respectively. Experimental observations seem to respect lepton number conservation in neutrino reactions. For instance, $\nu_\mu$ and $\bar{\nu}_\mu$ are produced through the decay of pions:

$$\pi^+ \to \mu^+ + \nu_\mu, \quad \pi^- \to \mu^- + \bar{\nu}_\mu$$  \hspace{1cm} (4.117)

and when they collide with nucleons, they produce $\mu^-$ (lepton number 1) and $\mu^+$ (lepton number -1)

$$\nu_\mu + n \to \mu^- + p, \quad \bar{\nu}_\mu + p \to \mu^+ + n$$  \hspace{1cm} (4.118)

in such a way as to conserve the lepton number. A $\nu_\mu$ does not produce a $\mu^+$ if it collides with a proton. However, the weak interaction acts only on particles with negative chirality. If the particle’s mass is zero, it is a Weyl particle and is in a pure helicity $(-)$ state, while the antiparticle is in pure helicity $(+)$ state. What was considered as lepton number conservation may have been nothing but a test for helicity conservation, although experimentally there is no evidence against lepton number conservation as far as the neutrino is concerned.

Let us assume that the neutrino is a Majorana particle, a name given to a fermion that does not have its own antiparticle. Conditions for the Majorana particle are

1. It satisfies the Dirac equation.
   $$\gamma^\mu i\partial_\mu - m) N(\alpha) = 0$$
2. The particle and its antiparticle are identical.
   $$N_\psi = N$$  \hspace{1cm} (4.119)

In the Weyl representation, we can put $\psi_L = \begin{bmatrix} \xi \\ 0 \end{bmatrix}$ and $\psi_R = \begin{bmatrix} 0 \\ \eta \end{bmatrix}$. Then define $N_1$ and $N_2$ as follows:

$$N_1 = \psi_L + (\psi_L)^c = \begin{bmatrix} \xi \\ -i\alpha_2\xi^* \end{bmatrix}$$  \hspace{1cm} (4.120a)

$$N_2 = \psi_R + (\psi_R)^c = \begin{bmatrix} i\alpha_2\eta^* \\ \eta \end{bmatrix}$$  \hspace{1cm} (4.120b)

They satisfy $N_{1\psi} = N_1$, $N_{2\psi} = N_2$. Applying the Dirac equation for $N_1$

$$\gamma^\mu i\partial_\mu - m) N_1 = \begin{bmatrix} -m & i\partial_0 + i\sigma \cdot \nabla \\ i\partial_0 - i\sigma \cdot \nabla & -m \end{bmatrix} \begin{bmatrix} \xi \\ -i\alpha_2\xi^* \end{bmatrix} = 0$$  \hspace{1cm} (4.121)

and rewriting the equation for two-component spinors, we obtain

$$\begin{align*}
(\partial_0 - \sigma \cdot \nabla)\xi &= -m(\alpha_2\xi^*) \\
(\partial_0 + \sigma \cdot \nabla)(\alpha_2\xi^*) &= m\xi
\end{align*}$$  \hspace{1cm} (4.122)

9) This is true only for charged current interactions, where $W^\pm$ is exchanged. The neutral current interactions act on both chiralities.
The two equations are not independent. The second equation can be derived from the first. It is indeed an equation for a two-component spinor. $N_{1}$ and $N_{2}$ have only two independent components despite their seemingly four-component structure. Similarly we can obtain the equation for $N_{2}$:

\[
\begin{align*}
(\partial_{0} + \sigma \cdot \nabla)\eta &= m(\sigma_{2}\eta^{*}) \\
(\partial_{0} - \sigma \cdot \nabla)(\sigma_{2}\eta^{*}) &= -m\eta
\end{align*}
\]

(4.123)

From the above argument, we see $\xi$ and $\eta$ satisfy two independent equations. In the limit of $m \to 0$, the equations become identical to the Weyl equation (4.9). If $m \neq 0$, the deviation from the Weyl solution is small for relativistic particles ($m \ll E$). $\xi$ is in a chirality minus state and is predominantly left-handed. Similarly, $\eta$ is predominantly right-handed. Therefore, we call $\xi$ ($\eta$) a left (right)-handed Majorana particle. Since they satisfy independent equations, their mass need not be the same in general ($m_{L} \neq m_{R}$).

\begin{quote}
Note, the Majorana particle is a quantum mechanical concept and there is no equivalent in classical mechanics. If we consider $\xi$ or $\eta$ as expressed by pure classical numbers, it is straightforward to show that Equation (4.122) has no solution. This can be seen by inserting $\xi^{\dagger} = (a, b)$ and $p = 0$ and solving the equation explicitly for $a$ and $b$. Another way to see this is to note that the Lagrangian to derive the mass term should contain a term like $\xi^{\dagger}i\sigma_{2}\xi^{*} = i\xi^{\dagger}i\sigma_{2}\xi^{\dagger}$. However, $\sigma_{2}$ is antisymmetric and it would vanish if $\xi^{\dagger}$ were an ordinary c-number field. In quantum field theory, $\xi$ is an anticommuting field operator and the contradiction does not appear.
\end{quote}

Let us see the effect of charge conjugation on the left- and right-handed Majorana neutrinos. After decomposing the four-component spinors $N_{1}$ and $N_{2}$ into $N_{L}$ and $N_{R}$ using the projection operators $(1 \mp \gamma^{5})/2$

\[
N_{L} = \frac{1}{2}(1 - \gamma^{5})N, \quad N_{R} = \frac{1}{2}(1 + \gamma^{5})N
\]

(4.124)

we apply the charge conjugation operation to them:

\[
(N_{L})^{c} = C N_{L}^{T} = i\gamma^{2}\gamma^{0}\bar{N}_{L}^{T} = i\gamma^{2}\frac{1}{2}(1 - \gamma^{5})N^{*} = \frac{1}{2}(1 + \gamma^{5})N^{c} = N_{R}
\]

(4.125a)

\[
(N_{R})^{c} = (N_{R})_{R} = N_{R} \quad \text{and} \quad (N_{L})^{c} = (N_{L})_{L} = N_{L}
\]

(4.125b)

The Majorana particle changes its handedness by the charge conjugation operation. No distinction exists between a particle and its antiparticle. The handedness can be