CP NONCONSERVATION AND
SPONTANEOUS SYMMETRY BREAKING

T.D. LEE

Columbia University, New York, N.Y. 10027, USA
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Contents:

1. CP nonconservation
   1.1. Present status
   1.2. Two fundamental questions
2. Spontaneous T violation
   2.1. A simple example
   2.2. A second example
   2.3. General classification
3. A generalized Georgi–Glashow model
   3.1. Lagrangian
   3.2. Vertices
   3.3. Lowest order diagrams
   3.4. e-parameter
   3.5. Electric dipole moment
Acknowledgement
Appendix
References

Abstract:

The observed CP violation is assumed to be due to the spontaneous symmetry-breaking mechanism; the Lagrangian is CP invariant but its particular solution is not. The general classification of such theories when coupled with different unified gauge models of the weak and electromagnetic interactions is given. All such theories lead naturally to a basically milliweak CP non-invariant solution. The possibility that for most weak transitions the result may resemble a superweak theory is analysed, and possible experiments to distinguish these two different types of theories are discussed. Detailed calculations for various CP violating amplitudes are carried out for a generalized Georgi–Glashow model.

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1. *CP* nonconservation

1.1. Present status

Although the theoretical possibility of *CP* nonconservation and time reversal asymmetry was raised [1] as early as 1956, at about the same time parity violation was suggested, the actual experimental discovery came much later. In 1964 Christenson, Cronin, Fitch and Turlay [2] observed that the long-lived neutral kaon $K^0_L$ can also decay into $2\pi$:

$$K^0_L \to \pi^+ \pi^-.$$  \hfill (1.1)

This is of great significance since $K^0_L$ is known to decay into $3\pi$ with the final pions predominantly in the $s$ states. The *CP* value of the $\pi^+ \pi^-$ final state is clearly $+1$, but that of the $\pi^+ \pi^- \pi^o$ is $-1$ (or, at least predominantly $-1$). The fact that a single non-degenerate state $K^0_L$ which has a definite lifetime can decay into states with different *CP* values establishes the violation of *CP* conservation.

At present, *CP* violations have been observed only in the various decay modes of the $K^0 - \overline{K}^0$ system; besides the $\pi^+ \pi^-$ decay, there are [3–7] the decay $K^0_L \to 2\pi^o$ and the charge asymmetry measurements [8, 9] in the $K_{\pi3}$ decay. Furthermore, from the detailed analysis [10] of various $K$ decay amplitudes and especially the recent upper bound [11]

$$\text{rate}(K^0_S \to 3\pi^o)/\text{rate}(K^0_L \to 3\pi^o) < 1.2,$$  \hfill (1.2)

one can conclude that the time reversal symmetry $T$ is also violated, while the product *CPT* symmetry remains valid.

From these experimental results, it follows that the state vectors of $K^0_L$ and $K^0_S$ are *not* eigenstates of *CP*. If one assumes *CPT* invariance, then these state vectors can depend only on one complex parameter [1]:

$$K^0_S = [2(1 + |\epsilon|^2)]^{-1/2} [(1 + \epsilon)K^0 + (1 - \epsilon)\overline{K}^0],$$  \hfill (1.3)

and

$$K^0_L = [2(1 + |\epsilon|^2)]^{-1/2} [(1 + \epsilon)K^0 - (1 - \epsilon)\overline{K}^0],$$  \hfill (1.4)

where $\overline{K}^0$ is defined to be the *CPT* conjugate of $K^0$. If *CP* invariance were true, then $K^0_L$ should be orthogonal to $K^0_S$, and therefore $\epsilon$ would be zero. Thus, the parameter $\epsilon$ characterizes the extent of *CP* nonconservation in the description of the $K^0_L$ and $K^0_S$ state vectors.

The phase angle $\text{arg}((1 - \epsilon)/(1 + \epsilon))$ depends on the relative phase between $K^0$ and $\overline{K}^0$ which can be arbitrarily chosen. If we adopt the phase convention that the transition amplitude $K^0 \to 2\pi$ in the isospin $I = 0$ final state is real, then the phase angle $\epsilon$ becomes approximately determined [12–14]

$$\text{arg} \epsilon \approx \tan^{-1} \{2(m_L - m_S)/(\gamma_S - \gamma_L)\},$$  \hfill (1.5)
where \( m_L, \gamma_L \) and \( m_S, \gamma_S \) refer to the mass and the width of \( K_L^0 \) and \( K_S^0 \) respectively. The magnitude of \( \epsilon \) can then be measured from the charge asymmetry results in the \( K_{\pi l3} \) decay. By using [1, 8–10]

\[
\frac{\text{rate}(K_S^0 \rightarrow \pi^- l^+ \nu_l)}{\text{rate}(K_S^0 \rightarrow \pi^+ l^- \bar{\nu}_l)} \approx \frac{\text{rate}(K_L^0 \rightarrow \pi^- l^+ \nu_l)}{\text{rate}(K_L^0 \rightarrow \pi^+ l^- \bar{\nu}_l)} \approx 1 + 4 \text{Re} \epsilon, \quad (1.6)
\]

one finds

\[
|\epsilon| \approx 2 \times 10^{-3}. \quad (1.7)
\]

The present experimental situation can best be summarized in terms of the simple phenomenological description that all observed \( T \) and \( CP \) violations are completely determined by the single small parameter \( \epsilon \neq 0 \); i.e., to a good approximation, one may regard the transition matrix elements between \( K^0, \bar{K}^0 \) and \( 2\pi, 3\pi, \pi l \nu, \text{etc.}, \) as \( T \) and \( CP \) invariant. For example, in this approximation one has

\[
\eta_{+-} \approx \eta_{oo} \approx \epsilon, \quad (1.8)
\]

where

\[
\eta_{ab} \equiv \text{amp}(K_L^0 \rightarrow \pi^a \pi^b)/\text{amp}(K_S^0 \rightarrow \pi^a \pi^b). \quad (1.9)
\]

Equation (1.8) is in good agreement with the present measurements of \( \eta_{+-} \) and \( \eta_{oo} \) [15, 16].

Since the \( K^0 \) and \( \bar{K}^0 \) system is determined not only by the weak interaction but also by the strong and the electromagnetic interactions, there are several alternative theoretical possibilities concerning the origin of the \( CP \) violation [17]:

(i) superweak theory [18]: the \( CP \) violating interaction is assumed to violate the hypercharge conservation by two units and its strength is \( \sim 10^{-9} \) \( G_F \), where \( G_F \) is the usual Fermi coupling constant for the weak interaction. Because of the extreme smallness of the superweak coupling constant, one may safely neglect any higher order effect. Equations (1.5) and (1.8) are then exact [17];

(ii) milliweak possibility: the \( CP \) violating interaction is assumed to violate the hypercharge conservation by one unit, and its strength is \( \sim 10^{-3} \) \( G_F \). We note that the deviation from (1.8) is given by [19]

\[
|\eta_{+-} - \epsilon| = 2^{-1/2} |A_2/A_0| \equiv |\epsilon'|, \quad (1.10)
\]

where \( A_0 \) and \( A_2 \) refer respectively to the transition amplitudes of \( K^0 \rightarrow 2\pi \) in the \( I = 0 \) and \( 2 \) states. At present, the origin of the usual \( |\Delta I| = \frac{1}{2} \) rule is quite unclear. If one assumes that the same rule applies to the \( CP \) violating amplitude as well, then any such milliweak theory would imply the magnitude of \( |\text{Im} A_2/A_0| \sim 10^{-3} \). Consequently, one has \( |\text{Im} A_2/A_0| \sim 5 \times 10^{-5} \), which is \( \ll |\epsilon| \) and is, therefore, consistent with (1.8). On the other hand, if the milliweak \( CP \) violating interaction does not satisfy the \( |\Delta I| = \frac{1}{2} \) rule, then an additional explanation is needed for (1.8);
(iii) millistrong possibility \([17, 20]\): the \(CP\) violating interaction strength is assumed to conserve the hypercharge and its strength is \(\sim 10^{-3}\) times the strong interaction strength. It is also possible that such a millistrong force may be electromagnetic in origin \([21]\). In any case, if such a millistrong force conserves isospin also, then one expects (1.8) to hold at least to the level where the \(|\Delta I| = \frac{1}{2}\) rule holds for the usual (\(T\) conserving) weak interaction.

