On the Theory of Hyperons and Heavy Mesons
by K. NISHIJIMA

§ 1. Introduction

During the past few years there has accumulated so much experimental information on hyperons and heavy mesons that some of the many puzzling questions on these strange particles are being resolved. Under such circumstances one is often motivated to seek for an ordering principle by which one can understand why there are such particles as being known, why they are of such nature as being observed. It is expected that the solution of this problem would be of great help to bring the present status of the theory of elementary particles much further. Besides such problems of a rather deep nature, we are interested in understanding the curious behaviours of the heavy unstable particles in a consistent manner and in predicting their implications to the pion-nucleon interactions.

In this report we shall confine ourselves to the latter problem. The theory of hyperons and heavy mesons is closely related to the pion theory on which we base our reasoning and arguments, and we have assumed that these particles are so strongly coupled to each other that one cannot treat them separately for quantitative calculations. Also it can readily be shown through tentative estimations of the strength of the nuclear interaction of these particles that the assumption mentioned above would certainly be reasonable. Hence the theory of these strongly coupled particles is sharply contrasted to quantum electrodynamics which is practically a closed theory. Since, however, the theory of pions as it stands is not yet fully successful in explaining the nature of pions at energies of the order of Gev, we cannot expect any quantitative theory of these particles for the time being. Still it is possible to draw some interesting conclusions on the nature of strange particles on which one can readily understand many of their striking behaviours observed so far. These arguments are based mainly on the general invariance principles in quantum field theory, which would certainly be more reliable than the present field
theory itself. Of them, the principle of charge independence is of the special importance on which most of the characteristic features of the heavy unstable particles have been interpreted. This report will be devoted mainly to the interpretation of these heavy unstable particles in terms of the charge independence hypothesis. Recently there is an increasing amount of experimental evidences that the rest masses and lifetimes of all kinds of K-mesons known so far are equal to each other within the presently attainable experimental precision. These evidences give rise newly to a very hard but interesting question of a mysterious nature, i.e. which of these K-mesons are identical and which of them are different. Till now this is an open question and undoubtedly one of the central problems in this field.

§ 2. Hypothesis of Pair Production

The nature of hyperons and heavy mesons appeared first very puzzling because of the following contradicting properties if simple detailed balancing arguments were applied:
(1) their long lifetimes as measured on a nuclear time scale, say $10^{-10}$ sec.
(2) their large abundance, i.e. more than 3% of the penetrating showers.

For the later convenience, we shall illustrate this contradicting nature by a simple example. Consider the decay process

\[ \Lambda^0 \rightarrow p + \pi^- \].

Let us assume the lifetime of $\Lambda^0$ to be $3 \cdot 10^{-10}$ sec, and the spin of $\Lambda^0$ to be $\frac{1}{2}$ and further that the following interaction gives rise to the above observed decay mode:

\[ H_{int} = i/\sqrt{2} \psi p \gamma_5 \psi_{\Lambda} \cdot \varphi_\pi + \text{hermitian conjugate}, \]

where $\psi_p$, $\psi_{\Lambda}$ and $\varphi_\pi$ are the field operators for proton, $\Lambda^0$ and pion, respectively. Then one can evaluate the magnitude of the coupling constant

\[ f^2/4\pi = 1.3 \times 10^{-11}. \]

This value is about $10^{-12}$ times as small as compared with the pseudoscalar coupling constant for the pion-nucleon interaction. Thus we can conclude that the interaction (2.2) is too weak to account for the observed production rate as far as the spin of $\Lambda^0$ is small. Thus our problem can be presented in the following form:

(1) Is it possible to make a model in which $\Lambda^0$ has a very high spin?
(2) Is it possible to introduce strong interactions responsible for the production of these unstable particles without violating the metastability of $\Lambda^0$?

(I) high spin model [FERMI-FEYNMAN (I)].

When $\Lambda^0$ had a very high spin, the relative orbital angular momentum between the secondary proton and pion which appear in the decay process (2.1) would be very large. The $Q$-value of this decay process is only $37 \pm 1$ Mev, and due to the barrier effect of the centrifugal force this process could occur only very slowly. Let $l$ be the relative orbital angular momentum be-
tween the decay products, then the transmission coefficient for the pion to leak through the barrier is given for large \( l \) by

\[
T_{l} \simeq \frac{4k}{K} v_{l} \simeq \frac{4k}{K} \left( \frac{x^2}{(2l - 1)!!} \right)^2,
\]

(2.4)

where \( x = kR \). \( K \) and \( k \) are the momenta of the pion inside and outside of the barrier respectively, and \( R \) the radius of the force (2). The expression \((2l - 1)!!\) is defined by the following formula and tabulated in the Table I:

\[
(2l - 1)!! = 1 \cdot 3 \cdot 5 \ldots (2l - 1)
\]

(2.5)

<table>
<thead>
<tr>
<th>( l )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>((2l - 1)!!)</td>
<td>1</td>
<td>3</td>
<td>15</td>
<td>105</td>
<td>945</td>
<td>10395</td>
<td>135135</td>
</tr>
<tr>
<td>([(2l - 1)!!]^2)</td>
<td>1</td>
<td>9</td>
<td>225</td>
<td>11025</td>
<td>893025</td>
<td>( \sim 10^8 )</td>
<td>( \sim 10^{10} )</td>
</tr>
</tbody>
</table>

The inside momentum of the pion \( k \) is about 100 Mev/c and \( x = kR \) would certainly be smaller than unity although we do not know the force range \( R \) precisely. From the above table one sees readily that for \( l = 6 \) or 7 the decay rate is much reduced so that the observed longevity of \( \Lambda^0 \) might be accounted for. On the other hand, the production of \( \Lambda^0 \) takes place at much higher energies than the decay process and the barrier effect would no more reduce the production rate. However, we face at once some objections against this model.

(1) If \( \Lambda^0 \) had really such a high spin, say \( 6^{1/2} \), then one has naturally to ask: Why are there no other metastable particles whose spins lie between \( 1/2 \) and \( 6^{1/2} \)? Were there no such particles, then one could hardly understand why nature liked such a special spin value. Anyhow this interpretation seems to be unnatural.

On the contrary, if there were other particles with spins ranging from \( 3/2 \) to \( 5^{1/2} \), \( \Lambda^0 \) had to undergo radiative decays into such particles much faster than the process (2.1), unless all other particles are heavier than \( \Lambda^0 \). The latter possibility appears again unnatural (3), (4).

(2) From the investigation on the non-mesonic decays of hyperfragments, to be discussed in §6, it can be concluded that it is impossible to assign so large a spin to \( \Lambda^0 \).

(II) two-meson theoretical model.

The difficulties we faced are very akin to those in the old meson theory. It is true that the cosmic ray mesons are very copiously produced, but still they have only very weak interactions. In order to overcome this difficulty, the two meson theory was invented. When one remembers this history, one might be inclined to favour a similar interpretation also for the heavy unstable particles. Suppose, for instance, that \( \Lambda^0 \) is a daughter of another heavy particle \( X \). \( X \) must have a strong nuclear interaction so as to be produced copiously and decays into \( \Lambda^0 \). Then \( \Lambda^0 \) is also produced copiously, but \( \Lambda^0 \) can interact only weakly
so that its decay into a proton and a meson takes place slowly through a very weak interaction. In this case, however, such a model does no more help as seen in what follows. Suppose that \( X \) is produced through the process

\[
\frac{\pi^-}{N} + N \rightarrow X + a + \ldots,
\]

(2.6)

and decays as

\[
X \rightarrow \Lambda^0 + \ldots
\]

(2.7)

The interaction \( G \) should be strong in order that \( X \) is copiously produced. Hence the particle \( a \) which is produced in association with \( X \) must have strong nuclear interaction at least through (2.6). The particle \( a \) is supposed to be a nucleon, a pion, or a photon. Hence \( X \) can decay inversely through the strong interaction \( G \)

\[
X \rightarrow \frac{\pi^-}{N} + N + \bar{a} + \ldots \rightarrow \Lambda + \pi \text{ or } \gamma,
\]

where \( \bar{a} \) is the anti-particle of \( a \).

In order that \( X \) can decay also into \( \Lambda^0 \), \( g \) should also be strong. Then, however, \( \Lambda^0 \) must decay rapidly through the strong interaction \( g \) (3).

**III** hypothesis of pair production.

As mentioned above, it is very hard to account for the nature of \( \Lambda^0 \) so far as one assumes that \( \Lambda^0 \) were produced singly. In order to avoid this difficulty, the hypothesis of pair production was proposed by Nambu and his collaborators (5), by Oneda (6) in 1951, and also by Pais (7) in 1952. Pais has formulated this idea in terms of the so-called even-odd rule. Since we shall later discuss more in detail, only a very brief sketch of this idea will be given here. Suppose that all interactions concerned with such phenomena are given in the following form:

\[
N_i N_j \pi_k.
\]

(2.8)

Provided that one assigns the subscripts suitably to heavy unstable particles, one can formulate the above idea by postulating

\[
(2.8) \text{ is strong if } i + j + k = \text{even},
\]

\[
\text{weak if } i + j + k = \text{odd}.
\]

(2.9)

\( N_i \) denotes the \( i \)-th hyperon, and \( \pi_k \) the \( k \)-th heavy meson. Especially the nucleon and pion have the subscript 0.

At that time many important experimental investigations were carried out on pion reactions, and we were convinced of the validity of the principle of charge independence for the pion-nucleon interaction. For instance, from Hildebrand's experiment (8), the following relation is ascertained:

\[
\frac{d\sigma(p + p \rightarrow d + \pi^0)}{d\Omega} / \frac{d\sigma(n + p \rightarrow d + \pi^0)}{d\Omega} = 2,
\]

(2.10)
which follows from the charge independence hypothesis. The experiments on the scattering of pions by proton were also of much help to establish this principle. Under such circumstances one may naturally ask for the connection between charge independence and heavy unstable particles. Pais has proposed his $\omega$-space theory (9), but without success to explain the cascade decay.

§3. Hypothesis of Charge Independence

Meanwhile many important experimental informations were accumulated and we had to investigate seriously the connection between the charge independence principle and the nature of unstable heavy particles (4, 9, 10, 11).

The problems that we are now concerned with might be formulated in the following way:

1. What is the connection between charge independence and curious behaviours of hyperons and heavy mesons?
2. How can one interpret the striking metastability of the cascade particle $\Xi$ which the even-odd rule failed to explain?
3. What is the reason for the positive excess of K-mesons found in nuclear emulsions and also in cloud chambers?
4. Why are the heavy unstable particles produced more easily in pion-nucleon collisions than in nucleon-nucleon collisions at energies of the order of Gev?

In fact these problems are not independent of each other, but intimately connected with each other, and the solution of the first problem provides us with the solution of other problems. Hence we shall start with discussing the first problem.

From the investigations on the levels of isobaric nuclei, nuclear reactions, and the nuclear interactions of pions it is now established that the principle of charge independence (abbreviated as C. I. hereafter) is valid for the pion-nucleon interaction and nuclear forces at low energies of the order of $10^2$ Mev. We here classify all the elementary interactions into three categories according to their orders of the strengths.

a) charge independent interactions

Of the three groups, the strongest interactions are the charge independent ones for which the isobaric spin $I$ is conserved. As is well known, the pion-nucleon interaction belongs to this class, and this is the only interaction for which the validity of C. I. is established. There cannot be any interaction that is as strong as the pion-nucleon interaction but is not charge independent, otherwise one could not recognize the C. I. principle for the pion-nucleon interaction.

Hence the nuclear interactions of hyperons and heavy mesons should obey the C. I. principle if they were comparatively strong with the pion-nucleon interaction.

From the cosmotron experiments (12), the cross-sections of the process

$$\pi^- + p \rightarrow \Lambda^0 + \theta^0, \text{ and } \Sigma^0 + \theta^0$$

(3.1)
for the incoming pion energy of about 1.5 Gev was measured to be of the
order of 1 mb. Furthermore it is also known that the ratio of this cross-
section to the elastic one is given by

\[ \frac{\sigma(\pi^- + p \rightarrow \Lambda^0 + \theta^0 \text{ and } \Sigma^0 + \theta^0)}{\sigma(\pi^- + p \rightarrow p + \pi^-)} \sim \text{one severalth.} \]  

This result shows directly that the nuclear interactions of the heavy unstable
particles at energies of the order of Gev is as strong as the pion-nucleon
interaction. The existence of hyperfragments in which \( \Lambda^0 \)'s are bound also
dicates that their nuclear interactions are very strong, at least much
stronger than the electromagnetic interaction. Thus we may be allowed to
take it for granted that the nuclear interaction of heavy unstable particles
is charge independent.

b) electromagnetic interactions

The principle of C. I. is not strict but approximate, otherwise the charge
multiplets, such as the positive, neutral and negative pions, would have not
been distinguished so that the principle itself would not have been called
under such a name. As far as we know, the slight deviations from the C. I.
at energies of the order of \( 10^3 \) Mev can be attributed to the electromagnetic
interaction and small mass differences among charge multiplets which might
also be of electromagnetic origin. Hence we may be allowed to assume that
the electromagnetic interaction is the strongest one among those that violate
the charge independence. For the electromagnetic interaction the total
isobaric spin is not conserved, but its third component \( I_3 \) is still conserved.
In connection with the discussions in a), this is another basis in favour of the
C. I. hypothesis for the heavy unstable particles, since the effects of these
particles are considered to be stronger than the electromagnetic interactions
even at low energies of the order of \( 10^2 \) Mev.

