
On Wentzel's Method in the Meson Theory. *

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Summary. As an adequate approximation in case of strong coupling between nucleons and mesons we have Wentzel's method in which solutions are obtained as a power series of the reciprocal of the coupling constant. This method has been discussed in detail by one of the authors (T. T.) who treated charged longitudinal mesons as an example. In this paper we apply this method to the symmetrical mesons. The principle of the method is the same as the case of the charged longitudinal mesons, we have presented only the main results of our calculation. Also the scattering of mesons by nucleons has been treated by this method.

§1. The fundamental equation and its transformations.

The Hamiltonian of the system of symmetric longitudinal mesons and a nucleon at the origin of the coordinates is written as follows:

\[ H = \frac{1}{2} \int (\mathbf{b}^2 + K^2 \alpha^2) \, d\mathbf{k} - G \int \mathbf{r} \cdot \alpha \varphi(\mathbf{k}) \, d\mathbf{k} \]  

(1)

Here \( \mathbf{r} \) means the isotopic-spin vector of the nucleon, \( G \) is the coupling con-

* These calculations were carried out several years ago, and the contents of this paper were read at the Annual Meeting of the Japan Physico-mathematical Society at Tohoku Imperial University, on July 17, 1943, but the wartime troubles prevented us from publishing it. In the meanwhile, the strong-coupling theory was extended by Pauli and his collaborators, and the same problems as ours were treated by Serber and Dancoff. (Phys. Rev., 63 (1944), 143. Though they worked out the calculations using the ordinary coordinate space, and ours are based on the momentum space, the essential contents are, of course, the same. But our calculations were also rigorously carried out to the vibrational states, and the scattering cross-section was obtained by our method.


(2) C. T. Miyazima: Journ. of Phys. Soc. 2 (1947), 58.
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stant, \( \varphi \) the function of interaction including a cut-off factor, and normalized as \( \int \varphi^{2} dk = 1 \), and \( a \) and \( b \) are the coordinate vector and its conjugate momentum of the field. \( K \) is \( (k^{2} + x^{2})^{1/2} \), and \( x \) the rest mass of the meson.

As easily seen, the part of the field interacting directly with the nucleon is \( \alpha \equiv \int \varphi a \, dk \). It is the component of \( a \) parallel to \( \varphi \) in the functional space, and the component perpendicular to \( \varphi \) interacts with the nucleon only through the non-diagonal elements of the first term of the equation (1). Therefore we separate these two parts in the following manner:

\[
\alpha \equiv \int a(k) \varphi(k) \, dk, \quad \beta \equiv \int b(k) \varphi(k) \, dk,
\]

(2)

\[
A(k) = a(k) - \alpha(k), \quad B(k) = b(k) - \beta(k),
\]

(3)

Then \( \alpha \) and \( \beta \) are conjugate variables with respect to each other. But \( A \) and \( B \) have the following commutation relations:

\[
[A_{m}(k), B_{n}(k')] = i \delta_{mn} \{ \delta(k - k') - \varphi(k) \varphi(k') \}, \quad m, n = 1, 2, 3.
\]

(4)

They are conjugate variables in the subspace perpendicular to \( \varphi \).

Corresponding to the above separation, the angular momentum in the isotopic-spin space can be separated as follows:

\[
\int a \times b \, dk = L_{t} + L,
\]

\[
L_{t} = \alpha \times \beta, \quad L = \int A \times B \, dk,
\]

(5)

where the \( s \)-component of \( L_{t} + L \) means the total charge of mesons. The part \( L_{t} \) is closely related to the charge of the nucleon, the \( s \)-component of which being called the charge of the bound mesons, and the \( s \)-component of \( L \) is the charge of free mesons.

As it is seen from the definition, \( L_{t} \) is perpendicular to \( \alpha \). This corresponds to the fact that, in case of a diatomic molecule, the rotational angular momentum is perpendicular to the axis itself. In fact, the total angular momentum of the nucleon and the bound mesons is \( J = \frac{1}{2} \tau + L_{t} \) where \( \frac{1}{2} \tau \) is parallel to \( \alpha \) by the following reason, and consequently we see that \( J \) is composed of the "electronic" angular momentum \( \frac{1}{2} \tau \), which
is parallel to the axis \( \mathbf{a}_1 \), and the "rotational" angular momentum \( \mathbf{L}_1 \), which is perpendicular to the axis. From this fact, we can obtain an expression analogous to a diatomic molecule to the energy of the nucleon as a function of \( J \). It is the purpose of the following calculation to justify this point of view by making use of the Wentzel's method.

