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DERIVATION OF STRONG INTERACTIONS FROM A GAUGE INVARIANCE

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Abstract: A representation for the baryons and bosons is suggested, based on the Lie algebra of the 3-dimensional traceless matrices. This enables us to generate the strong interactions from a gauge invariance principle, involving 8 vector bosons. Some connections with the electromagnetic and weak interactions are further discussed.

1. Introduction

Following Yang and Mills [1]), two new theories deriving the strong interactions from a gauge invariance principle have been published lately, by Sakurai [2]) and by Salam and Ward [3]). Sakurai's treatment is based on three separate gauges — isospin, hypercharge and baryonic charge — unrelated from the point of view of group-theory; Salam and Ward postulate one unified gauge, an 8-dimensional rotation gauge, combining isospin and hypercharge through Tiomno's [4]) representation.

One important advantage of the latter theory is the emergency of Yukawa-like terms, allowing for the production of single π or K mesons. Such terms do not arise normally from the boson-currents, and it is through the reintroduction of the σ scalar isoscalar meson [5]), and the assumption that it has a non vanishing vacuum expectation value, that they now appear in ref. [3]). On the other hand, boson-current terms with no σ factor then lead to weak interactions, as it is the creation and re-absorption of these σ mesons that generates the strong coupling. A 9-dimensional version, with a gauge based on restricted rotations, involves 13 vector bosons, of which only seven mediate the strong interactions; the remainder would generate weak interactions — though no way has been found to induce parity non conservation into these without affecting the strong interactions as well. The seven vector bosons of the strong interactions look like a K set and a π set; in Sakurai's theory they are replaced by a π set and two singlets.

The following treatment is an attempt to formulate a unified gauge, while reducing the number of vector bosons. It does, indeed, generate a set of 8 mediating fields, seven of which are similar to the above seven, the eighth is rather like Sakurai's B_y singlet. Still, one important factor is missed, namely, there is no room for the σ meson, and thus there are no single-pion terms.

To minimise the number of parameters of the gauge, and thus the number of vector bosons it will generate, we have adopted the following method: we abandoned the usual procedure of describing fields as vector components in a Euclidean isospace, and replace it by a matrix-algebra manifold. Fields still form vectorial sets only in the space of the group operators themselves, invariance of the Lagrangians being achieved by taking the traces of product matrices.

We have also abandoned rotations and use a group first investigated by Ikeda, Ogawa and Ohnuki [6]) in connection with the construction of bound states in the Sakata model. Our present use of this group is in an entirely different context, as our assumptions with regard to the representation of the fermions do not follow the prescriptions of the model.

2. Matrix Formalism

We use an 8-dimensional linear vector space P spanned by the semisimple Lie algebra of the 3×3 matrices X_{ij} of ref. [6]). We have excluded the identity transformation and use as basis the 8 linearly independent $\mathbf{u}^i \in \mathbf{U}$ given by the following formulae:

$$\mathbf{U}\begin{cases} \mathbf{u}^1 = \frac{1}{2}\sqrt{2}(X_{(31)}-iX_{[31]}), & \mathbf{u}^4 = \frac{1}{2}\sqrt{2}(X_{(31)}+iX_{[31]}), \\ \mathbf{u}^2 = \frac{1}{2}\sqrt{2}(X_{(23)}-iX_{[23]}), & \mathbf{u}^3 = \frac{1}{2}\sqrt{2}(X_{(23)}+iX_{[23]}), \\ \mathbf{u}^5 = \frac{1}{2}\sqrt{2}(X_{(12)}+iX_{[12]}), & \mathbf{u}^6 = \frac{1}{2}\sqrt{2}(X_{(12)}-iX_{[12]}), \\ \mathbf{u}^7 = \frac{1}{2}(X_{11}-X_{22}), & \mathbf{u}^8 = \frac{1}{6}\sqrt{3}(X_{11}+X_{22}-2X_{33}), \end{cases} \tag{1}$$

$$X_{ij}^{\alpha\beta} = \tfrac{1}{2}\delta_{i\alpha}\delta_{j\beta}(1-i)+\tfrac{1}{2}\delta_{i\beta}\delta_{j\alpha}(1+i),$$

$$X_{(ij)} = \tfrac{1}{2}(X_{ij}+X_{ji}), \qquad X_{[ij]} = \tfrac{1}{2}(X_{ij}-X_{ji}),$$