In the lowest order for the electromagnetic interaction, the total isobaric
spin is no more conserved but obeys the following selection rule:

\[ \Delta I = 0, \pm 1. \]  

(3.3)

c) weak interactions

As is clear from the above classification, interactions other than the above
two groups should be weaker than they are. In general neither the total isobaric
spin nor its third component \( I_3 \) is conserved for such interactions. The inter-
actions responsible for various observable decay processes and Fermi inter-
actions belong to this third group. When leptons take part in, the isobaric
spin itself can no more be defined.

What is interesting is the fact that the interactions in this category are much
weaker than the first group about twelve orders. We do not know yet the
true reason why there is so large a gap, but it might partly be due to the
experimental difficulties in detecting the interactions of intermediate
strength if there were any, since they would lead to too small cross-sections
for production or scattering processes on one hand, and too short lifetimes for
decay processes on the other hand. Or, one might suppose that the large gap is real since there seems to be regularities in the strengths of the interactions between Fermion-Fermion (universal Fermi interactions) and Boson-Fermion (universal weak Boson-Fermion interactions) (13). In this report we shall spare the discussions on this point.

Based on the above discussions on the classification of elementary interactions, we shall assign the isobaric spins to hyperons and heavy mesons.

(I) hyperons

In assigning the isobaric spin to $\Lambda^0$, one must be concerned with the problem whether there is a charged counter particle to $\Lambda^0$ or not. If it were to exist, its mass should approximately be equal to that of $\Lambda^0$ and it would have to be produced as frequently as $\Lambda^0$. Experimentally charged hyperons $\Sigma^\pm$ are known, but they are much heavier than $\Lambda^0$.

$$m(\Lambda^0) = 2181 \pm 2m_e, \quad m(\Sigma^+) = 2327 \pm 3m_e, \quad m(\Sigma^-) - m(\Sigma^+) \approx 10m_e.$$  

The mass difference between $\Lambda^0$ and $\Sigma^\pm$ are too large to be understood as being due to the difference of the electromagnetic self-energies, and this fact leads us to conclude that these charged hyperons cannot be the counter particles to $\Lambda^0$. Hence we assign the isobaric spin to $\Lambda^0$ as

$$I(\Lambda^0) = 0. \quad (3.4)$$

It may be natural to assume that $\Sigma^+$ and $\Sigma^-$ belong to the same charge multiplet, which requires inevitably the existence of their neutral counter particle $\Sigma^0$. Since no doubly charged hyperon has ever been observed experimentally, we assign

$$I(\Sigma) = 1. \quad (3.5)$$

According to the selection rule (3.3), $\Sigma^0$ must undergo a rapid radiative decay into $\Lambda^0$, as suggested in (4, 9, 11)

$$\Sigma^0 \rightarrow \Lambda^0 + \gamma. \quad (3.6)$$

Hence $\Sigma^0$ cannot be observed directly in agreement with our experiences. The cosmotron experiments presented some indirect evidences in favour of the existence of $\Sigma^0$, since in some cases the process

$$\pi^- + p \rightarrow \Lambda^0 + \theta^0 \quad (3.7)$$

had to be reinterpreted as being due to the process

$$\pi^- + p \rightarrow \Sigma^0 + \theta^0 \rightarrow \Lambda^0 + \gamma \quad (3.8)$$

on the energy-momentum balance (12, 14).

Also there are already some direct evidences in favour of (3.6). It must be noticed that the decay of $\Sigma$ into $\Lambda$ through

$$\Sigma^{\pm, 0} \rightarrow \Lambda^0 + \pi^{\pm, 0} \quad (3.9)$$
is energetically forbidden, since
\[ m(\Sigma^+) - m(\Lambda) \approx 146 m_e < m(\pi). \] (3.10)

Otherwise \( \Sigma \) particles would have never been directly observed.

The discussions on the cascade hyperon \( \Xi \) will be reserved till next section.

(II) heavy mesons

Utilizing the isobaric spin assignment to hyperons, one can determine the isobaric spins of heavy mesons. As argued before, we may take it for granted that the isobaric spin is conserved for the process (3.1). Hence the isobaric spin of \( \ell \)-meson should be either \( \frac{1}{2} \) or \( \frac{3}{2} \) as seen from

\[
\begin{align*}
\pi^- + p &\rightarrow \Lambda^0 + \theta^0. \\
I_1 & 1 \quad \frac{1}{2} \quad 0 \quad \frac{1}{2} \text{ or } \frac{3}{2} \\
I_2 & -1 \quad \frac{1}{2} \quad 0 \quad -\frac{1}{2}
\end{align*}
\] (3.11)

The latter choice results in the existence of a doubly charged heavy meson of the comparable abundance with \( \theta^0 \), which is excluded experimentally.

Hence we have
\[ I(\theta) = \frac{1}{2}. \] (3.12)

The charged counter particle to \( \theta^0 \) corresponding to \( I_2 = \frac{1}{2} \) should be of charge \( +e \), since \( I_2 \) assumes \( -\frac{1}{2} \) for \( \theta^0 \). These \( \theta^+ \) and \( \theta^0 \) mesons form a charge doublet just as proton and neutron.

In this report, we shall denote the \( \theta \)-mesons as \( K \), but a \( K \)-meson is called \( \theta \) only when it undergoes the characteristic two pion decay, namely this \( K \)-meson might also decay exhibiting other modes. In general the process (3.1) can be written in the form
\[
\pi^- + p \rightarrow \Lambda + K,
\rightarrow \Sigma + K.
\] (3.13)

By observing the decay modes of the \( K \)-mesons which are produced through the above reaction, one can see which mesons have isobaric spin \( \frac{1}{2} \). Some examples of the associated production are given below.

**Table II**

<table>
<thead>
<tr>
<th>group</th>
<th>hyperon</th>
<th>heavy meson</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bristol</td>
<td>( \Sigma )</td>
<td>( \kappa^+ ) ((\rightarrow \mu \rightarrow e))</td>
</tr>
<tr>
<td>Göttingen</td>
<td>( \Sigma )</td>
<td>( \tau^+ ) ((\rightarrow 3 \pi))</td>
</tr>
<tr>
<td>Göttingen</td>
<td>( \Sigma )</td>
<td>( K^0_\mu (g^* &amp; p\beta) )</td>
</tr>
<tr>
<td>Bristol</td>
<td>( \Sigma )</td>
<td>( \theta^+ (g^* &amp; p\beta) )</td>
</tr>
</tbody>
</table>

\( Y^- \) denotes an unidentified hyperon.

From this table we see that \( \kappa^+ \), \( \tau^+ \), \( K^0_\mu \), and \( \theta^+ \), or in another nomenclature, \( K_{\mu}, K_{\pi}, K_{\nu}, \) and \( K_{\pi} \), have isobaric spins \( \frac{1}{2} \). As we shall see later, \( K^0 \) and
its anti-particle \( \bar{K}^o \) should be distinguished from one another. The negative heavy mesons denoted as \( \bar{K}^- \) are also observed. They suffer nuclear absorptions like

\[
\bar{K}^- + \bar{p} \rightarrow \Lambda^o + \pi^o \\
\rightarrow \Sigma^+o + \pi^o + \gamma.
\]  

(3.14)

\( \bar{K}^- \) is considered to be the anti-particle of \( K^+ \), and forms another charge doublet together with \( \bar{K}^o \) (15, 16).

§ 4. Consequences of the Charge Independence Hypothesis

In the previous section, we have assigned the isobaric spins to hyperons and heavy mesons based on the hypothesis of C.I. In this section we shall discuss the consequences of this hypothesis.

(I) Hyperons and heavy mesons cannot be composed of nucleons and pions. Since both the ordinary and isobaric spins are half-integral for a nucleon, and integral for a pion, both spins of a system composed of them should be either integral or half-integral. However, it is not the case for hyperons and heavy mesons as seen from the isobaric spin assignments in the previous section and their decay modes:

\[
\begin{align*}
\text{hyperons} & : \\
\Lambda^o & \rightarrow p + \pi^- + (37 \pm 1) \text{ Mev} \\
\Sigma^+ & \rightarrow p + \pi^o + (110 \pm 5) \text{ Mev} \\
\Sigma^- & \rightarrow n + \pi^- \\
\Xi^- & \rightarrow \Lambda^o + \gamma \\
\text{heavy neutral mesons} & : \\
\theta^o & \rightarrow \pi^+ + \pi^- + (214 \pm 5) \text{ Mev} \\
\tau^o & \rightarrow \pi^+ + \pi^- + (215 \pm 5) \text{ Mev} \\
\text{heavy charged mesons} & : \\
K^+ & \rightarrow \mu^+ + \nu \\
K^+_\pi & \rightarrow \pi^+ + \pi^o \\
K^+_\tau & \rightarrow 2\pi^+ + \pi^- \\
K^+_\tau & \rightarrow 2\pi^- + \pi^+ \\
K^+_\mu & \rightarrow \mu^+ + \pi^o + \gamma \quad \text{(or } \chi^+, \theta^+) \\
K^+_e & \rightarrow e^+ + \pi^o + \gamma \quad \text{(or } \tau^+) \\
\end{align*}
\]

(4.1)

Anyhow there is no contradiction in assuming that all hyperons are Fermions and that all heavy mesons are Bosons.

The statement here does not mean that all hyperons and heavy mesons are elementary particles, but some of them might be composite (17).

(II) The \( K^o \) meson is described by a complex wave function, so that it must be distinguished from its charge conjugate particle \( \bar{K}^o \). Since \( K^+ \) and \( K^o \) form a charge doublet, their transformation properties in charge space are identical with those of proton and neutron, i.e. the wave function of the K-meson is a spinor in charge space. Hence \( K^o \) as well as \( K^+ \) should be described by
complex wave functions, and $K^0$ must be distinguished from its charge conjugate particle $\bar{K}^c$ in contrast with other neutral Bosons such as a neutral pion and a photon.

In the case of the pion, there are three charge states, positive, neutral, and negative, forming a charge triplet ($I = 1$), and under rotations in charge space the three bases

$$\pi^+, \pi^0, \pi^-$$

are mutually transformed irreducibly. Furthermore there are representations in which $\pi^-$ can be described by real a wave function so that the charge conjugate particle of $\pi^-$ is $\pi^-$ itself.

In the case of $K$-meson, on the contrary, two sets of bases

$$K^+, K^0, \bar{K}^-,$$

are not mixed under rotations, but they form separately irreducible sets of bases. These two sets are transformed into one another through the operation of charge conjugation. These relationships would schematically be understood in Fig. 1.

In the above figure, the underlines combine bases which are mutually transformable under rotations in charge space, and $C$ and $T$ represent the operations of charge conjugation and charge symmetry, respectively. The charge symmetry operation $T$ is defined as the rotation of $180^\circ$ around the "2" axis in charge space as shown in Fig. 2.

