is correct, this decay can be used to test the con-
served vector current hypothesis.

The authors would like to thank Professors
R. E. Peierls and G. Alaga for reading the manu-
script.

References
3) V. M. Lobashov and V. A. Nazarenko, J. Exptl. Theoret.
6) D. L. Pursey, Phil. Mag. 42 (1951) 1193.
7) T. Ahrens and E. Feenberg, Phys. Rev. 86 (1952) 64.
communication.
9) E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. 60
(1941) 308.
10) A. M. Smith, Phys. Rev. 82 (1951) 955.
118.
15) I. P. Deutsch and P. Lipnik, Brussels Coll. for Low
Energy Nuclear Physics, September 1962.

SOME CONSEQUENCES OF UNITARY SYMMETRY MODEL *

S. OKUBO
Department of Physics and Astronomy,
University of Rochester, Rochester, New York

Received 4 February 1963

The purpose of this note is to investigate further
consequences of the unitary symmetry model 1,2),
based upon invariance of the theory under the three
dimensional unitary-unimodular group SU(3) or un-
der the three dimensional unitary group U(3). In
this connection, it may be worthwhile to mention
the equivalence ** of U(3) and SU(3), if we restrict
ourselves to certain irreducible representations as
we indeed do in this paper. Hence, we shall use

* This work has been supported in part by the U.S.
Atomic Energy Commission.
** If we specify irreducible representations of U(3) as
U3(f1,f2,f3) (f1 > f2 > f3), then the unimodular restric-
tion of U(3) gives the condition that irreducible repre-
sentations U3(f1+e,f2+e,f3+e) for any integers e are
exactly the same as U3(f1,f2,f3). In our paper, we
restrict ourselves to representations satisfying the
condition f1 + f2 + f3 = 0 and hence this redundancy of
representations is immaterial for our case. This can
be seen also as follows. In tensor analysis, the differ-
ence between U(3) and SU(3) consists of the fact that
the completely antisymmetric constant tensors eλμν
and ελαμμ are invariant under SU(3) but not so under
U(3). However, in all problems treated in this paper,
ελαμμ, if appearing, should always be in the following
combination:

\[ ε^{αβγ} ε_{λμν} = δ^α_λ [δ^β_μ δ^γ_ν - δ^β_ν δ^γ_μ] - δ^β_λ [δ^α_μ δ^γ_ν - δ^α_ν δ^γ_μ] + δ^γ_ν [δ^α_λ δ^μ_ν - δ^α_μ δ^μ_ν] . \]

Therefore, the presences of ελαμμ and ε^{αβγ} are ir-
relevant.

U(3) rather than SU(3) throughout this paper, since
the former is more simple to manage.

In a previous paper 2), we have derived the fol-
lowing mass formula for a given irreducible repre-
sentation of U(3)

\[ M = a + b Y + c \left[ \frac{1}{4} Y^2 - I(U + 1) \right] . \] (1)

This formula has been proven to the lowest order
of perturbation with respect to any U3-violating
mass splitting interaction H1 with a transformation
property T_{3/2} (of a tensor T^{3/2}_1), but to all orders
with respect to U3-invariant interactions. If we take into
account the second order perturbation correction
with respect to the U3-violating interaction H_2
(= T^{1/2}_1), then we must compute matrix elements of
the second order interaction, having the transfor-
mation property T_{5/2} (of a tensor T^{5/2}_1). We can
evaluate this matrix element in a way analogous†
to the derivation of the mass formula eq. (1) to get

\[ M = a + b Y + c \left[ \frac{1}{4} Y^2 - I(U + 1) \right] + d Y^2 + e \left[ \frac{1}{4} Y^2 - I(U + 1) \right] Y + f \left[ \frac{1}{4} Y^2 - I(U + 1) \right]^2 . \] (2)

This second order mass formula contains six para-

† Similar to the derivation given in ref. 2), we can show
that a tensor T^{5/2}_{3/2} in a given irreducible representa-
tion of U(3) can be expressed as a linear combination of
the following terms: 1, A_{1/2}^1, A_{1/2}^2 (A_{1/2}^1), [A_{1/2}^1 (A_{1/2}^2)],
[(A_{1/2}^1 A_{1/2}^2)], and [A_{1/2}^1 (A_{1/2}^2)]. The rest follows exactly in the same
way as in ref. 2).
meters, and does not give any relations among particles belonging to the eight-dimensional representation $U_3(1,0,-1)$ in contrast to the first order mass formula eq. (1). Now, let us consider the ten-dimensional representation $U_3(2,-1,1)$, which may consist of $N^*(I = {3 \over 2}, Y = 1)$, $Y_1^*$ ($I = 1, Y = 0$), $\Xi^*$ ($I = {1 \over 2}, Y = -1$), and $\Omega$ ($I = 0, Y = -2$). In this representation, we have a special relation

$$I = {3 \over 2} + 1$$

(3)

