Meson-Baryon Couplings in a Quark Model*

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The widths for decay of low-lying baryons of negative parity into baryon plus pseudoscalar meson are determined in a quark model and extensively compared with experiment. One conclusion is that \( \pi^*(1816) \) is not in an octet with \( N^*(1518) \) and \( Y^*_1(1660) \). A second prediction is a kinematical factor in \( s \)-wave decay enhancing the decay into high-mass mesons \((K \otimes \eta)\), which provides a qualitative reason for \( \eta \) peaks at threshold. Properties of the many missing baryon resonances are discussed. Channels appropriate for the search for some of these are indicated.

I. INTRODUCTION

In this article we calculate the widths for strong decay of low-lying baryons, especially those of negative parity, into pseudoscalar meson plus baryon. Our baryon model is the three-quark system.† We first discuss the model and the form of the calculation, then we present the results and discuss implications of the current data.

Baryon states are assumed to be Gell-Mann-Zweig triplets of quarks governed by nonrelativistic dynamics.‡ It is assumed in conformity with Dalitz's general analysis that the baryon forms bases for representations of the group \( SU(6) \times O_3 \), with the lowest-lying representations being:

- (a) \((56)\) even-parity states of \( L=0 \) corresponding to the well-known \( \frac{1}{2}^+ \) octet and \( \frac{3}{2}^+ \) decuplet;
- (b) \((70)\times(3)\) odd-parity states of \( L=1 \) corresponding to two \( \frac{1}{2}^- \) octets whose \( N^* \) components are known, two \( \frac{3}{2}^- \) octets and a \( \frac{5}{2}^- \) decuplet with \( N^*(T=\frac{3}{2})(1518) \), \( Y^*_1(1660) \), and \( \Xi^*(1816) \) known, a \( \frac{1}{2}^- \) decuplet whose \( N^* \) component is known \((?\) \), a \( \frac{3}{2}^- \) octet with known \( N^* \) and \( Y^*_1(1675) \), and the known \( \frac{1}{2}^- \) and \( \frac{3}{2}^- \) singlets \( Y^*_1(1405) \) and \( Y^*_8(1520) \), respectively.

These known resonances and the assignments we would like to make on the basis of our results are shown in Table I.

In a previous paper we demonstrated that this model can be justified by explicit dynamical calculation to determine the lowest-lying states. The main assumptions in this calculation were parastatistics§ and operation of \( s- \) and \( p \)-wave forces in \( Q-Q \) pairs. The most attractive states separated into three sets:

- (a) and (b), above; and
- (c) \((20)\times(3)\) even-parity states of \( L=1 \) corresponding to \( \frac{1}{2}^+ \), \( \frac{3}{2}^+ \), \( \frac{5}{2}^+ \) singlets. Possible candidates for this multiplet are \( \frac{1}{2}^+ \), \( T=\frac{3}{2} \) \( N^*(1450) \) (or Roper resonance), and \( \Xi^*(1705) \) of unknown \( J^P \). There are also \( Y^* \) possibilities. We will not present an extensive discussion of couplings to \((20)\times(3)\) states.

We will show below that, in contrast to the mass spectrum, the baryon partial widths are essentially independent of the choice of quark statistics. The results as presented here, where certain spatial integrals are treated as parameters rather than being evaluated in detail, are the same for Fermi statistics and parastatistics and depend only on the assignments of the baryon states.

II. THE MODEL

Within each set \((a), (b), (c)\), the states are degenerate in the presence of the spin- and unitary-spin-independent forces. The actual mass splittings are not of interest to us here: We assume that the wave functions are not influenced by the splitting.


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8 In most cases our data are taken from A. H. Rosenfeld et al., Rev. Mod. Phys. 37, 633 (1965). For recently discovered \( N^* \) states we have referred mainly to R. H. Dalitz (Ref. 5) and to P. Barea et al., Phys. Letters 18, 342 (1965), obtaining widths by our own analysis.
The wave functions are sums of products of spatial functions $\psi(p_1, p_2, p_3)$, intrinsic functions $X(1, 2, 3)$, and $SU(3)$ functions $\phi(1, 2, 3)$. We distinguish different functions by their permutation symmetries. There are four permutation symmetries for functions of three variables: symmetric under any permutation labeled, e.g., $\psi'$, antisymmetric under any odd permutation labeled, e.g., $\psi''$, of mixed symmetry but symmetric with respect to, say, the 2, 3 pair of coordinates labeled, e.g., $\psi''$, and of mixed symmetry but antisymmetric with respect to the 2, 3 pair labeled, e.g., $\psi'$. The rules for constructing products of these functions with definite permutation symmetry are well known. With particles of intrinsic spin ½ only states $X$ (total spin $\frac{1}{2}$), and $X'$ and $X''$ (each with total spin $\frac{3}{2}$) can be formed. The $SU(3)$ states $\phi$, $\phi'$, $\phi''$, and $\phi^*$ correspond to $SU(3)$ representations $(1)$, $(8)$, $(8')$, and $(10)$, respectively.

Consider first the case of parastatistics where among the three quarks Bose statistics are obeyed. The $(56)$ is characterized by a symmetric spatial wave function $\psi$ with total angular momentum $L=0$. The overall symmetric states which can be constructed are

\[
\psi = \psi(X'X' + X''X''),
\]

\[
\psi = \psi X'X' \psi^a.
\]

For the $(70, 3)$ the spatial function is of mixed symmetry with $L=1$. Thus $\Psi$ contains terms

\[
(\sum_{MSL} [L M_L] M_S M_\rho \psi_L M_\rho \psi_X M_\sigma M_\pi \phi^a \phi^b)
\]

\[
(\sum_{ML} \int dp_1 dp_2 dp_3 \psi_{L=0} (p_1 - \frac{2}{3} q, p_2) [\psi - \alpha_{M_Q} M_\rho \psi']_{L=0, M_L} (p_3, p_4) \cdot S_{ML},
\]

where $S$ is the spin-$SU(3)$ matrix element. This spatial integral which occurs here has two distinct forms corresponding to $s$- and $d$-wave decays. Meanwhile, for all $p$ wave decays within the $(56)$ and within the $(70 \times 3)$ the spatial integral involving the direct term $q$ (which presumably dominates in this case) is just the normalized integral for the spatial wave function and can be done. The $s$, $p$, and $d$ wave decays thus correspond to three distinct types of transitions. Within each type of transition the ratios of the coupling constants, or partial widths are meaningful. We do not need to calculate the spatial integrals. In $(56) \rightarrow (56) + \Pi$, where the standard $SU(6)$ coupling results apply,11 and $(70, 3)\rightarrow
\[ \Gamma(N^*(1238) \rightarrow N+\pi) = 120 \text{ MeV}. \]