Namely, $T$ is defined by

$$T = \exp (i\pi I_2).$$

This operator was first introduced by KROLL and FOLDY in connection with the selection rules in nuclear reactions (18). In their definition, $I_1$ was used instead of $I_2$, but in connection with the meson theory the present choice is more convenient. They have further introduced the concept of "charge parity" which must be distinguished from "the parity under charge conjugation". If a system is composed of $A$ nucleons, we have

$$T = \exp \left( \frac{i\pi}{2} \sum_{s=1}^A \tau_2^{(s)} \right) = i^A \prod_{s=1}^A \tau_2^{(s)}.$$
This operation is anti-commutative with the third component of the total isobaric spin $I_3$

$$TI_3 = -I_3T.$$  \hspace{1cm} (4.5)

Hence, if and only if $I_3 = 0$, $T$ can be diagonalized, i.e. if the nucleus is composed of the same number of protons and neutrons. In this case we have

$$T^2 = (-1)^A = 1. \quad (A : \text{even})$$  \hspace{1cm} (4.6)

Hence $T$ can assume either $+1$ or $-1$ as the eigenvalues, and this sign is the charge parity. For light nuclei for which one can safely assume C. I., the charge parity is a good quantum number. Strictly speaking, the charge parity is a good quantum number if the nuclear forces are charge symmetric. If one assumes C. I., the charge parity is determined by the isobaric spin of the nucleus, i.e.

$$T = (-1)^J.$$  \hspace{1cm} (4.7)

An example of the application of the conservation of charge parity is seen in the forbiddenness of the following reaction:

$$d + O^{16} \rightarrow N^{*14} + He^4,$$

where $N^{*14}$ is the excited state of $N^{14}$ with the excitation energy 2.3 Mev. The nuclei $d$, $O^{16}$, and $He^4$ have even charge parity, while $N^{*14}$ is of odd parity, and this reaction is forbidden.

**Introduction of "strangeness"**

In developing the C. I. theory of hyperons and heavy mesons, the concept of "strangeness" or $\eta$-charge is convenient (II), (4). As is well known there is always an intimate relationship between the third component of the isobaric spin $I_3$ and charge $q$ of a given particle which has a definite isobaric spin, $q = I_3 + 1/2$ for a nucleon and $q = I_3$ for a pion. In general we may write

$$q = I_3 + 1/2 + s_a/2,$$  \hspace{1cm} (4.8a)

for a hyperon $N_a$, and

$$q = I_3 + s_b/2,$$  \hspace{1cm} (4.8b)

for a heavy meson $K_b$. For an anti-hyperon $N'_a$, one must change the sign of the term $1/2$ in the right hand side of (4.8a), i.e.

$$q = I_3 - 1/2 + s'_a/2.$$  \hspace{1cm} (4.8a')

These relations are generally written in the form

$$q = I_3 + n/2 + s/2,$$  \hspace{1cm} (4.9)

where $n$ is the so-called "nucleon number" which is known to be strictly conserved so that the matter can remain to be stable.

From the definition, the strangeness is zero for a nucleon and a pion. For hyperons and $K$-mesons, we have

$$s(\Lambda) = s(\Sigma) = -1,$$

$$s(K) = 1, \quad s(\bar{K}) = -1.$$  \hspace{1cm} (4.10)
An anti-particle, i.e. charge conjugate particle, has an opposite strangeness of the particle as is the case for the electric charge, namely

$$sC = -C s, \quad qC = -C q,$$  \hspace{1cm} (4.11)

where $C$ is the operation of charge conjugation.

Let us consider a system of particles with definite isobaric spins. Then the total charge $q$ of this system is expressed with reference to (4.8) by

$$q = I_3 + n/2 + s/2,$$  \hspace{1cm} (4.12)

where $I_3$ is the third component of the total isobaric spin of the system, $n$ is the number of hyperons minus the number of anti-hyperons and $s$ is the total strangeness of the system, i.e.

$$n = \sum_a n(N_a), \quad s = \sum_a s_a n(N_a) + \sum_b s_b n(K_b),$$  \hspace{1cm} (4.13)

where $n()$ is the number of particles minus the number of anti-particles.

From the stability of matter, $n$ should be a strict quantum number. For the production and scattering processes in which only charge independent and electromagnetic interactions are effectively operating, $q$ and $I_3$ are conserved. In this case $q$ is the total charge of particles with strong nuclear interactions but does not involve the charge of other particles like $\mu$-mesons and electrons. From the conservation law of $q$, $I_3$ and $n$, and the eq. (4.12), follows the conservation theorem of strangeness.

(III) The total strangeness of a system is conserved for processes caused by charge independent and electromagnetic interactions.

Making use of this conservation law, we can account for various properties of hyperons and heavy mesons as we shall see in what follows.

(IV) The even-odd rule is a direct consequence of the strangeness conservation law.

Let the group of elementary particles with strangeness $\pm s$ be $G_s$, then elementary particles with strong nuclear interactions are divided into several groups

$$G_0, G_1, G_2, \ldots.$$  \hspace{1cm} (4.14)

The pion and nucleon belong to $G_0$, and $\Lambda, \Sigma$ and $K$ to $G_1$. Since there is no evidence for the existence of other charge states of the cascade hyperon $\Xi^-$ which undergoes the cascade decay, we may assume

$$I = \frac{1}{2} \text{ for } \Xi^0 \text{ and } \Xi^-,$$  \hspace{1cm} (4.15)

or

$$I = 0 \text{ for } \Xi^-.$$  \hspace{1cm} (4.15)

In the above cases the strangeness of $\Xi$ is given, respectively, by

$$s(\Xi) = -2, \quad \text{or} \quad s(\Xi) = -3,$$  \hspace{1cm} (4.16)

and $\Xi$ belongs to $G_2$ or $G_3$. Since there is an example of $\Xi^-$ production associated with two $\theta^0$-mesons (19), i.e.

$$\text{cosmic ray } + \text{ nucleus } \rightarrow \Xi^- + 2 \theta^0 + \cdots,$$  \hspace{1cm} (4.17)

we shall assume in this report that its strangeness is equal to $-2$. 
Let us consider a strong interaction of the form

$$N_t N_j \pi_k$$ \hspace{1cm} (4.18)

for which the strangeness is conserved. \(N_t\) denotes the hyperon of the group \(G_t\) and \(\pi_k\) the heavy meson of the group \(G_k\). Then the strangeness conservation law requires

$$i + j + k = \text{even},$$ \hspace{1cm} (4.19)

as a necessary condition. Conversely, interactions of the form

$$i + j + k = \text{odd}$$ \hspace{1cm} (4.20)

should be very weak since they violate the conservation law. We shall call these interactions even and odd respectively.

With (4.19) and (4.20) the copious production of hyperons and heavy mesons is reconciled with their striking stability.

Processes allowed by the even-odd rule

$$\Lambda \rightarrow N + \bar{K}, \quad (N_1 \rightarrow N_0 + \pi_1),$$
$$\Sigma \rightarrow \Lambda + \pi, \quad (N'_1 \rightarrow N_1 + \pi_0),$$ \hspace{1cm} (4.21)

are forbidden energetically. Hence hyperons and heavy mesons should decay slowly through odd interactions

$$\Lambda \rightarrow N + \pi, \quad (N_1 \rightarrow N_0 + \pi_0),$$
$$K \rightarrow \pi + \pi, \quad (\pi_1 \rightarrow \pi_0 + \pi_0).$$ \hspace{1cm} (4.22)

It is characteristic of the even-odd rule that unstable particles should be produced in pairs, which was supported by the cosmotron experiments (12).

(V) The parities of particles of odd strangeness cannot be determined through strong interactions alone.

Strong interactions involve always even number of particles of odd strangeness, so that the theory for strong interactions is invariant against the simultaneous change of parities of particles of odd strangeness. The weak interactions responsible for decay processes are not invariant against this transformation and the parities can be determined only through the investigation of decay processes.

(VI) The strangeness selection rule can forbid many processes that are not forbidden by the even-odd rule.

(i) The cascade decay of \(\Xi^- (20)\)

$$\Xi^- \rightarrow \Lambda^0 + \pi^-$$
$$\downarrow \quad p + \pi^-$$ \hspace{1cm} (4.23)

cannot be accounted for by the even-odd rule.

If \(\Xi^-\) were odd the process (4.23) takes place through an even interaction so that its observed metastability could not be guaranteed. If on the contrary \(\Xi^-\) were even, the process

$$\Xi^- \rightarrow n + \pi^-$$ \hspace{1cm} (4.24)
would have to be fast. In either case the metastability of \( \Xi^- \) could not be proved. The strangeness selection rule can forbid both processes to occur rapidly as seen from

\[
\Xi^- \to \Lambda^o + \pi^- , \quad \Xi^- \to n + \pi^- .
\]  
\( (4.25) \)

While the process allowed by the strangeness selection rule

\[
\Xi^- \to \Lambda^o + K^-
\]
\( (4.26) \)

is forbidden again due to the energy conservation law.

Since \( \Xi^- \) forms a charge doublet together with \( \Xi^0 \), the observability of \( \Xi^0 \) must be discussed. It is natural to assume that \( \Xi^0 \) may decay as

\[
\Xi^0 \to \Lambda^o + \pi^o
\]
\( (4.27) \)

as compared to (4.23), but the decay can hardly be observed. If, on the other, \( \Xi^0 \) decayed as

\[
\Xi^0 \to p + \pi^-, \quad (4.28)
\]

it would have to be observed experimentally. So far as we know, there has been no evidence in favour of the decays (4.24) and (4.28), so that it will be instructive to impose a new selection rule upon the odd interactions in the form

\[
\Delta s = 0, \pm 1,
\]
\( (4.29) \)

which is an analogue of the selection rule (3.3). Then the processes

\[
\Xi^0 \to p + \pi^- ,
\Xi^- \to n + \pi^- ,
\quad -2 \quad 0 \quad 0
\]

are forbidden by the postulate (4.29) since \( \Delta s = 2 \) for the above processes.

(ii) It is characteristic of the cosmotron experiments that hyperons are more copiously produced in pion-nucleon collisions (12) than in nucleon-nucleon collisions (21). This can be attributed to the fact that the process of the lowest threshold energy

\[
N + N \to \Lambda + \Lambda
\]
\( (4.30) \)

is forbidden by the strangeness selection.

In nucleon-nucleus collisions the threshold energies of the production processes allowed by the selection rule are lower than those in nucleon-nucleon collisions so that hyperons can more easily be produced in nucleon-nucleus collisions than in nucleon-nucleon collisions in conformity with the experimental material (22).

We shall tabulate here some typical processes allowed and forbidden by the strangeness selection rule.
(iii) It has been remarked that comparatively long lived S-particles or some of the K-particles observed in photographic emulsions exhibit a large preponderance of positive ones over negative (23, 24). This tendency can also be understood on the basis of the strangeness conservation law provided that these particles are identical with the K-mesons discussed before. Since K-mesons have the strangeness $+1$ and $\bar{K}$-mesons and hyperons $\Lambda$ and $\Sigma$ have the strangeness $-1$, $K^-$ and $K^0$ can be produced in association with hyperons, say $\Lambda^0$, while $K^+$ and $K^0$ can be produced only with K-mesons. Since hyperons are more easily produced than the $\bar{K}$-mesons because of the lower excitation energy to transform a nucleon into a hyperon, i.e., $\sim 180$ Mev for a $\Lambda^0$ and $\sim 270$ Mev for a $\Sigma$, than the energy required to create a heavy meson $\bar{K}$, i.e., $\sim 490$ Mev, the production of heavy particles would occur in such a manner as to make the number of hyperons $n(\Lambda, \Sigma)$ as large as possible in comparison with the number of $\bar{K}$-mesons $n(\bar{K})$, i.e.

$$n(\Lambda, \Sigma) \gg n(\bar{K}).$$

From the strangeness conservation law, follows the relation

$$n(K^+, K^0) \sim n(\Lambda, \Sigma) \gg n(\bar{K}^-, \bar{K}^0), \quad (4.31)$$

namely the positive heavy mesons are more copiously produced than the negative ones. This argument is valid only at low energies of the order of 10 Gev where most experiments were carried out so far. Furthermore it is also worth while to notice that the production of $\bar{K}$-mesons results always at least three particles in the final state which also makes the production of K-mesons unlikely for small available energies.

(VII) There are some reactions by which one can directly test the C. I. hypothesis for hyperons and heavy mesons (11), (25).

The discussions made so far are based on the strangeness selection rule in which only the conservation of the third component of the isobaric spin is
used. And one needs only the conservation of the third component in order to account for the main qualitative behaviours of these particles and also it is only this weaker conservation law that is experimentally verified till now. However, if one assumes further that the isobaric spin is a good quantum number as we have done, there must be some reactions by which this stronger assumption can be tested as (2.10) in the case of pion-nucleon interaction. Some reactions can be utilized only to test the “charge symmetry”, i.e.

\[ \sigma (\pi^- + p \to \Lambda^0 + K^0) = \sigma (\pi^+ + n \to \Lambda^0 + K^+) \]  

(4.32)
or by adding some particles to the both sides of the above reaction, for instance, a deuteron which is isotropic in charge space, one gets a practically observable reaction

\[ \sigma (\pi^- + \text{He}^3 \to *\text{H}^3 + K^0) = \sigma (\pi^+ + \text{H}^3 \to *\text{H}^3 + K^+) \]  

(4.32')

where *\text{H}^3 is a hyperfragment.

