu + i \sigma_\mu \gamma_\nu g V \not\gamma. \quad [3]$$

If one assumes that the quarks in a meson have an effective mass $\frac{1}{2}M_{\text{meson}}$ [5] one has to attribute to them an anomalous magnetic moment $\mu_q = \mu_p - e/M_{\text{meson}}$ in order to retain the "observed" total quark moment $\mu_q = \mu_p$ ($\mu_p$ being the total proton magnetic moment). This causes the $\sigma_\mu\gamma$-term in (3).

From eqs. (1)-(3) one derives immediately

$$f_{\rho\gamma} = f_{\omega\gamma} = f_{\phi\gamma} = 3:1:-\sqrt{2}c \quad c = 0.58 \quad [4]$$

where $c$ is a correction factor for the different anomalous magnetic moment of the quarks in the $\phi$-state due to its rather different mass. $\omega_i$ and $\phi_i$ are the "ideally mixed" particles with mixing angle arctan $1/2$. With $c$ set equal to unity (4) is of course the SU3 result. Recalling that $f_{\rho\gamma}$ is not dimensionless, one adds another mass correction to obtain

$$(f_{\phi_i\gamma}/m_\phi^2) = (f_{\omega_i\gamma}/m_\omega^2) \times 0.59 \quad [5]$$

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a value which is in rather good agreement with the fit to isoscalar nucleon form factors [6].

Writing \( f_{\omega \gamma} = (2m_\omega/3r_0^2)^3 \times 0.8 \) (6)

where the factor 0.8 is the correction due to the \( \sigma_{\mu\nu} \)-term in eq. (3). The coupling-constant (6) is related to the partial width for decays into lepton-pairs by the well-known formula [7]

\[
\Gamma_V - n^- = m_V(4\pi a^2/3)(f_{\omega \gamma}/m_V^2 + O(m_t^2/m_V^2)) .
\]

(7)

Experimental data and the resulting coupling constants are shown in table 1.

Relying on ref. 9 we shall use in what follows eq. (4) and

\[ f_{\omega \gamma} = m_\omega^2/20 . \]

(8)

Though this value is too small to give a good fit to the isoscalar form factors [6] it predicts \( \tau = 10^{-13} \) cm, which is a rather reasonable value.

Turning to \( 2\gamma \) decays of mesons one observes that in a non-relativistic picture the emission of the first \( \gamma \) should just straighten out the two quark spins and then the quark - antiquark system can transform into the second \( \gamma \). Hence the process should go through \( P - V + \gamma - 2\gamma \). The second vertex is given by eq. (1) and the \( VP\gamma \)-

\[ L_{VP\gamma} = 2\mu_{VP}(P|\mu_{\gamma}V_{\mu\nu}|V_{\mu\nu}V_{\mu\nu}) \]

(9)

where \( \mu_{VP} \) is the transition moment defined by

\[ \mu_{VP} = \langle P|\mu_{\gamma}V_{\mu\nu}|V_{\mu\nu}V_{\mu\nu} = 0 \rangle \).

(10)

The relevant transition moments as obtained from the quark model [4] are summarized in table 2.

The decay widths are again obtained from a standard formula [10]

\[ \Gamma(P - 2\gamma) = \alpha(\mu_{VP}^2(f_{\omega \gamma}/m_V^2)^2 m_P^4) \]

(11)

where the index \( V \) in (11) refers to the intermediate vector meson. Inserting (4), (8) and the values from table 2, one computes the following widths

\[ \Gamma(\pi^0 - 2\gamma) = 37 \text{ eV} \quad (12a) \]

\[ \Gamma(\eta - 2\gamma) = 1.7 \text{ keV} \quad (12b) \]

\[ \Gamma(X \rightarrow 2\gamma) = 27 \text{ keV} \quad (12c) \]

where the subscript \( r \) on \( X \) and \( \eta \) denotes "real" particles for which \( \eta - X \) mixing has again been taken into account. Eq. (12a) is by about a factor

| \( \eta^- \rightarrow \gamma \gamma \) | \( | \gamma \gamma / | \gamma \gamma \) | Ref. | \( f_{\omega \gamma}/m_V^2 \) |
|---|---|---|---|
| \( \omega^{+}e^{-} \) | 2 \times 10^{-4} | \( \omega \) | 1/8.2 |
| \( \omega^{-}e^{+} \) | 1 \times 10^{-4} | \( \omega \) | 3/11.7 |
| \( \rho^{+}e^{-} \) | 0.65 \times 10^{-4} | \( \rho \) | 3/13.6 |
| \( \rho^{-}e^{+} \) | 0.3 \times 10^{-4} | \( \rho \) | 3/20 |

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<th>( \eta )</th>
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<td>( \rho )</td>
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<td>( \omega )</td>
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<td>( \phi )</td>
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3 bigger than the biggest experimental number [11]. With eq. (12c) we can predict

\[ \Gamma(\eta - 2\gamma)/\Gamma(\eta - 2\gamma) \approx 6.3 \]

(13)

in addition to the total width *. To check eq. (12b) one calculates the decay \( \eta - 2\pi + \gamma \) through the sequence \( \eta - \rho \gamma - \pi \gamma \). Here, one needs the VPP'-vertex which is given by

\[ L_{VPP'} = f_{VPP'}(P|\mu_{\pi}P - P - PP';) \]

(14)

The relevant coupling constant \( f_{VPP'} \) is obtained from the measured \( \rho \)-width to be \( f_{VPP'}^2/m_P^4 = 0.6 [1] \). Straight-forward computation yields

\[ \Gamma(\eta - 2\pi + \gamma) = \mu_{\eta \pi \gamma}^2 f_{\pi \gamma}^2 (m_P^6/m_\eta^6) \]

(15)

where \( \mu_{\eta \pi \gamma} \) is the transition moment defined by

\[ \mu_{\eta \pi \gamma} = \langle P|\mu_{\pi}P - P - PP'; \rangle \]

(16)

which agrees with the data [2] within the 20% error.