We express \( \mathbf{a}_1 \) by the polar coordinates \( r, \theta, \varphi \) and then introduce the rotating coordinates \( \xi, \eta, \zeta \) moving with \( \mathbf{a}_1 \). \( \zeta \) is in the direction of \( \mathbf{a}_1 \), \( \xi \) is directing to the south along the meridian passing through \( \mathbf{a}_1 \), and \( \eta \) is to the east along the latitude. In this coordinates system the Hamiltonian (1) is written as follows:

\[
\mathcal{H}^{-1} = \frac{1}{2} \mathbf{P}^2 + \frac{1}{2r^2 \sin^2 \theta} \mathbf{P}_\theta^2 \sin \theta \mathbf{P}_\varphi^2 + \frac{1}{2r^2 \sin^2 \theta} \mathbf{P}_\zeta^2 - G r \tau_z
\]

\[
+ \frac{1}{2} \int \mathbf{B} \cdot d\mathbf{e} + \frac{1}{2} \int \mathbf{E} \cdot d\mathbf{h} + \frac{1}{2} \int \mathbf{K} \cdot (\mathbf{A}_1 + \mathbf{A}_2) d\mathbf{e} + \frac{1}{2} \int \mathbf{K} \cdot (\varphi + \mathbf{A}_2 \zeta) d\mathbf{e},
\]

where

\[
\mathbf{P}_\xi = \mathbf{P}_\theta - L_\eta, \quad \mathbf{P}_\eta = \mathbf{P}_\varphi - \cos \theta L_\zeta + \sin \theta L_\xi
\]

In these equations \( \mathbf{P}_r, \mathbf{P}_\theta, \mathbf{P}_\varphi \) are the conjugate momenta of \( r, \theta, \varphi \), and \( \tau_z \) in the equation (6) is the \( \zeta \)-component of \( \mathbf{\tau} \) and it is not diagonal. It is transformed into diagonal form by the following operator:

\[
S = \begin{pmatrix}
\cos \frac{\theta}{2} e^{i \varphi/2} & \sin \frac{\theta}{2} e^{-i \varphi/2} \\
-\sin \frac{\theta}{2} e^{i \varphi/2} & \cos \frac{\theta}{2} e^{-i \varphi/2}
\end{pmatrix}
\]

Then the transformed Hamiltonian \( \mathcal{H}' = S \mathcal{H} S^{-1} \) has the same form as (6), by only replacing \( \mathbf{P}_\xi, \mathbf{P}_\eta, \mathbf{P}_\varphi \) with the following ones respectively:

\[
\mathbf{P}_\xi' = \mathbf{P}_\theta - \left( \frac{1}{2} \tau_\eta + L_\eta \right),
\]

\[
\mathbf{P}_\eta' = \mathbf{P}_\varphi - \cos \theta \left( \frac{1}{2} \tau_\zeta + L_\zeta \right)
\]

and the spin operator \( \tau_z \) is now represented by a diagonal matrix.

In the equation (6), the term which contains the spin operator \( \tau_z \) is

\[-G r \tau_z.\] This means that the states \( \tau_z = \pm 1 \) have the potential energies
Therefore the equilibrium positions in these two states lie at $r \sim \pm G$ and the potential energies are then $\mp G^2$ respectively. The energy difference between them is large, and it is the order of $G^2$. We can therefore separate these two states in the zeroth approximation when $G$ is large enough; in fact, as Wentzel showed, the perturbation energy of the mutual coupling between them does not exceed the order $G^{-2}$. Hence, in the limit of approximation of the order $G^{-2}$, we are allowed to separate these two levels completely and to put $\tau = 1$ or $-1$ in these two states respectively. As we are interested only in the lower stationary state, we must take the former. In this state the nucleon spin is parallel to $\alpha_t$. This is in the same circumstance as the separation of electronic terms in a diatomic molecule, as easily seen from the above discussion.