Reactions by which C. I. can be tested are illustrated by

\[ \sigma (\bar{K}^- + n \to \Lambda^0 + \pi^-) = 2\sigma (\bar{K}^- + p \to \Lambda^0 + \pi^0) \]  

(4.33)
or

\[ \sigma (\bar{K}^- + d \to \Sigma^- + p) = 2\sigma (\bar{K}^- + d \to \Sigma^0 + n) \]  

(4.34)

From (4.33) one can derive other formulas as we have already done in the above, namely

\[ \sigma (\bar{K}^- + \text{He}^3 \to *\text{H}^3 + \pi^-) = 2\sigma (\bar{K}^- + \text{He}^3 \to *\text{H}^3 + \pi^0) \]

\[ \sigma (\bar{K}^- + \text{H}^3 \to \Lambda^0 + d + \pi^-) = 2\sigma (\bar{K}^- + \text{He}^3 \to \Lambda^0 + d + \pi^0) \]  

(4.33')

(VIII) FUKUDA-MIYAMOTO’S theorem (26) and heavy unstable particles

(i) Let \( \varphi_A \) be the field operator of a neutral Boson \( A \) which is an eigenstate of the charge conjugation \( C \), then one has

\[ C \varphi_A C^{-1} = \varepsilon_A \varphi_A, \quad \varepsilon_A = \pm 1. \]

We call \( A \) even or odd according as \( \varepsilon_A = 1 \) or \( -1 \), e.g. \( \pi^0 \) is even and photon is odd. We can sometimes speak of the parity under charge conjugation not only for elementary Bosons but also for composite ones, for instance a positronium is even in the \( 1S \) state and odd in the \( 3S \) state.

The transition

\[ A + B + \cdots \to C + D + \cdots \]  

(4.34)
among such neutral Bosons is forbidden by the conservation of parity under charge conjugation, if

\[ \varepsilon_A \varepsilon_B \varepsilon_C \varepsilon_D \cdots = -1. \]  

(4.35)

This theorem was first given in the form:

\[ n(v) + n(t) = \text{odd} \]

is forbidden, where \( n(v) \) and \( n(t) \) denote the numbers of neutral Bosons participating in the process which couple to the Fermion fields with vector and tensor couplings, respectively.
To our regret this selection rule cannot be applied to the decays of neutral heavy mesons since they are not the eigenstates of the operator $C$ unless $s = 0$ as seen from (4.11). However, for weak interactions the strangeness is no more a quantum number, and we must prepare another argument which will be discussed in § 7.

(ii) In a similar way one can apply the conservation of charge parity to charge independent interactions. Because of the anti-commutativity between charge parity $T$ and the third component of the isobaric spin $I_3$, we can apply this conservation law only to such particles whose $I_3$ are equal to zero. The selection rule assumes the following form: The reaction

$$A + B + \cdots \to C + D + \cdots$$

is forbidden, if

$$T_A T_B T_C T_D \cdots = -1, \quad (4.36)$$

where $T_A, T_B, \cdots$ are the charge parities of the particles $A, B, \cdots$. If we remember the formula (4.7), (4.36) can be rewritten in the form

$$I_A + I_B + \cdots = \text{odd}. \quad (4.36')$$

This theorem was first stated in the form that reactions for which

$$n(t_3) = \text{odd} \quad (4.36'')$$

is forbidden, where $n(t_3)$ denotes the number of neutral mesons which couple to Fermions through the isobaric spin matrix $t_3$.

Since heavy mesons cannot have vanishing $I_3$, this theorem cannot be applied to the decays of heavy mesons. An example of this selection rule is that the reaction

$$\Lambda^0 + d \to d + \Sigma^0. \quad (4.37)$$

is forbidden, or by adding a deuteron to the both sides one gets another forbidden reaction

$$^*H^3 + d \to ^*He^4 + \Sigma^0. \quad (4.37')$$

(iii) A particle which is neither the eigenstate of $C$ nor that of $T$ is sometimes the eigenstate of the product $CT$, for instance, it is the case for charged pions ($CT = -1$). A selection rule similar to (i) and (ii) can be derived by referring to the operator $G = CT = C e^{i\pi I_3}$, which is known in a special case by the statement that the reaction for which

$$n(v) + n(t) + n(t_3) = \text{odd}$$

is forbidden. Again this selection rule cannot be applied to heavy particles since they are not the eigenstates of $G$ as seen from Fig. 1. However, this rule is very useful in discussing the annihilation of anti-protons (27).

§ 5. Weak Interactions

As has been discussed in § 3, all interactions are classified according to their strengths into three categories, i.e. a) charge independent interactions, b) electromagnetic interactions, c) weak decay interactions. For the former two, a) and b), the strangeness is conserved, i.e. $\Delta s = 0$, but for the last

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one, c) the strangeness obeys the selection rule $\Delta s = 0, \pm 1$, which is also valid to decays involving leptons in the decay products, provided that one assigns $s = 0$ to leptons. In this report we present a further specified classification of decay interactions into two classes.

**c') isospinor decay interactions**

When all particles participating in a decay process have definite isobaric spins, one might naturally be interested in the transformation properties of the decay interactions in charge space. In such a case the selection rule $\Delta s = 0, \pm 1$ can be rewritten as $\Delta I_3 = 0, \pm \frac{1}{2}$, and we may assume in most cases that the decay interactions are spinors in charge space. This assignment is consistent with the transformation properties of stronger interactions in charge space, i.e. a) scalar, and b) scalar + (vector)$_g$, in the sense that weaker interactions cannot be composed of stronger interactions because of the difference of the transformation properties in charge space. If we take the hypothesis for granted, the selection rule governing such decays are given by $\Delta I = \pm \frac{1}{2}$ and $\Delta I_3 = \pm \frac{1}{2}$, which has previously been proposed by GELL-MANN and PAIS (4). We further assume here that such interactions lead to lifetimes of the order of $10^{-10}$ sec for two-body decays. For three-body decays the lifetimes will be considerably longer (28).

**c'') weaker decay interactions**

It is clear, however, that not all decays are governed by the above selection rule, e.g. decays involving photons or leptons cannot be covered. Hence there must be weaker decay interactions which may lead to lifetimes of the order of $10^{-8}$ sec for decay processes.

In what follows, we shall exhibit the results derived on the hypothesis of "isospinor decay interactions".

(1) **Hyperons**

<table>
<thead>
<tr>
<th>Process</th>
<th>$I_i$</th>
<th>$I_f$</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda^0 \rightarrow p + \pi^-$</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>$(\Lambda^0 \rightarrow p + \pi^-) = 2$</td>
</tr>
<tr>
<td>$\Lambda^0 \rightarrow n + \pi^0$</td>
<td></td>
<td></td>
<td>$(\Lambda^0 \rightarrow n + \pi^0)$</td>
</tr>
<tr>
<td>$\Sigma^+ \rightarrow p + \pi^0$</td>
<td>1</td>
<td>$\frac{1}{2}$, $\frac{3}{2}$</td>
<td>$3 \tau(\Sigma^-) \geq \tau(\Sigma^+)$</td>
</tr>
<tr>
<td>$\Sigma^- \rightarrow n + \pi^-$</td>
<td>1</td>
<td>$\frac{3}{2}$</td>
<td></td>
</tr>
</tbody>
</table>

$I_i$: isobaric spin in the initial state

$I_f$: isobaric spin in the final state

All these hyperons must decay with lifetimes of the order of $10^{-10}$ sec in conformity with experiments.

(2) **$\theta$-Mesons**

The discussions differ drastically according to whether the $\theta$-meson has even spin and even parity or odd spin and odd parity.

(i) case of even spin and even parity
As seen in the above table, the decay of $\theta^+$ into two pions through $c')$ is forbidden so that the decay of a $\theta^+$ must take place through the weaker interaction $c''$ which obeys in this case the selection rule $\Delta I = \pm \frac{1}{2}$. Hence the lifetime of $\theta^+$ must be of the order of $10^{-8}$ sec which is also consistent with experimental information.

(ii) case of odd spin and odd parity

As is well known, the lifetime of $\theta^0$ is much shorter than that of $\theta^+$ in contradiction to the above result $\tau(\theta^0) = \tau(\theta^+)$.

Thus the hypothesis of “isospinor decay interactions” is reconciled with experiments only when $\theta$-meson has even spin and even parity. Recently the decay $\theta^0 \rightarrow 2\pi^0$ was observed supporting the above conclusion (29).

(3) $\tau$-Mesons

The branching ratio has been calculated by Dalitz (30) under the assumption $I_t = I_f = 1$ and our result agrees with his one in spite of the different choice of $I_t$, since we are not concerned with $\tau^0$ here.

Application of the unitarity of the $S$-matrix

Let us consider a scattering process, then as the energy increases the rate of inelastic scattering will also increase. It is known, however, that the cross section for the elastic scattering does never vanish even if the inelasticity becomes very large because of the occurrence of the shadow scattering. This is most simply seen in the unitarity condition of the $S$-matrix. There is a similar situation in the decay of an unstable particle with competing modes of decay, which we shall formulate by exploiting the unitarity condition.
The unitarity condition of the S-matrix can be expressed as

$$SR^\dagger = - R$$  (5.1)

where $R = S - 1$. Now let us take, for example, the matrix elements of the above equation corresponding to

$$\Sigma^+ \rightarrow p + \pi^+,$$  (5.2)

then one obtains equations of the form

$$\langle \pi^0 p | S | p \pi^- \rangle \langle \pi^0 p | R^\dagger | \Sigma^+ \rangle + \langle \pi^0 p | S | n \pi^+ \rangle \langle \pi^+ n | R^\dagger | \Sigma^+ \rangle = - \langle \pi^0 p | R | \Sigma^+ \rangle,$$

$$\langle \pi^+ n | S | p \pi^- \rangle \langle \pi^+ p | R^\dagger | \Sigma^+ \rangle + \langle \pi^+ n | S | n \pi^+ \rangle \langle \pi^+ n | R^\dagger | \Sigma^+ \rangle = - \langle \pi^+ n | R | \Sigma^+ \rangle.$$  (5.3)

Now let us employ an representation in which the parity, total angular momentum and isobaric spin of the final state are diagonalized, as done by KAWAGUCHI and MINAMI (31). Then the eq. (5.1) takes the following simple form:

$$e^{\delta_1} R_i^\dagger = - R_i,$$  (5.4)

$$e^{\delta_2} R_j^\dagger = - R_j,$$

where $\delta_1$ and $\delta_2$ are the phase shifts for the pion-nucleon scattering in $I = \frac{1}{2}$ and $I = \frac{3}{2}$ states, respectively at the energy corresponding to the $Q$-value for the $\Sigma$-decay. The parity and orbital angular momentum of the states are specified subject to the spin and parity of the $\Sigma$-particle. $R_2$ and $R_3$ are defined by

$$R_a = \langle \pi^0 p | R | \Sigma^+ \rangle = \sqrt{2/3} R_2 - \sqrt{1/3} R_1,$$

$$R_b = \langle \pi^+ n | R | \Sigma^+ \rangle = \sqrt{1/3} R_3 + \sqrt{2/3} R_1.$$  (5.5)

combining (5.4) and (5.5), one obtains

$$\frac{|R_2|^2}{|R_b|^2} = \frac{2 - 2\sqrt{2} x + x^2}{1 + 2\sqrt{2} x + 2x^2}.$$  (5.6)

where $x = \pm |R_1|/|R_3|$ and $Q = \cos(\delta_3 - \delta_1)$. (32), (33).

If we know the value of $Q$, one can readily derive the upper and lower limits of the branching ratio. Similarly we can apply the above method to $\Lambda^+$-decays, but we can derive no physically sensible results. Now let us apply the hypothesis of isospinor decay interactions to $\Sigma$-decay

$$\Sigma^- \rightarrow n + \pi^-,$$  (5.7)

then the transition matrix for this process would be equal to $\sqrt{3} R_3$, so that we have

$$\tau(\Sigma^-) = \frac{|R_3|^2}{3 |R_3|^2} = \frac{1 + x^2}{3}.$$  (5.8)

Hence if the branching ratio of the $\Sigma^+$ decays (5.6) and the ratio of the lifetime of $\Sigma^-$ to that of $\Sigma^+$ (5.8) are measured, it is possible in principle to determine $Q$ so that we can deduce the spin and parity of the $\Sigma$-particles.
§ 6. Hyperfragments

One of the most remarkable phenomena recently established is the heavy nuclear fragments or the hyperfragments. In 1953, Danyasz and Pniewski (34) have found the spontaneous disintegration of a boron nucleus emitted as a fragment from the disintegration of a silver or a bromine nucleus. In spite of its rather large Q-value of about 50 MeV, it decayed after reaching the end of its range within a time $\sim 3 \times 10^{-12}$ sec. If it were a highly excited state of the boron nucleus it would undergo a spontaneous decay within a quite short lifetime, say $10^{-12}$ sec and could not survive as long as $10^{-12}$ sec. Therefore they considered that a $\Lambda^0$-particle substitutes a nucleon in the fragment. Recently many evidences have been accumulated in favour of this viewpoint, which we shall describe in what follows.

