Symmetries of the Strong Interactions

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An attempt is made to study the symmetry properties of the strong baryon-meson couplings without using arguments concerning the origin of the baryon mass differences. It is shown that too high a symmetry is incompatible with associated production experiments. The argument is independent of perturbation theory. It is assumed that the $\Sigma$ and $\Lambda$ have the same spin, that the $(\Sigma, \Lambda)$ parity is even, and that the usual isotopic spin assignments are correct. The general conclusions may have to be revised if it would turn out that the commonly assumed baryon spectrum is incomplete.

I. INTRODUCTION

Several theoretical approaches have been made recently to a somewhat more detailed dynamics of the strong interaction of baryons with $\pi$ and $K$ mesons. The common idea is to make assumptions stronger than charge independence: one postulates coupling constant equalities which are more restrictive than charge independence implies.

In these attempts a certain emphasis is laid on the notion that in the absence of some of the strong couplings there exist what may be called supermultiplets. For example, one assumes that in the absence of all strong $K$ couplings the baryons are completely mass degenerate and then arrives at inequalities between the $K$-coupling constants to account for the large mass splits. Such arguments are perhaps plausible but not entirely convincing, as a satisfactory interpretation of mass differences is beyond the techniques of present field theories and, at least to some extent, it may be beyond its scope. It is the purpose of this paper to show that similar conclusions can be arrived at by arguments which are not in themselves tied to the interpretation of mass differences.

We shall begin by assuming that there exists an equality between the $[\Lambda, \Sigma, \pi]$ and the $[\Sigma, \Lambda, \pi]$ coupling constants. This is a weaker assumption on $\pi$ interactions than the one made in the mentioned papers. We shall furthermore assume that the $[\Lambda, \Sigma, N]$ and $[\Sigma, \Lambda, N]$ have equal coupling strengths and likewise for $[\Xi, \Lambda, K]$ and $[\Xi, \Sigma, K]$. The relative magnitude of the $\pi$ versus $K$ couplings is immaterial to the argument. Then the following result will be proved, independent of perturbation theory:

To the extent that one may neglect the $(\Sigma, \Lambda)$ mass difference in dynamical calculations (not in the kinematics), the above assumptions are incompatible with the present experimental information on associated production in $\pi$-nucleon collisions. It should be emphasized that the neglect of $m_\Sigma - m_\Lambda$ means the following: relations will be derived between the transition prob-

 abilities of certain processes. These relations are valid up to terms of relative order $\delta$, where

$$\delta = (m_\Sigma - m_\Lambda)/m_\Lambda \sim 0.067,$$

and in some instances they are valid up to order $\delta^2 \sim 0.005$.

In this way one is led to recognize that within the realm of the strong interactions there occur "breaks in symmetry" irrespective of arguments concerning the hyperon mass spectrum. Of course, one cannot say so far whether they occur in the $\pi$ or in the $K$ couplings. It should be noted at once, however, that it is essential to the present reasoning that one may consider the commonly assumed baryon spectrum to be complete. In particular the above conclusion might have to be revised if it would turn out that there exist "excited $\Lambda^0$ states," i.e., hyperon states with $I=0$, $S=-1$ but with higher mass than the $\Lambda^0$ (see Sec. III). If this were the case, a high symmetry is not necessarily ruled out.

The method described in the next section is based on the recognition that the mentioned relations between coupling constants make it possible to define auxiliary quantum numbers by means of which one quickly arrives at the stated result. In Sec. III further comments are made. Section IV deals with some applications of the present method to specific cases of lower symmetry.

II. METHOD

We ask if the following set (I)–(IV) of assumptions are compatible with experiment (the discussion of a fifth assumption is deferred till Sec. III):

(I) The $\Sigma$ and $\Lambda$ spin are equal. Indications are that both spins are $\frac{3}{2}$. For convenience all baryon spins are taken to be $\frac{3}{2}$ in what follows and the $K$ spin is assumed to be zero, as it probably is. However, it will become evident later that the value of the cascade and of the $K$ spin are immaterial to the argument.

(II) The $(\Sigma, \Lambda)$ parity is even. Possible means to determine this parity experimentally have been discussed recently. The argument will be independent of

4 Of course, the effects of electromagnetic and weak couplings are ignored as well in making such arguments.

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the parity of the $K$ relative to (nucleon, $\Lambda^0$) and of the 
($\Sigma$, nucleon) parity.

If $\Sigma$ and $\Lambda$ would have either different spin or odd relative parity, the subsequent argument in which further assumptions are put to a test would be irrelevant.