Thanks to this constraint, eq. (2) reduces to

$$M = a' + b' Y + c' Y^2$$

(4a)

which is a generalisation of the corresponding first order mass formula eq. (1):

$$M = a' + b' Y$$

(4b)

As it has been pointed out by several authors $5, 6$), eq. (4b) gives the equal spacing rule for the masses among $N^*$, $Y_1^*$, $\Xi^*$, and $\Omega$. In the case of the second order mass formula eq. (4a) we have one relation of the following form

$$M(\Omega) = M(N^*) + 3 [M(\Xi^*) - M(Y_1^*)]$$

(5)

which predicts $M(\Omega) \approx 1685$ MeV. This value of $M(\Omega)$ differs little from that $5, 6$) computed by means of eq. (4b), since the equal spacing rule is fairly well satisfied experimentally among $M(N^*)$, $M(Y_1^*)$ and $M(\Xi^*)$. Therefore, up to the second order perturbation with respect to $H_1$, we expect an existence of $\Omega$ with the mass around 1685 MeV. Experimental verification of its existence will be a crucial test of our theory.

We may give another application of our formula eq. (2) in the 27-dimensional representation $U_3(2,0,-2)$, although we have no particular evidence of any particles belonging to this representation at the moment. If we use the notation $M(I, Y)$ for masses of particles with the isotopic spin $I$ and hypercharge $Y$, then we shall have the following three relations in $U_3(2,0,-2)$

$$M(2,0) + 24 [M(1,1) + M(1,2)] = 28M(0,0)$$

$$+ 9M(1,0) + 6 [M(1,2) + M(1,2)]$$

(6a)

$$M(1,0) + 6M(1,0) + 3 [M(1,2) + M(1,2)]$$

(6b)

$$[M(2,1) + M(2,1)] + 11 [M(1,1) + M(1,2)]$$

$$= 12M(0,0) + 6M(1,0) + 3 [M(1,2) + M(1,2)]$$

(6c)

Next, we shall derive a similar formula for the magnetic moment of baryons. In the zeroth order in $H_1 = T_1^3$, we have obtained relations $2, 7$) like $\mu(\Sigma^+) = \mu(\Sigma^0)$, $\mu(\Xi^0) = \mu(n)$ etc., assuming that the electro-magnetic current $j_\mu$ has a transformation property $S_1^1$ of a tensor $S_1^1$ with respect to $U(3)$ (for simplicity, we omit the vector suffix $\mu$ of the Lorentz space). If we take account of the mass-splitting interaction $H_1$ up to the first order perturbation, then we have to compute matrix elements of a tensor $S_1^2$ $= S_1^1 T_3^2$. Although we can proceed in the same way as in the derivation of the mass formula, we shall use a more direct method. From invariance, we can set

$$\langle S_1^1 + S_1^1 \rangle = a M_{1/2}^2 \mu_{1/2}^2 + b M_{3/2}^2 \mu_{3/2}^2 + c M_{1/2}^2 \mu_{1/2}^2$$

$$+ d M_{1/2}^2 N_{1/2}^2 + e M_{1/2}^2 N_{1/2}^2 + f M_{1/2}^2 N_{1/2}^2 + g M_{1/2}^2 N_{1/2}^2$$

$$h M_{1/2}^2 N_{1/2}^2 + j M_{1/2}^2 N_{1/2}^2$$

(7)

where we use the same notations as in the previous paper 2), and repeated indices mean summations over 1, 2 and 3. In this case, we have nine parameters $^\dagger$ but we get the following two relations

$$\mu(\Sigma^0) = {1 \over 2} [\mu(\Sigma^+) + \mu(\Sigma^-)]$$

(8a)

$$\mu_T(\Lambda - \Sigma^0) = {1 \over 2 \sqrt{3}} [\mu(\Sigma^0) + 3 \mu(\Lambda) - 2 \mu(\Xi^0) - 2 \mu(n)]$$

(8b)

where $\mu_T(\Lambda - \Sigma^0)$ is the transition magnetic moment between $\Lambda$ and $\Sigma^0$. In this derivation, it is unnecessary to assume the traceless condition $S_{13} = 0$ for $S_{13}$. Eq. (8a) is known to be a consequence $8$) of the charge independence alone. Note that we have no longer simple relations like $\mu(p) = \mu(\Sigma^+) \mu(n)$ etc., which are valid in the zeroth order in $H_1$. Similarly, we may remark that the branching ratio $2, 4$) of about 15% for $Y_1^* - \Sigma + \pi$ against $Y_1^* - \Lambda + \pi$, which is valid in the zeroth order of $H_1$, does not hold any more in the first order of $H_1$. Nor is the relation of electro-magnetic mass differences $7$)

$$M(\Xi^0) - M(\Xi^0) = M(\Xi^0) + M(p) - M(n)$$

valid if we take account of $H_1$. In this case, we shall have the following relation up to the first order in $H_1$