(1) The hyperfragments are as metastable as a $\Lambda^0$-particle, i.e. the lifetimes of them are of the same order with that of the $\Lambda^0$.

(2) The Q-values of hyperfragments are nearly equal to that of a $\Lambda^0$-particle. The slight deviations, being of the order of MeV, are attributed to the binding energies of the $\Lambda^0$-particle in the hyperfragments. In some cases the pion being the decay product of the $\Lambda^0$ in the fragment is absorbed by the residual nucleus without being really observed. Examples of light hyperfragments are

\[ *\text{H}^3 \rightarrow \text{He}^8 + \pi^- \]
\[ \rightarrow d + p + \pi^- , \]
\[ *\text{He}^4 \rightarrow \text{He}^8 + p + \pi^- \]
\[ \rightarrow 2p + 2n , \]
\[ *\text{H}^4 \rightarrow \text{He}^4 + \pi^- \]
\[ *\text{He}^5 \rightarrow \text{He}^4 + p + \pi^- . \]  

(6.1)

Decays involving a real pion in the decay products are called "mesonic decays", whereas those involving no real pions are called "non-mesonic decays".

(3) Hyperfragments are sometimes observed in association with another heavy meson (35) and sometimes produced through the absorption of negative heavy mesons or hyperons by nuclei (36), e.g.

\[ \text{cosmic ray} + \text{nucleus} \rightarrow *\text{H}^3 + \tau^+ + \cdots , \]
\[ \text{K}^- + \text{nucleus} \rightarrow \text{hyperfragment} + \cdots , \]
\[ \Sigma^- + \text{nucleus} \rightarrow \text{hyperfragment} + \cdots . \]

In view of the strangeness conservation law, the hyperfragments must have strangeness $-1$. Assuming that hyperfragments are such excited states of nuclei that a $\Lambda^0$ is substituted for a nucleon, we shall discuss the nature of hyperfragments qualitatively.

(1) The hyperons $\Sigma$ and $\Xi$ cannot be bound to nuclei to form hyperfragments. We know that $\Sigma$ and $\Xi$ are metastable when they are isolated, but in the presence of nuclear matter they are no more metastable. As known from the cosmotron experiments (3.1), the interaction

\[ \Sigma \rightarrow \Lambda + \pi \]  

(6.2)
is strong although it cannot give rise to a real process. In the presence of nuclear matter, however, the virtual pion in (6.2) can be absorbed by another nucleon so that the following process occurs very rapidly

\[ \Sigma + N \rightarrow N + \Lambda^0. \]  

(6.3)

In nuclei, the following processes are considered to take place rapidly with Q-values of the order of 80 MeV:

\[ \Sigma + \text{nucleus} \rightarrow \Lambda^0 + \text{nucleus}, \quad \text{for } \Delta I = 0 \]
\[ \Sigma + \text{nucleus} \rightarrow \Lambda^0 + \text{nucleus} + \gamma, \quad \text{for } \Delta I = \pm 1. \]  

(6.4)

In a similar way, \( \Xi \) is unstable against the reaction

\[ \Xi + N \rightarrow 2\Lambda^0, \]  

(6.5)

with the Q-value of approximately 30 MeV.

In short, heavier hyperons than \( \Lambda^0 \) melt down very fast in the nuclear matter.

In this connection, it is worthy noticing that a K-meson (not \( \bar{K} \)-meson) is capable of forming a hyperfragment of strangeness +1, and some hyperfragments of anomalously large Q-values might allow this interpretation.

If two \( \Lambda^0 \)-particles are bound to a nucleus to form a hyperfragment of strangeness \(-2\), it will be distinguished from ordinary fragments by exhibiting a characteristic two-step (cascade) decay.

(II) Many properties of nuclei resulting from the principle of charge symmetry or C. I. hold also to the hyperfragments.

For instance, by applying the principle of charge symmetry, one can predict the existence of mirror hyperfragments in some cases. In fact, DALITZ (38) predicted the existence of *He* based on the existence of its mirror fragment *He*, and soon later it was really observed. Apart from slight deviations caused by Coulomb force, nuclear physics is invariant against the charge symmetry operation \( T \) by which proton, neutron and \( \Lambda^0 \) are transformed as

\[ p \rightarrow n, \quad n \rightarrow p, \quad \Lambda^0 \rightarrow \Lambda^0. \]

A fragment \( F^* \) is called mirror to \( F \), if \( F^* \) is obtained from \( F \) by means of this operation. By applying the principle of C. I., one can further extend the idea of mirror fragments to charge multiplets.

(III) The range of the \( \Lambda-N \) force must not be longer than about half of the pion Compton wave length, i.e. about half of the range of N-N force (39).

This conclusion is reached at by imposing the principle of C. I. as seen below. Let us consider FEYNMAN-diagrams which give rise to the \( \Lambda-N \) force.

If a directed line starts with \( \Lambda^0 \) and ends again with \( \Lambda^0 \), as shown in Fig. 3a, \( \Lambda^0 \) and N should exchange at least two mesons either pions or heavy mesons since \( I = 0 \) for \( \Lambda^0 \) and there is no neutral meson \( (I = 0) \) as light as a pion.

On the contrary, if a directed line starts with \( \Lambda^0 \) but ends with N, as shown in Fig. 3b, \( \Lambda^0 \) and N should exchange at least one quantum of strangeness,
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i.e. a heavy meson, as is obvious from the conservation of strangeness for the strong interactions. For both cases, the above statement is justified provided that the energy of the system is sufficiently low. In this connection, we shall investigate the strength of the $\Lambda$-$N$ force based on the evidence for the existence of the hyperfragment $^4H^3$, the lightest one known at present. The Hamiltonian for this system is given, neglecting the three body force, by

$$H_f = T_n + T_p + T_\Lambda + V_{np} + V_{\Lambda p} + V_{\Lambda n},$$  \hspace{1cm} (6.6)

where $T'$'s denote the kinetic energies of a neutron, a proton and a $\Lambda^-$-particle constituting the hyperfragment $^4H^3$ and $V'$'s are the potentials between them. Then the wave function $\Psi_f$ representing the fragment at rest satisfies the SCHRÖDINGER-equation

$$H_f \Psi_f = -(B_d + B_\Lambda) \Psi_f,$$  \hspace{1cm} (6.7)

where $B_d$ is the binding energy of a deuteron and $B_\Lambda$ the binding energy of the $\Lambda^-$ in this fragment. Let us denote the lowest eigenvalue of an operator $A$ by $\text{Min}(A)$, then we have

$$\text{Min}(A + B) \geq \text{Min}(A) + \text{Min}(B).$$  \hspace{1cm} (6.8)

With this notation we have the following equations:

$$\text{Min}(H_f) = -(B_d + B_\Lambda),$$

and

$$\text{Min}(H_d) = \text{Min}(T_n + T_p + V_{np}) = -B_d.$$  \hspace{1cm} (6.9)

Combining these relationships, we have

$$-(B_d + B_\Lambda) = \text{Min}(H_f) > \text{Min}(H_d) + \text{Min}(T_\Lambda + V_{\Lambda n} + V_{\Lambda p})$$

or

$$-B_\Lambda > \text{Min}(T_\Lambda + V_{\Lambda n} + V_{\Lambda p}) > \text{Min}(\frac{1}{2} T_\Lambda + V_{\Lambda n}) + \text{Min}(\frac{1}{2} \cdot T_\Lambda + V_{\Lambda p}).$$

With the help of charge symmetry, the above inequality is reduced to

$$\text{Min}(\frac{1}{2} \cdot T_\Lambda + V_{\Lambda n}) < -B_\Lambda/2,$$  \hspace{1cm} (6.10)

or

$$\text{Min}(p^2/4M + V_{\Lambda n}) < -B_\Lambda/2,$$

where $M$ is the rest mass of a $\Lambda^-$-particle. The Hamiltonian for a system composed of a nucleon and a $\Lambda^-$-particle is given in the centre of mass system by

$$H = (m + M) \frac{p_\Lambda^2}{2mM} + V_{\Lambda n},$$  \hspace{1cm} (6.11)

where $m$ is the nucleon rest mass. Comparing (6.10) and (6.11), we see that if the potential $V_{\Lambda n}$ is multiplied by $2(m + M)/m$, there exists always a bound state for this system. Hence we have

$$s > m/2 (m + M) \sim 0.22,$$  \hspace{1cm} (6.12)
where $s$ is the well depth parameter ($4\theta$) of the potential $V_{AN}$. Rigorously speaking, it would be better to replace the inequality by a stronger one

$$s \gg 0.22,$$  \hspace{1cm} (6.13)

as seen from the above derivation. So far as we know at present there is no definite evidence for the existence of hyper di-nucleon, and for the time being we shall assume the non-existence of such fragments, then we have an upper limit to the well depth parameter

$$1 > s.$$  \hspace{1cm} (6.14)

In order to obtain a more intuitive information on the potential $V_{AN}$, we shall investigate the allowed regions of the well depths for square well and Yukawa potentials.

(1) Square well potential

In this case the well depth parameter is given in terms of the well depth $V_0$ and force range $b$ by

$$s = \left(\frac{4}{\pi^2}\right) \mu V_0 b^2,$$  \hspace{1cm} (6.15)

where $\mu = 2mM/(m+M)$ and we employ the natural units $\hbar = c = 1$ throughout this report.

(2) Yukawa potential

We write the Yukawa potential in the form

$$V = -V_0 \exp\left(\frac{-r}{b}\right),$$  \hspace{1cm} (6.16)

then the well depth parameter is given by

$$s = 0.59531 \mu V_0 b^2.$$  \hspace{1cm} (6.17)

From the inequalities (6.13) and (6.14), one can derive inequalities for the well depths for two choices of the force range $b$ which are given in the Table IV. The range $b$ is chosen very short in accordance with the principle of C. I.

<table>
<thead>
<tr>
<th>Table IV</th>
<th>Allowed Regions for the Well Depths in Mev</th>
</tr>
</thead>
<tbody>
<tr>
<td>shape</td>
<td>$b$</td>
</tr>
<tr>
<td>Square well</td>
<td>360 $\gg V_0 \gg 80$</td>
</tr>
<tr>
<td>Yukawa</td>
<td>227 $\gg V_0 \gg 54$</td>
</tr>
</tbody>
</table>

It is very important to investigate more precisely the hyper-triton problem by adopting a specific model. If we assume that $\theta$ is scalar and that $\Lambda^o$ has spin $1/2$ and even parity, then the contribution from the lowest order Feynman diagram in the Fig. 3 b leads to a Heisenberg type exchange force (II).

$$V_{AN} = (g^2/4\pi) P_x P_\sigma e^{-m_\sigma r}/r.$$  \hspace{1cm} (6.18)
Hence in the $1S$ state, the well depth parameter $V_0$ defined by (6.16) is given by

$$V_0 = \left(\frac{g^2}{4\pi}\right) m_\theta = \frac{g^2}{4\pi} \times 494 \text{ MeV}. \quad (6.19)$$

Then noticing $b = m\bar{\phi} = 0.4 \times 10^{-18} \text{ cm}$, we have with reference to the Table IV the inequalities

$$0.85 > \frac{g^2}{4\pi} > 0.19. \quad (6.20)$$

In deriving (6.18), we have adopted an interaction of the form

$$H_{\text{int}} = g\bar{\psi}_N \Phi_\theta \psi_\Lambda + \text{hermitian conjugate}, \quad (6.21)$$

where $\psi_N, \Phi_\theta$, and $\psi_\Lambda$ are the field operators for nucleon, $\theta$-meson and $\Lambda$, respectively, and $g\bar{\psi}_N \Phi_\theta$ is a scalar product of two spinors in charge space. It is interesting that the interaction (6.21) with the coupling constant lying in the region in (6.20) gives the right order of magnitude for the cross-section of $(3.1) (41)$. This result might be regarded as another evidence in favour of the viewpoint on the hyperfragment adopted here.

