Anomalous Regeneration of $K_{10}$ Mesons from $K_{E0}$ Mesons*

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(Received 13 March 1963; revised manuscript received 27 August 1963)

A beam of 1.0-BeV/c $K^+$ mesons passing through liquid hydrogen in a bubble chamber was seen to generate $K^0$ mesons with the momentum and direction of the original beam. The intensity of $K^0$ production was far greater than that anticipated from conventional mechanisms, and the suggestion is made that the $K^0$ mesons are produced by coherent regeneration resulting from a new weak long-range interaction between protons and $K$ mesons.

INTRODUCTION

The fundamental interactions or forces which are now known are commonly divided into four classes: the strong nuclear interactions, the electromagnetic interaction, the weak or beta-decay interaction, and the gravitational interaction. These are simply differentiated by their different magnitudes and different symmetry properties. The assumption that there is a unique, well-defined, largely separate description for each of these classes of forces is attractive; and the investigation of the axioms for the strong interactions and for the weak interactions is an important subject of present experimental and theoretical researches. Rather less attention has been paid to fundamental connections between these classes of forces, and there has been almost no consideration of the possibility of the existence of other forces unrelated to the four known classes, or at least as little related to them as they are to each other.

It is not difficult to conceive of interactions which might exist but might not be observed or recognized. The effect of the existence of such a class of interaction need not be observed directly. Consider, for example, the importance of electromagnetic forces in elementary particle interactions. Even if $e^2/\hbar c$ were equal to $10^{-20}$ instead of $1/137$, and the electromagnetic forces were as a practical matter unobservable experimentally, isotopic spin would still have a meaning and an importance: e.g., reactions such as $D+D \rightarrow \alpha + \pi$ which do not conserve isotopic spin, would still be forbidden.

The experiment reported here serves to investigate some such possible interactions, in particular, weak long-range interactions between protons and $K$ mesons which differentiate between $K^0$ and $\bar{K}^0$. The possibility of doing this derives from a suggestion* by M. L. Good, who has pointed out that coherent effects from macroscopic volumes of material may have important observable consequences on the constitution of neutral $K$ meson beams.

Consider a plane wave of $K^0$ mesons where we write $|K^0\rangle = 1/\sqrt{2}(|K^0\rangle - |\bar{K}^0\rangle)$, in a medium of scattering centers which scatter the $K^0$ and $\bar{K}^0$ with different amplitudes in the forward direction. This will result in a change in the relative magnitude and phase of the amplitude of $K^0$ and $\bar{K}^0$ in the beam, equivalent to the introduction of a small amplitude of $|K^0\rangle = 1/\sqrt{2}$ $\times(|K^0\rangle + |\bar{K}^0\rangle)$. Within a region $\Delta X$, where $\Delta X$ is sufficiently small so that the generated $|K^0\rangle$ does not get sensibly out of phase with the $|K^0\rangle$ as a result of their difference in mass, and the $K^0$ amplitude is not too much reduced by the decay to two pions, these contributions to the $K^0$ amplitude from intervals along the $K^0$ beam will be coherent and a large intensity of $K^0\bar{K}^0$ will be built up. The size of this region will be such that $\Delta X \leq \hbar/\Delta M$, where $\Delta M$ is the $K^0\rightarrow K^0$ mass difference, and $\Delta X \leq \pi \hbar c/E$, where $\pi$ is the mean life of the $K^0$ and the other quantities refer to the $K$ beam.

Typically, $\Delta X$ is limited by the mass difference, which is about $1.5 \pi \hbar c^2$, to a few cm. The $K^0$ intensity can be detected by observation of the decay $K^0\rightarrow \pi^+ + \pi^- - \cdots$.

Since the intensity of $K^0\bar{K}^0$ produced by the coherent generation process is dependent only on the forward amplitude produced by the scattering centers, it is easily seen that even an extremely weak interaction can be important if the range is large. The scattering amplitude for Coulomb forces, neglecting recoil, is: $\frac{1}{3}M(E/c^2)(\epsilon'/\hbar c)\sin^2\theta(2)/2$ or for small angles and high momenta; $\frac{1}{2}A(e'^2/hc)/\Delta p/2p^2$, where $\Delta p$ is the transverse momentum transfer, $p$ is the beam momentum, and $\Delta p > \hbar/\pi$. But $\Delta p$ cannot be smaller than $\hbar/a$ where $a$ is the range of the force; then, in analogy with Coulomb forces, the forward scattering amplitude for an interaction of range $a$ and coupling strength $(e'/\hbar c)$, will be $= (e'/\hbar c)a^2/\lambda$. If $a$ is very large, the forward scattering amplitude will be appreciable even for very small interaction strengths, and if the interaction differentiates between $p-K$ and $p-K\bar{K}^0$, its existence may be significant.
result in an anomalous regeneration of $K^0$ mesons in a $K^0$ meson beam.