the indices α and β denoting the matrix elements. The X_{ij} are hermitian, whereas the basis matrices $\mathbf{u}_i$ are not, with the exception of $\mathbf{u}^7$ and $\mathbf{u}^8$, both diagonal. $\mathbf{U}$ can contain only two linearly independent diagonal elements, and the 2-dimensional sub-space $P_{\mathrm{d}} \subset P$ spanned by the set of all diagonal elements can be represented by a real Euclidean 2-space. In this 2-space, $\mathbf{u}^7$ and $\mathbf{u}^8$ are orthogonal: not only do they commute with each other, as any $[\mathbf{u}'_{\mathrm{d}}, \mathbf{u}''_{\mathrm{d}}]=0$ for $\mathbf{u}'_{\mathrm{d}}, \mathbf{u}''_{\mathrm{d}} \subset P_{\mathrm{d}}$; each also commutes with a 3-rotation consrructed by taking the other as an M_z. In the set (1), U_a ($\mathbf{u}^5$, $\mathbf{u}^6$, $\mathbf{u}^7$) forms such a 3-rotation, and

$$[\mathbf{u}^8, \mathbf{u}^a] = 0. \tag{2}$$

We also use a basis $\mathbf{U}'$ differing from $\mathbf{U}$ only in P_{d},

$$\mathbf{U}'\begin{cases} \mathbf{u}^{i\prime} = \mathbf{u}^i, \qquad i = 1, 2, \ldots, 6, \\ \mathbf{u}^{7\prime} = -\frac{1}{2}(\mathbf{u}^7-\sqrt{3}\,\mathbf{u}^8), \\ \mathbf{u}^{8\prime} = \frac{1}{2}\sqrt{3}(\mathbf{u}^7+\frac{1}{3}\sqrt{3}\,\mathbf{u}^8), \end{cases} \tag{3}$$

where again $\mathbf{u}^{7'}$ and $\mathbf{u}^{8'}$ are orthogonal, $\mathbf{u}^{8'}$ commuting with the 3-rotation U_b $(\mathbf{u}^2, \mathbf{u}^3, \mathbf{u}^{7'})$:

$$[\mathbf{u}^{8'}, \mathbf{u}^b] = 0. \tag{4}$$

We now define a metric g_{ij} in P space,

$$g_{ij} = \begin{vmatrix} & & & 1 & & & & \\ & & 1 & & & & & \\ & 1 & & & & & & \\ 1 & & & & & & & \\ & & & & & 1 & & \\ & & & & 1 & & & \\ & & & & & & 1 & \\ & & & & & & & 1 \end{vmatrix}, \tag{5}$$

such that

$$\sum_{i=1}^{8} g_{ij}\mathbf{u}^i = \mathbf{u}_j = \tilde{\mathbf{u}}^j. \tag{6}$$

Note that

$$2\mathrm{Tr}\{\tilde{\mathbf{u}}^i\, \mathbf{u}^i\} = 2\mathrm{Tr}\{\mathbf{u}^i\, \tilde{\mathbf{u}}^i\} = 1. \tag{7}$$

Thus

$$\mathbf{A}\cdot\mathbf{B} = 2\mathrm{Tr}\{\sum_{i,j=1}^{8} g_{ij}A^i\mathbf{u}^iB^j\mathbf{u}^j\} \equiv \sum_{i,j} g_{ij}A^iB^j \tag{8}$$

is a scalar product in P.

When using our algebra for unitary transformations, we shall take the hermitian set V as a basis for the infinitesimal operators,

$$\mathbf{V}\begin{cases} \mathbf{v}^{14} = \frac{1}{2}\sqrt{2}(\mathbf{u}^1+\mathbf{u}^4), & \mathbf{v}^{41} = \frac{1}{2}i\sqrt{2}(\mathbf{u}^1-\mathbf{u}^4), \\ \mathbf{v}^{23} = \frac{1}{2}\sqrt{2}(\mathbf{u}^2+\mathbf{u}^3), & \mathbf{v}^{32} = -\frac{1}{2}i\sqrt{2}(\mathbf{u}^2-\mathbf{u}^3), \\ \mathbf{v}^{56} = \frac{1}{2}\sqrt{2}(\mathbf{u}^5+\mathbf{u}^6), & \mathbf{v}^{65} = -\frac{1}{2}i\sqrt{2}(\mathbf{u}^5-\mathbf{u}^6), \\ \mathbf{v}^7 = \mathbf{u}^7, & \mathbf{v}^8 = \mathbf{u}^8, \end{cases} \tag{9}$$

so that

$$\sum_{k=1}^{8} A_V{}^k B_V{}^k = \sum_{i=1}^{8} A_U{}^i B_{U,i}, \tag{10}$$

i.e. the scalar product (8) is Euclidean in the V system.