(IV) The high frequency of the non-mesonic decay of hyperfragments suggests us that the spin of the $\Lambda^0$ is not very large, say $s \leq \frac{3}{2}$ (39). The increase of the charge of the hyperfragment results in the increase of the rate of non-mesonic decay modes. The hyperfragment $^4\text{He}$ is known to decay with approximately equal probability mesonically and non-mesonically, from which one can deduce some information on the spin of $\Lambda^0$. The radius of $^4\text{He}$ is supposed to be of the order of pion Compton wave length or more precisely

$$R \sim 1.45 \times \sqrt{\frac{3}{4}} \times 10^{-18} \text{ cm} = 2.5 \times 10^{-18} \text{ cm}. \quad (6.22)$$

The non-mesonic decay of a hyperfragment corresponds to the picture such that a $\Lambda^0$-particle in the fragment decays virtually into a pion and a nucleon and subsequently the pion is absorbed by a nucleon. While it is known experimentally that at low energies a pion interacts strongly with a nucleon in the $p$-states and less strongly in the $s$-states. Therefore, the high probability of the non-mesonic modes of decay indicate that the virtual pions are emitted mainly in $p$- or $s$-states from the sink. Since, however, the radius of the fragment is comparable with the impact parameter of the emitted pion, say around $2 \times 10^{-18} \text{ cm}$ for $l = 1$ and much larger for $l \gg 1$, the pions should be emitted mainly in $p$- or $s$-states from the source $\Lambda^0$. The probability of the non-mesonic modes of decay is given by

$$P_{\text{non}} = \frac{(\text{non-mesonic})}{(\text{mesonic}) + (\text{non-mesonic})} \sim 0.5. \quad (6.23)$$

Let the absorption probability of pions in the nucleus be $p_{\text{abs}}$, then

$$P_{\text{non}} = p_{\text{abs}} \cdot f, \quad (6.24)$$

where $f$ is the fraction of the $p$- and $s$-waves from the sink involved in the virtual pion wave emitted from the source $\Lambda^0$. The absorption probability $p_{\text{abs}}$ is given by

$$p_{\text{abs}} \sim 1 - \exp \left(-\frac{R}{\lambda_{\text{abs}}}\right), \quad (6.25)$$
where $\lambda_{abs}$ is the mean free path for the absorption. From (6.23), (6.24) and (6.25), one gets

$$f = \frac{P_{non}}{P_{abs}} = 0.5/P_{abs} \geq 0.5.$$  \hspace{1cm} (6.27)

Let $l_t$ and $l_f$ be the relative angular momentum of the virtual pion from the source and sink, respectively. Then from (6.27) and the argument above, we have for a low energy virtual pion

$$\Delta l = l_f - l_t = 0, +1$$

based on an intuitive classical geometrical consideration about the impact parameter. The relative rarity of the transition $\Delta l = -1$ compared with $\Delta l = +1$ is for a statistical reason.

If we assume as suggested from experiments

$$l_f = 0 \text{ or } 1,$$  \hspace{1cm} (6.28)

we reach at the following interesting conclusion

$$l_t = 0 \text{ or } 1.$$  \hspace{1cm}

This means that the pion emitted from $\Lambda^0$ by the process (4.1) is in the $s$ or $p$ state. Hence we may conclude that the spin of $\Lambda^0$ cannot be larger than $\frac{3}{2}$. With the help of an analogy to the internal conversion the above argument can be refined (42).

§ 7. Problem of Neutral Heavy Mesons

As has been discussed in § 4, the neutral heavy mesons as $\theta^0$ and $\tau^0$ are described by complex wave functions in contrast to other neutral Bosons so far known such as a neutral pion and a photon. And it is important to investigate the characters of the newcomers in the light of various invariance principles.

(I) quantities characterizing particles

The problem to be discussed here is how one can characterize a particle either elementary or composite. There are various quantities by which different particles can be distinguished and of them we must first cite the following quantities.

(a) mass, spin, parity, charge and nucleon number

All these quantities are intimately related to strict conservation laws in quantum field theory so that they can be regarded to be characteristic of each particle. These conservation laws are namely the conservation of energy-momentum, of angular momentum, of charge and of nucleon number. Of these the first one, the rest mass of a particle, can strictly be defined only when the particle is stable, but when a particle is unstable the rest mass is defined within a certain uncertainty, which is connected to the lifetime of the particle.
(b) lifetime

The lifetime can also distinguish among various unstable particles. When the lifetime of an unstable particle is very short so that the uncertainty of the rest mass is very large, the particle will lose its particle picture. Such a particle will be of sense only as an intermediate state of certain kinds of reactions.

(c) parity against charge conjugation

The parity against charge conjugation is conserved strictly but it can be used only to a restricted class of neutral particles.

(d) isobaric spin

To particles with strong nuclear interactions, one can assign their isobaric spins which is of importance for strong interactions. When two particles are identical to one another, all such quantities as mentioned above should necessarily be equal. One must notice, however, that not all of the above quantities can be employed to characterize a particle, since they are not always commutative with each other so that one must suitably choose a maximal set of observables. In general there are various ways to choose a maximal set of observables, and the problem is how to choose a suitable set.

For example, the charge $q$, charge conjugation $C$ and isobaric spin $\vec{I}$ satisfy the following commutation relations

$$[\vec{I}, q] = 0, \quad Cq = -qC, \quad CI = -IC,$$

so that one cannot take $I_3, q$ and $C$ simultaneously as the characteristic quantities of a particle unless $I_3 = q = 0$. Since the nucleon number $n$ satisfies similar commutation relations with other quantities, one can say that $C$ can be an attribute of a given particle only when $I_3 = q = n = 0$. Perhaps the lepton number should also be taken into account if the neutrino is of the Dirac type. Notice, however, $CI^2 = \vec{I}C$ in this connection.

We must stress here that a sensible maximal set of observables must always involve the charge $q$ and perhaps similarly the nucleon number $n$ in view of the superselection rule (43). It must also be mentioned that in the usual measurements by means of macroscopic instruments the charge operator is almost diagonalized.

(i) charged particles

The operators $q$, $n$, and $I_3$, if any, should be diagonalized, and $C$ cannot be diagonalized.

(ii) neutral particles

If $q = n = I_3 = 0$, one can speak of the parity of the particle against charge conjugation. For example, a photon, a positronium in $^3S$ state are
odd and a neutral, pion, a positronium in $^1S$ state are even. If, on the other hand, $q = n = 0$ but $I_3 = 0$, the discussion must be made carefully. The definition of such particles depend on the process by which they are observed. If a neutral heavy meson is observed to decay exhibiting the following mode:

$$0^o \rightarrow \pi^+ + \pi^-,$$

(7.2)

we must choose $C$ as being characteristic of this particle, since the final state is an eigenstate of $C$. One the other hand, if it is observed through a nuclear absorption

$$\bar{\psi} \rightarrow \Lambda^o + \pi^+,$$

(7.3)

we must choose $I_3$ rather than $C$, since the final state is an eigenstate of $I_3$ and $I_3$ is conserved for this process.

Since the problem of charge conjugation is very important in connection with the problem of neutral mesons, we shall give a brief sketch of the theory of charge conjugation.

\section*{(II) theory of charge conjugation}

Let us make a Gedankenexperiment. Can one recognize if all the charge in the universe changed sign simultaneously? The answer is clearly "No!!" This consideration provides us with the basis for an invariance character of the field theory against the sign change of the charge, the so-called charge conjugation. This transformation is expressed by a unitary transformation $C$, by which a state vector $\psi$ is transformed as

$$\Psi \rightarrow C \Psi.$$

(7.4)

By requiring that the whole theory be invariant against $C$, we shall prescribe the nature of this transformation.

Let us consider the equation of motion of an electron with charge $e$

$$[\gamma_\mu (\partial_\mu - i e A_\mu) + m] \psi = 0.$$  

(7.5)

We require that the field operator $C \psi C^{-1}$ satisfies the equation of motion of a positron, i.e.

$$[\gamma_\mu (\partial_\mu + i e A_\mu) + m] C \psi C^{-1} = 0.$$  

(7.6)

Comparing (7.5) and (7.6), we have $C e A_\mu C^{-1} = - e A_\mu$,

or taking into account that $e$ is a c-number, we have

$$C A_\mu C^{-1} = - A_\mu.$$  

(7.7)

In the quantized theory, $\psi$ corresponds to the destruction of an electron or the creation of a positron, and $C \psi C^{-1}$ is of the reversed nature. Hence we may write

$$C \psi C^{-1} = \psi,$$  

(7.8)
On the Theory of Hyperons and Heavy Mesons

where $c$ is a non-singular Dirac matrix. Inserting (7.8) into (7.8), one has

$$[\gamma_\mu (\partial_\mu + i e A_\mu) + m] c \bar{\psi} = 0. \quad (7.9)$$

By comparing the above equation with the equation of motion for $\bar{\psi}$, i.e.

$$[\gamma_\mu^T (\partial_\mu + i e A_\mu) - m] \bar{\psi} = 0, \quad (7.10)$$

one gets by multiplying $c^{-1}$ to (7.9) from the left

$$c^{-1} \gamma_\mu c = - \gamma_\mu^T, \quad \text{or} \quad c \gamma_\mu c^{-1} = - \gamma_\mu, \quad (7.11)$$

where $\gamma_\mu^T$ is the transposed matrix of $\gamma_\mu$.

Since $\gamma_\mu$'s and $\gamma_\mu^T$'s satisfy the same commutation laws, one can choose an unitary matrix $c$ out of many possibilities.

$$c^\dagger c = 1. \quad (7.12)$$

From the transposed equation of (7.11), follows the relation

$$- \gamma_\mu = (c^T) \gamma_\mu^T (c^T)^{-1} = - c^T c^{-1} \gamma_\mu c (c^T)^{-1} = - (c^T c^{-1}) \gamma_\mu (c^T c^{-1})^{-1},$$

which means that $c^T c^{-1}$ commutes with all other Dirac matrices. Hence

$$c^T c^{-1} = c\text{-number},$$

by adopting a special representation, one finds

$$c^T = - c. \quad (7.13)$$

This equation holds for all representations of the Dirac matrices.

In a similar way, one arrives at

$$C \bar{\psi} C^{-1} = c \bar{\psi}, \quad C \bar{\psi} C^{-1} = c^{-1} \psi. \quad (7.14)$$

The eqs. (7.7) and (7.14) determine completely the transformation properties in quantum electrodynamics against charge conjugation, and these equations hold in all representations. The transformation $C$ leaves the equation of motion invariant as is clear from the above derivation. Next let us examine the invariance of the commutation relations under charge conjugation, which will be carried out in the interaction representation.

(i) electron field

$$\{ C \varphi_\alpha(x) C^{-1}, \ C \bar{\varphi}_\beta(y) C^{-1} \} = \{ c_{\alpha \lambda} \bar{\varphi}_\lambda(x), \ c_{\mu \lambda}^{-1} \varphi_\mu(y) \}$$

$$= c_{\alpha \lambda} c_{\mu \lambda}^{-1} \{ \bar{\varphi}_\lambda(x), \ \varphi_\mu(y) \}$$

$$= - i c_{\alpha \lambda} c_{\mu \lambda}^{-1} S_{\mu \lambda} (y - x)$$

$$= i [c^{-1} S(y - x) c]_{\beta \sigma}.$$  

By exploiting the expression $S(x) = (\gamma_\mu \partial_\mu + m) A(x)$, one finds

$$c^{-1} S(x) c = - S(-x)^T,$$
and hence the above quantity reduces to
\[-iS_{\alpha\beta}(x - y) = \{\psi_{\alpha}(x), \psi_{\beta}(y)\}.

(ii) electromagnetic field

\[ [CA_\mu(x) C^{-1}, CA_\nu(y) C^{-1}] = [-A_\mu(x), -A_\nu(y)] = [A_\mu(x), A_\nu(y)]. \]

The invariance of the S-matrix can also be proved without difficulty. We start with the following formula:

\[ CP(q_1, q_2, \cdots) C^{-1} = P(Cq_1 C^{-1}, Cq_2 C^{-1}, \cdots), \quad (7.15) \]

where \( P \) is Dyson's chronological symbol and \( q \)'s are arbitrary field operators. Utilizing Dyson's expression for the S-matrix, one sees

\[ CSC^{-1} = CP \exp \left[ -i \int H_{\text{int}}(x) (d^4x) \right] C^{-1} \]

\[ = P \exp \left[ -i \int CH_{\text{int}}(x) C^{-1} (d^4x) \right] \]

\[ = P \exp \left[ -i \int H_{\text{int}}(x) (d^4x) \right] = S. \quad (7.16) \]

In the above proof, we utilized the equation

\[ CH_{\text{int}}(x) C^{-1} = H_{\text{int}}(x), \quad (7.17) \]

which must be proved separately. For this purpose, we calculate

\[ C\bar{\psi}(x) O \psi(x) C^{-1}, \quad (7.18) \]

where \( O \) is an arbitrary Dirac matrix and the bilinear form in the electron field operators should be understood in the sense of normal product.

