On the Universality of the Weak Interaction

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With the aim to understand the weak interactions occurring in nature from the unified point of view, we shall propose one possible model in this note. In our model, new charged Bose fields are introduced and are assumed to interact universally with all Fermi particles. We shall also discuss about the difference between ours and the other viewpoint in which all weak interactions are assumed to come from the universal Fermi interaction.

§ 1. Introduction

Recently, Pais-Nishijima-Gell-Mann\textsuperscript{1)} theory has given a clear explanation on the transition processes of the particle families which have both the strong and the weak interaction, including new unstable particles. According to this theory, the strongly interacting particles are all characterized by their isotopic spins and \( \gamma \)-charges which are recognized by their conservation in the strong interaction processes. On the other hand, there has been found no strong interaction in the transition processes accompanied with the lepton family including electron, neutrino and \( \mu \)-meson. Owing to this fact, we cannot anticipate a useful meaning of the isotopic spin or \( \gamma \)-charge in the lepton family.

Now, paying our attention only to weak interactions realized in nature, we shall find some peculiar regularity among them which has already been noticed in a previous paper.\textsuperscript{2)} The coupling constants of these interactions take a value of the unique order of magnitude \((\simeq 10^{-14})\) in the natural unit\textsuperscript{*}, independently of the coupling type or the sorts of the particles concerned. From this feature, we may expect the following:

\((a)\) The weak interaction occurs not only among the nucleon and \( \pi \)-meson families\textsuperscript{3)} which have the strong interaction too, but also among the lepton family. Within the frame work of Pais-Nishijima-Gell-Mann theory, the weak interaction comes out destroying the conservation of the isotopic spin or \( \gamma \)-charge, while the weak interaction itself originates from another root with wider validity.

\((\beta)\) The unique value of the coupling constants above mentioned indicates that all weak interactions come from a single base.

If these conjectures are correct, we may also anticipate the clue to the weak interaction in the processes participated by the lepton family. As the key to it, we shall take the following fact:

\textsuperscript{*} Through the paper, we take this unit as \( \hat{\theta} = c = r_0 (\simeq 10^{-13} \text{cm}) = 1 \), where \( \simeq \) means the equality in the order of magnitude.
When the lepton-family takes part in the decay (or capture) processes, we can always find neutrino, namely, there is no lepton process without neutrino.*

The purpose of this paper is to introduce one possible model, in order to understand the above characteristic features of the weak interactions in a unified manner. In this section, we shall present some outline of this idea. Paying our attention to the universal character of the weak interaction, we shall follow the example of the electromagnetic interaction

\[ e A_\mu \sum_a j_\mu (a) \]  

which appears most universally among all families. In the expression (I), \( e \) is the charge constant and \( j_\mu (a) \) is the current of particle \( a \). After the electromagnetic field, we shall introduce charged Bose fields \( B \)'s, which have the following type of interaction

\[ g \sum_a \sum_{\alpha, \beta} \bar{\psi}_a O_{\alpha \beta}(a, b, \alpha, \beta) \psi_b \cdot K_{\alpha \beta}(a, b, \alpha) B_a \]  

with all Fermi particles. This is the basic idea of this paper. The interaction (II) is assumed to conserve charge and nucleon-number and also to be Lorentz invariant. Further, from our view of the family,\(^3\) \( B \)'s are assumed to belong to the separate family from the \( \pi \)-meson family, namely, \( \pi, \theta \) and \( \tau \), etc., because of the difference in their characteristic interactions.

Compared with (I), the interaction (II) has some defect in the simplicity and symmetry. As we shall see below, \( B \)'s cannot be restricted in one type, and all combinations \((a, b)\) of Fermions do not interact in common with a \( B \)-field. Apart from these points, the interaction (II) can qualitatively satisfy the above features \((\alpha), (\beta)\) and \((\gamma)\) of the weak interactions. Leaving the precise discussion to the following sections, we shall give some brief remarks. First, all weak interactions come out through the interaction (II) with the coupling constant \( g \). From this fact we can understand the universality and the regularity of the weak interaction. Secondly, because \( B \)'s are charged, the decay interactions originated from (II) are necessarily charge-dependent, and in the coupled source with \( B \)-field, there are one neutral and one charged Fermion. This last feature is compatible with \((\gamma)\).