1.2. Two fundamental questions

At present, the full implications of \(CP\) nonconservation and \(T\) violation are far from being understood. There are at least two fundamental questions:

1. Why should \(T\) symmetry and \(CP\) symmetry be violated?
2. Why should the violation be so small?

In order to provide some possible answers to these questions, we shall examine in this paper a general class of theories \([22]\) in which the total Lagrangian (which includes all interactions, strong, electromagnetic and weak) is assumed to satisfy:

(a) invariance under \(CP\) and \(T\),
(b) renormalizability, and
(c) invariance under certain weak and electromagnetic gauge transformations.

We note that if (a) holds, then by identifying time reversal with \(T\) and space inversion with \(CP\) (not \(P\)), the full Poincaré group of space-time transformations, including these discrete ones, may be regarded as exact symmetries of all interactions. The dynamical equation is completely symmetrical with respect to \(T\) and \(CP\), though its particular solution can be non-invariant. Such a mechanism will be referred to as the spontaneous \(T\) violation, or spontaneous \(CP\) violation. In physics, it is a familiar practice to assume that the dynamical equation is symmetric under a relatively large group of transformations, while its particular solution does not have to be an invariant under the same transformations. For example, rotational invariance of the physical law by no means implies that all physical solutions should be spherically symmetrical. From an esthetical point of view, it appears pleasing to assume a \(CP\)-invariant and \(T\)-invariant interaction, but to attribute all observed \(CP\) violations merely to the asymmetrical nature of the particular solution. Such a possibility gives an answer to our first question.

As we shall see, the requirement (b) of renormalizability makes all such \(CP\) and \(T\) noninvariant effects of the solution finite and therefore, at least in principle, computable. In addition, as we shall also show, because of the condition (c) the magnitude of \(CP\) violation must be quite small at low energy. This then provides a possible answer to our second question.

We emphasize that the above three conditions (a), (b), and (c) are logically independent. For example, as discussed by Pais \([23]\), \(CP\) violation can be introduced into a gauge theory \([24, 25]\) without assuming the spontaneous \(CP\) violation mechanism. However, since the entire approach of the unified gauge theory of the electromagnetic and weak interactions \([26–30]\) is based on the spontaneous symmetry-breaking mechanism, it seems natural, and also esthetically more appealing, to include \(CP\) violation as an integral part of the same mechanism.
2. Spontaneous $T$ violation

2.1. A simple example

To illustrate the phenomenon of spontaneous $T$ violation, let us first examine a simple example which satisfies our conditions (a) and (b), but without the condition (c). In this way, we may also demonstrate the logical independence of the spontaneous $T$ violation from the well-known mechanism of spontaneous gauge-symmetry violation [24–26].

This simple example consists of a spin $\frac{1}{2}$ Dirac field $\psi$ and a spin 0 Hermitian field $\phi$. The Lagrangian density is assumed to be invariant under $T$, $C$ and $P$

$$L = -\frac{1}{2} \left( \frac{\partial \phi}{\partial x^\mu} \right)^2 - V(\phi) - \psi^\dagger \gamma_\mu \left( \gamma_\mu \frac{\partial}{\partial x^\mu} + m \right) \psi - ig \psi^\dagger \gamma_\mu \gamma_5 \phi,$$

where on account of renormalizability the most general form of the potential $V(\phi)$ is a quartic function, given by

$$V(\phi) = -\frac{1}{2} \lambda \phi^2 + \frac{1}{4} \kappa \phi^4.$$ 

From Hermiticity, the parameters $m$, $g$, $\lambda$ and $\kappa$ must be real. It can be readily verified that under $T$, $C$ and $P$, $\phi$ transforms according to

$$T\phi(r, t)T^{-1} = -\phi(r, -t),$$

$$C\phi(r, t)C^{-1} = \phi(r, t),$$

and

$$P\phi(r, t)P^{-1} = -\phi(-r, t).$$

To generate a spontaneous $T$ violation, we assume that the (renormalized) coupling constants $\lambda$ and $\kappa$ satisfy

$$\lambda > 0, \quad \text{and} \quad \kappa > 0.$$ 

Thus, as shown in fig. 1, the vacuum expectation value of $\phi$ is not zero;

$$\langle \phi \rangle_{\text{vac}} = \rho \neq 0.$$ 

Since according to (2.2), $\phi$ is of $T = -1$, $CP = -1$ and $P = -1$, a non-zero vacuum expectation value of $\phi$ implies a spontaneous violation of $T$ (and also of $CP$ and $P$). The $T$ symmetry of the Lagrangian requires that if $\langle \phi \rangle_{\text{vac}} = \rho$ is a solution, then $\langle \phi \rangle_{\text{vac}} = \rho$ must also be a solution. These two solutions transform into each other under $T$, but by itself either solution is not invariant under $T$. (It is also not invariant under $CP$ and $P$, though it is invariant under $C$ and $CPT$.)
In the tree approximation, \( \rho \) is determined by the minimum of the c. number function \( V \). Therefore

\[
\rho^2 = (\lambda / \kappa).
\]

Because of quantum effects, the field \( \phi \) fluctuates around its vacuum expectation value. We may write

\[
\phi = \rho + \delta \phi.
\]

In terms of \( \delta \phi \), the potential \( V \) becomes

\[
V = V_0 + \frac{1}{2} \mu^2 (\delta \phi)^2 + \kappa \rho (\delta \phi)^3 + \frac{1}{4} \kappa (\delta \phi)^4,
\]

where \( V_0 = -\frac{1}{4} \kappa^{-1} \lambda^2 \) and \( \mu^2 = 2 \lambda \). In this case, since \( T \) is a discrete symmetry and there is no spontaneous gauge-symmetry violation, one does not encounter any Goldstone boson [24]. The mass \( \mu \) of the field fluctuation \( \delta \phi \) is not zero.

In order to exhibit more clearly the \( T \)-violating character of the solution, we may perform a unitary transformation under which \( \phi \) is unchanged but

\[
\psi \rightarrow [\exp(-\frac{1}{2} i \gamma_5 \alpha)] \psi.
\]

Therefore, the quadratic expression

\[
\psi^\dagger \gamma_4 (m + i \rho \gamma_5) \psi \rightarrow \psi^\dagger \gamma_4 \rho \psi,
\]

where

\[
M = (m^2 + g^2 \rho^2)^{1/2},
\]

provided that the angle \( \alpha \) is given by \( \tan \alpha = m^{-1} g \rho \). Correspondingly, the Lagrangian density \( L \) becomes
Fig. 2. A $T$-violating scattering diagram due to the exchange of the quantum of $\delta \phi$, where $\delta \phi = \phi - \langle \phi \rangle_{\text{vac}}$.

$$-\frac{1}{2} \left[ \frac{\partial}{\partial x_\mu} (\delta \phi) \right]^2 - V - \psi^\dagger \gamma_4 \left( \gamma_\mu \frac{\partial}{\partial x_\mu} + M \right) \psi - g \psi^\dagger \gamma_4 (\sin \alpha + i \gamma_5 \cos \alpha) \psi \cdot (\delta \phi).$$

Since the operator $\psi^\dagger \gamma_4 \psi$ is of $P = 1$, $C = 1$ and $T = 1$ while the operator $i \psi^\dagger \gamma_4 \gamma_5 \psi$ is of $P = -1$, $C = 1$ and $T = -1$, any exchange of the $\delta \phi$-quantum would give an interference term between these two operators that violates $T$, $P$ and $CP$; but the product $CPT$ remains invariant. For example, the $T$ and $CP$ violating amplitude $A_-$ of the diagram given in fig. 2 is

$$A_- = g^2 \sin \alpha \cos \alpha (k^2 + \mu^2)^{-1},$$

where $k$ denotes the 4-momentum transfer.