Under a unitary transformation $E^{(m_V)} = \exp(i\varepsilon^{m_V}\mathbf{v}^{m_V})$ (m_V is the single or double index in $\mathbf{V}$), the component $A^k\mathbf{u}^k$ transforms like

$$\sum_{l=1}^{8} \delta_{(m_V)} A^l\mathbf{u}^l = i\varepsilon^{m_V}A^k[\mathbf{v}^{m_V}, \mathbf{u}^k] = i\varepsilon^{m_V}A^k \sum_{l=1}^{8} f^l_{m_V,k}\mathbf{u}^l$$

and for

$$E = \exp(i\sum_{m_V} \varepsilon^{m_V}\mathbf{v}^{m_V}) \tag{11}$$

we get variations

$$\delta A^l = i \sum_{m_V} \varepsilon^{m_V} \sum_{k=1}^{8} f^l_{m_V,k} A^k. \tag{12}$$

The $f^l_{m_V,k}$ define an 8×8 representation of our algebra in P space,

$$C^{l,k}_{m_V} = f^l_{m_V,k}, \tag{13}$$

so that (12) becomes in P

$$\delta A^l = i \sum_{m_V=1}^{8} \varepsilon^{m_V} \sum_{k=1}^{8} C^{l,k}_{m_V} A^k,$$

or

$$\delta \mathbf{A} = i \sum_{m_V} \varepsilon^{m_V} C_{m_V} \mathbf{A} = i \sum_{i,j} g_{ij} \varepsilon^i C^j \mathbf{A}, \tag{14}$$

where we have returned to the basis **U** or **U′**.

3. Fields and Interactions

We define the quantum operators

$$\mathbf{I}(C_5, C_6, C_7), \qquad I_Z = C^7, \qquad Q = \tfrac{2}{3}\sqrt{3}\, C^{8'}, \qquad Y = \tfrac{2}{3}\sqrt{3}\, C^8, \tag{15}$$

and write the fields as vectors in P space

$$\begin{array}{ll} \psi(\mathrm{p}, \mathrm{n}, \Xi^0, \Xi^-, \Sigma^+, \Sigma^-, \Sigma^0, \Lambda), & \overline{\psi}(\overline{\Xi^-}, \overline{\Xi^0}, \bar{\mathrm{n}}, \bar{\mathrm{p}}, \overline{\Sigma^-}, \overline{\Sigma^+}, \overline{\Sigma^0}, \bar{\Lambda}) \\ \varphi(\mathrm{K}^+, \mathrm{K}^0, \overline{\mathrm{K}^0}, \overline{\mathrm{K}^-}, \pi^+, \pi^-, \pi^0, \pi^{0'}), & \overline{\varphi} = \varphi, \end{array} \tag{16}$$

or in matrix form

$$\psi = \tfrac{1}{2}\sqrt{2} \left| \begin{array}{c|c|c} \tfrac{1}{2}\sqrt{2}\Sigma^0 + \tfrac{1}{6}\sqrt{6}\Lambda & \Sigma^+ & \mathrm{p} \\ \hline \Sigma^- & -\tfrac{1}{2}\sqrt{2}\Sigma^0 + \tfrac{1}{6}\sqrt{6}\Lambda & \mathrm{n} \\ \hline \Xi^- & \Xi^0 & -\sqrt{\tfrac{2}{3}}\Lambda \end{array} \right|,$$

$$\overline{\psi} = \tfrac{1}{2}\sqrt{2} \left| \begin{array}{c|c|c} \tfrac{1}{2}\sqrt{2}\,\overline{\Sigma^0} + \tfrac{1}{6}\sqrt{6}\,\bar{\Lambda} & \overline{\Sigma^-} & \overline{\Xi^-} \\ \hline \overline{\Sigma^+} & -\tfrac{1}{2}\sqrt{2}\overline{\Sigma^0} + \tfrac{1}{6}\sqrt{6}\,\bar{\Lambda} & \overline{\Xi^0} \\ \hline \bar{\mathrm{p}} & \bar{\mathrm{n}} & -\sqrt{\tfrac{2}{3}}\,\bar{\Lambda} \end{array} \right|, \tag{17}$$