For the \( s \)- and \( d \)-wave decays, the same spatial integrals occur for all transitions within each type, so within each type one of the partial widths \( \Gamma_i \) can be chosen as input. The spatial integrals are energy-dependent. It is only here, and in the phase space, that we take into account the actual symmetry-violating masses of the baryons and mesons. The partial widths \( \Gamma_i \) are thus proportional to the square of a Yukawa coupling constant \( g_i \) where this is the spin-\( SU(3) \) matrix element, times an energy dependence which would arise from the spatial integration. The \( g_i \)'s are algebraic quantities which could be readily calculated using appropriate group theory. The energy dependence is, however, intimately associated with the quark model and contains a surprise. We must examine both the direct and recoil contributions in the spatial integral in the \((70) \times (3) \rightarrow (56) \) matrix element [see Eq. (3)]. For the quark mass we can assume only that \( \omega / M_q \lesssim 1 \), since some believe that the effective mass in a bound system is as low as \( M_N/3 \). Meanwhile we can assume that \( (qR)^< 1 \), where \( R^{-1} \) is the typical expectation value of a quark momentum operator. The order of magnitude of the direct and recoil terms for \( s \)- and \( d \)-wave decays is given in terms of these parameters in Table II.\(^{12} \) It thus seems reasonable that the recoil term dominates the \( s \)-wave decay and the direct term dominates the \( d \)-wave decay. This is what we assume. The partial widths are then taken to have the forms:

\[
\begin{align*}
\text{s wave:} \quad \Gamma &= e_6 g^6 q^2, \\
\text{p wave:} \quad \Gamma &= e_6 g^6 q^2 / (1 + \alpha q^2), \\
\text{d wave:} \quad \Gamma &= e_6 g^6 q^2 / (1 + \alpha q^2)^2.
\end{align*}
\]

Here \( \alpha = (600 \text{ MeV})^{-2} \) introduces the finite range of the angular momentum barrier.\(^{13} \) In fact, the direct and recoil terms are very different for the \( s \) wave case, and we have the surprising prediction of a factor \( e_6^6 \) which is very sensitive to the mass of the emitted meson for a low-\( Q \) decay. In the \( d \)-wave case the difference between direct and recoil contributions is not large. One could not discriminate between these dependences by current experiment alone.

To complete the description of the calculation note that for all but the \((8) \times (8) \rightarrow (8) \) transitions, the quantity \( g_i \) is given by the Wigner-Eckart theorem for \( SU(3) \), which expresses it as a "reduced width" (independent of \( I, F \)), times the appropriate isoscalar factor, viz.,

\[ g_i = X_i (\Pi B | B \rangle). \]

For the \((8) \times (8) \rightarrow (8) \) transition, on the other hand, \( g_i \) depends on two reduced widths, \( X_1 \) and \( X_2 \), according to the formula

\[ g_i = X_1 \langle 8 | 8 \rangle + X_2 \langle 8 | 8 \rangle. \]

Let us digress to see that the partial widths will be the same whether we use Fermi statistics or Bose statistics in constructing the baryon wave functions. The over-all antisymmetric wave functions are, for the \((56) \) (where the spatial state has \( L=0 \) and is antisymmetric):

\[
\begin{align*}
(8): \quad &\Psi = \psi^s (x' \phi' + x'' \phi''), \\
(10): \quad &\Psi = \psi^x x \phi'.
\end{align*}
\]

Meanwhile for the \((70,3) \) (where the spatial states have \( L=1 \) and mixed symmetry) we have essentially

\[
\begin{align*}
(1): \quad &\Psi = \phi' \phi'' \\
(8): \quad &\Psi = \phi'(\phi' - \phi'') \\
&\quad - (\phi' \phi'' + \phi'' \phi') \\
&\quad + b(\phi' \phi'' - \phi' \phi'') \\
(10): \quad &\Psi = (\phi' \phi'' - \phi' \phi'').
\end{align*}
\]

Consider, for example, the transition from a \((1) \) in the \((70,3) \) to an \((8) \) in the \((56) \). The factorization of the matrix elements into spatial\times spin-\( SU(3) \) part has the form

\[
\begin{align*}
\text{Bose:} \quad & (\psi^s, O_{\text{B}} \psi') \cdot (x' \phi', O_{\text{B}} x \phi'), \\
\text{Fermi:} \quad & (\psi^s, O_{\text{F}} \psi') \cdot (x' \phi', O_{\text{F}} x \phi').
\end{align*}
\]

Here the operator has been factored into space and spin-\( SU(3) \) parts using the notation \( O_{\text{B}} \cdot O_{\text{F}} \). We see that the spin-\( SU(3) \) matrix elements are identical. This result holds for all cases. Indeed if we denote the spatial function of various symmetries by \( A, S, M', M'' \), in obvious notation, the only modifications that the spatial integrals would suffer in going from parasitistics to Fermi statistics is \( S \rightarrow A, A \rightarrow S, M' \rightarrow M'', M'' \rightarrow M' \). Within a given type of transition the spatial integral is the same for the transitions. The only difference between Bose and Fermi statistics is then that these spatial factors are formally different. Since we treat a given spatial integral as proportional to a free parameter (except in Sec. VI, below), our results are independent of the choice of statistics.

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\(^{13}\) These estimates do not take into account the degree of overlap on the spatial integral. This will depend on the dynamics and could easily give rise to results at variance with Table II. In particular, in the \( d \)-wave case the quark dynamics of Ref. 6 suggest that the overlap may be much greater in the recoil than direct term.

\(^{14}\) The choice of \( \alpha \) effectively determines the scale for the \( f_{p1}^*(1520) \) decays. The results are not, in fact, very sensitive to \( \alpha \), considering the experimental errors.
### Table III. Partial widths for decay of low-lying baryons. There are three major types of decay: The top group is $d$-wave decays. The second group is $s$-wave decays. The bottom group is $p$-wave decays. Masses are chosen to check the assignment of known particles. The plus superscript indicates a mass arbitrarily introduced for illustrative purposes in the case of an undiscovered particle. Other notations are explained in the text.