This simple example illustrates the basic mechanism of a spontaneous $T$ violation. One assumes that the ground state of the system (called the vacuum state) has a nonzero expectation value $\langle \phi \rangle_{\text{vac}}$, where $\phi$ is a $T = -1$ spin 0 field; thus, the vacuum state is noninvariant under $T$, even though the interaction satisfies $T$ invariance. The fact that $\langle \phi \rangle_{\text{vac}} \neq 0$ also makes the vacuum state resemble more closely a "medium", and this is why the fermion field $\psi$ in moving through this "medium" acquires a $(\text{mass})^2$-shift $\Delta^2$, given by

$$\Delta^2 \equiv M^2 - m^2 = g^2 \langle \phi \rangle_{\text{vac}}^2.$$  

For small $\Delta$, and at zero momentum transfer ($k = 0$), the $T$ violating amplitude (2.8) becomes simply

$$A_- \approx g^2 \Delta/\mu^2 M.$$  

The $T$ invariance of the interaction implies that the vacuum state must have a double degeneracy. It is interesting to examine the barrier penetration between these two degenerate ground state solutions: $\langle \phi \rangle_{\text{vac}} = \rho$ and $-\rho$. By assuming the entire system to be in a finite volume $\Omega$ and by following the standard procedure of quantization, one can easily show that as $\Omega \to \infty$, this barrier penetration probability becomes zero exponentially. On the other hand, in any realistic cosmological model the volume $\Omega$ should be finite. In particular, if one assumes an oscillating cosmological model in which $\Omega$ changes periodically, then when $\Omega$ is of a microscopic dimension, the system should undergo transitions between these two solutions $\langle \phi \rangle_{\text{vac}} = \rho$ and $-\rho$. At different cycles of such $\Omega$ oscillations, the system may acquire different solutions, sometimes with $\langle \phi \rangle_{\text{vac}} = \rho$ and sometimes $-\rho$. An examination of such cosmological consequences is extremely interesting, but lies outside the scope of the present paper.
In this simple example, the magnitude of the \( T \) violating amplitude \( A_- \) is, of course, entirely arbitrary. As we shall see, this arbitrariness may be limited if the spontaneous \( T \) violation is coupled with the spontaneous gauge-symmetry violation of the weak and electromagnetic interactions.

### 2.2. A second example

In order to illustrate how a discrete symmetry, such as \( T \) or \( CF \), may be combined with a continuous gauge-symmetry so that both symmetries can be broken spontaneously and simultaneously, let us consider another simple example. While this second example is also *not* realistic, it nevertheless does contain most of the essential features. (All the so-called "realistic" gauge models [26–30] of weak and electromagnetic interactions are unfortunately rather complicated; these will be discussed in the next section.)

We assume that the Lagrangian density \( L \) of all interactions satisfies the three conditions (a), (b) and (c), stated in section 1.2. In this simple model, the gauge-symmetry group is assumed to be the \( SO_3 \) group, like that in the Georgi-Glashow model [27]. The generators of the group are

\[
Q^{wk}, \quad Q^\gamma \quad \text{and} \quad (Q^{wk})^\dagger,
\]

which are related to the weak interaction current \( J^{wk}_\mu \) and the electromagnetic current \( J^\gamma_\mu \) by

\[
Q^{wk} = \int J^{wk}_0 \, d^3r, \quad Q^\gamma = \int J^\gamma_0 \, d^3r, \tag{2.12}
\]

and \( \dagger \) denotes the Hermitian conjugate. Corresponding to these three generators there is a triplet of spin 1 gauge fields \( W_\mu \):

\[
W^+, \quad W_0^0, \quad W^-;
\]  

(2.13)

where \( W_0^0 \) is the usual electromagnetic field \( A_\mu \). In addition, there is a triplet of spin 0 Hermitian fields \( \phi \):

\[
\phi^+, \quad \phi^0, \quad \phi^-;
\]

and a triplet of spin \( \frac{1}{2} \) hadron fields

\[
\psi = \begin{pmatrix} p^+ \\ n^0 \\ q^-
\end{pmatrix}.
\]

(The realistic Georgi-Glashow model [27] would require many additional fields.)

The Lagrangian density is

\[
L = L_\phi + L_{W\psi} + L_{\psi\phi}, \tag{2.14}
\]
where

\[ L_{\phi W} = -\frac{1}{4} W_{\mu\nu}^2 - \frac{1}{2} \left( \partial_\mu \phi \right)^2 - V(\phi), \tag{2.15} \]

\[ L_{w\psi} = -\psi^\dagger \gamma_4 (\gamma_\mu \partial_\mu + m) \psi, \tag{2.16} \]

and

\[ L_{\psi\phi} = -ig \psi^\dagger \gamma_4 \gamma_5 I \psi \cdot \phi, \tag{2.17} \]

in which the components of \( I \) form the \((3 \times 3)\) Hermitian matrix representation of the generators of the \( SO_3 \) group,

\[ W_{\mu\nu} = \frac{\partial_\nu}{\partial x_\mu} W_{\mu} - \frac{\partial_\mu}{\partial x_\nu} W_{\nu} + e(W_{\mu} \times W_{\nu}), \tag{2.18} \]

\[ \partial_\mu \phi = \left( \frac{\partial}{\partial x_\mu} + eW_{\mu} \times \right) \phi, \tag{2.19} \]

\[ \partial_\mu \psi = \left( \frac{\partial}{\partial x_\mu} - ieI \cdot W_{\mu} \right) \psi, \tag{2.20} \]

and

\[ V(\phi) = -\frac{1}{2} \lambda \phi^2 + \frac{1}{4} \kappa (\phi^2)^2. \tag{2.21} \]

In this Lagrangian, all constants \( e, g, \lambda \) and \( \kappa \) are real because of Hermiticity. One can easily verify that the above Lagrangian is invariant under \( T, C, P \) and the \( SO_3 \) transformation, and is also renormalizable [31, 32].

Just as in the previous example, we require the renormalized constants \( \lambda \) and \( \kappa \) to be both positive definite, so that \( \langle \phi \rangle_{\text{vac}} \) is not zero. Defining the direction of \( \langle \phi \rangle_{\text{vac}} \) to be the neutral direction, we may write

\[ \langle \phi^0 \rangle_{\text{vac}} \neq 0, \]

and

\[ \langle \phi^\pm \rangle_{\text{vac}} = 0. \tag{2.22} \]

Since \( \phi^0 \) is the third component of a vector, any solution with \( \langle \phi^0 \rangle_{\text{vac}} \neq 0 \) clearly is not invariant under a general \( SO_3 \) transformation. In addition, because of the pseudoscalar interaction in \( L_{\psi\phi} \), \( \phi^0 \) is of \( T = -1, P = -1 \) and \( C = +1 \); such a solution is also not invariant under \( T, P \) and \( CP \).

So far as the Lagrangian \( L_{\phi W} \) is concerned, the structure is identical to that of the usual Higgs mechanism [25]. Since \( L_{\phi W} \) is an even function of \( \phi \), the usual Goldstone-Higgs treatment makes no distinction between \( \phi^0 \) of \( T = -1 \) or \( +1 \). To understand the Higgs mechanism, let us consider the specific term \( -\frac{1}{2} (\partial_\mu \phi)^2 \) in the Lagrangian (2.15). By using (2.19) and the fact that \( \langle \phi \rangle_{\text{vac}} \neq 0 \),
Let us examine the first term in the above expression: because of the vector product, only the components of $W_\mu$ that are perpendicular to the direction of $\langle \phi \rangle_{\text{vac}}$ are involved. One sees that these components acquire a mass $m_\omega$ given by

$$m_\omega^2 = e^2 \langle \phi \rangle_{\text{vac}}^2,$$

while the component of $W_\mu$ that is parallel to $\langle \phi \rangle_{\text{vac}}$ remains massless and is identified to be the photon field $A_\mu$; therefore, the direction of $\langle \phi \rangle_{\text{vac}}$ is indeed the neutral direction, as given by (2.22). The second term $\langle \phi \rangle_{\text{vac}} \cdot (W_\mu \times \partial \phi / \partial x_\mu)$ couples only the charged components $W_\mu^\pm$ with $\partial \phi^\pm / \partial x_\mu$. It is straightforward to verify that $\partial \phi^\pm / \partial x_\mu$ now combines with $W_\mu^\pm$ (which has only transverse components) to form a single massive spin 1 charged intermediate boson field; $\partial \phi^\pm / \partial x_\mu$ now becomes the longitudinal mode of the spin 1 field. Let $G_F$ be the Fermi weak interaction constant in this simple example; one has

$$e^2 / 4m_w^2 = G_F / \sqrt{2}.$$  

(The realistic Georgi-Glashow model [27] would have an additional factor $\sin^2 \beta$ on the left-hand side.)