Now, in the resultant expression, the interaction (II) can be brought into Fermi interaction among Fermi particles. In this sense, there is no phenomenological difference between ours and the viewpoint of "primary Fermi interaction" about weak interactions. Under these circumstances, some detailed discussion will be necessary as to our standpoint. As was already noted,\(^3\) to understand all weak interactions on a single base, the view point of "primary Fermi interaction" will also serve to the purpose. Then, where does the difference between ours and the viewpoint of "primary Fermi interaction" lie? In precise analysis of the transition processes, there will be found some differences in the selection rule. However, in view of the present stage far from the final determination by the experimental test, we should like to emphasize rather the difference in the standpoint of the theory. If the

\(^*\) Such higher order processes as double \( \beta \)-decay are excluded.
view of Fermi interaction succeeds in the clarification of the phenomena, then it will be necessary to clarify the reason why Fermi interaction destroys the charge independence and why it maintains such feature as \((\gamma)\). In the present stage of our theory, however, there occurs no such problem, while the problems to be inquired are, for instance, the distinction of three characteristic Boson families, that is, the electromagnetic field, \(\pi\)-meson fields (including \(\theta, \tau\)) and \(B\)-fields, and also the origin of their interactions. Further, we are following the view-point in which the interactions among the particles (Fermions) are all assumed to be mediated by the existence of the fields (Bosons).

§ 2. The nature of \(B\)-fields

In this paragraph, we shall present a brief summary of the nature of \(B\)-fields and its interactions. First, all weak interactions are mediated by \(B\)-fields. For instance, the natural decay of free neutron takes place through

\[ N \rightarrow P + B(\rightarrow e + \nu) \rightarrow P + e + \nu. \]

The scalar type Fermi interaction thus brought abut is given by

\[ g^2 \langle \bar{\psi}_f \psi_f \rangle \left( \frac{1}{m_B} \right)^2 \tag{1} \]

where \(m_B\) is the mass of \(B\)-fields. Accordingly, our coupling constant \(g\) is correlated with Fermi coupling constant \(f\), as

\[ (g/m_B)^2 \approx f = 10^{-40} \text{ erg cm}^3 \tag{2} \]

and in the natural unit

\[ (g/m_B)^2 \approx 10^{-7} \tag{2'} \]

As is clear from this, the strength of the coupling of Fermions with \(B\)-fields lies between the strong interaction \((G^2 \approx 1)\) and the weak interaction \((\approx 10^{-14})\). For this reason, it is very improbable to detect \(B\)-particles in the nuclear interactions, and the observed \(K\)-mesons cannot be identified with \(B\)-particles. On the other hand, when \(B\)-particles are created, then they decay into leptons in very short interval \((\approx 10^{-17} \text{ sec})\).

Now, if the mass \(m_B\) of \(B\)-particles were less than that of the unstable \(K\)-meson or the mass difference between hyperon's and nucleon's, the decay of \(K\)-meson or hyperon would be very fast. To avoid this difficulty, the minimum of \(m_B\) is restricted as

\[ m_B > m_K \approx 1000 m_e. \tag{3} \]

The mass value of \(B\)-particles would, in other words, designate the maximum mass of the \(K\)-mesons, whose life-time lies within the present technics of the experiment \((\approx 10^{-10} \text{ sec})\). Further, if there exists another heavy meson with mass \(> m_B\), we shall observe such phenomenon as the direct production of leptons in the nuclear interaction, because this heavy meson created will rapidly decay into \(B\)-particle and \(B\)-particle also into leptons.

Because of their electric charge, \(B\)-fields are also accompanied with the electromagnetic interaction. However, the effect of \(B\)-field to the ordinary electromagnetic phenomena
would not be so large, in virtue of its large mass. The pair creation of \( B \)-particles by the high energy \( \gamma \)-ray is also hardly detectable, because the pair creation of electron and \( \mu \)-meson would rise up simultaneously and cover the \( B \)-field phenomena.