* It is interesting to note that from the observed branching ratios of \( X^0 \) into the various charged modes of the \( \eta \) system [12] and from the known ratio

\[ \eta - \pi^0 + \pi^0 \gamma / \eta - \pi^0 + \pi^0 \gamma \] one can predict the fraction of completely neutral decay products. Subtraction of this fraction from the observed neutral fraction which, obviously, includes \( X^0 \rightarrow 2\gamma \) decays, yields \( X^0 \rightarrow \rho^0 \gamma / X^0 \rightarrow 2\gamma \approx 3 \pm 2 \). We are indebted to Dr. B. Buschbeck-Czapp for discussions on this point.
Regarding the discrepancy of the $\pi^0$ lifetime one may argue that the $\pi$ may have an anomalously large $r$ and hence also the virtual $p$ and $w$ in the chain $\pi^0 \rightarrow V + \gamma \rightarrow 2\gamma$. This would decrease $f_V$, and bring the theoretical $\Gamma(\pi^0 \rightarrow 2\gamma)$ down. However, if all other predictions of the model, in particular the large $\eta$-width (2.6 keV) and the $X \rightarrow \rho\gamma/X \rightarrow 2\gamma$ ratio are experimentally verified one may also question the experimental evidence on the $\pi^0$ lifetime.

The quark model also provides a means to estimate the absolute rate of the weak decays $\eta^+ \rightarrow \mu^+ + \nu_\mu$ and $K^+ \rightarrow \mu^+ + \nu_\mu$ (in addition to their ratio!) In analogy with eq. (1) one defines a constant $c_P$ by

$$\langle P| A^+_{\mu}(\sigma) | 0 \rangle = (m_\rho \Omega)^{-\frac{1}{2}} c_P q \mu e^{i q x} . \quad (17)$$

It is connected to the lifetime by the classical formula

$$1/\tau_P = (G/2\pi)^2 m_\mu^2 m_\rho \left[ 1 - (m_\mu/m_\rho)^2 \right] . \quad (18)$$

The empirical values are

$$c_\eta = m_\eta \cdot 0.69, \quad c_K = c_\tau/3.87 . \quad (19)$$

Evaluating (17) in the same way as (1) yields [4]

$$c_\eta^2 m_\eta = 0.69 \cos \theta \sqrt{2} \phi(0) \quad (20)$$

where $\theta = (0.06) \frac{1}{2}$ is the Cabbibo-angle, which gives $r_\eta = [\phi(0)]^{-\frac{1}{2}} = 1.75 \times 10^{-13}$ cm, $r_K = 1.1 \times 10^{-13}$ cm. Here again one faces a rather large value for the quark wave function in the pion.

Possible applications of the quark model to more complicated strong decays are limited by the fact that any selection of particular intermediate states well reflect the true situation only very crudely. Thus there remains only one decay, namely $\phi \rightarrow \rho + \pi$. The VVP-vertex can be calculated from an effective quark-meson interaction

$$L = \sqrt{2} (g/\mu_\sigma) q^*(\sigma \cdot \Psi) (\lambda \cdot P_1) q . \quad (21)$$

by evaluating $\langle V|L_1|V \rangle$. $\mu_\sigma$ is some "mean meson mass" [14], $g/\mu_\sigma$ is related to the static pion-nucleon coupling constant $f^2/4\pi = 0.08$ by

$$g/\mu_\sigma = \frac{3}{2} (g/m_\eta) . \quad (22)$$

Straight-forward computation with the relativistic VVP vertex $(4g/\mu_0)e^{\mu_0}\Phi_{\mu_\nu}P_1 \Phi \pi$ yields

$$\Gamma(\Phi \rightarrow \rho + \pi) = (g/\mu_0)^2 \sin^2 \lambda \left[ 1 - (m_\rho m_\pi/m_\Phi)^2 \right] \times$$

$$\left[ 1 - (m_\rho - m_\pi/m_\Phi)^2 \right]^{3/2} m_\rho^2 m_\Phi / \mu_0 . \quad (23)$$

where $\lambda$ is the difference between the real and the ideal mixing angles. The factor $\sin^2 \lambda$ enters because the ideal $\Phi$ state is completely decoupled from the $\rho\pi$-system. The experimental partial width [3] requires $\lambda \approx 40^0$ thus shifting the total mixing angle to about $40^0$ in good agreement with the mass formula.

With the VVP-vertex one can estimate the decay $X^0 \rightarrow 4\pi$ via the chain $X^0 \rightarrow 2\rho - 4\pi$ to be of the order eV, and thus to be entirely negligible.

We would like to thank J. Kuti for interesting discussions.

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