In order to exhibit more clearly the $T$ violating character of the solution, let us concentrate on the $\phi^0$-interaction part of $L_{\psi\phi}$. By using (2.17), one may write

$$L_{\psi\phi} = -ig(p^+ \gamma_4 \gamma_5 q - q^+ \gamma_4 \gamma_5 p)\phi^0 + ... .$$

Just as in the first example, similarly to (2.4) we may expand

$$\phi^0 = \langle \phi^0 \rangle_{\text{vac}} + \delta\phi^0.$$  

The transformation (2.5) is slightly modified because of the $SO_3$ property

$$\psi \rightarrow \exp(-\frac{1}{2}i\gamma_5 \alpha I_3) \psi;$$

i.e., $p \rightarrow \exp(-\frac{1}{2}i\gamma_5 \alpha) p$, $n \rightarrow n$ and $q \rightarrow \exp(\frac{1}{2}i\gamma_5 \alpha) q$ where $\tan \alpha = m^{-1}g\rho$. The masses of $p$, $n$ and $q$ become

$$m_p^2 = m_q^2 = m_n^2 + (g\rho)^2,$$

and $m_n = m$. Correspondingly, after the transformation, the $\delta\phi^0$-part of the interaction is given by

$$-\frac{1}{2}(\partial_\mu \phi)^2 = -\frac{1}{2} e^2(W_\mu \times \langle \phi \rangle_{\text{vac}})^2 - e\langle \phi \rangle_{\text{vac}} \cdot (W_\mu \times \partial \phi / \partial x_\mu) + ... .$$

(2.23)
Thus, just as in the previous example, the exchange of the $\delta \phi^0$ quantum leads explicitly to $T$, $P$ and $CP$ violations. In the single $\delta \phi^0$-exchange diagram the $T$-violating amplitude $A_-$ remains as given by (2.8). However, because the spontaneous $T$ violation is coupled to the spontaneous gauge-symmetry violation of the weak interaction, one finds that by using (2.24), (2.25) and (2.28), at $k = 0$

$$A_- \approx 2^{3/2} G_F \cdot \Delta^3/\mu^2 m_p,$$

where $\mu$ is the mass of the neutral spin 0 boson and $\Delta^2 = m_p^2 - m_n^2$. By definition, the ratio $\Delta/m_p$ is $< 1$.

Although the magnitude of $A_-$ still depends on the unknown mass ratio $(\Delta^3/\mu^2 m_p)$, certain limits can already be estimated. The mass shift $\Delta$ must reflect in some way certain observed hadron mass differences; therefore $\Delta$ cannot be too large. In addition, independently of the question of $T$ invariance, in order that the usual weak interaction be of the $V-A$ character, it must be dominated by the exchanges of the intermediate spin 1 boson, not the spin 0 boson; consequently, $\mu$ must be fairly large. Thus, it seems reasonable to assume $(\Delta^3/\mu^2 m_p) \ll 1$, and therefore the $T$-violating amplitude $A_-$ is much smaller than the Fermi constant $G_F$.

This simple example illustrates the main feature of combining the spontaneous $T$ violation with the spontaneous gauge-symmetry violation of the weak and electromagnetic interactions. Because $\langle \phi \rangle_{\text{vac}} \neq 0$, the vacuum acts as a "medium"; it gives the hadron field a mass shift $\Delta = g \langle \phi^0 \rangle_{\text{vac}}$ and the intermediate boson field a mass $m_W = e \langle \phi^0 \rangle_{\text{vac}}$, which then leads the $T$ violating amplitude $A_-$ from eq. (2.10) of our first example to the above expression (2.30). Furthermore, since $g \langle \phi^0 \rangle_{\text{vac}}$ contributes to the mass matrix of the hadrons, any exchange of $\delta \phi^0$ quantum satisfies the same selection rule as that particular piece of the mass matrix. As we shall see in subsequent sections, in a more realistic model this implies the selection rule $\Delta Y = 0$ where $Y$ is the hypercharge. If one assumes

$$\Delta^3/\mu^2 m_p \sim 10^{-3},$$

then it gives a milliweak $\Delta Y = 0$ $T$-violating interaction.

In order to make a "realistic" model, with the inclusion of the Cabibbo angle, the usual $V-A$ current, etc., one must at least increase the number of hadron fields to eight [27, 28], and the number of lepton fields also to eight. In addition, one must at least double the neutral spin 0 Hermitian fields; otherwise, there will be difficulties. For example, among the lepton fields, there is a triplet representation

$$\psi_i = \begin{pmatrix} X^+ \\ X^0 \\ e^- \end{pmatrix}. $$

If there is only a single spin 0 triplet $\phi$, the interaction between $\psi_i$ and $\phi$ would have the same
form as $L_\psi\phi$ given by (2.17). By going through the same argument that leads to (2.28), one would have obtained the unacceptable conclusion that the heavy electron $X^+$ should have the same mass as that of the electron $e^-$. To avoid this difficulty, we must have at least two neutral spin 0 hermitian fields $\phi_1^0$ and $\phi_R^0$; one of them, say $\phi_1^0$, has a pseudoscalar interaction, and the other, $\phi_R^0$, a scalar interaction. Therefore, $\phi_1^0$ is of $T = -1$ and $\phi_R^0$ is of $T = +1$. We assume

$$\langle \phi_1^0 \rangle_{\text{vac}} = \rho_1 \neq 0$$

and

$$\langle \phi_R^0 \rangle_{\text{vac}} = \rho_R \neq 0. \quad (2.31)$$

Under $T$, $\langle \phi_1^0 \rangle_{\text{vac}}$ becomes $-\rho_1$, but $\langle \phi_R^0 \rangle_{\text{vac}}$ remains the same. The $T$ invariance of the interaction requires both solutions $+\rho_1$ and $-\rho_1$ to exist; but, as before, either solution is not $T$ invariant, and it leads to a spontaneous $T$ violation.

Under the $SO_3$ transformation, $\phi_1^0$ and $\phi_R^0$ must belong to two different representation spaces, which may or may not be of the same dimension. For simplicity we may consider the special case where $\phi_1^0$ belongs to a triplet $\phi_1$, and $\phi_R^0$ to another triplet $\phi_R$, different from $\phi_1$. The field $\phi_1$ may interact with the fermions through a pseudoscalar interaction like that in (2.17), and $\phi_R$ through a scalar interaction like that in the original Georgi-Glashow model [27, 28]. The discussion of such a “realistic” Georgi-Glashow model with spontaneous $T$ violation is straightforward, but somewhat complicated because of the relatively large number of fields involved. The details will be given in section 3.

2.3. General classification

Because of the large variety of different gauge models of weak and electromagnetic interactions available, in this section we shall try to study only the general question: how can each of these models be enlarged to include a spontaneous $T$ violation, and what would be the general pattern of such a $T$ violation? We assume the total Lagrangian to satisfy the three conditions (a), (b) and (c), stated in section 1.2. Let $G$ be the relevant gauge-symmetry group. The neutral spin 0 field $\phi^0$ must belong to a representation of $G$.

We distinguish two different cases:

(i) $\phi^0$ belongs to a real representation, and

(ii) $\phi^0$ belongs to a complex representation.

In case (i), as illustrated in our second example, a single hermitian $\phi^0$ with a non-zero vacuum expectation value is sufficient to generate a spontaneous $T$ violation (although in the “realistic” Georgi-Glashow model, as mentioned before, because of other requirements there are actually two such hermitian $\phi^0$ fields).

