Muons and Electron Number Nonconservation in a $V-A$ Gauge Model

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We analyze muon and electron lepton-number nonconservation in a pure $V-A$ gauge model. The rates for $\mu - e\gamma$, $\mu - e\bar{e}e$, and $K_L - \mu\bar{e}$ are computed for this model. We find that for a reasonable range of neutral heavy-lepton mass these rates are in accord with, but not extremely small compared to, present experimental bounds. We comment on the nonorthogonality of $\nu_e$ and $\nu_{\mu}$, and interesting features of the $L^-$ decays.

Some time ago we discussed a six-quark model with only left-handed currents. This is a minimal extension of the "standard" four-quark Weinberg-Salam SU(2)$\times$U(1) gauge model which allows CP nonconservation to be incorporated. The alternatives are right-handed currents or proliferation of Higgs bosons. Such a model leads to approximate superweak (or microweak) predictions for CP nonconservation. The model also includes a pair of leptons ($L', L$), both massive, coupled to the W's through a left-handed current, in order to cancel anomalies. The $L'$ can be tentatively identified with the heavy lepton of mass $=2$ GeV reported at SPEAR and corroborated at DORIS. This model gives the same predictions for atomic-physics parity nonconservation as the Weinberg-Salam model.

The general form of the leptonic current is

$$J_{\mu} = \bar{l}_e \gamma_{\mu}(1 - \gamma_5)U_{\mu e}$$

(1)

where $l_e = (\bar{e}, \mu^-, L')$ and $l_\nu = (\nu_1, \nu_2, L_\nu)$. $U$ is a general unitary matrix. The massless neutrino produced in association with the electron, $\nu_e$, is given by

$$\langle U_{11}^2 + |U_{12}|^2 \rangle^{1/2} \nu_e = U_{11} \nu_1 + U_{12} \nu_2;$$

mutatis mutandis for the muon neutrino $\nu_\mu$:

$$\langle U_{21}^2 + |U_{22}|^2 \rangle^{1/2} \nu_\mu = U_{21} \nu_1 + U_{22} \nu_2.$$ 

The known limits on hadron-lepton universality, $\mu e$ universality, and nonorthogonality between $\nu_e$ and $\nu_\mu$, imply that

$$|U_{13}^* U_{23}| < 0.055.$$ 

If $|U_{13}^* U_{13}|$ is nonzero, there are several interesting consequences.

$\mu - e\gamma$ decay.—The diagram which contributes in leading order to the decay $\mu - e\gamma$ is shown in Fig. 1. We have calculated this amplitude to be

$$M(\mu - e\gamma) = i\epsilon (G_F/\sqrt{2})(m_\mu/32\pi^2) \epsilon U_{23}^* U_{13} \bar{e} \gamma_{\alpha} \sigma_{\alpha \beta}(1 + \gamma_5) \mu \epsilon \gamma_\beta,$$

(3)

where $\epsilon = (m_e \epsilon)^2/m_\mu^2$. The branching ratio of $\mu - e\gamma$ to $\mu - e\bar{e}\mu$ is then

$$B(\mu - e\gamma) = (3\epsilon/32\pi^2) \epsilon^2 |U_{23}^* U_{13}|^2.$$ 

(4)

If we take $|U_{23}^* U_{13}|^2$ to be $0.3 \times 10^{-2}$ (see below) and $m_\mu \approx 60$ GeV, we find $m_L \approx 12-30$ GeV for $B = 10^{-9}$.
FIG. 1. One-loop diagram contribution in $\mu \rightarrow e \gamma$ via $L_0$.

Such a value for $B$ can be tested very soon by an experiment in progress at the Swiss Institute for Nuclear Research.\(^{10}\) The angular distribution of the decay of the polarized muon is given by 1 + cos $\theta$, where $\theta$ is the angle between the direction of the electron momentum and the direction of the muon polarization. This is due to the left-handedness of our weak currents.

$\mu \rightarrow e e \bar{e}$—In the SU(2)$\otimes$U(1) gauge theory, there are three classes of diagrams contributing to this process: the photon exchange, the $Z$ exchange, and the $W^+ W^-$ exchange; the calculation involved is very similar to that previously performed for the process $s \rightarrow d \mu \bar{\mu}$.\(^{11}\) The final result is

$$M(\mu \rightarrow e e \bar{e}) = \frac{G_F^4}{\sqrt{2}} \left( \frac{1}{9} \ln U_{13} U_{23} \bar{e} \gamma \left( \frac{1}{2} - \gamma^5 \right) \right) e \bar{e}$$ (5)

leading order in $\ln \epsilon$. From this we calculate a branching ratio

$$\frac{\Gamma(\mu \rightarrow e e \bar{e})}{\Gamma(\mu \rightarrow e \nu}) = \frac{3\alpha^2}{16\pi^3 \epsilon^3} \ln^2 \epsilon \left| U_{13} \right| \left| U_{23} \right|^2.$$ (6)

For $m_L = 10$ GeV, $m_\mu = 60$ GeV, and $\left| U_{13} \right| \left| U_{23} \right| \approx 0.055$, this branching ratio is equal to $0.3 \times 10^{-10}$, safely smaller than the experimental limit $6 \times 10^{-9}$. It is interesting to observe that although this result depends sensitively on the parameter of the theory, the ratio

$$\frac{\Gamma(\mu \rightarrow e e \bar{e})}{\Gamma(\mu \rightarrow e \nu}) = (2\alpha/\pi) \ln^2 \epsilon$$ (7)

varies only slowly as one changes $m_L^2$. For the values of $m_L$ and $m_\mu$ given, this ratio is equal to 0.06, somewhat larger than the level $\sim \alpha/\pi$ which one might, a priori, expect. The reason for this is the $\ln(1/\epsilon)$ term in the $Z$-exchange contribution to $\mu \rightarrow e e \bar{e}$.