<table>
<thead>
<tr>
<th>$SU(3)$ transition $B' \rightarrow B$</th>
<th>Reduced width $\lambda_\pi$, (8) $\times$ (8) = (8) $(X_3, X_2)$</th>
<th>Transition $\Pi^*$</th>
<th>For $\frac{g^2}{\lambda_{\pi}^2}$ octets $^a(8); ^b(8)$</th>
<th>$\Gamma_{\pi}$ (MeV) Theory</th>
<th>$\Gamma_{\pi}$ (MeV) Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>(8)$<em>{\pi}^-$ $\rightarrow$ (8)$</em>{\pi}^+$</td>
<td>$(-\sqrt{\frac{8}{3}}, -\sqrt{\frac{8}{3}})$</td>
<td>$\Pi^*(1690) \rightarrow N^+ + \pi^-$</td>
<td>( \frac{1}{6} )</td>
<td>17</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Pi^*(1765) \rightarrow \Delta^+ + \pi^+$</td>
<td>(-\frac{1}{9})</td>
<td>6</td>
<td>( \leq 2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\Lambda^+ + \pi^-)</td>
<td>4/9</td>
<td>29</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Xi^*(1930)^+ \rightarrow \Sigma^+ + \pi^+ )</td>
<td>1/2</td>
<td>0</td>
<td>not observed</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\Sigma^+ + \pi^-)</td>
<td>1/6</td>
<td>11</td>
<td>not observed</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\Xi^+ + \pi^-)</td>
<td>(-2/3)</td>
<td>27</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\Sigma^+ + \Xi^-)</td>
<td>1/6</td>
<td>11</td>
<td>0</td>
</tr>
<tr>
<td>(8)$<em>{\pi}^-$ $\rightarrow$ (10)$</em>{3/2}^+$</td>
<td>$\sqrt{(35/12)}$</td>
<td>$\Pi^*(1690) \rightarrow N^+ + \pi^-$</td>
<td>7/18</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Pi^*(1385) + \pi^-)</td>
<td>7/4</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Xi^<em>(1930)^+ \rightarrow \Xi^</em>(1930) + \pi^+)</td>
<td>7/12</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>(1)$<em>{3/2}^- \rightarrow$ (8)$</em>{3/2}^+$</td>
<td>$\sqrt{5}$</td>
<td>$\Pi^*(1520) \rightarrow N^+ + \pi^-)</td>
<td>(-\frac{5}{4})</td>
<td>5</td>
<td>5</td>
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<tr>
<td></td>
<td></td>
<td>(\Sigma^+ + \pi^-)</td>
<td>15/8</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\Lambda^+ + \pi^-)</td>
<td>5/18</td>
<td>36</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\Xi^* (1816) \rightarrow \Xi^+ + \pi^-)</td>
<td>5/72</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\Xi^* (1990)^+ \rightarrow \Xi^* (1990)^+ + \pi^-)</td>
<td>5/18</td>
<td>6</td>
<td>1</td>
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<tr>
<td></td>
<td></td>
<td>(\Lambda^+ (1518) \rightarrow \Lambda^+ + \pi^-)</td>
<td>(-\frac{1}{10/9})</td>
<td>(-1/36)</td>
<td>52</td>
</tr>
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<td></td>
<td></td>
<td>(\Lambda^+ + \pi^-)</td>
<td>(-\frac{5}{8})</td>
<td>0</td>
<td>...</td>
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<tr>
<td></td>
<td></td>
<td>$\Gamma_{\pi}^*(1660) \rightarrow N^+ + \pi^-)</td>
<td>(-5/108)</td>
<td>(-2/27)</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\Lambda^+ + \pi^-)</td>
<td>125/108; (1/54)</td>
<td>29</td>
<td>0.5</td>
</tr>
<tr>
<td>(8)$<em>{3/2}^- \rightarrow$ (8)$</em>{3/2}^+$</td>
<td>(\sqrt{\left(\frac{1}{2}\right)^3}) (\frac{1}{0(3 + \lambda)^2})</td>
<td>$\Pi^*(1660) \rightarrow N^+ + \pi^-)</td>
<td>(-5/72)</td>
<td>(1/36)</td>
<td>3</td>
</tr>
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<td></td>
<td></td>
<td>(\Sigma^+ + \pi^-)</td>
<td>5/4</td>
<td>0</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\Lambda^+ + \pi^-)</td>
<td>(-5/24)</td>
<td>(-1/12)</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\Xi^* (1816) \rightarrow \Xi^+ + \pi^-)</td>
<td>5/72</td>
<td>1/9</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\Xi^* (1990)^+ \rightarrow \Xi^* (1990)^+ + \pi^-)</td>
<td>10/9</td>
<td>(-1/36)</td>
<td>7</td>
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<tr>
<td></td>
<td></td>
<td>(\Lambda^+ - \pi^-)</td>
<td>0(1 MeV)</td>
<td>0(1 MeV)</td>
<td>0(1 MeV)</td>
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<tr>
<td>(8)$<em>{3/2}^- \rightarrow$ (10)$</em>{3/2}^+$</td>
<td>(\sqrt{(35/6)}) (\frac{1}{a + b^2}) (\frac{1}{a + b^2}) (\frac{1}{a + b^2}) (\frac{1}{a + b^2})</td>
<td>(\Pi^*(1405) \rightarrow N^+ + \pi^-)</td>
<td>(-2/3)</td>
<td>(-1/6)</td>
<td>21 input</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\Lambda^+ + \pi^-)</td>
<td>1/18</td>
<td>...</td>
<td>...</td>
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<tr>
<td></td>
<td></td>
<td>(\Lambda^+ + \pi^-)</td>
<td>(-1/81)</td>
<td>48</td>
<td>...</td>
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<td></td>
<td></td>
<td>(\Sigma^+ + \pi^-)</td>
<td>(-1/81)</td>
<td>0.06q</td>
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<td></td>
<td></td>
<td>(\Sigma^+ + \pi^-)</td>
<td>1/243</td>
<td>19</td>
<td>0.04q</td>
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<td></td>
<td></td>
<td>(\Lambda^+ + \pi^-)</td>
<td>(-1/162)</td>
<td>17</td>
<td>0.038q</td>
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<td></td>
<td></td>
<td>(\Sigma^+ + \pi^-)</td>
<td>1/162</td>
<td>0.038q</td>
<td>...</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\Xi^* (1820) \rightarrow \Xi^+ + \pi^-)</td>
<td>(-1/162)</td>
<td>26</td>
<td>not observed</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\Xi^+ + \pi^-)</td>
<td>(-1/162)</td>
<td>13</td>
<td>not observed</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\Xi^+ + \pi^-)</td>
<td>(-1/162)</td>
<td>19</td>
<td>not observed</td>
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<td></td>
<td></td>
<td>(\Xi^+ + \pi^-)</td>
<td>1/162</td>
<td>0.038q</td>
<td>not observed</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\Omega(?) \rightarrow \Xi^+ + \pi^-)</td>
<td>2/81</td>
<td>not observed</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>(\Xi^+ + \pi^-)</td>
<td>(-1/81)</td>
<td>240</td>
<td>60</td>
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<td></td>
<td></td>
<td>(\Xi^+ + \pi^-)</td>
<td>(-2/81)</td>
<td>0.17q</td>
<td>0.17q</td>
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<td></td>
<td></td>
<td>(\Xi^+ + \pi^-)</td>
<td>1/18</td>
<td>0.27q</td>
<td>0</td>
</tr>
</tbody>
</table>