**EXPERIMENTAL PROCEDURES AND RESULTS**

For the purpose of investigating the reactions and decays of $K^0$ mesons, a $K^0$ beam of high intensity, reasonably small energy spread, and reasonable freedom from neutron contamination was desired. A beam which fulfilled these requirements adequately was produced as follows: A 3-BeV external proton beam from the Brookhaven Cosmotron struck a steel target. Pions produced in the forward direction were momentum analyzed such that the central momentum was about 1.5 BeV/c and the width about 7%, and focused onto a polyethylene target. Neutral $K$ mesons were produced in the polyethylene through reactions such as $\pi^+ + p/n \rightarrow K^0 + \Sigma/\Lambda + \text{pions}$. Those neutral $K$ mesons produced near the forward direction passed through a magnetic field designed to eliminate charged particles, into a liquid hydrogen bubble chamber about 2.5 m from the target. The mean $K$-meson momentum was found to be about 1.0 BeV/c with a width of about 300 MeV/c. The average external proton flux was about $8 \times 10^7$ protons/pulse, the pion flux was about $3 \times 10^6$ pions/pulse, and the $K^0$ intensity at the chamber was found to be about $(2.5 \pm 0.6) \times 10^3$/cm$^2$/pulse. Approximately 90,000 useful pictures were taken, scanned, and measured.

An effective fiducial in the chamber region was defined for most measurements. The chamber was cylindrical, 35 cm in diameter, 20 cm deep, with the axis parallel to the floor. The boundaries of the fiducial region were defined as the cylindrical surface 4 cm along the beam line from the chamber wall in the upstream direction, planes 9.5 cm above and below the median beam line, planes 7.5 cm to the left and right of the beam line, and a cylindrical surface 12 cm along the beam line from the chamber wall in the downstream direction. This region comprised a volume of about 4.4 liters, less than 25% of the total chamber volume. In this volume 252 events were found which fitted $K^0$ decay kinematics. More than twice as many events, identified as $K^0$ events in the scanning process, were found throughout the chamber, but those events found outside of the fiducial region were subject to biases resulting from imperfect measurements which served to distort most results beyond the purely statistical errors.

It is then from the events in the fiducial region that the flux is estimated using as further input data a $K^0$ lifetime of $6.1 \times 10^{-8}$ sec, a scanning efficiency of 90% derived from double scanning comparisons, and a $K$ momentum of 1.0 BeV/c. The total uncertainty is quite large, primarily as a result of the uncertainty in the $K^0$ lifetime.

The beam momentum was estimated by measuring the $K^0$ momentum for each well-determined interaction event; events such as $K^0 + p \rightarrow \Lambda + \pi$, $\Sigma + 2\pi$, $\Lambda + 2\pi$, and $K^0 + p$ were used. The plot of Fig. 1 represents the momentum of 182 such events. Though some error will be introduced by a possible variation of reaction cross sections over the beam spread, this estimate of the momentum structure is probably not seriously in error.

A "normal" source of generation of $K^0$ mesons is the reaction $K^0 + p \rightarrow K^0 + p$. A plot of the differential cross section, as measured in this experiment, is shown in Fig. 2. This is based on 47 events found in the fiducial region, identified by the $K^0$ decay and by the proton

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**Fig. 1.** Momentum distribution of all reaction events and all forward $K^0$ decays. The dashed histogram represents the distribution in momenta of the incident $K^0$ meson which produced a measurable event of the type $\rightarrow \Lambda + \pi$, $\Lambda + 2\pi$, $2\pi + \pi$, and $K^0 + p$. The shaded histogram represents the momentum distribution of the events identified as $K^0$ decays according to the criteria of Fig. 2. Those events such that $0.997 \leq \cos \theta \leq 1.00$ are plotted.