By applying the formula (7.14), one finds

\[ \bar{\psi}(x) cO^T c^{-1} \psi(x) = \varepsilon \bar{\psi}(x) O \psi(x), \quad (7.18) \]

where \( \varepsilon \) is a sign factor corresponding to the Dirac matrix \( O \) and tabulated in Table V.

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>( \gamma_\mu )</th>
<th>( \gamma_\mu \gamma_\tau )</th>
<th>( \gamma_6 \gamma_\mu )</th>
<th>( \gamma_9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon )</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Hence eq. (7.17) is ready for being proved as follows:

\[ CH_{\text{int}}(x) C^{-1} = C[-i \varepsilon \bar{\psi}(x) \gamma_\mu \psi(x) A_\mu(x)] C^{-1} \]

\[ = -i \varepsilon C \bar{\psi}(x) \gamma_\mu \psi(x) C^{-1} CA_\mu(x) C^{-1} \]

\[ = -i \varepsilon [-\bar{\psi}(x) \gamma_\mu \psi(x) (-A_\mu(x))] \]

\[ = H_{\text{int}}(x). \]
Next we shall be concerned with the eigenvalues of the operator $C$. From (7.7) follows at once

$$C^2 A_{\mu} C^{-2} = A_{\mu},$$  
(7.19)

and also from (7.14) follows the equation

$$C^2 \psi_a C^{-2} = C(c_{a\beta} \bar{\psi}_\beta) C^{-1} = c_{a\beta} C \bar{\psi}_\beta C^{-1} = c_{a\beta} c_{\beta\gamma} \psi_\gamma = \psi_a,$$  
(7.20)

and similarly

$$C^2 \bar{\psi}_a C^{-2} = \bar{\psi}_a.$$  
(7.21)

Hence $C^2$ is commutative with any other operator. Now let us fix the arbitrary phase factor involved in the definition of the operator $C$ by imposing

$$C \Omega = \Omega,$$  
(7.22)

where $\Omega$ is the vacuum state. From (7.19), (7.20), (7.21) and (7.22), one gets

$$C^2 \text{(operators)} \Omega = \text{(operators)} \Omega.$$  

Since any state vector is spanned by applying a suitable combination of operators on the vacuum state, one may rewrite the above relation as

$$C^2 \Psi = \Psi, \text{ for any } \Psi.$$  
(7.23)

Thus we can conclude that the operator $C$ assumes the eigenvalues either $+1$ or $-1$.

**Application — Furry's theorem**

If a state $\Psi_i$ is the eigenstate of $C$ with the eigenvalue $-1$, then the state $\Psi'_i = S \Psi_i$ is also an eigenstate of $C$ with the same eigenvalue. As seen from (7.7), a state consisting of an odd number of photons belongs to the negative eigenvalue, and the transition from such a state to a state consisting of an even number of photons is forbidden, since the latter has the positive eigenvalue.

**Extension to the meson theory**

As illustrated by the Furry's theorem, the concept of parity against charge conjugation is often utilized to derive selection rules for the transitions among Bosons. For this purpose, we must further extend the above theory to meson theories.

(a) Proton

When one is concerned with the electromagnetic properties of the proton, it is clear that the field operator for proton should be transformed in the same way with that of electron against charge conjugation.

(b) Neutron

Let us consider the number of "nucleons" which is equal to the number of nucleons and hyperons minus the number of anti-nucleons and anti-hyperons, then $n$ can be rewritten by

$$n = \frac{1}{2} \cdot (n + C n C^{-1}) + \frac{1}{2} (n - C n C^{-1}).$$
Since both $n$ and $C$ commute with the total Hamiltonian, the two terms in the right hand side of the above equation are conserved separately, and they are odd and even, respectively, against charge conjugation. Therefore we may assume in the sense of superselection rule that the number of nucleons should be expressed by either

$$n = \frac{1}{2}(n + CnC^{-1}) \text{ or } n = \frac{1}{2}(n - CnC^{-1}). \quad (7.24)$$

While the number of protons which is involved in $n$ is given by

$$n(p) = i \int \overline{\psi}_p \gamma_\mu \psi_p d\sigma_\mu = \int \overline{\psi}_p \gamma_\mu \psi_p d^3x,$$

and is odd against charge conjugation, so that we take

$$n = \frac{1}{2}(n - CnC^{-1}) \text{ or } n = -CnC^{-1}. \quad (7.25)$$

Hence the field operator for the neutron must be transformed as

$$C(\overline{\psi}_n \gamma_\mu \psi_n) C^{-1} = -\overline{\psi}_n \gamma_\mu \psi_n,$$

and it must be transformed in the same way as that of the proton. Rigorously speaking the transformation property of the neutron field operator is determined apart from some redundant phase factor, which should suitably be fixed. The same argument applies on other hyperons.

(c) $\pi$-mesons.

Now let us investigate how the following expression is transformed against charge conjugation:

$$\overline{\psi} O \tau \psi,$$

where $\psi$ is the nucleon field operator, $O$ a Dirac matrix and $\tau$ an isobaric spin matrix. With reference to the formula (7.14), one derives readily

$$C(\overline{\psi} O^T \psi) C^{-1} = \overline{\psi} (cO^T c^{-1}) \tau^T \psi.$$

We shall choose $\gamma_5$ for $O$, then the above expression reduces to

$$C(\overline{\psi} \gamma_5 \tau \psi) C^{-1} = \overline{\psi} \gamma_5 \tau^T \psi. \quad (7.26)$$

In order that the meson-nucleon interaction

$$H_{\text{int}} = ig\overline{\psi} \gamma_5 \tau_a \psi \cdot \varphi_a$$

be invariant against charge conjugation, it is necessary to impose

$$\tau^T_a \cdot C \varphi_a C^{-1} = \tau_a \varphi_a,$$

or

$$C \varphi_a C^{-1} = \begin{cases} \varphi_a, & \text{if } \tau^T_a = \tau_a, \\ -\varphi_a, & \text{if } \tau^T_a = -\tau_a. \end{cases} \quad (7.27)$$

If we choose the representation of the isobaric spin matrices as

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

(7.29)
then we can write

\[
C \varphi_a C^{-1} = \begin{cases} 
\varphi_a & \text{for } \alpha = 1,3 \\
-\varphi_a & \text{for } \alpha = 2.
\end{cases} \tag{7.30}
\]

By introducing the field operator \( \varphi \) and \( \varphi^* \) corresponding to \( \pi^+ \) and \( \pi^- \) in the following way

\[
\varphi^* = \left( \frac{1/\sqrt{2}}{1/\sqrt{2}} \right) (\varphi_1 - i \varphi_2), \quad \varphi = (1/\sqrt{2}) (\varphi_1 + i \varphi_2), \tag{7.31}
\]

one can rewrite (7.30) as

\[
C \varphi C^{-1} = \varphi^*, \quad C \varphi^* C^{-1} = \varphi, \quad C \varphi_2 C^{-1} = \varphi_3. \tag{7.32}
\]

From (7.32) we see that \( \pi^0 \) is even under charge conjugation so that the decay of a \( \pi^0 \) into odd number of photons is forbidden.

If we confine ourselves only to charge independent interactions, the charge symmetry operation \( T \) is also commutative with the Hamiltonian, and one can discuss the transformation properties of the theory under \( CT \), or \( G \) as given in § 4 (VIII) (iii). The utility of this transformation consists in the formula

\[
G \varphi_a G^{-1} = - \varphi_a, \tag{7.33}
\]

which is simpler than (7.30).

In the case of pions for which the interaction is given by (7.27), the field operators are transformed as (7.32), but in some cases the transformations are given by

\[
C \varphi' C^{-1} = - \varphi'^*, \quad C \varphi'^* C^{-1} = - \varphi', \quad C \varphi_2' C^{-1} = - \varphi_3'.
\]

Now let \( \Phi \) be the field operator for the \( \theta^0 \) meson, then one has

\[
C \Phi C^{-1} = \pm \Phi^*, \quad C \Phi^* C^{-1} = \pm \Phi. \tag{7.34}
\]

One can also define \( \Phi_1 \) and \( \Phi_2 \) from \( \Phi \) and \( \Phi^* \) as done in (7.31), then

\[
C \Phi_1 C^{-1} = \pm \Phi_1, \quad C \Phi_2 C^{-1} = \mp \Phi_2. \tag{7.35}
\]

Let us consider the decay \( \theta^0 \to \pi^+ + \pi^- \), then the final state is an eigenstate of the operator \( C \), since in the rest system of \( \theta^0 \), the state vector for the \( \pi^+ + \pi^- \) system \( \Psi(\vec{x}) \) is transformed as

\[
C \Psi(\vec{x}) = \Psi(-\vec{x}) = (-1)^l \Psi(\vec{x}),
\]

where \( \vec{x} \) is the relative coordinates between positive and negative pions, and \( l \) is the relative orbital angular momentum and equal to the spin of the \( \theta^0 \) meson which is known to be even as mentioned in § 5. Hence we have

\[
C \Psi(\vec{x}) = \Psi(\vec{x}). \tag{7.36}
\]

For simplicity we shall choose the upper signs in eqs. (7.34) and (7.35), then only the \( \theta_1^0 \) meson corresponding to \( \Phi_1 \) can decay into two pions but the decay of \( \theta_2^0 \) into two pions is forbidden. As discussed in the former part of this section, \( \theta_1^0 \) and \( \theta_2^0 \) should be regarded as true particles rather than \( \theta^0 \) and \( \bar{\theta}^0 \) themsel-
ves. $\theta_1$ and $\theta_2$ have their own lifetimes different from one another. $\theta_2$ will decay perhaps with a rather longer lifetime than $\theta_1$ does. Strictly speaking, not only the lifetimes but also the rest masses are different but the difference is so small that one cannot expect it experimentally, say of the order of $\lambda/\tau_0 \sim 10^{-5}$ ev.

Since $\theta$ involves $\theta_1$ and $\theta_2$ with equal probability as seen from (7.31) and $\theta_2$ cannot decay into two pions, one can conclude that not more than half of all $\theta_0$'s can undergo a spontaneous disintegration into two pions (44). This striking feature of the $\theta$ meson will partly be responsible for the long standing contradiction between the pair production hypothesis and experiments. After a long time the constitution of $\theta$ mesons will change because of the difference of lifetimes between $\theta_1$ and $\theta_2$ so that finally only $\theta_2$'s will be involved in the $\theta$ beam.

If a $\theta_2$ meson could be bound to a nucleus to form a hyperfragment as suggested by the observation of anomalous hyperfragments, it can no more decay with two distinct lifetimes but it must decay with a single lifetime, since in this case $n$ and $q$ do not vanish so that $C$ can no more be diagonalized.

§ 8. Problem of Charged Heavy Mesons

The problem of charged heavy mesons is one of the most difficult and mysterious open questions in this field. There are many different modes of decay as given in (3.4) and all mesons corresponding to these different decay modes have the same rest mass, say $\sim 965 m_e = 494$ Mev, and the same lifetime $1.2 - 1.3 \times 10^{-8}$ sec. within the presently available experimental precision. It is remarkable that all experimental results so far made are consistent with the strangeness selection rule, i.e. in no case the $K^+$-mesons suffered nuclear absorption but they are only scattered. On the other hand, $\bar{K}^-$-mesons are often absorbed emitting hyperons, i.e.

$$
\bar{K}^- + p \rightarrow \Sigma^{\pm,0} + \pi^{\mp,0},
\rightarrow \Lambda^0 + \pi^0,
$$

the former modes being as three times as probable over the latter.

The cross-section for the nuclear interaction of negative K-mesons is geometrical, while that for the positive ones is one third of the geometrical cross-section (45).

The problem of charged heavy mesons consists of the following questions: Is there only a single kind of K-mesons or are there two or more different kinds of K-mesons? That all K-mesons have the same rest mass and the same lifetime is a necessary condition for that there is only a single kind of them, but it is not a sufficient condition. If, on the contrary, there are two or more different kinds of K-mesons, why do they have the same mass and the same lifetime?