Consequently we use, instead of $H$, $\tilde{H}$ which is obtained by replacing all the terms in $H$ related to the spin by the average values in the state $\tau = 1$. Thus we have

$$\tilde{H}_t = \frac{1}{2} \mathbf{p}^2 + \frac{1}{2} \mathbf{r}^2 \left( \frac{1}{\sin^2 \theta} - \frac{1}{\sin \theta} \sin \theta \frac{1}{\sin \theta} \left( \frac{1}{2} \cos \theta \right) + \frac{1}{2} \right) - Gr$$

$$+ \frac{1}{2} \int B^2 d\mathbf{k} + \frac{1}{2} \int K^2 (A^2_t + A^2_\mathbf{k}) d\mathbf{k} + \frac{1}{2} \int K^2 (\varrho r + A^2_\mathbf{k}) d\mathbf{k} \quad (9)$$

Then we remove the origin to the centre of vibration $GY_t$. By the following transformation

$$r = q_{1t} + GY_t, \quad p_r = \mathbf{p}_t, \quad A_t = \varrho_t + Gy, \quad B_z = \mathbf{p}_t, \quad (10)$$

we get

$$\tilde{H}_t = -\frac{1}{2} G^2 Y_t + \frac{1}{2} \left( \mathbf{p}^2 + \int \mathbf{p}_t^2 d\mathbf{k} \right) + \frac{1}{2} \int K^2 (\varrho q_{1t} + q_{1t}^2) d\mathbf{k}$$

$$+ \frac{1}{2} \int (B^2_t + K^2 A^2_t) d\mathbf{k}$$

$$+ \frac{1}{2} \int (B^2_\mathbf{k} + K^2 A^2_\mathbf{k}) d\mathbf{k} + \frac{1}{2} \int \left( \frac{1}{\sin^2 \theta} - \frac{1}{\sin \theta} \sin \theta \frac{1}{\sin \theta} \left( \frac{1}{2} \cos \theta \right) + \frac{1}{2} \right) \varrho_\mathbf{k}^2 \sin \theta \mathbf{p}_\mathbf{k}^2$$

$$+ \frac{1}{\sin^2 \theta} \left( \frac{1}{2} \cos \theta \right)^2 + \frac{1}{2} \right) \{ \mathbf{p}_t^2 \}, \quad (11)$$
where

\[ Y_1 = \frac{\varphi^2}{K^2} \, d\kappa, \quad f = \frac{\varphi}{K^2} - Y_1 \varphi \]  

(12)

The factor \( GY_1 + q_t \) in the denominator of the last term in (11) means that the axis changes its length in the course of rotation, but we can neglect such a small interaction between the vibration and the rotation as in the case of a diatomic molecule. The correction, if necessary, can be treated as perturbation. Then the last term in (11) includes the terms of order \( G^0, G^{-1} \) and \( G^{-2} \). To calculate these quantities, we call attention to the fact that the angular momentum \( \mathbf{L} \), by the transformation (10), has the following form

\[ \mathbf{L} = \mathbf{L} + G \int f \beta \, d\kappa, \quad \mathbf{L} = \mathbf{L} + G \int f \beta \, d\kappa, \quad \mathbf{L} = \mathbf{L} \]  

(13)

where \( \mathbf{L} \) is given by the definition (5) replacing \( A_t, B_t \) with \( q_t, \rho_t \). If we use the following notation

\[ \mathcal{F} = \int \mathcal{F}(\kappa) \varphi(\kappa) \, d\kappa, \]  

(14)

we obtain

\[ \mathcal{H}_t = -\frac{1}{2} \left( G^2 Y_1 + \frac{1}{2} \int (B_t^2 + K^2 A_t^2) \, d\kappa + \frac{1}{2 Y_1} \sum \mathcal{F}_t \right) \]

\[ + \frac{1}{2} \int (B_t^2 + K^2 A_t^2) \, d\kappa + \frac{1}{2 Y_1} \sum \mathcal{F}_t \]

\[ + \frac{1}{2} \left( \int \varphi \, d\kappa + \rho_\eta \right) + \frac{1}{2} \int K^2 (q_t^2 + q_t^2) \, d\kappa \]

\[ - \frac{1}{2 Y_1} \left\{ \mathcal{F}_t (\rho_t - L_t^2) + (\rho_t - L_t^2) \mathcal{F}_t + \frac{\cot \theta}{i} \mathcal{F}_t \right\} \]

\[ + \frac{1}{\sin \theta} \left\{ (\rho_t - L_t^2) \varphi - \frac{1}{2} \cos \theta - L_t \cos \theta + L_t \sin \theta \right\} \mathcal{F}_t + \frac{1}{\sin \theta} \mathcal{F}_t (\rho_t - L_t^2) \]

\[ + \frac{1}{2 Y_1 G^2} \{ (\rho_t - L_t^2) + \frac{\cot \theta}{i} (\rho_t - L_t^2) \} \]
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\[ \frac{1}{\sin^2 \theta} \left( \rho - \frac{1}{2} \cos \theta - \cos \theta L_i + \sin \theta L_i \right)^2 + \frac{1}{2} \right). \quad (15) \]

In this equation we have omitted all higher terms of \( G^{-1} \).

\[ \frac{1}{2} \int B^2 \, dk + \frac{1}{2Y^2} B^2 + \frac{1}{2} \int K^2 A^2 \, dk \]

for \( \xi \)-component and the similar expression for \( \eta \)-component.