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**Fig. 2.** Differential cross section for the process $K^0 + p \rightarrow K^0 + p$. The empty regions in the forward and backward directions are regions where the detection efficiency is low or zero. The histogram is based on 47 events. The absolute error in scale is 25%.

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3 This is further discussed by R. K. Adair, Rev. Mod. Phys. 33, 406 (1961).
recoil. Corrections are made for events produced in the forward direction which are not observed, as they are not associated with an observable recoil, and events produced backwards in the center-of-mass system, which are likewise not easily seen as the $K^0$ decays, has a low momentum and is moving backwards in the laboratory. Scanning efficiency has been shown to be low for such events. A decay ratio of 0.33 to $\pi^0+\pi^0$ is assumed in accord with experimental results and the $\Delta t=\frac{1}{2}$ rule.

Since $K^0$ mesons produced by coherent processes transfer no appreciable momentum to protons of the bubble chamber, they will be observed by their decays as $V$ particles unassociated with a recoil particle and can be separated from the leptonic $K^0$ decays only by detailed analysis. In particular their $Q$ value, measured under the assumption that both charged particles are pions, must be 218.6 MeV within experimental error. Since $K^0$ decays such as $K^0\rightarrow \mu+\pi+\nu$ and $K^0\rightarrow e^+\pi^+\nu$, if the lepton is mislabeled a pion, simulate two pion $Q$ values which extend to values even greater than 218 MeV, true $K^0$ decays can be separated only statistically. Coherently produced $K^0$ mesons should result in $K^0$ decays consistent with the assumption that the particles which have so decayed, as determined by the momentum of the decay products, were traveling in the beam direction. Such events should have the same momentum as the original beam, and the measured $Q$ value distribution should center about 218.6 MeV. The angular distribution about the beam direction of those events for which the observed momentum is consistent with the beam momentum, and those for which the $Q$ values (assuming always that the charged particles are pions) are consistent with 218 MeV, are shown in Fig. 3. The events in the very sharp peak in the forward direction are interpreted as resulting from coherent production. The probability that the peak arises purely as a statistical fluctuation is $=10^{-6}$. The experimental resolution is estimated to be about 1.5° and the width of the peak is consistent with zero within this error. The solid line, representing the contribution to be expected from $K^0$ leptonic decays, is the result of a Monte Carlo calculation following the formulae of Pais and Treiman for a pure $V-A$ interaction. There is but a small contribution from the incoherent process $K^0+p\rightarrow K^0+p$. From the results shown in Fig. 2 we expect about one event per interval $\Delta \cos\theta=0.003$ from this source, where $\theta$ is the angle between the total momentum and the beam direction. The main walls of the cylindrical chamber are of copper, 1.6 cm thick. Guided by the work of R. Good et al.\textsuperscript{4} who

\textsuperscript{4} A. Pais and S. B. Treiman, Phys. Rev. 105, 1616 (1957). For $\pi\nu\nu$ decay the ratio of the matrix elements $j_{\nu}^2/f_{\nu}$, in their notation, was taken as $m_{\nu}/m_{\pi}$. The results are not much changed for smaller values, and are less in magnitude and peaking for larger values. Branching ratios are taken from D. Laiers, I. S. Mittra, W. J. Willis, and S. S. Yamamoto, Phys. Rev. Letters 7, 253 (1961).


\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig3.png}
\caption{Angular distribution of events which have a $2\pi$ decay $Q$ value consistent with $K^0$ decay, and a momentum consistent with the beam momentum. $\theta$ is the angle between the total visible momentum and the incident beam. All events are plotted for which 180 MeV $\leq Q \leq 270$ MeV, $\rho \geq 800$ MeV/c. The black histogram presents the events the front wall of thin window. The solid curve presents the contribution expected from $K^0$ decays.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig4.png}
\caption{Histogram of events produced from $K^0$ decay, and a momentum consistent with the beam momentum. A square with $Q$ values consistent with $K^0$ decay, and a momentum consistent with the beam momentum. $\theta$ is the angle between the total visible momentum and the incident beam. All events are plotted for which 180 MeV $\leq Q \leq 270$ MeV, $\rho \geq 800$ MeV/c. The black histogram presents the events in the front wall of thin window. The solid curve presents the contribution expected from $K^0$ decays.}
\end{figure}