$$M(\Xi^0) - M(\Xi^0) = M(\Xi^0) + M(p) - M(n)$$

$^\dagger$ Actually the time-reversal invariance requires $d = e$ in eq. (7), thus reducing the number of parameters from nine to eight. This can be seen if we compare both terms after expressing them in terms of $h$ and $\Sigma^0$. ** This relation can also be derived by a direct tensor calculation as follows: The expectation value of $T^3_1 + T^3_3$ in the $U_3(2,1,-1)$ representation will be expressed as

$$\langle T^3_1 + T^3_3 \rangle = a F_{\mu \nu}^3 F_{\mu \nu}^3 + b F_{\mu \nu}^3 F_{\mu \nu}^3 + c F_{\mu \nu}^3 F_{\mu \nu}^3$$

$$+ d F_{\mu \nu}^3 F_{\mu \nu}^3 + e F_{\mu \nu}^3 F_{\mu \nu}^3$$

where $F_{\mu \nu}^3$ satisfies the conditions $F_{\mu \nu}^3 = F_{\mu \nu}^3 = - F_{\mu \nu}^3$ $F_{\mu \nu}^3 = 0$ and $F_{\mu \nu}^3 = [F_{\mu \nu}^3]^*$. If we express $F_{\mu \nu}^3$ in terms of $N^*$, $Y_1^*$, $\Xi^*$ and $\Omega$ as in ref. 2), the above equation results in eq. (6) after some calculations.

* In refs. 4, 5) the symbol $Z^*$ is used instead of $\Omega$.
where \( M_{\Gamma}(\Lambda - \Sigma^0) \) is the transition mass between \( \Lambda \) and \( \Sigma^0 \) due to the electromagnetic interaction. The proof of eq. (9) can be carried out almost exactly in the same way as that of eq. (8b). Note that the right hand side of eq. (9) will vanish due to the first order mass formula eq. (1) when we switch off the electromagnetic interactions.

Finally, let us investigate electromagnetic decay modes of bosons. In the zeroth order in \( H_I \), we can similarly show the validity of the following relations among transition matrix elements of corresponding processes

\[
M(p^+ \pi^0 + \gamma) = M(p^0 \pi^+ + \gamma) = M(K^+ \pi^- + \gamma) = -\frac{1}{\sqrt{3}} M(p^0 \pi^- + \gamma)
\]

In the derivation of eq. (10), we have utilised the charge-conjugation invariance of the theory together with the traceless condition \( S_{\mu}^M = 0 \). Again, if we take into account the mass-splitting interaction \( H_I = T_4^I \) to the first order of perturbation, then eqs. (10) and (11) do not hold any longer; instead we have the following weaker relation

\[
\frac{1}{\sqrt{3}} [M(\omega - \pi^0 + \gamma) + M(\rho^0 - \eta + \gamma)] = M(\omega - \eta + \gamma)
\]

Some of these relations may be tested experimentally.

The author would like to express his thanks to Prof. R.E. Marshak for many stimulating discussions. He is also grateful to Professor E.C.G. Sudarshan for reading this manuscript and also for various suggestions.

After having completed this work, it has come to the author's attention that eq. (11) has also been derived by R. Gatto in his lecture given at Trieste 1962.

References

1) Y. Yamaguchi, unpublished (1960).
2) Y. Ne'eman, Nuclear Phys. 26 (1961) 222.

TWO-PHOTON DECAY RATES OF THE \( \pi^0 \) AND \( \eta \) MESONS

Barbara BARRETT and G. BARTON
The Clarendon Laboratory, Oxford

Received 29 January 1963

In their paper on neutral pion decay Goldberger and Treiman \(^1\) assumed that the process \( \pi^0 \rightarrow 2\gamma \) occurs predominantly through a primary virtual dissociation of the pion into baryon-antibaryon (BB) pairs, but in the actual calculation they took into account only the nucleon-antinucleon pair contribution. They obtained a value for the rate which is roughly three times that subsequently observed \(^2\) \( \ast \).

We have extended their calculation by including all BB pairs; as a rough guide, until experimental values become available, we have used the predictions of the eightfold way \(^3,4\) \( \dagger \) for the \( \pi^0 \text{BB} \) coupling constants \(^3\) and the hyperon magnetic moments \(^5\). The resultant estimate is in good agreement with experiment.

Thus encouraged, we report the result of an exactly analogous calculation for the \( \eta \rightarrow 2\gamma \) decay rate which turns out to be appreciably smaller than other values that have recently been proposed on somewhat different theoretical grounds \(^6\). The discrepancy may be relevant to the feasibility of measuring this rate, which is currently being considered \(^7,8\).

\( \ast \) For a full list of references see footnote 8 of the paper by Andersen et al. \(^8\).

\( \dagger \) In ref. \(^3\) the \( \eta \) meson is denoted by \( x^0 \).