Next, we want to get the normal vibrations of the vibrational term, that is, the term of the order \( G^2 \). Let us call mesons polarizing in \( \xi, \eta \)-directions as "tangential" mesons, and those in \( \zeta \)-direction as radial mesons. The normal vibrations of the radial mesons are plane waves, so the radial mesons are not scattered in the approximation of \( G^2 \). The vibrational term of the tangential mesons is

\[ (16) \]

for \( \xi \)-component and the similar expression for \( \eta \)-component.

We can obtain the normal mode of the tangential mesons as stationary solutions of the following equations of motion:

\[ \dot{B} = -K^2 A + \varphi \int K^2 \varphi Adk, \]

\[ \dot{A} = B + \frac{1}{Y^2} \int fB \, dk. \quad (17) \]

By eliminating \( B \) from these equations, we obtain the following equation for a stationary solution \( A = \varphi e^{\pi i} \):

\[ (K^2 - P^2) K^2 \varphi = \frac{1}{Y} \varphi(k) \int \varphi K^2 \varphi aK. \quad (18) \]

Its solution can be written as a sum of a plane wave and a diverging spherical wave:

\[ K^2 \varphi(k) = \delta(k - p) + \frac{\varphi(k) \varphi(p)}{Y(1 - K - iW)} \left( \frac{1}{K - p} + i\pi \delta(K - p) \right), \quad (19) \]

where
From the equation (19) we get the scattering cross section of the tangential mesons by nucleons:

$$\sigma = \frac{2\pi}{p^3} \left( \frac{2W_f}{1-K^2} \right)^2 \frac{\pi}{p^3} \left[ \frac{4\pi^2 \phi(p)p^2}{(Y_1 p^2 + [2\pi^2 \phi(p)^2 p^2])} \right].$$

(21)

On the contrary, the scattering cross section of the radial mesons are zero.

Let us now transform the vibrational term into the normal form by the following transformation from $A(k), B(k)$ to $F(p), G(p)$. Let us define the real transformation-function $(k | p)$ by the following equation:

$$(K^2 - \rho^2)(k | p) = \frac{\phi(k)}{Y_1} \int (k | p) \phi(k) d\kappa , \text{ and } \int \phi(k) K^2 \phi(k) d\kappa = 0.$$  (22)

The orthogonality relation is obtained from this equation by a simple calculation, and we normalize it by the relation

$$\int (k | p) (k' | p') d\kappa = \delta(p - p').$$  (23)

Then we transform $A, B$ by the following relations:

$$K^2 A(k) = \int P^2 F(p)(k | p) d\p ,$$

$$B(k) = \int \left[ 1 + Y_1 (P^2 - K^2) \right] G(p)(k | p) d\p.$$  (24)

From the latter of these equations and (22) we have

$$\overline{B} = \int \overline{P} d\kappa = -Y_1 \int \phi(k) G(p) d\kappa d\p ,$$

* The inverse orthogonality relation is

$$\int (k | p)(k' | p) d\p = \delta(k - k') - \frac{1}{Y_1} \frac{\phi(k)}{K^2} \frac{\phi(k')}{K'^2} .$$  (23)

...
\[ K^2(B(k) + Y_r^* f(k) \overline{B}) = \int P^2(k \mid p) G(p) \, dp. \]  

(25)

Then the new variables \( F, G \) are calculated from the old ones as follows:

\[ P^2 F(p) = \int K^2 A(k \mid p) \, dk, \]
\[ P^2 G(p) = \int K^2 B(k \mid p) \, dk + \frac{1}{V^2} \int f(k) K^2(k \mid p) \cdot \int f(k) B(k) \, dk. \]  

(26)