§ 3. Decay interactions mediated by \( B \)-fields

In our theory, all weak interactions are brought about through the interaction (II) by two steps. In the resultant expression for weak interactions, however, there is no distinction from Fermi interaction. Therefore we shall take, for a moment, Fermi interaction as the primary of the weak interactions.

Now, about the weak interaction realized in nature, we know three kinds, that is, Fermi interaction, Boson-Fermion and Boson-Boson interactions. To derive the latter two from Fermi interaction, the strong Boson-Fermion (e. g., \( \pi-N \)) interaction is necessary. They are represented, for instance, by the following diagrams:

\[
\begin{align*}
\text{Diagram 1} & \quad \text{Diagram 2}
\end{align*}
\]

where \( b \) and \( f \) indicate Boson and Fermion respectively, and \( F \) is Fermi interaction and \( G \) the strong Boson-Fermion interaction. When all weak interactions are derived from a single weak interaction as above, an important suggestion is presented by the regularity of coupling constants mentioned in § 1.

Let \( \tau_a \) be the lifetime of particle \( a \), then the general form of \( \tau_a \) is given, in the unit \( \hbar = c = 1 \), as

\[
1/\tau_a = (2\pi)^4 \frac{\hbar}{E_a} |I| \int \frac{dE_i}{(2\pi)^4 E_i} \delta (E_a - \sum E_i) \delta (k_a - \sum k_i)
\]

(4)

where \( E_i \) and \( k_i \) are the energy and momentum of final particle \( i \), and

\[
I = \sqrt{E_a} \langle k_a|H'|k_1\ldots k_n \rangle \prod_i \sqrt{E_i}
\]

is an invariant transition matrix. Extracting Fermi coupling constant \( f \), we may express \( I \) as

\[
I = fM(G, m_a)
\]

where \( M \) would be a complicated function of, for instance, the strong coupling constant \( G \) and the masses of the particles in virtual and real state. Under this preliminary, rewriting the expression (4) in the unit \( \rho_0 (\simeq 10^{-13} \text{ cm}) = 1 \), we shall obtain

\[
1/\tau_a = 3 \times 10^{23} (\text{sec}^{-1}) \cdot f^4 \cdot |M|^4 \cdot \rho_\rho^*
\]

(5)

from the dimensional consideration. \( \rho_\rho \) is the final density including \( (2\pi)^4 \) in (4). As

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* The numerical factor is nothing but \( (c/\rho_0) \).
is clear from the expression (5), if $p_F \approx |M|^2 \approx 1$ and $f^2 \approx 10^{-14}$ in the natural unit, it results $\tau \approx 10^{-10}$ sec. For the various unstable particles now observed, namely, $\tau (\rightarrow 3\pi)$, $\theta (\rightarrow 2\pi)$ and $\Lambda^0 (\rightarrow P + \pi^-)$, etc., we shall see $p_F \approx 1$ and we are also informed of the lifetime of these particles to be $10^{-9} - 10^{-10}$ sec. These facts indicate that $|M|^2$ is not seriously dependent on the transition scheme and takes a value $\approx 1$ in the natural unit.

In virtue of this, we shall take the following assumption as a first step:

(A) Except in the case where some particular selection rules play a role, the contribution $M(G, m_c)$ of the strong interaction to the decay matrix takes a value $\approx 1$ in the natural unit.

This assumption is not so drastic because the practical calculations so far have not found the serious deviations. In view of the difficulty to perform the correct estimation of the strong interaction, we shall rather intend to leave the problem to be studied in future on the above assumption. This assumption will be used without notices in the following.

When the above assumption is accepted, the absolute lifetime of each decay process is not a serious problem to be inquired. We shall only mention that the lifetimes of various unstable particles now observed are all elucidated in its order of magnitude. More precise estimation of the lifetime, —for instance the difference between $\theta^0 \rightarrow \pi^+ + \pi^-$ and $\theta^* \rightarrow \pi^0 + \pi^0$—will necessitate more profound analysis on the strong interactions. The problems to be studied in our theory will rather be the following:

(a) the selection rules for transition processes under the existence of the various interactions,
(b) the dynamics due to the interaction with $B$-fields and
(c) the final density.