*Notes: $a$ and $b$ are coefficients determined by the isospin and charge quantum numbers of the particles, $\lambda_{\pi}$ is the partial width for the decay of the $\pi$-meson, and $\Gamma_{\pi}$ is the partial width for the decay of the $\pi$-meson.*
TABLE III (continued)

<table>
<thead>
<tr>
<th>SU(3) transition</th>
<th>Reduced width $X$ for $(8) \times (8) = (8): (X_1, X_2)$</th>
<th>Transition</th>
<th>For $\frac{g^2}{\Gamma}$ octets $\Gamma_\text{Theory}$ $\Gamma_\text{Experiment}$</th>
<th>$\Gamma_\text{Theory}$ $\Gamma_\text{Experiment}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N^*(1700) \rightarrow N^+\pi$</td>
<td>as above</td>
<td>410</td>
<td>100</td>
<td>$\approx 200$</td>
</tr>
<tr>
<td>$N^-\pi$</td>
<td>as above</td>
<td>90</td>
<td>90</td>
<td>...</td>
</tr>
<tr>
<td>$\Sigma^+\bar{K}$</td>
<td>as above</td>
<td>80</td>
<td>0</td>
<td>...</td>
</tr>
<tr>
<td>$\Sigma^+\bar{K}$</td>
<td>1/162; 8/81</td>
<td>0.03q</td>
<td>0.48q</td>
<td>...</td>
</tr>
<tr>
<td>$Y_3^*(1750)^+ \rightarrow N^+\bar{K}$</td>
<td>-1/243; -16/243</td>
<td>19</td>
<td>300</td>
<td>...</td>
</tr>
<tr>
<td>$Y_3^*(1670) \rightarrow N^-\bar{K}$</td>
<td>1/9</td>
<td>370</td>
<td>0</td>
<td>...</td>
</tr>
<tr>
<td>$Y_3^*(1670) \rightarrow N^-\bar{K}$</td>
<td>-1/162; 2/81</td>
<td>17</td>
<td>68</td>
<td>...</td>
</tr>
<tr>
<td>$\Xi^-\bar{K}$</td>
<td>32/243; -8/243</td>
<td>0.03q</td>
<td>0.15q</td>
<td>...</td>
</tr>
<tr>
<td>$\Xi^-\bar{K}$</td>
<td>-2/81</td>
<td>102</td>
<td>102</td>
<td>...</td>
</tr>
<tr>
<td>$F^*(1880)^+ \rightarrow \Sigma^+\bar{K}$</td>
<td>1/162; 8/81</td>
<td>13</td>
<td>210</td>
<td>...</td>
</tr>
<tr>
<td>$\Xi^-\bar{K}$</td>
<td>1/18</td>
<td>0</td>
<td>0.34q</td>
<td>0</td>
</tr>
</tbody>
</table>

$(8)_{1/2}^- \rightarrow (8)_{1/2}^+$ $4/9$

$N_1^*(1585)^+ \rightarrow N^*(1328)^+\pi$ | 10/81 | 70 | ... |

$(8)_{1/2}^- \rightarrow (10)_{3/2}^+$ $\frac{\sqrt{10}}{3\sqrt{3}}$

$N_1^*(1518) \rightarrow N^*(1328)^+\pi$ | 8/81; -20/81 | 30 | 74 | $\approx 50$ |

$(10)_{3/2}^- \rightarrow (10)_{3/2}^+$ $\frac{\sqrt{10}}{3\sqrt{3}}$

$N_1^*(1518) \rightarrow N^*(1328)^+\pi$ | 8/81; -20/81 | 30 | 74 | $\approx 50$ |

$(8)_{1/2}^- \rightarrow (1)_{1/2}^+$ $\frac{\sqrt{2}}{3\sqrt{3}}$

$Y_3^*(1765) \rightarrow Y_3^*(1520)^+\pi$ | -2/27 | 16 | ... |

$(8)_{1/2}^- \rightarrow (1)_{1/2}^+$ $\frac{\sqrt{2}}{3\sqrt{3}}$

$Y_3^*(1765) \rightarrow Y_3^*(1520)^+\pi$ | -2/27 | 16 | ... |

* See C. Michael, Phys. Letters 21, 93 (1966); A. W. Hendry and R. G. Moorhouse, ibid., 18, 171 (1965); A. T. Davies (private communication).

** This guess is based on data of D. Bierie et al., Phys. Rev. Letters 15, 641 (1965).**

III. RESULTS

The results of the calculations of partial widths $\Gamma_\text{I}$ are given in Table III. For $(8)_{3/2}^-$ and $(8)_{1/2}^-$ states (the notation is: $SU(3)$ multiplicity), $\rho^2$, the predictions are given for the doublet and quartet quark intrinsically spin $(S)$ states. These octets are labeled $^S\langle S \rangle$ and $^Q\langle Q \rangle$, respectively. The reduced widths are given in the doublet and quartet amplitudes, $a$ and $b$, i.e., the wave function is

$$\Psi = a\Psi(S=\frac{1}{2}) + b\Psi(S=\frac{3}{2}),$$

where the $\Psi'$s are normalized. The threshold production of K's and q's is given in the form: partial width $\Gamma = \rho g$, where $q$ and $\Gamma$ are both in MeV. The sign of $g$ is entered before the values of $\rho^2$. This column can be used to evaluate rates in case of mixtures.\footnote{Relative values of these signs can also be observed directly. See, e.g., A. Kerman and W. M. Smart, Phys. Rev. Letters 17, 18, 171 (1965).}

First some general remarks about the results: (1) The recoil factor $\omega^2$ in the $(70, 3) \rightarrow (56)$ $s$-wave rate strongly enhances $K$ and $q$ relative to $\pi$ for low-$Q$ decays. This means that the $K$ and $q$ partial widths will be much larger than indicated by $SU(3)$ and phase space, which

\footnote{Relative values of these signs can also be observed directly. See, e.g., A. Kerman and W. M. Smart, Phys. Rev. Letters 17, 18, 171 (1965).}
can explain the prominence of the \( N+\eta \) and \( \Lambda+\eta \) peaks near threshold.