To answer the first question, it is instructive to recall DALITZ' analysis on the $\tau$-mesons to mind.
I) DALITZ’ analysis on the $\tau$-meson decays

In order to investigate whether or not all kinds of K-meson decays represent the competing modes of decay of a single kind of K-mesons, it is necessary to get insight into the transformation properties of K-mesons.

a) $\theta$-mesons

The $\theta$-mesons undergo spontaneous decays into two pions

$$0^\circ \rightarrow \pi^+ + \pi^-, \quad \theta^\circ \rightarrow \pi^0 + \pi^0.$$  \hspace{1cm} (8.2)

Let the relative angular momentum between two product pions be $l$, then the spin of a $\theta$ meson $s$ is equal to $l$, since pions have spin 0. The parity of the $\theta^\circ$-meson $\Pi$ is then given by

$$\Pi = (-1)^l \times (-1)^s = (-1)^s.$$  \hspace{1cm} (8.3)

Thus a $\theta$-meson has either even spin and even parity or odd spin and odd parity. Recent observation of the process

$$\theta^\circ \rightarrow 2\pi^0$$

lends itself to conclude that $l = s$ be even, so that $\theta$-mesons have even spin and even parity.

b) $\tau$-mesons

Then the question is if the $\tau$-meson is identical with the $\theta$-meson. In order to answer this question one must first examine the energy distribution of the secondary pions resulting from the $\tau$-meson decay so that one can draw some conclusions on the type of the $\tau$-meson.

Let $\vec{p}_1$, $\vec{p}_2$, and $\vec{p}_3$ be the momenta of two positive and one negative pions which are the decay products of a positive $\tau$-meson in the rest system. For the sake of convenience we choose $|\vec{p}_1| \leq |\vec{p}_2|$. From the energy-momentum conservation law, one gets in the non-relativistic approximation

$$1/2 \; m \cdot (\vec{p}_1^2 + \vec{p}_2^2 + \vec{p}_3^2) = Q = 75 \text{ Mev}, \quad \vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0.$$  \hspace{1cm} (8.4)

DALITZ (46) suggested a triangular diagram to represent each experimental data as a point in this diagram. The distance of the point $P$ from the basis $BC$ is proportional to the energy of the negative pion $E_3$, and the distance from another side $AC$ is proportional to $E_1$. (Fig. 4). The semi-circle encloses the region in which the points are restricted by the eqs. (8.4).

By putting

$$\vec{p}_1 = -1/2(\vec{p} + \vec{q}), \quad \vec{p}_2 = -1/2(\vec{p} - \vec{q}), \quad \vec{p}_3 = \vec{p},$$  \hspace{1cm} (8.5)

the momentum conservation law is automatically satisfied. In terms of these new variables, we have

$$2mQ = \frac{3}{2}p^2 + \frac{1}{2}q^2,$$  \hspace{1cm} (8.6)

$$\cos \theta = (\frac{\vec{p}_3}{p} \cdot \frac{\vec{p}_2}{q})/pq,$$

where $\theta$ is the angle between $\vec{p}_3$ and $\vec{p}_2 - \vec{p}_1$. 

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The differential probability for the meson decay is given by

$$dw = \text{const.} \left| M \right|^2 \frac{d\vec{p}_1 d\vec{p}_2}{dE}. \quad (8.7)$$

where $M$ is the matrix element for this decay process.

The volume element in the phase space is thus proportional to

$$p^2 q dq d(\cos \theta). \quad (8.8)$$

It is worth noticing that (8.8) is proportional to the surface element in the DALITZ' diagram, and from the distribution of the experimental points which fall in DALITZ' diagram one can derive $|M|^2$. Now let the relative angular momentum between two positive pions and that between the negative pion and the centre of mass of two positive pions be $l'$ and $l$, respectively.

Fig. 5.

(i) case of $s = 0$ ($s$: the $\tau$-meson spin)
In order to get $s = 0$, it is necessary to take $l = l'$, since $s = l + l'$. Hence the parity of the $\tau$-meson should be odd, as seen from

$$\Pi = (-1)^{l' + l + s} = -1. \quad (8.9)$$

Thus, in this case, the $\tau$-meson cannot be identical with $\theta$-meson.

(ii) If the density of the experimental points near the point $D(E_1 \ll Q)$ is not small compared with other regions in the triangle, one gets

$$\Pi = (-1)^{s+1}, \quad (8.10)$$

which follows from arguments on the $\theta$-meson and barrier effects of centrifugal forces on the positive pion of lower energy.

(iii) If the density of the experimental points near the point $B(E_3 \ll Q)$ is not small, one arrives at

$$l = 0, \ l' = \text{even}. \quad (8.11)$$

From the Bose statistics, one can infer that $l'$ should always be even. Hence, in this case, one concludes

$$s = \text{even}, \ \Pi = -1. \quad (8.11)$$

The above arguments, for instance the assumption $l = 0$, are based on the assumption that the pion wave length is much larger than the force range,
between pions so that the barrier effects be sensible. If one takes the Compton wave length of the K-meson as the force range, the ratio is given by

$$\frac{\lambda_\pi}{\lambda_K} = \sqrt{\frac{m_K}{m_\pi}} \cdot \sqrt{\frac{m_K}{E}} > 3.5 \gg 1,$$

and the assumption can be justified.

If the above assumption were right and the density at the corners is comparable with that of the inside region, the $\tau$-mesons cannot be identical to the $\theta$-mesons. Experimentally a negative meson of a very low energy of the order of 0.1 Mev has been observed suggesting that the eq. (8.11) might really be the case.

At low energies, the energy dependence of the matrix element $M$ is considered to be given by

$$M \sim \left(\frac{p R}{\hbar}\right)^i \left(\frac{q R}{\hbar}\right)^i \quad \text{(for } p R/\hbar, q R/\hbar \ll 1) \quad (8.12)$$

where $R$ is the force range between pions.

The experiments so far made suggest us that $M$ is approximately constant so that pseudoscalar (0-) is most favourable. There are of course many doubts and ambiguities against this method since one must base such an argument on many plausible assumptions.

(II) nuclear interactions of positive heavy mesons

One of the most natural ideas to distinguish between different kinds of K-mesons is to exploit the possible difference of the nuclear interactions between them. If a K-meson beam consists of two or more different kinds of heavy mesons, the branching ratio of this beam might change after nuclear interactions provided that different kinds of K-mesons have different nuclear cross-sections. Invoking that this idea might help, the branching ratios of the K-beam before and after the collisions were measured. The results are given in Table VI.

<table>
<thead>
<tr>
<th>Decay Modes</th>
<th>Percentage</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>before collisions</td>
<td>after collisions</td>
</tr>
<tr>
<td></td>
<td>Rochester</td>
<td>Paris</td>
</tr>
<tr>
<td>$K_{\mu}$</td>
<td>59</td>
<td>54</td>
</tr>
<tr>
<td>$K_{\mu}(\theta)$</td>
<td>21</td>
<td>26</td>
</tr>
<tr>
<td>$K_{\mu}(\tau)$</td>
<td>5.2</td>
<td>7</td>
</tr>
<tr>
<td>$K'_{\pi}(\tau)$</td>
<td>3.5</td>
<td>1</td>
</tr>
<tr>
<td>$K_{\tau}$</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>$K_{\eta}$</td>
<td>5</td>
<td>8</td>
</tr>
</tbody>
</table>

Rochester: Cosmotron experiments using 2.9 Gev proton (47).
Berkeley, and Göttingen: as above (49, 50).
In the experiments carried out by the Berkeley group, the relative frequencies of decay modes were determined for $K^+$-mesons which had suffered a nuclear scattering in the same target in which they had been created. The Göttingen group scanned two stacks of Ilford G 5 emulsions exposed to the $K^+$-beam of the Bevatron for interactions from which the meson is reemitted and coming to rest in the stack.

From the above table, one can see that the branching ratio of $K^+$-mesons does not vary sensibly under different experimental conditions. This fact means, if there were different kinds of $K$-mesons, that not only the rest masses and lifetimes but also the nuclear cross-sections of different kinds of $K$-mesons should be equal to each other.

The interpretation of this mysterious degeneracy of $K$-mesons has not yet been given. Although there are many theories on this subject, none of them is satisfactory.

**(III) theories of $K$-mesons**

Various theories have been proposed to interpret the mysterious degeneracy of $K$-mesons, about which we shall briefly discuss here.

**(i) Lee-Orear theory**

Lee and Orear (51) suggested that the lifetime identity might be due to a genetic relationship between the two particles. For instance the decay diagram may be of the type given by Fig. 6.

If the true lifetime of $\theta$ is short compared with $10^{-8}$ sec, but that of the $\tau$ is the observed one, e. g. $10^{-8}$ sec., then a few meters from the source, the $\theta'$ s, if originally produced, would all have decayed and one would observe only a single lifetime for both modes of decay. On this theory one can also understand why the branching ratio is constant. The defects of this theory consist in the facts that the experimental conclusion seems to be against the existence of $\gamma$-rays of energy higher than 1 Mev (29), and that short-lived $\theta$-mesons are not yet observed among the $K$-mesons scattered by a nucleus (50). Theoretically, if $\tau$ and $\theta$ are $0^-$ and $0^+$ particles, and if their mass difference is not as large as 1 Mev, the radiative transition, say double $\gamma$-emission, would become so slow that this explanation would not be tenable. (Between two spinless particles, single $\gamma$-emission is strictly forbidden.)
(ii) **Weinstein theory**

Weinstein (52) suggested that when the mass difference is small, say $< 10^{-3}$ ev, the two states may be mixed in passing through matter, so that the measured lifetimes become identical. For instance, if $\tau$ and $0$ are $2^{-}$ and $2^{+}$ respectively, then there would be a static electric dipole strength between the two states. It arises from processes like Fig. 7.

The strength of such electric dipoles is expected to be $\sim e\hbar/mc$ on dimensional grounds. We take $m$ to be the rest mass of the heavy meson, then this field causes an energy split $\sim (e\hbar/mc)E \sim 10^{-3}$ ev in an atomic electric field. This is larger than the mass difference, so that the two split states are complete mixtures of the two states $0$ and $\tau$. Their relative phase would change with time as $(10^{-3}$ ev $)\tau/\hbar \sim t/10^{-12}$ sec. If the field were uniform, the particles would then be completely mixed in $10^{-12}$ sec. If this theory is really operative, two different methods of lifetime measurements, namely at rest and in flight, should be examined. However, if one takes two most likely assignments $(0^{-}, 0^{+})$ and $(0^{-}, 2^{+})$, the coupling with the electromagnetic fields is so weak that no mixing occurs in $10^{-8}$ sec, and this explanation for equal lifetimes would no more work.

(iii) **Yand-Lee theory**

If one supposes that the degeneracies of rest masses and of nuclear cross-sections are not accidental but can be attributed to a certain symmetry property of the strong interactions, then it follows that all particles whose strangeness is odd must exist in two states of opposite parity (53). Assuming that $\tau$ and $0$ are $0^{-}$ and $0^{+}$ particles, we would then have two $\Lambda_{1}^{0}$, $\Lambda_{1}^{*}$ and $\Lambda_{2}^{0}$ of opposite relative parity such that

\[
\pi^{+} + n \rightarrow \Lambda_{1}^{0} + \theta^{*}, \\
\pi^{+} + n \rightarrow \Lambda_{2}^{0} + \tau^{*},
\]

(8.13)
occur with equal amplitude. This symmetry must extend to all fast interactions so that one can define the simultaneous switching of $\theta$ and $\tau$, and of $\Lambda_{1}^{0}$ and $\Lambda_{2}^{0}$, etc. as an operation that commutes with the strong part of the Hamiltonian. This operation is called parity conjugation and denoted by $C_{P}$. All ordinary particles are eigenstates of $C_{P}$ with $C_{P} = +1$. All particles with odd strangeness would exist as a parity multiplet, i.e. two particles with opposite parity that switch to each other under $C_{P}$. From this theory one can readily prove that $\tau$- and $0$-mesons are produced always with equal abundance, and that they have equal nuclear cross sections. Although this theory provides us with a tenable scheme for strong interactions, it is still unsuccessful to account for the lifetime degeneracy.

(iv) **Is the parity a strict quantum number?**

Under such circumstances, one might suppose that there is only a single kind of K-mesons and that the parity is no more conserved for weak interactions. Then all degeneracies can reasonably be interpreted. If one stands on such a point of view, one must examine if such a hypothesis contradicts any of our experience.
Lee and Yang (54) have investigated if such an idea can be reconciled with the phenomena of beta-decay and found that so far as we know there is no evidence against this hypothesis. It is also important to investigate to what extent this hypothesis can be reconciled with other invariance properties in quantum field theory, e.g. charge conjugation and time reversal and so on.

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Göttingen, Max-Planck-Institut für Physik, Böttingerstr. 4

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