These problems play an important role in the determination of lifetime or the competition among the various decay modes.

About the problem (c), only remembering the fact that the large difference of the lifetime between $\mu - e$ decay and free neutron decay is due to the final density, we shall rather draw our attention to (a) and (b). These problems will be studied with a special notice to the difference between ours and the viewpoint of primary Fermi interaction. For this example, we shall take the decay process of $K$-meson into leptons. From each viewpoint, the decay process will be given by the following diagrams:

As is already pointed out by K. Nishijima, the theorem of Fukuda-Miyamoto is not valid in the transition processes accompanied with unstable $K$-mesons or hyperons. Accordingly, when Fermi interaction is of scalar and vector type, scalar $K$-meson cannot be

* Because of the mediation of charged $B$-field, we can expect the existence of the difference in their decay life time.
forbidden to decay into lepton.* In our theory, on the other hand, scalar K-meson can decay, only when B-field is of scalar type. The above mentioned distinction lies just in this point, namely, that in the view of “primary Fermi interaction”, the selection rule in the transition process is controled by the interaction type, while in our case, it is rather ruled by the type of B-field concerned. At this moment, however, this distinction cannot be examined because the established facts are too meagre to find the distinction in selection rule successfully.

§ 4. Decay processes

In this section, to show a way of application of our theory to the decay phenomena, we shall give some explanations, although the following discussion is not necessarily decisive. To apply our theory to the decay phenomena, we need the identification of the particles. Because τ-meson is seemingly of pseudo-scalar type, it cannot be identified with θ-meson which decays into two π-mesons. In the following, assuming θ-meson to be scalar or vector, we shall take only τ and θ as unstable K-mesons. On the hyperons, we shall take Σ⁺ and Λ en bloc and represent them by Λ, because Σ⁺ is able to change into Λ by the strong interaction (Σ⁺ → Λ + π⁺). In virtue of the lack of the information about various competing decay processes, we shall exclude Σ⁻ particle for the moment. Λ is assumed to be spin 1/2 particle.

In studying the decay processes under the above identifications, we will select the following phenomena. Of course we should take account of the possibility that some of the following facts would be denied by the future development of the experiment.

(i) $\theta \rightarrow \mu + \nu$ has been found but $\theta \rightarrow e + \nu$ has not. We assume $K_{\mu \nu}$ to be identical with $\theta$, but not with $\tau$, tentatively.

(ii) $\tau \rightarrow e + \nu$ has not been detected also.

(iii) $\tau$ or $\theta \rightarrow \mu + \nu + \pi^0$ and seemingly $\rightarrow e + \nu + \pi^0$ are found.⁸

(iv) $A^0 \rightarrow p + e + \nu$ is not yet detected.

Now we shall study the predictions from our theory. Owing to the fact (iii), we cannot forbid (iv) absolutely. To ensure (iv), we may take $(A, \gamma_5 p) (e, \gamma_5 \nu)$ as the resultant interaction derived through B-field, because $\gamma_5$-interaction reduces the transition probability by a factor $\sim (M_\Lambda - M_p)^2 / M_\Lambda^2$ in comparison with other types of interaction.*** $\gamma_5$ is $\gamma_5$ or 1 corresponding to the same or opposite parity of $A^0$ to that of proton.****

The B-field to bring this interaction must be of pseudo-scalar type or scalar type, according to that $A^0$ and proton have the same or different parity. Because $\tau \rightarrow e + \nu$ is allowed in the case of pseudo-scalar type B-field, B-field here should be scalar. In this case, vector θ-meson is also forbidden to decay into electron and neutrino. $\tau \rightarrow e + \nu + \pi^0$ is allowed and $\theta \rightarrow e + \nu + \pi^0$ will be expected to be damped from the kinematical consideration. $\theta \rightarrow \mu + \nu$...
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becomes comprehensible under the introduction of vector type $B$-field between $(A^0 P)$ and $(\mu \nu)$. In this case, $\theta (and \tau \rightarrow \mu + \nu + \pi^0)$ can be realized. In the case of scalar $\theta$, the solution is not able to be found in our theory. The limitation for the particle as this follows from the facts (i) ... (iv). When one of these facts, for instance, (iv) is neglected, the possibility extends considerably.