calculate and measure the diffraction production of $K^0$ from 660 MeV/c $K^0$ mesons on iron, we estimate that we might expect only a few events from this source in an interval of 0.001 of $\cos\theta$. From so thin a source the contribution of the normal coherent production to the forward peak should not exceed two events. This is discussed in more detail in the next section. The solid histogram presents events in front of a 0.015-in. thick stainless steel window. No contamination of this small sample from production in the wall is possible. The number of events at large angles is consistent with that to be expected from the sum of three body decays of $K^0$ mesons, incoherent production of $K^0$ mesons, and a few $\Lambda^0$ decays which could not be distinguished from $K_0$ decays with certainty.

A plot of the $Q$ values of the events in the interval $0.997<\cos\theta<1.000$ is shown in Fig. 4, together with a similar plot for events such that $0.985<\theta<0.997$. The solid curve represents the shape of the pseudo-$Q$ value distribution anticipated from leptonic $K^0$ decays which fulfill the selection criteria. The distribution for the events in the forward interval is consistent with that to be expected from an almost pure sample of $K_0$ decays. The other distribution shows no important peaking at $Q=218$ MeV and is consistent with that to be expected from a sample to which $K^0$ decays and incoherently produced $K^0$ contribute about equally.

The possibility of interpreting the events as two-pion decays of $K^0$, which would be allowed after $CP$ invariance were violated, is excluded by the results of observation of 411 $K^0$ decays in cloud chambers,\textsuperscript{5,6} none of which


were consistent with two-pion decays. The low density of the gas in the cloud chambers excludes any substantial $K^0$ regeneration by coherent $K^0$ interactions in those experiments.

The momentum distribution of the events in the forward direction, together with the distribution from the measurements of identified reactions is shown in Fig. 1.

Altogether the data support the conclusion that $K^0$ mesons have been generated by the $K^0$ beam; the $K^0$ mesons having the same direction and momentum as the $K^0$ beam.

CONVENTIONAL INTERPRETATIONS OF THE DATA

It is possible to make an estimate of the magnitude of the differential cross section in the forward direction for the reaction $K^0 + p \rightarrow K^0 + p$ using the optical theorem and the information available concerning the $K^+$, $K^-$-nucleon total cross sections.

Since the interactions, and therefore the scattering, of the strangeness (+1) $|K^0\rangle$ state and the strangeness (−1) $|\bar{K}^0\rangle$ state will be generally different, the scattering of the CP eigenstate $|K^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle)$, with the eigenvalue (−1), will result in a production of the eigenstate $|K^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle)$ with eigenvalue (+1). Writing the scattering amplitude for the scattering of the $K^0$ mesons by protons at a specific angle as $A^+$, and $\bar{K}^0$ mesons as $A^-$, we have for the amplitude of $|K^0\rangle$ in the scattered wave: $A^+ = \frac{1}{2}(A^+ - A^-)$. This process, involving a change in the CP eigenvalue, is formally very like spin-flip scattering or charge-exchange scattering; we call it, imprecisely, CP-change scattering.

This discussion neglects spin and isotopic spin. The latter is relevant only to the relations of the neutral $K$ to charged $K$ interactions which will allow us to estimate the forward scattering. However, the amplitude represents an amplitude where the proton spin has been reversed, and an amplitude in which the spin is unchanged, which contribute incoherently to the scattered intensity. The spin-flip term goes to zero, however, in the forward direction where we can make use of the optical theorem to find the imaginary part of the amplitude: $\text{Im} A(0) = \frac{k_0}{4\pi} |A(0)|$. The differential cross section for $K^0$-production in the forward direction then will be: $\frac{d\sigma}{dQ} = \frac{1}{2} (A^+ - A^-) \geq \frac{1}{2} \frac{(k^0/64\pi)^2 (\sigma^+ - \sigma^-)^2}{(k^0/64\pi)^2 (\sigma^+ - \sigma^-)^2}$, where $\sigma^+$ and $\sigma^-$ represent the total $K^0 - p$, and $\bar{K}^0 - p$ cross sections respectively. While these are not known, it is instructive to consider the related charge particle cross sections. For strangeness +1, the $K^+ - n$ and $K^0 - p$ total cross sections for 1000 MeV/c $K$ mesons are about 20 mb while the $K^- p$ strangeness −1 cross section is about 45 mb. Using these values for $\sigma^+$ and $\sigma^−$, respectively, we find $d\sigma/dQ = 2.5$ mb/sr in the laboratory system, or 0.65 mb/sr in the center-of-mass system. The close agreement with the results of our measurement of $0.67 \pm 0.17$ mb/sr suggests that no anomalous effects are important.