(2) At present the sensitivity of experiments is such that many of the resonances will be unobserved. Many states have very small width and/or high inelasticity or are very broad such that they may not be observed for some time. It would be a mistake to assume that lack of observation of a resonance necessarily implies higher mass.

One final remark before detailed discussion of the results. There are three negative-parity \( Y^* \)'s, \( Y_1^* \), and \( Z^* \)'s both for spin \( \frac{3}{2} \) and spin \( \frac{1}{2} \). Some of these probably overlap. For this reason presently quoted experimental partial widths for \( Y_1^* \)'s at 1660 and 1766 (as well as those for the \( N^* \)'s recently discovered in partial wave analyses) must be regarded as preliminary. For purposes of discussion we will, however, take the present numbers quite seriously.

A. Detailed Discussion of Table III

1. Decays of \( \frac{1}{2}^- \) Baryons

The \( (8)_{1/2}^- \) decays are in reasonable agreement with the 6 experimental numbers (including \( p \)-wave decay at bottom of Table III). The \( d \)-wave decay of \( (8)_{1/2}^- \) baryons (into \( (8)_{1/2}^+ \) and \( (10)_{3/2}^+ \) plus pseudoscalar mesons) are the same as given by Coyne et al.\(^3\) in their study of the (1134) representation of \( SU(3) \). We determine the scale factor for \( d \)-wave decays \( c_d \) of Eq. (4) from the input width \( \Gamma_{1/2}^+(1765) \to \Lambda\pi \). Two members of the (8)\( _{1/2}^- \), \( Y^* \) and \( Z^* \), have not been observed. They are discussed below. \( \{ \)The decay of the (1)\( _{3/2}^- \), the \( Y_6^* \) (1520), is not the same as in the (1134) as given by Coyne et al., but this fact is of little significance because of the factor \( \alpha \) [see Eq. (4)] which can be adjusted to give the correct relation between the low-\( Q \) \( Y^* \) decays and the high-\( Q \) \( Y_1^* \) decays.\( \} \) The agreement with current data for \( Y_1^*(1765) \) and \( N^*(1690) \) is satisfactory.

2. Decays of \( \frac{3}{2}^- \) baryons

All other (70,3) states except the \( N \) decuplets have 2- or 3-fold degeneracy. Assuming \( SU(3) \) is good and that the octet states divide into \( \frac{3}{2} \) and \( \frac{1}{2} \) we have to choose which of the assignments most suits experiment.

For known \( \frac{3}{2}^- \) baryons we have one of our most interesting results: they are not in the same octet.\(^4\) The \( N^*(1518), \ Y^*(1660) \) (assumed \( \frac{3}{2}^- \)), and \( Z^* \) (1816) are fitted semi-quantitatively by placing \( N^* \) and \( Y^* \) in the \( \frac{3}{2} \) and \( Z^* \) in the (10). An excellent fit to data in the latter case can be obtained with 10\% \( Y^* \) \( \frac{3}{2} \) mixing. The state \( (9/10)^2 [1/(1/2)^{1/2}) \) \( Z^* \) yields satisfactory widths: \( \Gamma_{\Lambda\pi}, \Gamma_{\Sigma\pi}, \Gamma_{\Sigma\pi}, \Gamma_{\Sigma\pi} = 2 \), \( 1 \), \( 0 \), \( 0 \), \( 4 \) MeV respectively. Small admixture to a dominant \( \frac{3}{2} \) state does not work if we believe that \( Z^* \) (1816) \( \to \Sigma + K \) is very small. For the \( Y^*_1 \) (1660) small admixture of \( \frac{1}{2} \) to the dominant \( \frac{3}{2} \) yields good agreement with current experiment. The many unobserved \( \frac{3}{2}^- \) states are discussed below.

3. Decays of \( \frac{1}{2}^- \) baryons

Among \( \frac{1}{2}^- \) baryons the \( Y_1^*(1405) \) \( \frac{1}{2}^- \) observed decay width is relatively large. It would be convenient for the model if the actual width were about \( \frac{1}{2} \) the quoted 35 MeV or if the theoretical widths were slightly damped compared with the expression (4a) for higher-\( Q \) decays.

The only other observed \( \frac{1}{2}^- \) states are \( N^* \) (1540) and \( Y_7^*(1670) \) associated with \( \eta \) decays, which should be placed in the \( \frac{3}{2} \) (8), the probable \( T = \frac{3}{2} \) \( N^* \) (1680) in the (10), and the very wide \( N^* \) (1700) which should be placed in the \( \frac{1}{2} \) (8). Only fair agreement is achieved. Possible mixing which could remedy the disagreements is exemplified by the \( Y_8^*(1670) \), whose observed narrowness is a serious difficulty. One needs to adjust the mixture sensitively. Just as an example,

\( \sqrt{(17/20)} \) \( \frac{1}{2} \) \( - 1 \sqrt{(10)} \) \( +(1/20\) \( \frac{3}{2} \)

yields \( \Gamma_{\Xi_{\eta}} = 3, \Gamma_{\Xi_{\eta}} = 12, \Gamma_{\Xi_{\eta}} = 0.25g \). One of the most interesting results is that the \( \frac{3}{2} \) \( \frac{1}{2} \) lies above \( \frac{1}{2} \). Several other \( \frac{1}{2}^- \) states are indicated to have reasonable widths so that they should be observable in spite of the low statistical weight. These are discussed in Sec. V below.

We do not present here any model of \( \Xi \Xi \) forces which would lead to the mixtures mentioned above.

The properties of the missing resonances in the (70,3) are summarized in Table IV. In this table “input width” refers to the formation channel of the resonance relevant in most experiments. The three resonances which should be most visible are not inconsistent with observation. There is a possible \( \Xi_{\pi} \) around 1700 MeV.\(^12\) There is a \( \Xi^* \) (1930) quite consistent with the 1870 state indicated in the table. Masses in the \( \Xi^* \) around 2040 MeV are not well explored.

IV. MISSING \( \frac{3}{2}^- \) AND \( \frac{1}{2}^- \) STATES

Now consider the as yet unobserved resonances in the (70,3) of spin \( \frac{3}{2} \) and \( \frac{1}{2} \). All reference below is to the \( \frac{3}{2} \) states except when explicitly stated. Very crude mass guesses are indicated for these particles in Table IV.