The commutation relation of \( F, G \) is obtained from the above equations by making use of the commutation relation (4) between \( A \) and \( B \), and the relation (22):

\[ P^2 P^{*n}[F(p), G(p')] \]
\[ = i \left\{ \int K^2 K'^n(k \mid p)(k' \mid p') \{ \delta(k - k') - \varphi(k) \varphi(k') \} \, dk \, dk' \right. \]
\[ + Y_r^{*n} K^2(k \mid p') \, dk \cdot \int \int K^2(k \mid p)f(k') \{ \delta(k - k') - \varphi(k) \varphi(k') \} \, dk \, dk' \]
\[ = i \left\{ \int K^2(k \mid p)(k' \mid p') \, dk - Y_r^{-1} \int \varphi(k) K^2(k \mid p) \, dk \cdot \int \varphi(k) K^2(k \mid p') \, dk \right. \]
\[ + Y_r^{-1} \int \varphi(k)(k' \mid p) \, dk \cdot \int \varphi(k)(k \mid p') \, dk \]
\[ = i \int \left\{ K^4 - K^2(P^2 - P'^2) - K^2(K^2 - P'^2) + (K^2 - P^2)(K^2 - P'^2) \right\} (k \mid p)(k \mid p') \, dk \]
\[ = iP^2 P^{*n} \delta(p - p'), \]
\[ = iP^2 P^{*n} \delta(p - p'), \]
\[ = i \delta(p - p'), \]  

(27)

that is to say \( F \) and \( G \) are conjugate with each other.

In the next place, we must transform the energy operator into new variables. The calculation is straightforward. In the first place, we have, by (22),
\[ \int K^2 A^2 dk = \int \int dp\, dp' P^\alpha P^\beta F(p)F(p') \cdot \int dk \frac{(k|p)(k|p')}{K^2} \]
\[ = \int dp\, dp' F(p)F(p') P^\alpha \delta(p-p') = \int P^\alpha F(p) d\, dp . \]

In the next place,
\[ \int \frac{F^2}{2} d\, dk + \frac{1}{Y_1} \bar{F}^2 \]
\[ = \int \int G(p)G(p') \left\{ 1 + Y_1 (P^\alpha - K^\alpha) \right\} \left\{ 1 + Y_1 (P^\alpha - K^\alpha)(k|p) \right\} dk\, dp\, dp' \]
\[ + \int \int \int G(p)G(p') \varphi(k) \varphi(k')(k|p)(k'|p') dk\, dk'\, dp\, dp' \]
\[ = \int G(p)^2 d\, dp \]

where we have repeatedly used the relation (22). Thus the transformed energy operator has the following normal form:
\[ \frac{1}{2} \int \left\{ F_\xi(p)^2 + F_\eta(p)^2 + F^2 [G_\xi(p)^2 + G_\eta(p)^2] \right\} d\, dp . \quad (28) \]

§ 3. The rotational term.

The Hamiltonian (15), expressed in the new variables \( F_\xi, F_\eta \) and \( G_\xi, G_\eta \) defined in the preceding section and \( F_\xi, G_\xi \) defined by
\[ F_\xi(p) = q_{\alpha\xi} \varphi(p) + q(p), \quad G_\xi(p) = \rho_{\alpha\xi} \varphi(p) + \rho(p), \quad (29) \]
has the following form:
\[ \bar{H}_\xi = -\frac{1}{2} G^2 Y_1 + \frac{1}{2} \int [G(p)^2 + K^2 F(p)^2] d\, dp \]
\[ - \frac{1}{2G Y_1^2} \left\{ \bar{B}_\xi (\rho - L_\eta') + (\rho - L_\eta)' B_\xi \right\} \]
\[ + \sin^{-1} \theta \left( \rho + L_\xi \sin \theta - L_\xi' \cos \theta - \frac{1}{2} \cos \theta \right) \]
\[ \times \frac{\mathrm{d}}{\mathrm{d}\theta} B_\eta \]

\[ * F_\xi \text{ and } G_\xi \text{ are canonical with each other, that is to say, satisfy} \]
\[ [F_\xi(p), G_\xi(p')] = \delta(p-p'). \]
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\[ + \sin^{-1} \theta \frac{\bar{E}_t}{\sin \theta} \left( P^2 + L'_t \sin \theta - L'_t \cos \theta - \frac{1}{2} \cos \theta \right) \]

where \( \bar{E}_t \) are given by (25) and the similar formula for \( \bar{E}_n \) and the angular momentum \( L' \) must be expressed in the new variables.