$$\varphi = \tfrac{1}{2}\sqrt{2} \left| \begin{array}{c|c|c} \tfrac{1}{2}\sqrt{2}\pi^0 + \tfrac{1}{6}\sqrt{6}\pi^{0'} & \pi^+ & \mathrm{K}^+ \\ \hline \pi^- & -\tfrac{1}{2}\sqrt{2}\pi^0 + \tfrac{1}{6}\sqrt{6}\pi^{0'} & \mathrm{K}^0 \\ \hline \overline{\mathrm{K}^-} & \overline{\mathrm{K}^0} & -\sqrt{\tfrac{2}{3}}\pi^{0'} \end{array} \right|,$$

The free field Lagrangians are

$$\mathscr{L}_\psi{}^0 = -\bar{\psi}\cdot(\gamma^\mu\partial_\mu+m_\psi)\psi, \qquad \mathscr{L}_\varphi{}^0 = -\tfrac{1}{2}(\partial^\mu\boldsymbol{\varphi}\cdot\partial_\mu\boldsymbol{\varphi}+m_\varphi{}^2\boldsymbol{\varphi}\cdot\boldsymbol{\varphi}). \quad (18)$$

We postulate the invariance of these Lagrangians under the unitary gauge transformation

$$E = \exp(i\sum_{m_\nu}\varepsilon^{m_\nu}(x)C^{m_\nu}) \quad (19)$$

and follow the now standardized technique of Yang and Mills [1]) and Utiyama [7]), recombining the C set in terms of the basis $\mathbf{U}$ of (1).

The total Lagrangian becomes

$$\mathscr{L}_{\text{total}} = \mathscr{L}_\psi{}^0+\mathscr{L}_\varphi{}^0+\mathscr{L}_\psi{}^{\mathscr{I}}+\mathscr{L}_\varphi{}^{\mathscr{I}}+\mathscr{L}_B{}^0, \quad (20)$$

$$\mathscr{L}_\psi{}^{\mathscr{I}} = -\sum_{i=1}^{8}\sum_{n=1}^{8}\bar{\psi}^n\gamma^\mu C_i\psi_n B_\mu{}^i, \quad (21)$$

$$\mathscr{L}_\varphi{}^{\mathscr{I}} = -\sum_{\substack{i=1\\j=1}}^{8}\sum_{n=1}^{8}(\partial_\mu\bar{\varphi}_n+C_i\bar{\varphi}_n B_\mu{}^i)C_j\varphi^n B^{j\mu}. \quad (22)$$

The $B_\mu{}^i$ is a set of 8 vector bosons, with the following isobaric and strangeness qualities:

$$\begin{array}{llll} B_\mu{}^1 \to K^+, & B_\mu{}^2 \to K^0, & B_\mu{}^3 \to \overline{K^0}, & B_\mu{}^4 \to \overline{K^-}, \\ B_\mu{}^5 \to \pi^+, & B_\mu{}^6 \to \pi^-, & B_\mu{}^7 \to \pi^0, & B_\mu{}^8 \to \pi^{0\prime}. \end{array} \quad (23)$$

We have here the same set of vector bosons Salam and Ward got out of the 8-dimensional rotation gauge — with an additional $\pi^{0\prime}$—like interaction. Denoting as in ref. [3]) the K-like set by Z_μ,

$$Z_\mu = (Z_\mu{}^+, Z_\mu{}^0), \qquad \bar{Z}_\mu = (\bar{Z}_\mu{}^-, \bar{Z}_\mu{}^0),$$

the π-like one by $V_\mu{}^i$ with

$$V_\mu{}^\pm = \tfrac{1}{2}\sqrt{2}(V_\mu{}^1 \pm iV_\mu{}^2), \qquad V_\mu{}^0 = V_\mu{}^3$$

and the $\pi^{0\prime}$-like by $B_\mu{}^8 = X_\mu{}^0$ we get

$$\mathscr{L}_B{}^0 = -\tfrac{1}{4}(\mathbf{F}_{\mu\nu}\cdot\mathbf{F}^{\mu\nu}), \quad (24)$$

$$\mathbf{F}_{\mu\nu} = \mathbf{H}_{\mu\nu}+\mathbf{G}_{\mu\nu}, \quad (24')$$

$$\mathbf{H}_{\mu\nu} = \partial_\mu\mathbf{B}_\nu-\partial_\nu\mathbf{B}_\mu, \quad (24'')$$