\( N^* \): There are a (10) and a \( \frac{3}{2} \) with \( I = \frac{3}{2}, \frac{1}{2} \), respectively. These \( N^* \)'s are relatively narrow and inelastic. The \( I = \frac{3}{2} \) \( N^* \), in particular, should have low mass if our assignment to \( Z^* \) (1816) is correct and is so narrow that it would likely be missed in the current mesh of accurate experiments.

\( Y^*_1 \): There are (10) and \( \frac{3}{2} \) \( Y^*_1 \)'s. They have very similar characteristics. They will be very difficult to observe, being highly inelastic. Perhaps indirect production, such as a missing-mass experiment, would reveal

\cite{17} J. D. Davies et al., Phys. Rev. Letters 18, 62 (1967).

there. The $\Delta \pi$ channel is also weak.\textsuperscript{18} A possible experiment is $K^-\pi \rightarrow Y_1^*\pi$.

$Y_0^*$: There are $^*(8)$ and $^*_2(8)$ and also the member of the $^*_1(8)$ octet. The $^*(8)$ is mainly $K\pi$ and should be seen in total cross sections and in elastic scattering.\textsuperscript{17} In a heavy liquid one could look at $K^-p \rightarrow Y_0^*\pi^0$ which is characteristic of $I=0$ (the $\pi\pi$ coupling is small). The $^*(8)$ presents no obvious method of detection. The $(8)_{1/2}$ has very small $K\pi$ coupling. It should be looked at in indirect production through the large $\Sigma\pi$ and $Y_1^*\pi$ modes.

$\Xi^*$: There are, as for $Y_0^*$, $^*(8)$, $^*_2(8)$, $(8)_{1/2}$ and $(8)_{1/2}$. The $^*(8)$ is characterized by the large $\Sigma K$ (and $\Lambda K$) mode (unless the mass is too low), while $\Xi$ is small. The $(8)_{1/2}$ is dominated by $\Xi^*\pi$, and $\Xi^*$ is significant. The $(8)_{1/2}$ is mainly $\Xi\pi$ and perhaps can be identified with the structure that has been observed\textsuperscript{5} at 1930 MeV.

V. DISCUSSION OF ENHANCED HEAVY MESON DECAY

The $\omega$ recoil factor more than overcomes the suppression due to phase space of the s-wave decays into $\pi$ or $\eta$ compared to $\pi$, once one is a few MeV above the relevant threshold. Let us examine whether this effect is confirmed by experiment. We will say an inelastic cross section has a “threshold peak” when in crossing the threshold (e.g., with increasing energy across the threshold in $\pi N \rightarrow \pi N$) the cross section rises (with infinite derivative as it must) to a large value and then drops down shortly above (say < 100 MeV) threshold. The following conditions lead to a threshold peak: (1) The resonance mass is appropriately placed in the vicinity of the threshold. (2) The resonance is not too wide in other than the threshold channel. (3) The branching ratio into threshold channel is large and (not too small into input channel). Let us review the predictions of our model to see when conditions (2) and (3) are met [with some comments on (1)]. We first discuss $\frac{1}{2}^-$ to $\frac{1}{2}^+$ decay.

$N^*$'s: The relevant thresholds are $N\eta(1488)$, $\Delta K(1611)$, and $\Sigma K(1689)$. The prediction for $^*(8)_{1/2}$ satisfies the conditions for (2) and (3) for $N\eta$. The $\Delta K$ branching ratio is very small. The $N\eta$ threshold peak is observed. The $(10)_{1/2}$ and $(8)_{1/2}$ are probably near the $\Sigma K$ threshold, but $^*(8)$ is too wide and both have small $\Sigma K$ branching ratio. A very accurate experiment would show a $\Sigma K$ threshold peak due to the $(10)$.

$Y_1^*$'s: The relevant thresholds are $\Sigma\eta(1742)$ and $\Sigma K(1814)$. The $(8)$ is too wide for a threshold peak. The $^*(8)$ is narrow except for the possible $\Sigma K$ channel, and the $\Sigma\eta$ branching ratio is significant. There could be an observable $\Sigma\eta$ threshold peak. If the mass is not too far below threshold a large $\Sigma K$ peak should be seen. Experimental work to check this possibility would be of great interest. The $(10)$ is narrow, and $\Sigma\eta$ branching ratio is large. There should be a $\Sigma\eta$ threshold peak if the mass is

