On the Theory of Leptons.

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In the last few years the theory of hyperons and heavy mesons made a great progress and it became clear that the invariance of the theory against rotations in isotopic space plays an important rôle. In this paper we would like to discuss a new invariance principle upon which many properties of leptons can be explained.

According to the well-known classification of interactions, all interactions are divided into three classes: (a) charge independent interactions, (b) electromagnetic interactions, and (c) weak interactions. Since leptons have no interactions of the type (a), let us first study if there is any invariance property in the electromagnetic interaction of leptons. We start with stating the following theorem:

Theorem 1. The following two sets of field equations (1) and (2) lead to the same $S$ matrix:

\begin{align}
(1) & \quad \left\{ \begin{array}{l}
\left[ \gamma_\mu (\partial_\mu - i e A_\mu) + m \right] \psi = 0, \\
\Box A_\mu = i e \gamma_\mu \psi.
\end{array} \right.

(2) & \quad \left\{ \begin{array}{l}
\left[ \gamma_\mu (\partial_\mu - i e A_\mu) + m \exp [2 i x_\gamma] \right] \psi = 0, \\
\Box A_\mu = i e \gamma_\mu \psi.
\end{array} \right. \\
& \quad (x: \text{real constant}).
\end{align}

(Proof.) It is clear that the set (1) is transformed into (2) under the transformation

$$
\psi \rightarrow \psi \exp [i x_\gamma], \quad \bar{\psi} \rightarrow \bar{\psi} \exp [i x_\gamma].
$$

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which leaves the canonical commutation relations invariant. Let us distinguish the operators appearing in eq. (1) from those in (2) by a subscript 0, then one can write

\[ \psi_0 = \exp [i \gamma_\mu \psi], \quad \bar{\psi}_0 = \bar{\psi} \exp [i \gamma_\mu]. \]

In deriving eq. (2) from eq. (1) through the transformation (4), we must notice the following relation:

\[ \exp [i \gamma_\mu] \exp [i \gamma_\nu] = \gamma_\mu \gamma_\nu, \]

or more generally

\[ \exp [i \gamma_\mu] O \exp [i \gamma_\nu] = \begin{cases} O, & \text{for } O = \gamma_\mu \gamma_\nu, \gamma_\mu \gamma_\nu, (V, A), \\ O \exp [2i \gamma_\mu], & \text{for } O = 1, \gamma_\mu, \sigma_\mu, (S, P, T). \end{cases} \]

The Dirac four spinors of free particles corresponding to eqs. (1) and (2) are related to one another through

\[ \bar{u}(p, \sigma) = \exp [-i \gamma_\mu] u_0(p, \sigma), \]

\[ u(p, \sigma) = \bar{u}_0(p, \sigma) \exp [-i \gamma_\mu]. \]

Thereby it must be noticed that both \( u \) and \( u_0 \) belong to the same eigenvalue of the spin operator, since the spin operator \( \sigma_\mu \) is commutative with \( \gamma_\mu \).

The transformation of the propagation function is given by

\[ S_F(x - y) = \langle \Omega, T[\psi(x), \bar{\psi}(y)]\Omega \rangle \]

\[ = \exp [-i \gamma_\mu] \langle \Omega, T[\psi_0(x), \bar{\psi}_0(y)]\Omega \rangle \exp [-i \gamma_\mu] \]

\[ = \exp [-i \gamma_\mu] S_F(x - y) \exp [-i \gamma_\mu], \]

where \( T \) is Wick's chronological symbol and \( \Omega \) the vacuum state. Now let us prove that the \( S \) matrix is independent of the choice of \( \alpha \).

The contributions of the spinor field to the \( S \) matrix consist of two kinds of Feynman diagram: closed loops and open polygons. The contributions from the closed loop are generally of the form

\[ \text{Tr} [\gamma_\mu S_F(p_1) \gamma_\mu S_F(p_2) \ldots \gamma_\mu S_F(p_n)]. \]

If we insert eq. (8) into the above expression, we get

\[ \text{Tr} [(\exp [-i \gamma_\mu] \gamma_\mu \exp [-i \gamma_\mu]) S_F(p_1)(\exp [-i \gamma_\mu] \gamma_\mu \exp [-i \gamma_\mu]) S_F(p_2) \ldots]. \]

according to eq. (5), the above expression is again reduced to

\[ \text{Tr} [\gamma_\mu S_F(p_1) \gamma_\mu S_F(p_2) \ldots \gamma_\mu S_F(p_n)]. \]
The result means that the contributions of the closed loops to the $S$ matrix are invariant against the transformation (3). The contributions from the open polygons are generally given by

$$\bar{u}(p_f, \sigma_i) \gamma_\mu S_F(p_1) \gamma_\mu S_F(p_2) \ldots S_F(p_n) \gamma_\mu u(p_f, \sigma_i).$$

Again inserting eqs. (7) and (8) into this expression, one arrives at

$$= \bar{u}(p_f, \sigma_i) \gamma_\mu S_F(p_1) \gamma_\mu S_F(p_2) \ldots S_F(p_n) \gamma_\mu u(p_f, \sigma_i).$$

Thus we have completed the proof of the statement that the $S$ matrix is invariant against the transformation (3). Even if we take account of the renormalization procedures and of the existence of bound states, Theorem I still holds.