In case (ii), the situation is quite different. The complex representation means that $\phi^0$ must be a complex field. The gauge invariance of the interaction implies that the time reversal operator $T$ is not defined up to a gauge transformation factor. Therefore, if there is only a single $\phi^0$ field, one has in general

$$T\phi^0 T^{-1} = e^{i\theta} \phi^0, \quad (2.32)$$
where $a$ is arbitrary. Let the vacuum expectation value of $\phi^0$ be

$$\langle \phi^0 \rangle_{\text{vac}} = \rho e^{ib},$$

where $\rho$ is real. By choosing $a = -2b$, one can always arrange to have

$$T\langle \phi^0 \rangle_{\text{vac}} T^{-1} = \langle \phi^0 \rangle^*_{\text{vac}} = \langle T \phi^0 T^{-1} \rangle_{\text{vac}}. \quad (2.33)$$

Therefore, with only a single complex $\phi^0$ field, it is not possible to generate a spontaneous $T$ violation. The simplest way to produce a spontaneous $T$ violation is to double the number of neutral spin 0 fields, so that there are two such complex fields, say $\phi_1^0$ and $\phi_2^0$. These two neutral fields must belong to two separate representation spaces, which may or may not be equivalent. In general, the relative phase between $\phi_1^0$ and $\phi_2^0$ is no longer arbitrary. For definiteness let us assume

$$T\phi_1^0 T^{-1} = e^{ia} \phi_1^0,$$

and

$$T\phi_2^0 T^{-1} = e^{ia} \phi_2^0$$

with the same phase angle $a$. The vacuum expectation values of $\phi_1^0$ and $\phi_2^0$ may be written as

$$\langle \phi_1^0 \rangle_{\text{vac}} = \rho_1 \exp \{i(\theta + b)\},$$

and

$$\langle \phi_2^0 \rangle_{\text{vac}} = \rho_2 e^{ib},$$

where $\rho_1$ and $\rho_2$ are both real and $> 0$. Just as in (2.32), the angle $a$ is arbitrary since the time reversal operator $T$ is not defined up to a gauge transformation factor. Furthermore, the absolute phase of $\phi_1^0$ and $\phi_2^0$ is not determined; therefore, the angle $b$ is also arbitrary. Thus, we can always set

$$a = b = 0,$$

and consequently

$$\langle \phi_1^0 \rangle_{\text{vac}} = \rho_1 e^{i\theta}, \quad \text{and} \quad \langle \phi_2^0 \rangle_{\text{vac}} = \rho_2. \quad (2.34)$$

These vacuum expectation values may be represented geometrically by the triangle given in fig. 3. Under $T$, $\theta \to -\theta$. If $\theta \neq 0$ or $\pi$, then the $T$ invariance of the interaction implies that both solutions $\theta$ and $-\theta$ must exist, but either solution is not $T$ invariant.

The quantum fluctuations of $\phi_1^0$ and $\phi_2^0$ about their vacuum expectation values correspond pre-
cisely to the vibrations of this triangle. Since there are three degrees of freedom: \( \delta \rho_1 \), \( \delta \rho_2 \) and \( \delta \theta \), there must be three normal modes, just like the vibrations of a triangular molecule. The virtual exchange of these vibrational quanta can lead to explicit \( T \) and \( CP \) violations, similar to the exchange of the \( \delta \phi^0 \)-quantum discussed before in the simple example.

As mentioned before, in the Georgi-Glashow model, although the spin 0 field belongs to a real representation (i.e., case (i)), because of other dynamical reasons, in order to construct a “realistic” theory, we also need to double the neutral spin 0 field; their vacuum values are given by (2.31), which can again be represented by a triangle, but with

\[
\theta = 90^\circ,
\]

and the corresponding fluctuation \( \delta \theta = 0 \).

If one wishes, one may also regard our first simple example of only one Hermitian neutral field \( \phi^0 \) (discussed in section 2.1) as a further degeneration of the triangle, with the length of one of its sides approaching zero. In this sense, the triangular representation offers the general characterization of the spontaneous \( T \) violation, applicable to all such cases. The detailed description of such a triangle is given in ref. [22].

In general, if the two sides of the triangle \( \rho_1 \) and \( \rho_2 \) are of comparable magnitude, one has

\[
e_{\rho_1} \approx e_{\rho_2} \approx m_w,
\]

\[
(e/m_w)^2 \approx G_F,
\]

and therefore, just as in (2.30), at low energy the \( T \) and \( CP \) violating amplitude is of the order of

\[
A_\perp \approx G_F \cdot \Delta^3/\mu^2 m,
\]

(2.35)

where \( \Delta^2 \) is some appropriate (mass)\(^2\)-difference between the hadrons (or leptons), \( m \) is the appropriate hadron (or lepton) mass, and \( \mu \) is the mass of the relevant spin 0 quantum. As noted before, in any reasonable model, \( \Delta \) cannot be too large because of the observed hadron spectrum, and \( \mu \) cannot be too small since the observed weak interaction is dominated by the exchange of an intermediate boson of spin 1, not spin 0. In addition, the ratio \( (\Delta/m) \) is by definition < 1. Thus, \( A_\perp \) must be quite small. If we assume \( \Delta \lesssim 1 \) GeV and \( \mu \gtrsim 10 \) GeV, then it seems reasonable to expect \( (\Delta^3/\mu^2 m) \sim 10^{-3} \) and therefore that the \( T \) violating amplitude \( A_\perp \) is milliweak.

A typical pattern of such a theory is that any exchange of spin 0 bosons leads to \( T \) and \( P \) violation while any exchange of spin 1 bosons between the usual \( V-A \) currents gives only \( C \) and \( P \) violation. This is because a transformation of the form \( \psi \rightarrow \exp(-\frac{1}{2} \gamma_5 \alpha) \psi \) leaves the vector
i \psi^\dagger \gamma_\mu \gamma^\mu \psi and the axial-vector i \psi^\dagger \gamma_\mu \gamma_5 \psi invariant, but not the scalar \psi^\dagger \gamma_\mu \psi and the pseudo-scalar i \psi^\dagger \gamma_\mu \gamma_5 \psi. To lowest order, \textit{T} violation can be produced via the exchange of the neutral massive spin 0 boson; as already mentioned in section 2.2, such a \textit{T} violation usually satisfies the selection rule \Delta Y = 0. In all the realistic models that we have examined, there is always the existence of some genuine (i.e., non-Goldstone) charged massive spin 0 bosons. Through the exchange of these charged bosons, \textit{T} violation can also be generated. As we shall see in the next section, such exchange typically satisfies the \Delta Y = 0 and \pm 1 rule. In both cases, the order of magnitude of the \textit{T} violating amplitude is given by (2.35). If we assume the ratio \(\Delta^3/\mu^2 m\) to be \(\sim 10^{-3}\), then one has a \textit{milliweak} \textit{P, T} violating theory which satisfies the rule:

\[ \Delta Y = 0, \quad \text{and} \quad \pm 1. \tag{2.36} \]

In principle, since all \textit{T} violating effects in the theory are finite, one can calculate, e.g., the \(\epsilon\) parameters in \(K^0_L\) and \(K^0_S\) states, and the electric dipole moment \(D_E\) of the neutron. However, in practice, there are many serious difficulties: there is no confidence that any of the presently available gauge theories is the correct one (especially when it is applied to hadrons). In any of these theories, one does not know how to properly include the strong interaction; reliable calculations can only be made for free quark-like hadron fields. The connection between a calculation for these free fields and that for the physical hadrons is extremely uncertain. Furthermore, the theoretical expressions for \(\epsilon\) and \(D_E\) are in terms of some appropriate mass difference \(\Delta\) of these quark-like hadrons and the mass \(\mu\) of the relevant spin 0 boson, which are all unknown. Nevertheless, it seems possible that the order of magnitude of, say, \(\Delta^3/\mu^2 m\) in (2.35) may be obtained by using the observed \(\epsilon\) parameter in the \(K^0_L - K^0_S\) system, and from that one may estimate the order of magnitude of \(D_E\) for the neutron. As remarked before, without any detailed calculation, one may well expect the resulting theory to be of the milliweak type. Just as in any milliweak theory, the order of magnitude of \(D_E\) may be estimated to be

\[ D_E \sim |(4\pi)^{-1} G_F e m_n \epsilon| \sim e \times 10^{-23} \text{ cm}, \tag{2.37} \]

which is consistent with the present experimental value [33]. To see how such a crude estimation may survive a more detailed calculation, we shall in the next section consider a specific model: the “realistic” Georgi-Glashow model, but generalized to include the spontaneous \textit{T} violation mechanism. Both \(\epsilon\) and \(D_E\) will be calculated explicitly.

Before discussing the details of such calculations, we note that in any “realistic” model, it is necessary to postulate the existence of some “charmed” hadrons [34]. If one wishes, one may arrange to have \textit{T} violation \textit{only} when there are either real or virtual “charmed” hadrons. (See, e.g., eq. (3.51) below.) In all observed lowest order weak transitions between known particles, it is a good approximation to neglect the “charmed” hadrons. Thus, even though the theory is basically of a milliweak character, so far as all known weak decays are concerned, the predictions can be almost identical to those of a superweak theory; i.e., in all lowest order \(\Delta Y = 0\) and \(\pm 1\) weak reactions the \textit{T} violating transition amplitudes can be much less than \(10^{-3}\) times the corresponding \textit{T} conserving amplitudes. However, as we shall see in section 3.5, the order of magnitude of the electric dipole moment of the neutron, or proton, should retain its milliweak character, and this may well offer the best experimental possibility for differentiating such a theory from a genuine superweak theory.
3. A generalized Georgi-Glashow model

In order to give a quantitative illustration of the general remarks made at the end of the previous section, we shall consider in some detail a specific model, which is chosen because of some of its relatively simple features. (Of course, none of these "realistic" gauge models [26–30] is truly simple, especially when applied to hadrons.)