<table>
<thead>
<tr>
<th>$Y_1^*$</th>
<th>$^{28+}(SU(3))_2$</th>
<th>Assumed mass (MeV)</th>
<th>$\Gamma$ (MeV)</th>
<th>Input widths (MeV)</th>
<th>Readily visible</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{1/2}^*$</td>
<td>$^*(8)_{1/2}$</td>
<td>1518*</td>
<td>75</td>
<td>$\Gamma_{NN}=1$</td>
<td>No</td>
<td>Highly inelastic</td>
</tr>
<tr>
<td>$N_{1/2}^*$</td>
<td>$(10)_{1/2}$</td>
<td>1465</td>
<td>25</td>
<td>$\Gamma_{NN}=4$</td>
<td>Perhaps</td>
<td>Narrow and inelastic</td>
</tr>
<tr>
<td>$Y_0^*$</td>
<td>$^*(8)_{1/2}$</td>
<td>1875*</td>
<td>800</td>
<td>$\Gamma_{NN}=750$</td>
<td>No</td>
<td>Too wide</td>
</tr>
<tr>
<td>$Y_0^*$</td>
<td>$^*<em>2(8)</em>{1/2}$</td>
<td>1705</td>
<td>90</td>
<td>$\Gamma_{NN}=45$</td>
<td>Yes</td>
<td>$K\pi$ decay dominant</td>
</tr>
<tr>
<td>$Y_0^*$</td>
<td>$^*(8)_{1/2}$</td>
<td>1660*</td>
<td>55</td>
<td>$\Gamma_{NN}=0$</td>
<td>No</td>
<td>Highly inelastic</td>
</tr>
<tr>
<td>$Y_0^*$</td>
<td>$(8)_{1/2}$</td>
<td>1900</td>
<td>110</td>
<td>$\Gamma_{NN}=0$</td>
<td>No</td>
<td>Highly inelastic</td>
</tr>
<tr>
<td>$Y_0^*$</td>
<td>$^*<em>2(8)</em>{1/2}$</td>
<td>1875*</td>
<td>450</td>
<td>$\Gamma_{NN}=30$</td>
<td>Perhaps</td>
<td>Large width mainly $\Xi^*\pi$</td>
</tr>
<tr>
<td>$Y_0^*$</td>
<td>$(10)_{1/2}$</td>
<td>1800*</td>
<td>550</td>
<td>$\Gamma_{NN}=400$</td>
<td>No</td>
<td>Too wide</td>
</tr>
<tr>
<td>$Y_0^*$</td>
<td>$(8)_{1/2}$</td>
<td>1650*</td>
<td>30</td>
<td>$\Gamma_{NN}=2$</td>
<td>Perhaps</td>
<td>Low statistical weight</td>
</tr>
<tr>
<td>$Y_0^*$</td>
<td>$(10)_{1/2}$</td>
<td>1640</td>
<td>20</td>
<td>$\Gamma_{NN}=1$</td>
<td>Perhaps</td>
<td>Low statistical weight</td>
</tr>
<tr>
<td>$\Xi^*$</td>
<td>$^*(8)_{1/2}$</td>
<td>1890</td>
<td>400</td>
<td>No</td>
<td>Too wide</td>
<td></td>
</tr>
<tr>
<td>$\Xi^*$</td>
<td>$(8)_{1/2}$</td>
<td>1800</td>
<td>125</td>
<td>No</td>
<td>Too wide</td>
<td></td>
</tr>
<tr>
<td>$\Xi^*$</td>
<td>$(10)_{1/2}$</td>
<td>1870</td>
<td>50</td>
<td>No</td>
<td>Too wide</td>
<td></td>
</tr>
<tr>
<td>$\Xi^*$</td>
<td>$^*<em>2(8)</em>{1/2}$</td>
<td>1800*</td>
<td>25</td>
<td>Yes</td>
<td>$\eta'K$ decay dominant</td>
<td></td>
</tr>
<tr>
<td>$\Xi^*$</td>
<td>$(8)_{1/2}$</td>
<td>2040</td>
<td>125</td>
<td>Yes</td>
<td>$\Sigma K$ width</td>
<td></td>
</tr>
<tr>
<td>$\Xi^*$</td>
<td>$(8)_{1/2}$</td>
<td>2205</td>
<td>150</td>
<td>No</td>
<td>Too wide</td>
<td></td>
</tr>
<tr>
<td>$\Xi^*$</td>
<td>$^{*2}(8)_{1/2}$</td>
<td>1900</td>
<td>10</td>
<td>Yes</td>
<td>Sensitive to $\Xi^*\pi$ threshold</td>
<td></td>
</tr>
</tbody>
</table>

\textsuperscript{18} A possible new $Y_1^*$ at 1680 produced indirectly at high energy has been observed by M. Derrick et al. (private communication). Since the enhancement is strong in the $\Delta\pi$ channel and appears rather wide, there is some doubt that this $Y_1^*$ could be associated with either state under discussion here.
right. A $^{\Sigma}$ threshold peak is observed. It could be either (10) or $^{\Sigma}(8)$. Y$: The relevant thresholds are $\Lambda\eta(1664)$ and $\Sigma K(1814)$. The $^{\Sigma}(8)$ satisfies the conditions for a $\Lambda\eta$ threshold peak which is observed. A little mixing is needed to fit the observation. The $^{\Sigma}(8)$ is too wide.

$\Xi$**: The relevant thresholds are $\Lambda\Xi(1611)$, $\Sigma\Xi(1689)$, and $\Xi\eta(1697)$. The $^{\Xi}(8)$ is rather wide but is strong in $\Sigma\Xi$. The $^{\Xi}(8)$ is narrow except for $\Sigma\Xi$ and $\Xi\eta$, which are quite strong; $\Lambda\Xi$ is significant. The (10) is narrow and relatively strong in these channels. Threshold peaks in one or more of these channels should be seen, depending on mass. Threshold peaks will be difficult to see, however, for $Y = -2$.

One can also consider the $s$ wave decays: $3^- \rightarrow 3^+$. The $N^* \rightarrow Y^* + K$ or $N^* \rightarrow Y^{-} + \pi$ thresholds are probably too high in mass. The decay $Y^* \rightarrow N^* + K$ (threshold $\approx 1730$) is predicted to be large if any of the $^{\Xi}(8)_{1/2}$ or $^{\Xi}(8)_{3/2}$ (or $K_{3/2}$) has enough high mass. Presumably, according to results discussed above, the $^{\Xi}(8)$ and (10) have too low a mass. Similarly, $\Xi^* \rightarrow Y^* + K$ would be large if any state overlaps the threshold.

We conclude that all threshold peaks seen are contained in the model. Further cases are predicted. We feel the evidence for the heavy meson enhancement is very good.

We should also discuss the theoretical justification of this recoil correction. The justification lies in the validity of the nonrelativistic quark model as opposed simply to the quark model. If quark model dynamics are taken seriously, the determining factor is the detailed form of the interaction. Roughly speaking, very strong $QQ$ forces at the origin lead to large relativistic effects, making our Galilean invariance argument unreliable. If, on the other hand, the forces are not singular, most of the quark wave function will involve low momenta, and the Galilean invariance argument is good.

**VI. DISCUSSION**

It is seen that we obtain a semiquantitative agreement with baryon-decay rates, especially the $d$-wave decays, which is quite encouraging at the present time. The quark model specific kinematical effect of a factor (meson energy)$^2$ in the $s$-wave rates is helpful in obtaining agreement with data. Further experiments are needed to obtain definitive widths, and theoretical work is needed to remove the degeneracies in order to properly test the model.

This discussion would be incomplete without reference to the possibly low-lying $1^\pm$ singlet $Y^*_{1/2}$ in the (20,3). Its mass depends on problematical symmetry-breaking effects. One estimates places it very low: 1280 $< M < 1350$ MeV. If $M < 1330$ it will decay into $\Lambda + \pi$. If $M > 1395$ it would decay predominantly into $\Lambda \pi$. The coupling of (70,3) baryons to this $Y^*_{1/2}$ involves several different radial integrals depending on dynamical details. We just remark that since coupling (56,1)$^+ \rightarrow (20,3)^* + \pi$ is small (it vanishes in our model), it is probably via decay of negative parity $Y^*_{3/2}$ produced in $KN$ reactions that the easiest experimental search could be made.