Coherent regeneration can be considered in liquid hydrogen in a particularly simple manner since the interaction mean free path is so long compared to the maximum interval of coherence, the $K^0$ decay length. A detailed derivation of coherent $K^0$ regeneration is presented by M. L. Good. The conclusions which are reached are suggested by simple considerations. Over a small interval $dx$ we can expect that the change in amplitude $\sigma$ of the $K^0$ beam, effected by the material through which it passes, can be expressed as a complex phase change: $d\sigma = a(ik)x dx$ where $K$ is a complex wave number representative of a complex optical potential of the bulk medium. For $K$ mesons passing through isotropically pure matter, $dI = (I - N\sigma_I)dx$, where $I$ is the $K^0$ intensity, $N$ the number of nuclei per cc, and $\sigma_I$ the total cross section for $K^0$ nucleon interaction. The imaginary part of $K$ is then equal to $\frac{1}{2}N\sigma_I$. Using the optical theorem, for spin-\frac{1}{2} or spin-0 nuclei, $\sigma_I = 4\pi Im A/k$, where $A$ is the forward scattering amplitude and $k$ the $K^0$ wave number, $lmK = 2\pi NIm A/k$. This relation is continuous such that $K^0 = 2\pi NA/k$ and $dK = a(2\pi NA/k)dx$. The incremental production of $|K^0\rangle$ from a $K^0$ beam, in an infinitesimal interval $dx$, will then be: $dK = a[2\pi(\lambda - \lambda^*)]dx$.

The amplitude of the $K^0$ state will not build up indefinitely as both the $K^0$ and $K^0$ amplitudes will be attenuated by decay and by nuclear interactions; furthermore, the phase of the $K^0$ beam will change with distance relative to the $K^0$ beam, and hence further $K^0$ increments, as a result of the small mass difference.
between the $K^p_1$ and $K^p_2$. At distance $l$ in a medium the total $K^p_1$ beam can be constructed as a variety of Cornu spiral such as is used in optics in discussions of Fresnel scattering. Contributions from intervals at a distance $x$ upstream along the $K^p_1$ beam will be reduced in amplitude by a factor $\exp(-x/2L)$, and rotated in phase by an angle $\delta x/L$, where $\delta = (M_1-M_2)c^2/\hbar$ is the dimensionless mass difference measured variously to be from 0.84 to 1.9 units.\(^{6-9}\)

$M_1-M_2$ is the $K^p_1-K^p_2$ mass difference, $\tau_1$ is the $K^p_1$ mean life, and $\ell$ is the $K^p_1$ mean decay distance, equal to $\tau_1\beta p/E$, where $p$ and $E$ are the momentum and total energy of the $K$. Neglecting the attenuation by nuclear interactions,

$$a_1(L) = a_2Nk^{-1/4}(A^+ - A^-) \int_0^L \exp(-x/2L) \exp(i\delta x/L) dx, \quad (1)$$

and for the intensity,

$$I_1 = I_2N^2\lambda k^{-1/4}(A^+ - A^-)^2(1+\delta^2)^{-1} \int \{\exp(-(L/2L)\cos(\delta x/L)-1)^2 \exp[-(L/\ell)\sin^2(\delta L/L)] \} \quad (2)$$

Since the absolute magnitude of the coherent regeneration peak is difficult to measure, it is attractive to consider the ratio of regeneration by single nucleon scattering and by the coherent process. The intensity of production into a solid angle $d\Omega$ by the nuclear interaction in the forward direction is

$$I_N(L) = \frac{dI}{d\Omega}N \int_0^L \exp[-(L-x)/\ell] dx, \quad (3)$$

where $dI/d\Omega = \frac{1}{4}(A^+ - A^-)^2$. Then

$$I_1(L) = \frac{dI}{d\Omega}I_1N^2(A^+ - A^-)^2[1 - \exp(-L/\ell)]. \quad (4)$$

Note that the ratio $I_1/I_N$ is now independent of the production cross sections, an independence which holds only for materials and distances such that plural scatterings can be neglected.