(III) All strong couplings are charge independent. In this framework we may consider the following well-known set of strong $\pi$-baryon interactions$^6$:

$$[N_1, N_1, \pi] = iG\gamma_{\nu N_1 \tau}N_1 \pi, \quad (1)$$

$$[\Sigma, \Lambda, \pi] = iG\left(\Sigma^+\gamma_\mu N_1 \tau + \Sigma^0\gamma_\mu N_1 \tau^{-} \right) + h.c., \quad (2)$$

$$[\Sigma, \Sigma, \pi] = iG\left(\Sigma^+\gamma_\mu N_1 \tau - \Sigma^0\gamma_\mu N_1 \tau^{-} \right) \pi^+ + (\Sigma^+\gamma_\mu N_1 \tau^{-} - \Sigma^{-}\gamma_\mu N_1 \tau) \pi^- \quad (3)$$

$$[N_4, N_4, \pi] = iG\gamma_{\nu N_4 \tau}N_4 \pi. \quad (4)$$

Here $N_1$ and $N_4$ are two-component fields:

$$N_1 = \left( \begin{array}{c} \bar{p} \\ \pi_0 \end{array} \right), \quad N_4 = \left( \begin{array}{c} \bar{\pi}_0 \\ \Sigma^0 \end{array} \right); \quad (5)$$

the upper (lower) component corresponds to $\tau_3 = +1 (-1)$. The $K$ couplings are

$$[\Lambda, N_1, K] = F_0\gamma_{\nu N_1 \tau}N_1 \cdot \Lambda K + h.c., \quad (6)$$

$$[\Sigma, N_1, K] = F_0\gamma_{\nu N_1 \tau}N_1 \cdot \Sigma K + h.c., \quad (7)$$

$$[N_4, \Lambda, K] = F_0\gamma_{\nu N_4 \tau}N_4 \cdot \Lambda K^+ + h.c., \quad (8)$$

$$[N_4, \Sigma, K] = F_0\gamma_{\nu N_4 \tau}N_4 \cdot \Sigma K^+ + h.c. \quad (9)$$

The notations are as follows: the dot to the right of $\bar{N}$ stands for the choice between 1 and $i\tau_5$ depending on whether the $(N_1, \Lambda, K)$ parity is even (odd). As the $(\Sigma, \Lambda)$ parity is assumed to be even, one has to make the same choice in Eqs. (6) and (7), and likewise in (8) and (9). As the $\Sigma$ parity is irrelevant for what follows, the choice for Eqs. (6) and (7) is so far independent from that for Eqs. (8) and (9). $K$ is a two-component field, $K$, its charge conjugate:

$$K = \left( \begin{array}{c} K^+ \\ K^- \end{array} \right), \quad K_\tau = \left( \begin{array}{c} -K^0 \\ K^+ \end{array} \right). \quad (10)$$

Finally we consider the following assumption:

(IV) There exist these relations between the coupling constants

$$G_2 = G_3 = G; \quad F_1 = F_2 = F_4; \quad F_3 = F_4 = F_{11}. \quad (11)$$

We now introduce the one dynamical approximation to be made (in the sense explained in Sec. I) which is the neglect of the $\Sigma, \Lambda$ mass difference, the smallest such difference in the baryon system. It will never be necessary in what follows to ignore any other isotopic multiplet splitting. Then all $\pi$ interactions can be collected as

$$[\pi] = iG\gamma_{\nu N_1 \tau}N_1 + G\left(\bar{N}_2\gamma_\mu N_2 \tau + \bar{N}_4\gamma_\mu N_4 \tau\right)$$

$$+ G\gamma_{\nu N_4 \tau}N_4 \pi, \quad (12)$$

and the $K$ interactions can be written as

$$[K] = F_0\gamma_{\nu N_1 \tau}N_1 + G\left(\bar{N}_2\gamma_\mu N_2 \tau + \bar{N}_4\gamma_\mu N_4 \tau\right)$$

$$+ F_{11}\gamma_\mu [\bar{N}_2 N_2 \tau + \bar{N}_4 N_4 \tau] + h.c., \quad (13)$$

where

$$N_2 = \left( \begin{array}{c} \Sigma^+ \\ \Sigma^- \end{array} \right), \quad N_3 = \left( \begin{array}{c} Z^0 \\ \Sigma^0 \end{array} \right). \quad (14)$$