3.1. Lagrangian

This model is a generalization of the Georgi-Glashow model [27, 28]. In this model, the basic gauge group of the weak and electromagnetic interaction is $SO_3$, with its three generators given by the usual weak interaction charge $Q_{\text{wk}}$, the electric charge $Q_e$ and the Hermitian conjugate $(Q_{\text{wk}})^\dagger$. Correspondingly, there is a triplet of spin 1 gauge field $W_\mu$, just as in (2.11) and (2.13). There are at least eight quark-like hadron fields grouped into two triplets $\psi, \psi'$ and two singlets $\chi, \chi'$. These fields are related to the physical (but still quark-like) states $p^+, n^0, \lambda^0, q^-$ and $p'^+, n'^0, \lambda'^0, q'^-$ as follows:

\[
\psi = \begin{pmatrix} p \\ n_c \sin \beta + n' \cos \beta \\ q \end{pmatrix}_L + \begin{pmatrix} \exp(i\alpha_p) \end{pmatrix} p \\ \begin{pmatrix} n' \\
\exp(i\alpha_q) \end{pmatrix} q \\
\end{pmatrix}_R,
\]

\[
\psi' = \begin{pmatrix} p' \\ \lambda_c \sin \beta + \lambda' \cos \beta \\ q' \end{pmatrix}_L + \begin{pmatrix} \exp(i\alpha_{p'}) \end{pmatrix} p' \\ \begin{pmatrix} \lambda' \\
\exp(i\alpha_{q'}) \end{pmatrix} q' \\
\end{pmatrix}_R,
\]

\[
\chi = (n_c \cos \beta - n' \sin \beta)_L + n_R,
\]

and

\[
\chi' = (\lambda_c \cos \beta - \lambda' \sin \beta)_L + \lambda_R,
\]

where

\[
n_c = n \cos \theta_c + \lambda \sin \theta_c,\\n\lambda_c = -n \sin \theta_c + \lambda \cos \theta_c,
\]

$\theta_c$ is the Cabibbo angle and $\beta$ is the Georgi-Glashow angle, the subscripts L and R refer respectively to the usual $\frac{1}{2} (1 + \gamma_c)$ and $\frac{1}{2} (1 - \gamma_c)$ projections of the Dirac spinors, and the angles $\alpha_p, \alpha_{p'}, \alpha_q, \alpha_{q'}$ are due to the spontaneous $T$ violation which will be discussed later. (See (3.19) below.)

There are two $SO_3$-triplets of spin 0 Hermitian field: $\Phi_R$ and $\Phi_I$. Under $T$, 


The total Lagrangian density can be written as

\[ L = L(W, h) + L(h, \phi_R) + L(h, \phi_I) + L(\phi, W), \]  

in which

\[ L(W, h) = -\psi^\dagger \gamma_4 (\gamma_\mu \partial_\mu + m_0) \psi - \psi'^\dagger \gamma_4 (\gamma_\mu \partial_\mu + m'_0) \psi' \]
\[ - \chi^\dagger \gamma_4 (\gamma_\mu \partial/\partial x_\mu + m_1) \chi - \chi'^\dagger \gamma_4 (\gamma_\mu \partial/\partial x_\mu + m'_1) \chi', \]  

where

\[ \partial_\mu = \partial/\partial x_\mu - ie I \cdot W_\mu, \]  

and \( I \) is the \( 3 \times 3 \) matrix representation of the \( SO_3 \) generators. The interaction Lagrangian \( L(h, \phi_R) \) between the hadron fields and \( \phi_R \) is identical to the usual one given in ref. [28]. We may write

\[ L(h, \phi_R) = -(g_R \psi^\dagger \gamma_4 I \psi + g'_R \psi'^\dagger \gamma_4 I \psi') \cdot \phi_R + ... \]

where ... contains all other \( CP \) invariant and \( SO_3 \) invariant couplings between \( \phi_R \) and the hadron fields, and in addition it also contains a simple quadratic coupling term between \( \chi \) and \( \chi' \), but without \( \phi_R \). The interaction between \( \phi_I \) and the hadron fields is assumed to be given by

\[ L(h, \phi_I) = -i(g_I \psi^\dagger \gamma_4 \gamma_5 I \psi + g'_I \psi'^\dagger \gamma_4 \gamma_5 I \psi') \cdot \phi_I, \]

where \( g_I \) and \( g'_I \) are real by Hermiticity. For simplicity, we assume \( \phi_I \) not directly interacting with the singlet fields \( \chi \) and \( \chi' \). The Lagrangian density \( L(\phi, W) \) is given by

\[ L(\phi, W) = -\frac{1}{4} W_{\mu \nu}^2 - \frac{1}{2} (\partial_\mu \phi_R)^2 - \frac{1}{2} (\partial_\mu \phi_I)^2 - V(\phi), \]

where

\[ W_{\mu \nu} = \frac{\partial}{\partial x_\nu} W_\mu - \frac{\partial}{\partial x_\mu} W_\nu + e(W_\mu \times W_\nu), \]
\[ \partial_\mu \phi_a = \left( \frac{\partial}{\partial x_\mu} + e W_\mu \times \right) \phi_a, \]

the subscript \( a \) denotes \( R \) or \( I \), and \( V(\phi) \) is the potential function between \( \phi_I \) and \( \phi_R \). The entire Lagrangian is assumed to be renormalizable and invariant under \( T, CP \) and \( SO_3 \) transformations; i.e., it satisfies the three conditions (a), (b) and (c) given in section 1.2. The most general form of \( V(\phi) \) is
\[ V(\phi) = -\frac{1}{2} \lambda_R \phi_R^2 - \frac{1}{2} \lambda_I \phi_I^2 + \frac{1}{4} A_R (\phi_R^2)^2 + \frac{1}{4} A_I (\phi_I^2)^2 + \frac{1}{4} B (\phi_R^2)(\phi_I^2) - \frac{1}{4} C (\phi_R \cdot \phi_I)^2, \]  

(3.11)

in which the six constants \( \lambda_R, \lambda_I, ..., C \) are real by Hermiticity.

To insure that \( V(\phi) \) has a minimum, we assume

\[ A_R > 0, \quad A_I > 0, \]

and

\[ A_R A_I > \frac{1}{4} (B - C)^2. \]  

(3.12)

In order that the minimum of \( V(\phi) \) occurs at the values of \( \phi_R \) and \( \phi_I \) given by

\[ \langle \phi_R \rangle_{\text{vac}} = \rho_R \neq 0, \]

and

\[ \langle \phi_I \rangle_{\text{vac}} = \rho_I \neq 0, \]  

(3.13)

we assume

\[ A_I \lambda_R > \frac{1}{2} (B - C) \lambda_I, \]

and

\[ A_R \lambda_I > \frac{1}{2} (B - C) \lambda_R. \]  

(3.14)

Consequently, at least one of the two constants \( \lambda_R \) and \( \lambda_I \) should be greater than zero. In addition, we impose

\[ C > 0, \]  

(3.15)

so that

\[ \rho_R \parallel \rho_I. \]

Due to the nature of the vector product \( W_\mu \times \phi_a \) in (3.10), among the three components of \( W_\mu \), only the two that are \( \perp \) to \( \langle \phi_a \rangle_{\text{vac}} \) acquire a mass \( m_w \), given by

\[ m_w^2 = e^2 (\rho_R^2 + \rho_I^2), \]  

(3.16)

where \( \rho_a \) (\( a = R \) and \( I \)) denotes the magnitude of \( \langle \phi_a \rangle_{\text{vac}} \). The \( \perp \) component of \( W_\mu \) remains of zero mass; therefore, it is the photon field \( A_\mu \). The direction of \( \langle \phi_a \rangle_{\text{vac}} \) then defines the neutral direction. As usual, all the above conditions (3.12), (3.14) and (3.15) refer to the renormalized con-
stants. Because $\phi^0_R$ is odd under $T$ and $CP$, (3.13) implies that our solution is not invariant under $T$ and $CP$; it is also not invariant under a general $SO_3$ transformation, since $\phi_I$ and $\phi_R$ are both vectors.