**Decay of $L^-$**—If the neutral lepton $L^0$ associated with the charged lepton $L^-$ of mass 2 GeV is indeed as massive as 10 GeV there are several unusual effects. Decays of $L^-$ such as $e^- \bar{\nu}_e L_0$ are forbidden. We have (summing over neutrino and antineutrino species)

$$\Gamma(L^- \rightarrow e^- \gamma) = (G_F^2/2\pi^3) \left( m_L^2 \right)^6 \left( U_{31}^2 + U_{32}^2 \right)^3 \Gamma(\mu \rightarrow e \nu) \left( m_L^2/m_\mu \right)^6 \left( U_{31}^2 + U_{32}^2 \right) \approx 10^{-2}$$ (8)

This rate is suppressed considerably by the smallness of the mixing angles; taking $\left( U_{31}^2 + U_{32}^2 \right)^3 \approx 10^{-2}$, we find $\tau(L^- \rightarrow e^- \gamma) \approx 10^{-10}$ sec.

The decays $L^- \rightarrow e^- \gamma$ and $\mu^- \gamma$ are expected.\(^{12}\) We have

$$\frac{\Gamma(L^- \rightarrow e^- \gamma)}{\Gamma(\mu \rightarrow e \gamma)} = \left( \frac{m_L}{m_\mu} \right)^5 \frac{U_{13}^2 U_{23}^*}{U_{13}^* U_{23}} \approx \left( \frac{m_L}{m_\mu} \right)^5 \left| U_{23} \right|^2.$$ (9)

Combining Eqs. (8) and (9), we deduce that

$$\frac{\Gamma(L^- \rightarrow e^- \gamma)}{\Gamma(\mu \rightarrow e \nu}) = \frac{1}{U_{23}^2 (U_{31}^2 + U_{32}^2)} \approx 10^{-5}$$ (10)

if $\Gamma(\mu \rightarrow e \gamma)/\Gamma(\mu \rightarrow e \nu)$ is about $10^{-5}$, and $U_{31} \approx U_{32}$.

**Neutrino Reactions.**—We predict a nonzero coupling of the muon neutrino to $e^-$ and $L^-$. For sufficiently high incident neutrino energies where the mass differences may be neglected, we get

$$\sigma(\nu_e N \rightarrow e^- X) \propto \sigma(\nu_e N \rightarrow e^- X) \propto (1 - \left| U_{23} \right|^2: \left| U_{13} \right|^2: \left| U_{33} \right|^2)^2 \approx (1 - \left| U_{23} \right|^2)^4: \left| U_{13} \right|^2: \left| U_{33} \right|^2.$$ (11)

The second reaction gives the upper bound for $\left| U_{23} U_{13} \right|^2$ which we estimate\(^ {10}\) as no bigger than $10^{-2}$.

The third reaction is very interesting, because the $L^-$ tracks may be observable in bubble-chamber experiments. This model does not give rise to a large high-$\gamma$ anomaly in the reaction $\nu_e N \rightarrow \mu^- X$.

In the version of the model presently discussed, there is no neutrino oscillation. However, it is possible to endow $\nu_1$ and $\nu_2$ with finite, nondegenerate masses in the model; in that case, there will be neutrino oscillations, as discussed in Ref. 8.
Other phenomena.—There are several classical effects discussed in the literature associated with muon number nonconservation, such as $\mu^N - e^N$ and $\mu\bar{e} - e\bar{\mu}$, but these effects are too small to have a chance for detection in this model. The decays $K_L - \mu\bar{e}$ (or $e\bar{\mu}$) or $K_L - \pi e\bar{\mu}$ are also difficult to detect; for the former, we have in the free-quark approximation

$$M(K_L - \mu\bar{e}) \sim \frac{G_F}{\sqrt{2}} \frac{\alpha}{4\pi} \left( \frac{m_e}{38 \text{ GeV}} \right)^2 \sin \theta_c \cos \theta_c U_{13}^* U_{23} f_K \left[ \mu\gamma^\mu \left( \frac{1 - \gamma_5}{2} \right) e \right] p_\mu,$$

where $f_K$ is the kaon decay constant and $p^\mu$ is the kaon four-momentum. This leads to the prediction in this model that

$$\Gamma(K_L - \mu\bar{e})/\Gamma(K_L - \mu\bar{\mu}) \simeq \left| U_{13} U_{23} \right|^2 \lesssim 10^{-2}.$$ (13)

Experimentally, $B(K_L - \mu\bar{e}) < 2 \times 10^{-9}$, a bound five times lower than the one on $B(K_L - \mu\bar{\mu})$.

Note added.—After the submission of this work, we received preprints by S. Glashow and H. Fritzsch on similar matters.

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11Swiss Institute for Nuclear Research Physics Report No. 1, 1976 (unpublished); described herein is an experiment in progress by W. Dey et al.
ERRATUM


There is a misprint in the last sentence of p. 937; "$m_{D} \approx 12-30 \text{ GeV for } B = 10^{-8}$" should read "$m_{D} \approx 12 \text{ GeV for } B = 10^{-8}$." Also, in Ref. 1, "J. Ellis, M. M. Gaillard, and D. V. Nanopoulos, to be published" should read "J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. B109, 213 (1976)."