Here the following convenient quantities$^5$ have been introduced:

$$Z^0 = 2^{-1}(\Lambda^0 + \Sigma^-), \quad Y^0 = 2^{-1}(\Lambda^0 - \Sigma^-). \quad (15)$$

From Eqs. (12) and (13), it is at once evident that the possibility exists of invariantly gauging $N_2$ and $K^0$ oppositely. Likewise and independently one may proceed for $N_4$ and $K^+$, the gauge of $N_4$ is then uniquely determined. Correspondingly one may assign two quantum numbers $S_1, S_2$ to each baryon and meson. An appropriate set of values of $S_1, S_2$ is given in Table I. We have

$$S = S_1 + S_2 \quad (16)$$

where $S$ is the usual strangeness, $S$ conservation is, of course, guaranteed to begin with by Eqs. (1–4) and (6–9). In the present situation, however, we have a stronger set of rules:

(A) The separate conservation of $S_1, S_2$. (Observe that we may accordingly assign $I = \frac{1}{2}$ to all baryons, $I = 0(1)$ to $K(\pi)$ mesons. Then the charge operator is

$$Q = I_3 + S_1 + N/2).$$

(B) The invariance for the following combined interchanges:

$$N_2 \rightarrow N_3, \quad K^+ \rightarrow K^0, \quad -K^0 \rightarrow K^+.$$
The two rules (A) and (B) make it a trivial matter to prove the statement made in Sec. I. Let us first consider
\[ \pi^- + p \rightarrow \Lambda^0 + K^0, \]  
\[ \pi^- + p \rightarrow \Sigma^+ + K^+. \]  
According to rule (A) a \( \pi^- \)-nucleon state can combine with a \( Y^0 K^0 \), but not with a \( Z^0 K^0 \) state:
\[ \langle Y^0 K^0 | \pi^- p \rangle \neq 0; \quad \langle Z^0 K^0 | \pi^- p \rangle = 0. \]  
From Eqs. (15) and (19) we have therefore
\[ \langle \Lambda^0 K^0 | \pi^- p \rangle = - \langle 2\Sigma^0 K^0 | \pi^- p \rangle, \]  
or
\[ d\sigma (\Lambda^0 K^0) \approx d\sigma (2\Sigma^0 K^0). \]  
The near-equality sign serves to remind one of the kinematical phase space differences. This relation is not unreasonable. Experiment indicates that the total cross sections for \( \Lambda^0 \) and \( \Sigma^+ \) production are not very different, while both reactions are characterized by a similar-looking backward peaking of the distribution in angle between the incoming \( \pi^- \) and the emerging hyperon (in the center-of-mass system). Considerable uncertainty seems to attach to the information regarding the \( \Sigma^+ \) reaction, however.

More precisely, Eq. (21) means that the transition probabilities for \( \Lambda^0 K^0 \) and \( 2\Sigma^0 K^0 \) production are equal up to terms of order \( \delta \). (An inspection of the problem in perturbation theory indicates that this estimate may be too cautious.)

Next consider the reactions
\[ \pi^- + p \rightarrow \Sigma^- + K^+, \]  
\[ \pi^- + p \rightarrow \Sigma^+ + K^-. \]  
We note first that the rule (B) implies in particular the interchange \( Y^0 \leftrightarrow \Sigma^- \), \( K^+ \leftrightarrow K^0 \). Hence it follows from Eqs. (15) and (20) that
\[ \langle \Sigma^- K^+ | \pi^- p \rangle = - \sqrt{2} \langle 2\Sigma^0 K^0 | \pi^- p \rangle, \]  
or
\[ d\sigma (\Sigma^- K^+) = 2d\sigma (2\Sigma^0 K^0). \]  
Experimentally the \( \Sigma^- \)-production reaction has a cross section which seems to be somewhat smaller than that for the \( \Lambda^0 \) case. The factor two in Eq. (25) is at any rate inadmissible. Even more striking is the discrepancy in angular distribution: the \( \Sigma^- \) is peaked forward, the \( \Sigma^0 \) backward.

Furthermore it follows from Eqs. (13) and (14) that
\[ \langle \Sigma^0 K^+ \pi^- p \rangle = 0, \]  
so that the \( \Sigma^+ \) reaction would be forbidden which it certainly is not. More precisely, Eq. (26) means that the cross section for \( \Sigma^+ K^+ \) production is zero to order \( \delta \). This statement in itself has little meaning as presumably many large coupling constants are involved. It seems reasonable to say, however, that the present assumptions would indicate a ratio of order \( \delta \) between \( \Sigma^- K^+ \) and \( 2\Sigma^0 K^+ \) production which is an experimentally inadmissible result.