By setting $\phi_R$ and $\phi_I$ equal to their vacuum expectation value and by using (3.5), (3.7) and (3.8), one finds the masses of $p^+$, $q^-$, $p'^+$ and $q'^-$ to be given by, respectively

$$m_p^2 = (m_0 + g_R \rho_R)^2 + (g_I \rho_I)^2,$$

$$m_q^2 = (m_0 - g_R \rho_R)^2 + (g_I \rho_I)^2,$$

$$m_{p'}^2 = (m'_0 + g'_R \rho_R)^2 + (g'_I \rho_I)^2,$$

$$m_{q'}^2 = (m'_0 - g'_R \rho_R)^2 + (g'_I \rho_I)^2,$$

where, because of (3.1), $m_0$ and $m'_0$ are also related to the masses of $n'$ and $\lambda'$ by

$$m_0 = m_n \cos \beta, \quad \text{and} \quad m'_0 = m_\lambda \cos \beta.$$  \hspace{1cm} (3.17)

The angles $\alpha_p$, ..., $\alpha_q$ in (3.1) are given by

$$\alpha_p = \sin^{-1}(g_I \rho_I / m_p), \quad \alpha_q = -\sin^{-1}(g_I \rho_I / m_q),$$

$$\alpha_{p'} = \sin^{-1}(g'_I \rho_I / m_{p'}), \quad \text{and} \quad \alpha_{q'} = -\sin^{-1}(g'_I \rho_I / m_{q'}).$$  \hspace{1cm} (3.18)

To find the normal modes of the spin 0 fields, we introduce new field variables

$$R^0 \equiv \phi^0_R - \langle \phi^0_R \rangle_{\text{vac}}, \quad I^0 \equiv \phi^0_I - \langle \phi^0_I \rangle_{\text{vac}},$$

$$H^\pm \equiv (\rho_R^2 + \rho_I^2)^{-1/2} [\rho_I \phi_R^\pm - \rho_R \phi_I^\pm],$$

and

$$G^\pm \equiv \mp i(\rho_R^2 + \rho_I^2)^{-1/2} [\rho_R \phi_R^\pm + \rho_I \phi_I^\pm].$$  \hspace{1cm} (3.19)

These fields represent the various quantum fluctuations of the two vector fields $\phi_R$ and $\phi_I$ around their respective vacuum expectation values $\rho_R$ and $\rho_I$, which are parallel. By definition, $R^0$ and $I^0$ denote the oscillations along the direction $\rho_a$ ($a = R$ or $I$). By decomposing $G^\pm$ to two Hermitian fields $G_1$ and $G_2$:

$$G^\pm = 2^{-1/2} (G_1 \mp i G_2),$$

one sees that $G_1$ and $G_2$ are the two nonzero components of the vector

$$G = (\rho_R^2 + \rho_I^2)^{-1/2} (\rho_I \times \phi_1 + \rho_R \times \phi_R).$$
which is \( I \) to \( R \). It is straightforward to verify that geometrically \( G_1 \) and \( G_2 \) represent the overall \( SO_3 \) rotations of \( \Phi_R \) and \( \Phi_I \) as an ensemble; they are the two Goldstone [24] degrees of freedom in this model. The fields \( H^+ \) and \( H^- \) represent the quantum fluctuations due to the relative rotation between \( \Phi_R \) and \( \Phi_I \).

Recalling that \( R^0 = I^0 = H^2 = 0 \) denotes the minimum of \( V(\phi) \), one finds

\[
V(\phi) = \text{constant} + \frac{1}{2} C(\rho_R^2 + \rho_I^2) H^+ H^- + A_R \rho_R^2 (R^0)^2 + A_I \rho_I^2 (I^0)^2 + (B - C) \rho_R \rho_I R^0 I^0 + ... \quad (3.21)
\]

where \( ... \) consists of only cubic and quartic expressions of these new field variables. Thus, the normal modes consist of two (genuine) neutral spin 0 bosons, called \( \phi_1^0 \) and \( \phi_2^0 \), whose \((\text{mass})^2\) are the eigenvalues of the matrix

\[
m^2_\phi = \begin{pmatrix}
2A_R \rho_R^2 & (B - C) \rho_R \rho_I \\
(B - C) \rho_R \rho_I & 2A_I \rho_I^2
\end{pmatrix}.
\]

(3.22)

Both \( \phi_1^0 \) and \( \phi_2^0 \) are coherent mixtures of \( R^0 \) and \( I^0 \) determined by the eigenstates of \( m^2_\phi \). There is also a (genuine) charged spin 0 boson \( H^\pm \) whose mass is

\[
m^2_H = \frac{1}{2} C(\rho_R^2 + \rho_I^2).
\]

(3.23)

As expected, the quadratic expression of \( V(\phi) \) is independent of \( G^\pm \). In accordance with the Higgs mechanism [25], \( G^\pm \) becomes the longitudinal mode of the spin 1 intermediate boson.

The inclusion of leptons is straightforward, just as in the usual Georgi-Glashow model [27]. Since our main interest lies in the \( K^0_L - K^0_S \) decay, the details of the lepton part will not be given here.

3.2. Vertices

It is convenient to group the eight quark-like hadron states \( p^+, n^0, ..., \lambda'^+, q'^- \) into four doublets

\[
P = \begin{pmatrix}
p^+ \\
p'^+
\end{pmatrix}, \quad Q = \begin{pmatrix}
q^- \\
q'^-
\end{pmatrix}, \quad N = \begin{pmatrix}
n^0 \\
\lambda^0
\end{pmatrix}, \quad N' = \begin{pmatrix}
n'^0 \\
\lambda'^0
\end{pmatrix}.
\]

(3.24)

The interaction between these hadron states and the other integer-spin normal modes \( W^\pm, H^\pm, \phi_1^0 \) and \( \phi_2^0 \) can be readily derived by using (3.4). In the following we list a few of these interaction vertices:

1. The interaction vertex between \( W^\pm \) and \( \bar{P}N \), or \( \bar{Q}N' \), is given by the Lagrangian density
\[ L_{\text{FNW}} = -i(\frac{1}{2} e \sin \beta) W^\mu_\mu P^\dagger \gamma_4 \gamma_\mu (1 + \gamma_5) c N + \text{h.c.} \]

and

\[ L_{\text{QNW}} = +i(\frac{1}{2} e \sin \beta) W^-_\mu Q^\dagger \gamma_4 \gamma_\mu (1 + \gamma_5) c N + \text{h.c.} \] (3.25)

where \( W^+ \) and \( W^- \) annihilate, respectively, the positively and the negatively charged intermediate boson, and \( c \) is the matrix for the Cabibbo rotation [35, 36]

\[ c = \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix}. \] (3.26)

One sees that, as expected, these interaction vertices conserve \( CP \) and \( T \).

2. The interaction vertices between \( G^\pm \) and these hadron states are determined by

\[ L_{\text{FG}} = +i \left( \frac{e\sin \beta}{2m_w} \right) G^+ P^\dagger \gamma_4 [(1 + \gamma_5) m_p c - (1 - \gamma_5) c m_N] N + \text{h.c.}, \]

and

\[ L_{\text{QG}} = -i \left( \frac{e\sin \beta}{2m_w} \right) G^- Q^\dagger \gamma_4 [(1 + \gamma_5) m_Q c - (1 - \gamma_5) c m_N] N + \text{h.c.}, \] (3.28)

where \( m_p, m_Q \) and \( m_N \) are the mass matrices,

\[ m_p = \begin{pmatrix} m_p & 0 \\ 0 & m_p \end{pmatrix}, \quad m_Q = \begin{pmatrix} m_q & 0 \\ 0 & m_q \end{pmatrix}, \quad \text{and} \quad m_N = \begin{pmatrix} m_p & 0 \\ 0 & m_\lambda \end{pmatrix}. \]

As it should, the \( G^\pm \)-transition amplitude between \( N \) and \( P \), or \( N \) and \( Q \), conserves \( CP \) and \( T \), like the corresponding \( W^\pm \)-transition amplitude.