As one can readily see in eq. (10), the Dirac matrix $\gamma_\mu$ plays the essential role to absorb the transformation factor $\exp \left[-i \gamma_\mu \right]$. One can adopt an equation of the form (2) to nucleons only at the cost of abandoning the parity conservation in strong interactions, since the pseudoscalar coupling between nucleon and pion fields destroys this invariance property. Thus we can apply the transformation (3) only upon leptons whose strongest interactions are electromagnetic. The only possibility to distinguish between eqs. (1) and (2) is therefore to examine the weak interactions. Instead of examining this possibility, however, one can require that the $S$ matrix be strictly invariant against such a transformation so that in principle one cannot distinguish between eqs. (1) and (2), at least for leptons.

Before entering into the discussion of weak interactions, let us make a short remark. Since quantum electrodynamics is clearly invariant against the transformation

$$\psi \rightarrow \exp \left[i \beta \right] \psi, \quad \bar{\psi} \rightarrow \bar{\psi} \exp \left[-i \beta \right] \quad (\beta: \text{real constant})$$

we can generalize the transformation (3) as

$$\psi \rightarrow \exp \left[i \gamma_5 \beta \right] \psi, \quad \bar{\psi} \rightarrow \bar{\psi} \exp \left[i \gamma_5 \beta \right],$$

which leaves the $S$ matrix in quantum electrodynamics again invariant.

Although quantum electrodynamics is invariant against a wide class of transformations of the form (12), it is no more the case in weak interactions as we shall see later. This situation has a strong resemblance to the charge independence in the sense that the strongest interactions of the class (a) are invariant against all rotations in isotopic space but the electromagnetic interactions (b) are invariant only against the rotations around the third axis in isotopic space.

Recently Lee and Yang (1) and also independently Salam (2) have proposed the so-called screwon theory of the neutrino to guarantee the vanishing rest mass of the neutrino. This theory is invariant against the following transformation of the neutrino field:

$$\psi \rightarrow \psi \gamma_5, \quad \bar{\psi} \rightarrow \pm \bar{\psi} \gamma_5.$$
Without loss of generality, we shall choose the upper sign and assume this invariance from now on.

Let us now investigate what kind of restrictions should be imposed upon the choice of the parameters $\epsilon$ and $\beta$ in order that the weak interactions be invariant against the transformation of the form (12) (See Note (1)).

To see this invariance requirement more closely, we shall discuss a simple example, the $\pi^{\pm}\mu$ decay. According to the screwon theory, the decay interaction is given by

$$H_{\text{decay}} = g_s \bar{\psi}_\mu (1 - \gamma_5) \psi_\tau \cdot q + g_v \bar{\psi}_\mu \gamma_\lambda (1 - \gamma_5) \psi_\tau \cdot \frac{\partial}{\partial x_\lambda} + \text{herm. conj.}$$

Through the transformation (12), $H_{\text{decay}}$ is transformed as

$$H_{\text{decay}} \rightarrow H'_{\text{decay}} = g_s \bar{\psi}_\mu \exp[i \gamma_5(\gamma_5 + 1)] (1 - \gamma_5) \psi_\tau \cdot q + g_v \bar{\psi}_\mu \gamma_\lambda \exp[i \gamma_5(\gamma_5 + 1)] \psi_\tau \cdot \frac{\partial}{\partial x_\lambda} + \text{herm. conj.}$$

From this result one can conclude that if $g_v = 0$ or $g_s = 0$ the theory is invariant against the transformations of the form

$$\gamma_\lambda \rightarrow \gamma_\lambda' = \gamma_\lambda \pm i \gamma_5 (\gamma_5 + 1)$$

and the invariance of the theory requires $g_s g_v = 0$.