Now, we shall present an example of $B$-fields and its interactions which are able to explain $\beta$-decay and $\pi - \mu$ decay, etc.

Some illustrations will be necessary for this diagram. To give the tensor-type interaction of the ordinary $\beta$-decay, $B$-field with spin 2 is introduced. For a mere derivation of the tensor-type interaction, we may take a vector $B$-field and assume its tensor interaction with Fermions. In this case, however, we cannot identify this $B$-field with the vector-type $B$-field which couples with $(A^0 P)$, because then $\theta \rightarrow e + \nu$ will be allowed, and further to maintain the strength of the resultant tensor interaction comparable with that of scalar interaction, we must enlarge the value of coupling constant $(2')$ by two units only for this case. For this reason, we have excluded the above possibility. This diagram, of course, follows from taking the facts (i) ... (iv) obstinately. We should expect the modification of this diagram with the future progress of the experiment. The knowledge of $\Xi$-particle will also offer another limitation. In this sense, the diagram indicated should not be taken too seriously.*

§ 5. Concluding remarks

In this paper, to understand the various features about weak interactions in a unified manner, we have proposed a possible model. In our viewpoint, all weak interactions are assumed to originate through the mediation of the charged Bose field $B$ newly introduced. While, in the phenomenological stage, our theory is not so different from the view of primary

* It should be noted that the present diagram does not necessarily forbid $\theta \rightarrow 3\pi$. 
Fermi interaction, we have found some disparities in the selection rules. However, in view of the present stage difficult to examine the existence of $B$-fields, we shall rather intend to take our theory as a possible indication for the origin of Fermi interactions which are to elucidate the weak interaction phenomena. Further, we should note one exceptional case for which our theory alone may not find solution, namely, the prohibition of the process $\mu \rightarrow e + \gamma$. To forbid this process, another restriction, for instance, the conservation of lepton-number will be necessary.\(^9\)

Finally, we shall present a brief consideration on the families and their interactions\(^3\). We know two Fermion—namely, nucleon and lepton-families of which we have nothing to be newly added. For Boson-families, we have now three characteristic families. These are the electromagnetic field, the $B$-fields and the $\pi$-meson-fields, the last of which is assumed to include $\pi$-meson, and $\beta$-meson etc. These families are also characterized by their intrinsic interactions which are, respectively, given by the interactions, (I), (II) and only the strong (in contrast with Pais-Nishijima-Gell-Man theory) interaction

$$G \sum_{a} \sum_{a} \bar{\psi} \psi O_{J}(a, b, \alpha) \psi_{b} \cdot K_{J}(a, b, \alpha) \phi_{a}$$

(III)

where $\phi$ is the wave function of the nucleon family and $\phi$ that of the $\pi$-meson family. Thus, from our viewpoint, we may say that Pais-Nishijima-Gell-Mann theory has given the systematics of what combinations $(a, b, \alpha)$ of the particles are realized in the interaction (III) and that all interactions occurring in nature originate from the existence of three characteristic Boson-families with their three intrinsic interactions (I), (II) and (III).

In conclusion, we should like to express our sincere thanks to Prof. K. Sakuma for his kind interest and continual encouragement throughout this work. We also take great pleasure in thanking Prof. S. Sakata, because this work has received many suggestions from his methodology.

References

3) For the original idea, see, S. Oneda, Prog. Theor. Phys. 9 (1953), 327.
4) This is independently noted by Wakasa, Hori and Oneda, Soryushion-Kenkyu (mimeographed circular in Japanese) 9 (1955), 559, and R. G. Sachs, Phys. Rev. 99 (1955), 1573.
6) For instance, see S. Nakamura, H. Fukuda, K. One, M. Sasaki and M. Taketani, Prog. Theor. Phys. 5 (1950), 740 and E. Fermi, Prog. Theor. Phys. 5 (1950), 570. \(R\) in the latter article seems to be compared to our \(r_{0}\).