$$\begin{aligned} G^Z_{\mu\nu} &= \tfrac{1}{2}\{Z_\mu(\mathbf{V}_\nu\cdot\boldsymbol{\tau}+\sqrt{3}\,X_\nu{}^0)-(\mathbf{V}_\mu\cdot\boldsymbol{\tau}+\sqrt{3}\,X_\mu{}^0)^\sim Z_\nu\}, \\ G^{\bar{Z}}_{\mu\nu} &= -\tfrac{1}{2}\{\bar{Z}_\mu(\mathbf{V}_\nu\cdot\boldsymbol{\tau}+\sqrt{3}\,X_\nu{}^0)^\sim-(\mathbf{V}_\mu\cdot\boldsymbol{\tau}+\sqrt{3}\,X_\mu{}^0)\bar{Z}_\nu\}, \\ G^V_{\mu\nu} &= i\mathbf{V}_\mu\wedge\mathbf{V}_\nu+\tfrac{1}{2}\{\bar{Z}_\mu\boldsymbol{\tau}Z_\nu-Z_\mu\boldsymbol{\tau}\bar{Z}_\nu\}, \\ G^X_{\mu\nu} &= \tfrac{1}{2}\sqrt{3}\,\{\bar{Z}_\mu Z_\nu-Z_\mu\bar{Z}_\nu\}. \end{aligned} \quad (24''')$$

In the 3-space of the matrix elements of U, we have the set

$$\mathbf{B}_\mu = \tfrac{1}{2}\sqrt{2}\begin{vmatrix} \tfrac{1}{2}\sqrt{2}\,V_\mu{}^0+\tfrac{1}{6}\sqrt{6}\,X^0 & V_\mu{}^+ & Z_\mu{}^+ \\ V_\mu{}^- & -\tfrac{1}{2}\sqrt{2}\,V_\mu{}^0+\tfrac{1}{6}\sqrt{6}\,X_\mu{}^0 & Z_\mu{}^0 \\ \bar{Z}_\mu{}^- & \bar{Z}_\mu{}^0 & -\sqrt{\tfrac{2}{3}}\,X_\mu{}^0 \end{vmatrix}. \quad (25)$$

4. Discussion

The fermion and boson interaction Lagrangians provide us with the full set of known strong interactions (plus the $\pi^{0'}$ set) through the current-current-like 2nd order terms — but with no Yukawa-like simple processes for π or K.

In its general features, our Lagrangian reflects a certain similarity with Sakurai's theory [2]). The V_μ is similar to the $B_T{}^\mu$ (isospin-current boson) of the latter, and the $X_\mu{}^0$ is similarly related to its $B_Y{}^\mu$ (the hypercharge-current boson singlet). On the other hand we have no $B_B{}^\mu$ (baryon-current singlet) and do have a Z_μ set which has no place in ref. [2]).

We note that we do get directly from our group structure a ratio between the couplings; for the V and X fields, this is $f_X = f_V\sqrt{3}$, a value that fits Sakurai's phenomenological conclusion (from KN and $\overline{\text{K}}$N at low energies) that $(1/4\pi m_X{}^2)f_X{}^2 \approx (3/4\pi m_V{}^2)f_V{}^2$ if we assume the masses of be similar. Our X^0 field does not interact with the (Σ, Λ) set, and V does not interact with Λ, so that we get a split (N Ξ), Σ, Λ but though the interactions of X and Z with N and Ξ have opposite signs, lacking $B_B{}^\mu$ we cannot repeat here Sakurai's simple interpretation of the origin of the N-Ξ mass split. The arguments explaining the π-N S-wave scattering exist in our gauge. We also note that $\mathscr{L}_B{}^0$ in (24) with its $G^i_{\mu\nu}G^{\mu\nu}_i$ provides us with effective mass terms (in the sense of the mass of $A_\mu{}^\pm$ in a former work [8]) of Salam and Ward) for V_μ, Z_μ and $X_\mu{}^0$ (from $G^Z_{\mu\nu}$ and $G^{\bar{Z}}_{\mu\nu}$), whereas ref. [2]) lacks such terms for the singlets B_Y and B_B. From $\mathscr{L}_\varphi{}^\mathscr{I}$ we see that provided the masses are sufficient, there exist fast decays

$$Z \to \text{K}+\pi,$$

$$V \to 2\pi \quad \text{or} \quad V \to \text{K}+\overline{\text{K}} \qquad \text{(the even } G \text{ combination)},$$

$$X \to \text{K}+\overline{\text{K}} \qquad \text{(odd } G \text{ combination)}.$$

Note the possibility that $m_X \approx m_V$.