One of the interesting results is in the absolute rates. From our fit we already have $e^{-1/2} = 0.6$ BeV. Presumably the baryon radius is $R = O(\alpha^{1/2})$. Quoting $q$, $\omega$, $\Gamma$ in BeV in the following, the empirically fitted rates in Table III obey

\[ s \text{ wave: } \frac{\omega}{M_Q} \int \frac{d^3 p_1 \cdot \hat{v}'}{R M_Q} \propto O \left( \frac{q Y_q(\delta)}{1 + \alpha q^2} \right) \]

\[ p \text{ wave: } q_s \int |\psi|^2 \hat{v}^2 d\tau \propto q Y_q(\delta) \left[ 3(1 + \alpha q^2) \right]^{1/2} \]

\[ d \text{ wave: } \int \psi^* \left[ \frac{\partial^2}{\partial \hat{q}^2} + \frac{\hat{q}^2}{4} - \frac{\hat{q} \cdot \hat{v}'}{4} \right] \psi' d\tau = O \left( \frac{R_{1/2} Y_{1/2}(\delta)}{1 + \alpha q^2} \right) \]

where $p_1$ is the momentum of quark 1, and $\psi'$ and $\psi''$ are spatial wave functions symmetric and of mixed symmetry (symmetric in quarks 2, 3), respectively, normalized:

\[ \int |\psi|^2 d\tau = \int \psi^* \cdot \hat{v}' d\tau = 1. \]

The relations (8b) and (10b) are exceedingly crude, but are chosen optimistically to obtain large $1/R$ and $M_Q$. Comparing right-hand sides of Eqs. (8) and (9) we have $O(M_Q R) = (42 \times 3^{1/2})$, and comparing (9) and (10) we have $O(1/R) = (42 \times 3^{1/2})$. Thus

\[ M_Q = 0(10 \text{ BeV}) \]

\[ 1/R = 0(5 \text{ BeV}) \]

These should be regarded as upper limits. If the overlap in both (8) and (10) is, say, 1/5 we would obtain $1/R = 1 \text{ BeV}$, $M_Q = 0.5 \text{ BeV}$. The significance of these results is not obvious and we will not discuss it.

Finally, let us review the problem of quark statistics. To distinguish between the two forms of statistics one
must resort to more specific dynamical considerations. In particular the symmetry-breaking mass differences and mixing of wave functions, and the structure of the radial function (form factors) must be considered.

For example, the role of the \( L \cdot S \) coupling is very different for the two. Clearly, a \( p \)-wave spin-orbit force, which one might expect to play the dominant role in bringing about the splitting among the various members of the (70,3) super multiplet, has the \( SU(3) \)-spin dependence \( P_\alpha^- \) for parastatistics and \( P_\alpha^+ \) for Fermi statistics, where \( P_\alpha^\pm \) are respectively the projection operators for the (6) and (3\*) states of a \( Q\bar{Q} \) system. Since, on the other hand, \( P_\alpha^- \) is a null operator for (10) states and likewise \( P_\alpha^+ \) for (1) states, one would expect a \( p \)-wave spin-orbit force to split the (1) states but not (10) under parastatistics and vice versa under Fermi statistics. This would immediately explain the splitting between the \( Y_9^0(1405) \) and \( Y_9(1520) \) states under parastatistics, but not under Fermi statistics.

Our results also indicate that the (10)\(_{1/2} \) and (10)\(_{3/2} \) states are strongly split. In parastatistics this splitting might come about through \( SU(3) \) violating terms, say of the form \( \lambda_6^{(3)} \lambda_8 \), in the \( Q\bar{Q} \) potential. The near degeneracy among several \( SU(3) \) multiplets makes this hypothesis rather attractive. We have not yet worked out the detailed consequences of such a hypothesis. The splitting of (8) states by an \( L \cdot S \) force is comparable under both forms of statistics.

A more interesting piece of evidence favoring parastatistics against Fermi statistics comes from the role of the positive parity states other than the (56). Dynamically \( A \) functions of \( L^\pi=1^+ \) have strongly attractive kernels under \( p \)-wave interaction. These functions give a total of (20,3) states of \( SU(6)\times O_4 \) under parastatistics and (56,3) under Fermi statistics. The spin-orbit force splits these states into various \( SU(3) \) multiplets, the lowest ones having \( J^P=\frac{3}{2}^- \). This leaves for the states \( J^P=\frac{1}{2}^+ \) of lowest energies, a singlet and an octet under parastatistics, and a decuplet and an octet under Fermi statistics. Experimentally, it is tempting to identify the 1450-MeV Roper resonance with the \( Y=1, I_3=\frac{1}{2} \) member of the above octet. Parastatistics therefore give a (more economical) prediction of a mere extra singlet, while Fermi statistics require a whole extra decuplet of low energy.

A third feature bearing on statistics concerns the shape of baryon form factors in relation to the kind of spatial symmetry (\( S \) or \( A \)) assumed.\(^{26} \) Thus an \( A \) function predicts nodal behavior for the form factor at \( q^2=20 \) \( F^{-2} \), in complete disharmony with experiment. An \( S \) function, on the other hand, predicts a smooth monotonic fall, which is at least consistent with experiment. This again favors parastatistics to Fermi statistics, as long as the (56) representation for baryons is not questioned.

ACKNOWLEDGMENTS

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\(^{26} \) A. N. Mitra and Rabi Majumdar, Phys. Rev. 150, 1194 (1966).

Errata

**Dynamics at Infinite Momentum, Steven Weinberg** [Phys. Rev. 150, 1313 (1966)]. The following should be added at the end of Ref. 3: The relation between the old rules and the Feynman rules has been extensively examined by A. Ramakrishnan and his colleagues. See A. Ramakrishnan, J. K. Radha, and R. Thunga, Proc. Indian Acad. Sci. 52(A), 228 (1960); J. Math. Anal. Appl. 4, 494 (1962); 5, 225 (1962); A. Ramkrishnan, K. Venkatesan, and V. Devanathan, ibid. 8, 345 (1964); and to be published.

**Renormalization Constants in the Extended Lee Model (\( W=\gamma+0 \)), Fei S. Chen-Cheung** [Phys. Rev. 152, 1408 (1966)]. J. B. Bronzan [Phys. Rev. 139, B751 (1965)] has given a similar demonstration of the renormalization constants. The work done by this author, however, was done independently [see Fei Shian Chen and C. M. Sommerfield, Bull. Am. Phys. Soc. 10, 61 (1965)].

**Possible Charge-Conjugation Noninvariance in the Photoproduction of Neutral \( \rho \) Mesons, Saul Barshay and Yuan Li** [Phys. Rev. 153, 1657 (1967)]. In an error subsequent to the proof, the printer erroneously duplicated Fig. 8 as Fig. 9, leaving out entirely the figure referred to as Fig. 9 in the Appendix. The correct Fig. 9 appears below.