We now can consider the contribution to the forward peak of $K^p_1$ mesons from these two processes: single nucleon scattering, and the coherent process associated with the nuclear scattering. From the curve of Fig. 2 we estimate a contribution to the region $0.999 < \cos\theta < 1.000$, of 0.3 events. Here $\delta I = 0.006$ sr. If the mass difference were zero and taking $L \to \infty$, we would find a value for the ratio of coherent to nuclear scattering of $4\Delta/\alpha^2 = 2.5$, and the contribution to the forward peak would be 0.75 events. A more nearly complete estimate using $\delta = 1.0$ gives about 0.2 events.

The 1.6-cm-thick copper chamber wall covering much of the entrance to the fiducial region may be a source of $K^p_1$ mesons. We estimate the direct production in the forward direction from the optical model: $d\sigma/d\Omega(\theta) \geq (k^4/64\pi^4)(\sigma_{+} - \sigma_{-})^2$. $\sigma_{+}$ will be of the order of $\pi a^2$ or 600 mb. We can expect the cross section for $K^p_1$ to be appreciably greater than for $K^1$ since the interaction with the individual nucleons is stronger. Let us assume then $|\sigma_{+} - \sigma_{-}| = 300$ mb, where $\sigma_{+}$ and $\sigma_{-}$ are the total cross sections for $K^1$ and $K^p_1$ mesons, respectively, an estimate consistent with the optical model.\(^{6}\) Then $d\sigma/d\Omega = 300$ mb/sr and in the interval $0.999 < \cos\theta < 1.000$ we might expect to find about 2 events. Such a diffraction pattern would not be extremely peaked forward, the intensity would drop to about half value at $\cos\theta = 0.995$. The value of the ratio of coherent to incoherent processes will be about 0.6 for zero mass difference, and is not much reduced by mass differences which are less than two units. This would give a contribution of about 1.5 events to the peak, and is the only contribution which is intrinsically sharply peaked. Each of these contributions from the copper should exhibit an exponential decay as a function of distance from the wall. In Fig. 5 the events in the interval $0.997 < \cos\theta < 1.000$ are plotted as a function of distance from the wall together with curves representing: (a) a probability independent of distance (which reflects the shape of the fiducial region), (b) an exponential decay from the wall with the mean decay distance of the $K^p_1$, and (c) a distribution representing coherent production in hydrogen with the dimensionless mass difference $\delta = 1.5$. The peak in the latter curve is the first maximum of the Cornu spiral. The observed distribution fits the
exponential decay poorly, suggesting that the peak intensity does not derive primarily from the wall. This, of course, does not exclude the possibility of some contribution from the wall. The fit to the coherent production is striking though not strongly significant statistically.

Altogether it would appear that a reasonable estimate of the contributions and characteristics of conventional mechanisms leads to the conclusion that they fail to account for the singular characteristics of the data. Further, except that an appropriate mechanism or interaction is lacking, the data are consistent with, and suggest, that the events are $K^0$ decays, which result from a coherent generation process from the $K^0$ beam in the liquid hydrogen.

**CONJECTURES AND DISCUSSION**

Since conventional interactions$^{10}$ do not appear to account for the $K^0$ peak, it is interesting to explore the characteristics required of an interaction which does. Such an interaction must result in a large CP-change differential scattering cross section in the forward direction but not be important at large angles, angles such that $\cos \theta \leq 0.997$, since production at larger angles does not seem to be abnormal. This immediately dictates a long range force with a range greater than $\approx 1.22 (\lambda/\Theta)^2 = 25 \beta$, where $\lambda$ is the $K$-meson wave length and $\Theta$ is the width of the peak taken in the laboratory system, from Fig. 2, as $\Theta \approx 0.03$. We can estimate the strength required of such an interaction simply using the forms of dispersion theory. We consider the interaction of range $a$ as if it were the result of the transfer of a virtual intermediate particle of mass $m=m_0/c^2$. The scattering amplitude will have the form:

$$A = \lambda \left( g^2/\hbar c \right) \left( X - X' \right),$$

where $X = \cos \theta$, and $X'$ is the position of the pole, $X_0 = 1 + (m_0/2 \sqrt{2})^2$, where $\lambda$ is the $K$-momentum, and $g^2/\hbar c$ is the dimensionless interaction strength. Assuming that the interaction has opposite signs for $K^0$ and $\bar{K}^0$, and as a value for the forward $CP$ change scattering cross section, $K^0 + p \rightarrow K^0 + p'$, $> 200$ mb, a value determined from the size of the coherent forward peak observed, we have:

$$I(0) = 2 \times 10^{-48} \text{ cm}^2 = \frac{1}{4} (A^+ - A^-)^2 \approx \chi^2 \left( g^2/\hbar c \right)^2 \times \left( 4 \lambda^2 / m \chi^2 \right) = 4 \left( \alpha^2 / X \right) \left( g^2 / \hbar c \right)^2,$$

or

$$\alpha^2 \left( g^2 / \hbar c \right)^2 \approx 2 \times 10^{-48} \text{ cm}^2.$$

As a particular example it is interesting to consider the possibility that the $K^0$ and $\bar{K}^0$ have small electric charges of opposite sign and magnitude $e'$. The interaction will be a shielded Coulomb interaction with a range $a$ of $5 \times 10^{-9}$ cm. Then $(ee'/\hbar c)$ will be about equal to $1.7 \times 10^{-40}$ and $\alpha^2 = 2.4 \times 10^{-8}$. In our chamber the angular separation induced on the $K^0$ and $\bar{K}^0$ by the magnetic field of 17.6 Kgauss, acting over the region of coherence of about 10 cm, will be only about $3 \times 10^{-4}$ rad.

Since this is small compared to $a_0/\lambda$, there should be no effect from this source. For interactions connecting charge and strangeness the results may be much the same as represented by the "charged" neutral $K$ mesons.

Consider a particle of mass $m$ acting as the quanta where all particles have a "paracharge" equal to $g(Q \pm S)$. (If each particle, including leptons, is assigned a paracharge $q$ according to this recipe, conservation of strangeness in strong interactions becomes conservation of charge and paracharge. (In weak interactions $\Delta Q = \pm 1, 0$.) If the mass $\leq m\sqrt{137}$, the range will still be $a_0$ determined by the electron shielding, and $(g^2/\hbar c) = 1.7 \times 10^{-40}$.

Conjectures which consider couplings such as $Y' \cdot Y''$, where $Y$ is the hypercharge, or $T_3 T'_3$, where $T_3$ is the third component of the $K^0$ proton interaction, lead to extremely strong coherent production by heavy nuclei, a result contradicted by the results of R. H. Good et al.$^4$

Coherent production would be proportional to the square of the density of matter if the interaction is proportional to $YY''$, and to the density squared times $\left( (N-P)^2 / (N+P)^2 \right)$, where $N$ and $P$ are the number of neutrons and protons in the nucleus, for an interaction of the form $T_3 T'_3$. In either case coherent production in Pb would be more than a thousand times that in hydrogen, contradicting observations.

Since the contribution to the forward intensity varies as $\alpha^2$, an interaction like a shielded Coulomb potential does not lead to such excessive production in heavier nuclei, and is not in obvious contradiction with the results of R. H. Good et al.$^4$ It is quite interesting to note that the coherent peak found in that experiment from which the mass difference $\delta = 0.84 \pm 0.29$ was deduced, cannot result from ordinary nuclear interactions if the mass difference is actually equal to 1.5 units as measured by Natali et al.$^7$ (1.5±0.2), or 1.9 units as determined by Fitch et al.$^8$ (1.9±0.3). If these mass differences are correct, another mechanism must be invoked to account for the peak found in the Berkeley experiment, supporting our result that an anomalous interaction exists.

In conclusion it appears to us that the results of this experiment strongly suggest the existence of anomalous coherent production of $K^0$ mesons from a $K^0$ beam. However, in view of the extraordinary consequences which may be required by such a result, it is necessary to emphasize that we cannot, at this time, completely exclude the possibility or even evaluate precisely the probability that the striking character of the data results from a combination of real effects underestimated by us together with strong statistical fluctuations.

$^{10}$ Electromagnetic interactions between $K^\pm$ and the protons or electrons are not important. G. Feinberg, Phys. Rev. 109, 1381 (1958); Ya. B. Zel'dovich, Zh. Ekperim. i Teor. Fiz. 36, 1381 (1959) [translation: Soviet Phys.—JETP 36, 9 (1959)].