From the point of view of the Lagrangian formalism, it seems preferable to us to have what is in fact one conservation law for a "charge" that behaves like a 2nd rank tensor in three dimensions, than three separate unrelated conservation laws; this is even more important in view of the necessity to bring in at some further stage the electromagnetic and the weak interactions. In Sakurai's theory, these seem to imply two new independently conserved quantities Q and l, though the relation $Q = I_Z + \frac{1}{2}Y$ seems to indicate that the interactions are not wholly independent (and so does the $|\Delta \mathbf{I}| = \frac{1}{2}$ rule of the weak interactions). We think that the "aesthetic' value of Sakurai's theory and the "Urschmiere" approach would be enhanced if there proved to be only one kind of "Urschmiere" instead of five.

Our gauge has not given us directly an additional electromagnetic and weak Lagrangian. Still, it is interesting to check the connection it may have with these. We can do that by using (3) and rewriting our gauge invariance and the vector bosons in that basis. We get

$$B_\mu^{8'} \to A_\mu, \tag{26}$$

$$\mathbf{B}'_\mu = \tfrac{1}{2}\sqrt{2} \begin{vmatrix} \sqrt{\frac{2}{3}}\,A_\mu & V_\mu^+ & Z_\mu^+ \\ V_\mu^- & \frac{1}{2}\sqrt{2}\,B_\mu^{7'} - \frac{1}{6}\sqrt{6}\,A_\mu & Z_\mu^0 \\ Z_\mu^- & Z_\mu^0 & -\frac{1}{2}\sqrt{2}\,B_\mu^{7'} - \frac{1}{6}\sqrt{6}\,A_\mu \end{vmatrix}. \tag{26'}$$

The interaction Lagrangian corresponding to (26) will be identical with the electromagnetic Lagrangian. It leads to the conditions

$$|\Delta \mathbf{I}| = 0,1 \qquad |\Delta I_Z| = 0, \qquad |\Delta Y| = 0, \tag{27}$$

though, of course, all quantum numbers are fully conserved when one adds the remaining interactions of P_d, mediated by $B_\mu^{7'}$. This last, with quantum numbers similar to (27), belongs with B_μ^2 and B_μ^3 (the K^0, $\overline{K^0}$-like vector bosons) to the subspace defined by $\mathbf{U}_b$ in (4). We note (4), from which we can see that $B_\mu^{8'}$ is the only matrix in P_d orthogonal to the $\mathbf{U}_b$ set. If, following Salam and Ward's [3]) treatment of weak interactions, we assume that the $|\Delta \mathbf{I}| = \frac{1}{2}$ law results from a non zero vacuum expectation value for the field K_1^0, our system could mediate the weak interactions through this B_μ^2, B_μ^3 subset; thus the $\mathbf{U}_b$ subset is apparently responsible for the generation of weak interactions as a secondary effect, with non conservation of parity — but whatever the mechanism involved, it cannot affect the electromagnetic interactions, generated by an orthogonal gauge. Still, we have no suggestion to explain why the strong interactions, mediated by a $\mathbf{U}$ gauge, should be accompanied by weaker interactions involving a change of basis into $\mathbf{U}'$.

One last remark, concerning the $\pi^{0'}$. From the group-structure aspect, it is related to the spinor-like subgroups (i.e. the K and not the π). From (2) we note that its matrix representation commutes with that of the pion. It has no direct interaction with the pions, while it does interact with all the kaons. If we assume a single parity for all the components of the φ vector, it is a pseudoscalar particle, with a fast decay into $K^0 + \overline{K}^- + \pi^+$ (mediated by Z_μ) or $K^+ + \overline{K}^- + \pi^0$ (through Z_μ again) etc., provided it has sufficient mass. On the other hand, if it were scalar, it could do for the medium strong interactions what the vacuum decay of the σ meson does for all strong interactions in ref. [3]. In fact it would then be identical with the σ' particle suggested in a variant of ref. [3].

I am indebted to Prof. A. Salam for discussions on this problem. In fact, when I presented this paper to him, he showed me a study he had done on the unitary theory of the Sakata model, treated as a gauge, and thus producing a similar set of vector bosons [9].

Shortly after the present paper was written, a further version, utilizing the 8-representation for baryons, as in this paper, reached us in a preprint by Prof. M. Gell Mann.

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