The purpose of calculation in this section is to obtain the expression for isobar energies as the function of the isobar charge, that is, the proper charge of the nucleon and the charge of bound mesons. For this purpose, we assume there are no free mesons present and calculate the expectation value of the energy to the order \( G^{-2} \). In doing so, the linear terms in (25) must either be treated as perturbation or eliminated by a suitable transformation. Let us here take the latter way. As the generating function of the transformation we take the following one:

\[ S = \exp \left[ \frac{i \rho^2}{GY_2} A_t + \frac{i}{GY_2} P - \frac{1}{2} \cos \theta \right] \]

The transformation caused by it become:

\[ F_t, F_n = F_t, F_n, F_t \]

\[ G_t \rightarrow G_t + \frac{1}{GY_2} \int K^2 f(k) |p| \, dk, \]

\[ G_n \rightarrow G_n + \frac{1}{GY_2} \int K^2 f(k) |p| \, dk \cdot \frac{P - \frac{1}{2} \cos \theta}{\sin \theta} \]

\[ \bar{E}_t \rightarrow \bar{E}_t + \frac{2}{GY_2} \rho \]

\[ \bar{E}_n \rightarrow \bar{E}_n + \frac{2}{GY_2} \rho \]

\[ \rho \rightarrow \rho - \frac{1}{GY_2} A_t \rho \cos \theta - \frac{1}{2} \frac{\cos \theta}{\sin \theta} \]

\[ \theta \rightarrow \theta - \frac{1}{GY_2} A_t \]

\[ L'_t \rightarrow L_t - \frac{\rho}{GY_2} \]

\[ L'_n \rightarrow L_n - \frac{\rho}{GY_2} \]

\[ L'_n \rightarrow L_n + \frac{1}{GY_2} \rho \frac{P - \frac{1}{2} \cos \theta}{\sin \theta} \]

(31)
Consequently we have

\[
\int G^2 d\mathbf{p} \rightarrow \int G^2 d\mathbf{p} + 2 \frac{\rho_0}{G Y_1^2} \vec{p}_t + \frac{\rho^2}{G^2} \left( \frac{1}{Y_2^2} - \frac{1}{Y_2^2} \right)
\]

\[
\int G^2 d\mathbf{p} \rightarrow \int G^2 d\mathbf{p} + 2 \frac{1}{G Y_1^2} \frac{\rho_0 - \frac{1}{2} \cos \theta}{\sin \theta}
\]

\[
+ \frac{1}{G^2} \left( \frac{\rho_0 - \frac{1}{2} \cos \theta}{\sin \theta} \right)^2 \left( \frac{1}{Y_2^2} - \frac{1}{Y_2^2} \right).
\]

Inserting these relations into (30), the term of the order \(G^{-1}\) becomes

\[
\frac{1}{G Y_1^2} \left\{ \vec{p}_t \vec{I}^t - \vec{p}_t \vec{L}^t + \cot \theta \vec{P}_t \vec{L}^t \right\},
\]

where the first two terms are functions of only free mesons and independent from the rotation. As we concern with the rotation, they can be omitted. The last term \(\cot \theta \vec{P}_t \vec{L}^t\) is not independent from the rotation, but its mean value in the state of vibration is zero. Moreover, the second order correction caused by its perturbation also reduces to zero, which results from the fact that the component of the angular momentum along the axis is a good quantum number as in the case of a diatomic molecule. Thus, all terms of the order \(G^{-2}\) are eliminated essentially by the transformation.

In the next place, the term of the order \(G^{-2}\) becomes

\[
\frac{1}{2G^2 Y_2^2} \left\{ \rho_0 + \cot \theta \rho_0 + \left( \frac{\rho_0 - \frac{1}{2} \cos \theta}{\sin \theta} \right)^2 + \ldots \right\},
\]

which contains many other terms, but, their mean values in the state of the vibration vanish. As we are here interested only in getting stationary energy of the isobars, these terms can be taken as irrelevant. The above equation shows that the moment of inertia is not \(G^2 Y_2^2\) but \(G^2 Y_3\).

The eigenvalues of the above operator was obtained by the method of Tamm,\(^{(5)}\) which was used in the theory of a diatomic molecule: According to his result, the eigenvalues of the operator (35) is given by

\(^{(5)}\) Ig. Tamm: ZS. f. Phys., 71 (1931), 141.
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\[ \frac{1}{2G^2 Y^2} \{ J(J+1) + \text{const.} \}, \quad (36) \]

As \( J = Q + \frac{1}{2} \), in the lower state, (36) becomes

\[ \frac{1}{2G^2 Y} \{ Q(Q+2) + \text{const.} \}, \quad (37) \]

where \( Q \) is the charge of the isobar.