In the above discussion one can see that the invariance of the theory is attained only when $H_{\text{decay}}$ is accompanied by the matrix $(1 - \gamma_5)$. Contrary to the case of quantum electrodynamics the Dirac matrix $(1 - \gamma_5)$ accompanied by the neutrino field plays the rôle of the absorber of the transformation factor $\exp[i \gamma_5(\gamma_5 + 1)]$. If we assume furthermore that the neutrino operators appear always only in the combination $(1 - \gamma_5)\psi_\mu$ or $\bar{\psi}_\mu(1 + \gamma_5)$ and that this is the only source of the parity non-conservation in lepton interactions, then it is not hard to prove the following theorem:

**Theorem II.** Weak interactions involving only a $\mu$-meson-electron pair cannot be invariant against transformations of the form (12).

Hence the invariance requirement forbids the following unwanted processes:

$$K \rightarrow \mu + e, \quad \pi^{\pm} \rightarrow \mu + e,$$

$$\mu \rightarrow e + \gamma, \quad \mu \rightarrow e + e + e,$$

$$\mu + N \rightarrow N + e,$$ etc.
It must be noticed that the emission of an electron pair or a $\mu$-meson pair is not forbidden by this theorem. But the process

$$(\text{spin } 0) \rightarrow \nu + \bar{\nu}$$

is strictly forbidden in the sservon theory.

If we apply the above argument to the $\beta$-decay, we get the following theorem:

**Theorem III.** The $\bar{\nu}$-decay interaction must be either of the $STP$ combination or of the $AI'$ combination.

To prove this theorem one has only to remember eq. (6).

If we now choose according to experiment the $STP$ combination, the theory is invariant against the transformation of the electron operators

$$(17) \quad \psi' \rightarrow \exp \left[ i x (\gamma_5 - 1) \right] \psi', \quad \bar{\psi}' \rightarrow \bar{\psi}' \exp \left[ i x (\gamma_5 + 1) \right].$$

By similar arguments we can prove through the process (*)

$$\mu \rightarrow e + \nu + \bar{\nu} \quad \text{or} \quad e + \nu + \bar{\nu} \quad \text{or} \quad e + \bar{\nu} + \bar{\nu},$$

that the theory is invariant against the transformation of the $\mu$-meson operators

$$(18) \quad \chi_{\mu} \rightarrow \exp \left[ i x' (\gamma_5 - 1) \right] \chi_{\mu}, \quad \bar{\chi}_{\mu} \rightarrow \bar{\chi}_{\mu} \exp \left[ i x' (\gamma_5 + 1) \right],$$

and that in all interactions the operators $\bar{\chi}_{\mu} O \chi_{\nu}, \bar{\chi}_{\nu} O \chi_{\nu}$ and their hermitian conjugates appear only in $STP$ combinations. On the contrary, the transformation of the neutrino field which leaves the theory invariant is given by

$$(19) \quad \chi_{\nu} \rightarrow \exp \left[ i x (\gamma_5 + 1) \right] \chi_{\nu}, \quad \bar{\chi}_{\nu} \rightarrow \bar{\chi}_{\nu} \exp \left[ i x (\gamma_5 - 1) \right].$$

Thus we have clarified the universality of $STP$ combination in weak interactions.

The present theory does not contradict the results of the analysis of the decay processes (3)

$$K_{\mu 3} \cdot \mu - \pi + \nu,$$

$$K_3 \cdot e - \pi + \nu,$$

that the $STP$ combination is likely in both interactions.

Finally it must be remarked that according to the present theory the decay interactions for

$$\pi - \mu + \nu, \quad K_{\mu 3} \rightarrow \mu + \nu$$

(*) In any case we assume the conservation of the lepton number for a suitable particle-antiparticle assignment to electron, $\mu$-meson, and neutrino.

are not of the AV type as believed so far to account for the small rates of 
\((\pi \to e + \nu)/(\pi \to \mu + \nu)\) and of \((K \to e + \nu)/(K \to \mu + \nu)\). These small rates must be attributed to another source.

Once the present theory were justified, we would be free to substitute an 
equation of the form \((\gamma \partial + m \exp [2i\gamma_5])\psi = 0\) for the ordinary Dirac equations of leptons.

**Note added in proof.**

For the sake of clarity it must be mentioned that by invariance we mean in this paper the invariance of the interaction Hamiltonian which is enough to guarantee that the \(S\) matrix is independent of the choice of the parameter \(\gamma\) appearing in the modified Dirac equation as seen in the discussion of quantum electrodynamics.

If the neutrino-antineutrino mode is granted for the \(\mu\)-decay, the interaction is given uniquely by

\[
H_{\text{decay}} = \bar{\psi} \gamma_\mu (1 + \gamma_5) \psi \cdot \gamma_\nu (1 - \gamma_5) \gamma_\nu + \text{herm. conj.}
\]

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