Thus we come to the conclusion stated in Sec. I that the assumptions (III) and (IV) are incompatible with experiment to the extent that one may rely on a theoretical argument in which \( \delta \) is neglected. It is easy to find further paradoxes. For example, the reaction
\[ K^0 + p \rightarrow K^+ + \pi^- \]  
is forbidden, as was noted by Barshay in a related context. Again the forbiddenness means a ratio \( \sim \delta^2 \) as compared to nonexchange scattering. Furthermore \( K^- + p \rightarrow \Sigma^- + \pi^- \) is forbidden, etc.

III. COMMENTS

(1) In the language of field theory, the neglect of the \( (\Sigma, \Lambda) \) mass difference is made only with respect to the virtual appearance of these particles (internal lines). To correct for this in a given order of approximation is simple but perhaps not too meaningful. One would expect such qualitative statements as Eq. (25) to be true within a 10% margin.

(2) It is readily verified that the present results also hold true if one replaces Eq. (11) by
\[ G_1 = -G_2, \quad F_1 = -F_2, \quad F_3 = -F_4. \]  
(3) It has been suggested by Gell-Mann and by Schwinger that the following \( G \) symmetry be imposed:
\[ G_1 = G_2 = G_3 = G_4. \]  
If this is true, at least one of the \( F \) symmetries of Eq. (12) is broken. These authors come, of course, to the same conclusion by considering the baryon mass splitting. Evidently neither the latter nor the present arguments are sufficient to decide which of the two (or both): \( G \) or \( F \) symmetry, is broken.

(4) A break in either \( F \) or \( G \) symmetry (or in both) destroys the validity of the rules (A) and (B) of Sec. II simultaneously. It is interesting to note that the retention of rule (A) combined with a violation of rule (B) would mean: first that the relation (21) between the \( \Lambda^0 \) and \( 2\Sigma^0 \) production cross sections is maintained, second that the unwanted relation (25) is no longer valid. Theoretically a situation of this kind corresponds to a violation of the assignments \( I = 1 \) for \( \Sigma \) and \( I = 0 \)

\[ G_i \]
\[ F_i \]  
\[ G_1 = G_2 = G_3 = G_4. \]  

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\( ^{10} \) See, for example, Vandevelde, Cronin, and Glaser (to be published). Note that the relations (25) and (26) are equivalent to the statement that the reactions in question go only via the \( I = \frac{1}{2} \) channel.

\( ^{11} \) S. Barshay, Phys. Rev. 109, 2160 (1958). This author also considers a coupling of the type \( K K \pi \). This interaction can likewise be dealt with by the present method.
for \( \Lambda^0 \). It would not affect in any way the meaning and the range of validity of the charge independence concept as applied to \( \pi \)-nucleon phenomena.

The reason for making this remark is that some doubt has been expressed recently\(^{9,10} \) about the validity of charge independence. There is some indication, although the evidence is not very firm, for a violation of one of the so-called triangle inequalities which relate the cross section for the reactions (18), (22), (23). This inequality follows from charge independence together with the notion that the \( \Sigma \) states form an \( I = 1 \) triplet. If further experiments would confirm the violation in question, one might consider a description of the (\( \Sigma, \Lambda \)) system as a (triplet, singlet) with some admixture\(^2 \) of (doublet, doublet). Consequences of this would be the existence of a contribution to the \( \Sigma^+ \Sigma^- \) and to the \( K^- \bar{K}^0 \) mass difference without the intermediacy of the electromagnetic field. The experimental magnitude of these differences\(^8 \) suggests perhaps that such admixtures should be small. At any rate, rule (A) must of necessity be broken to avoid the null result of Eq. (26).

(5) There is another way, however, in which apparent violations of charge independence could come about: Let us assume for the moment that there exists a particle \( \Lambda^0 \) which, like the \( \Lambda^0 \), has \( I = 0, S = 1 \). As long as \( m(\Lambda^0) - m(\Sigma^0) < m(\pi^0) \), \( \Lambda^0 \) would be stable against \( \pi \) emission into either \( \Sigma \) or \( \Lambda^0 \); the latter transition would be forbidden as an isotopic \( 0 \rightarrow 0 \) reaction. On the other hand, the reactions \( \Lambda^0 \rightarrow \Lambda^0 + \pi^- \) [and \( \Lambda^0 \rightarrow \Sigma^0 + \pi^- \) if \( m(\Lambda^0) > m(\Sigma^0) \)] would be allowed and would generally be of comparable speed to \( \Sigma^0 \rightarrow \Lambda^0 + \pi^- \). Thus a hypothetical \( \Lambda^0 \) would introduce an "anomalous \( \Sigma^0 \) effect" which would necessitate a reinterpretation of the experimental information that bears on the triangle inequalities.

Conversely, if a \( \Lambda' \) were to exist, it can readily be seen that all the arguments of Sec. II would need a thorough revision. The symmetry implied by Eq. (11) could then not be ruled out on such general grounds. For the present we shall merely state that the results of Sec. II can only be maintained if one moreover makes the following assumption:

(V) The commonly known baryon spectrum is complete.

(6) The indications of a possible hierarchy of symmetries within the strong interactions are reminiscent of the developments in attempts to view the isotopic spin and strangeness rules jointly within a four-dimensional isotopic framework. The initial attempt in this direction\(^{12} \) failed as the degree of symmetry invoked was too high (full four-dimensional invariance for all

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\( ^{12} \) This can be achieved dynamically in many ways. For example, one could generalize Eq. (12) to \( \mathbf{F}^2 = \sum_{\pi} \mathbf{G} \cdot \mathbf{A} \); \( \mathbf{G} \) could differ slightly from \( \mathbf{G} \).


yields

\[ H_0 \rightarrow H_0', \quad H_1 \rightarrow -H_1. \] (34)

This means that with regard to \( H_1 \) we have something like a Furry theorem. For example, it follows immediately that

(a) To the extent that one neglects \( \delta \) in dynamical calculations, a mass displacement between \( \Lambda \) and \( \Sigma \) comes about because of contributions odd in \( H_1 \). It follows trivially that to the extent of validity of the Gell-Mann identity\(^{17}\) for the baryon masses, the \( \Lambda \) and \( \Sigma \) masses remain degenerate under the present conditions.

(b) The matrix elements for the reactions (22) and (27) are odd in \( H_1 \) (barring terms of relative order \( \delta \)).

\[ G_2 = -G_3, \quad F_1 = F_2, \quad F_3 = F_4. \] (35)

The analysis and results are substantially the same as in the previous case.

(c) \( G_2 = G_3, \quad F_1 = F_3, \quad F_3 = -F_4. \)

Again the rules (A) and (B) are broken but now the dynamics looks entirely different. Here the lack of symmetry is one between the nucleon-doublet and the cascade-doublet. Put

\[ H = H_0' + H_1', \] (36)

\[ H_1' = iF \sum_i (\bar{F} \bar{R}^+ - \bar{F} \bar{R}^0) \Lambda^0 \]

\[ + (\bar{F} \bar{R}^+ + \bar{F} \bar{R}^0) \Sigma^0 \] + h.c. (36)

With respect to \( H_0' \) one can use the assignments of Table I and the conclusions of Sec. II. With respect to the substitution

\[ N_1 \rightarrow N_1, \quad N_2 \rightarrow -N_2, \quad N_3 \rightarrow N_3, \quad N_4 \rightarrow N_4, \] (37)

\[ \pi \rightarrow \pi, \quad K \rightarrow \tau_3 K, \quad K_\alpha \rightarrow \tau_3 K_\alpha, \]

one has

\[ H_0' \rightarrow H_0', \quad H_1' \rightarrow -H_1', \] (38)

and thus one obtains a Furry theorem for \( H_0' \) with similar kinds of applications as were mentioned above. The remaining sign combinations for the constants form an analogous pattern.

Thus the separation of \( H \) into a 0 and a 1 part seems useful to get an over-all insight in this complicated dynamical situation. It is to be hoped that other arguments of a rather qualitative kind may yield further clues about the maze of baryon-meson interactions.

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Note added in proof.—In connection with symmetry considerations one sometimes finds in the recent literature the statement that baryon self-energies are even functions of the coupling constants.\(^{18}\) In general this is not the case, however. For any baryon the most general expression for the self-energy is

\[ W = W^{(1)} + F_1 F_4 G_2 W^{(2)} + F_3 F_4 G_2 W^{(3)} + F_1 F_3 F_4 W^{(4)}, \]

where the four functions \( W^{(i)} \) are all even in \( F_i \), \( i = 1, \ldots, 4 \) and in \( G_2 \).\(^{18}\)

\( ^{17} \) See reference 2, footnote 13.