3. The interaction vertices between \( H^\pm \) and these hadron states are given by

\[ L_{\text{FH}} = \frac{1}{2} \left( \frac{e\rho_1}{m_w \rho_R} \right) \sin \beta \quad H^+ P^\dagger \gamma_4 [(1 + \gamma_5) M_p c - (1 - \gamma_5) c m_N] N + \text{h.c.} \]

and

\[ L_{\text{QH}} = \frac{1}{2} \left( \frac{e\rho_1}{m_w \rho_R} \right) \sin \beta \quad H^- Q^\dagger \gamma_4 [(1 + \gamma_5) M_Q c - (1 - \gamma_5) c m_N] N + \text{h.c.} \] (3.29)

where

\[ M_p = m_p - i \rho_1^{-1}(\rho_1^2 + \rho_R^2) \begin{pmatrix} g_1 \exp(-i\alpha_p) & 0 \\ 0 & g_1' \exp(-i\alpha_p) \end{pmatrix}. \]
and

\[ M_Q = m_Q + i \rho_1^{-1}(\rho_1^2 + \rho_R^2) \begin{pmatrix} g_1 \exp(-i\alpha_{I}) & 0 \\ 0 & g_1' \exp(-i\alpha_{I}) \end{pmatrix}. \] (3.30)

Thus, the \( H^\pm \)-transition amplitude violates \( T \) and \( CP \) conservation, reflecting the \( T \) noninvariant character of our solution.

4. The interaction between the neutral spin 0 fields \( \phi_1^0, \phi_2^0 \) and the charged hadrons also violates \( T \) and \( CP \). For example,

\[
L_{\bar{p}p\phi^0} = -p^+ \gamma_4 [\exp(-i\alpha_p \gamma_5)] (g_R^0 + ig_1 \gamma_5 I^0) p \\
- p'^+ \gamma_4 [\exp(-i\alpha'_p \gamma_5)] (g'_R^0 + ig'_1 \gamma_5 I^0) p',
\] (3.31)

\[
L_{\bar{q}Q\phi^0} = + q^+ \gamma_4 [\exp(-i\alpha_q \gamma_5)] (g_R^0 + ig_1 \gamma_5 I^0) q \\
+ q'^+ \gamma_4 [\exp(-i\alpha'_q \gamma_5)] (g'_R^0 + ig'_1 \gamma_5 I^0) q',
\]

where \( R^0 \) and \( I^0 \) are linear combinations of the normal modes \( \phi_1^0 \) and \( \phi_2^0 \), depending on the matrix (3.22). In contrast, the interaction between \( \phi_1^0, \phi_2^0 \) and the neutral hadrons \( n \) and \( \lambda \) conserves \( T \) and \( CP \):

\[
L_{\bar{N}N\phi} = -(\rho_1^{-1} \sin^2 \beta) R^0 (m_n n^+ \gamma_4 n + m_\lambda \lambda^+ \gamma_4 \lambda).
\] (3.32)

This is because in (3.8), for the sake of simplicity, we assume \( \phi_1 \) coupled only to \( \psi \) and \( \psi' \).

If one defines the hypercharge \( Y \) for \( \lambda \) to be \(-1\) and the others zero, then the \( T \) and \( CP \) violating matrix elements (3.29) via \( H^\pm \)-transitions satisfy the \( \Delta Y = 0 \) and \( \pm 1 \) rule, while those via \( \phi_1^0 \) and \( \phi_2^0 \) transitions satisfy the \( \Delta Y = 0 \) rule, in agreement with the general conclusion given in section 2.3.

5. The interaction between the neutral charmed hadrons \( n', \lambda' \) and \( W_\mu^\pm \) violates \( T \) and \( CP \) conservation. We find

\[
L_{\bar{p}N'W} = -\frac{1}{2} i e W_\mu^+ p^+ \gamma_4 \gamma_\mu [((1 + \gamma_5) \cos \beta + (1 - \gamma_5) \exp(-i\alpha_p)] N' + h.c.,
\]

and

\[
L_{\bar{q}N'W} = +\frac{1}{2} i e W_\mu^- q^+ \gamma_4 \gamma_\mu [((1 + \gamma_5) \cos \beta + (1 - \gamma_5) \exp(-i\alpha_q)] N' + h.c.,
\] (3.33)

where

\[
\alpha_p = \begin{pmatrix} \alpha_p & 0 \\ 0 & \alpha'_p \end{pmatrix}, \quad \text{and} \quad \alpha_Q = \begin{pmatrix} \alpha_q & 0 \\ 0 & \alpha'_q \end{pmatrix}.
\] (3.34)
In the above expression, the intermediate boson \( W^\pm \) interacts with a rather unfamiliar looking current: consists of a \((V+A)\), as well as a \((V-A)\), current.

### 3.3. **Lowest order diagrams**

In order to evaluate the weak interaction transition amplitudes for the observed hadrons, one faces a serious defect of the model: the absence of the strong interaction. Strictly speaking, our calculations are limited to the scattering matrix of free quark-like hadrons which have only electromagnetic and weak interactions. All extensions to the observed hadrons are therefore extremely tentative; at best, they are only order of magnitude estimates.

In the following, we regard the usual SU\(_3\) octet mesons as the appropriate bound states of \( \pi \), \( \rho \), etc., consisting only of \( p, n, \lambda \) and their conjugate states. For example, we have the familiar expressions

\[
\begin{align*}
    K^0 &= (n\bar{\lambda}), \\
    \bar{K}^0 &= (\lambda\bar{n}), \\
    \pi^+ &= (p\bar{n}),
\end{align*}
\]

Because these quark-like hadrons are of integer-charges, the known SU\(_3\)-octet baryons are assumed to be bound states of these octet mesons and another neutral quark-like hadron, called \( Z^0 \), where \( Z^0 \) is a singlet under SU\(_3\) and it has a unit baryon number, a unit hypercharge and a half-integer spin. Equivalently, the physical baryons may also be regarded as three-body bound states of these quark-like hadrons; for example,

\[
\text{physical neutron} = (Z^0\bar{\lambda}n),
\]

\[
\text{physical proton} = (Z^0\bar{\lambda}p),
\]

For simplicity, we assume \( Z^0 \) to be the ninth quark-like hadron, different from \( n' \) and \( \lambda' \); it has no weak and electromagnetic interaction. (The alternative assumption that \( Z^0 \) is either \( n' \) or \( \lambda' \) does not change any calculations of the lowest order transition amplitudes for these known hadrons. However, for higher order amplitudes, since \( n' \) and \( \lambda' \) have some rather unusual interactions such as those given by (3.33), one would encounter unnecessary complications which might be best avoided.)

For practical computations it is convenient to adopt a gauge in which \( W^\pm \) and \( G^\pm \) are treated as independent degrees of freedom. The propagator of \( W^\pm_\mu \) has the same form as that used in the \( \xi \)-limiting process [37]

\[
S_{\mu\nu} = -i(k^2 + m_w^2)^{-1}(\delta_{\mu\nu} + \frac{m_w^2}{m_w^2} k_\mu k_\nu) + i(k^2 + \xi^{-1}m_w^2)^{-1}(m_w^{-2}k_\mu k_\nu).
\]

The propagator of \( G^\pm \) is given by

\[
D = -i(k^2 + \xi^{-1}m_w^2)^{-1}.
\]

There is no mixing between \( W^\pm_\mu \) and \( G^\pm \) in the propagator. The parameter \( \xi \) is an arbitrary positive constant. The \( S \)-matrix is independent of \( \xi \), and this independence serves as a useful test for the
correctness of the matrix elements. Formally, this gauge can be achieved by adding to the original Lagrangian, the integral of (3.4), an additional term

$$-\int \frac{\xi}{2} \left[ \frac{\partial W_\mu}{\partial x_\mu} + \xi^{-1} e(\rho_R \times \phi_R + \rho_I \times \phi_I) \right]^2 d^4x - i \text{ trace } (I + e \Delta^{-1}N),$$

(3.38)

where $I$ is the unit matrix given by

$$\langle i, x | I | j, y \rangle = \delta_{ij} \delta^4(x-y),$$

(3.39)

in which $i$ and $j$ vary from 1 to 3 denoting the coordinates of the SO$_3$-transformation space, $x$ and $y$ are the usual 4-dimensional space-time coordinates, and $\delta_{ij}$ is the usual Kronecker symbol. The matrices $\Delta$ and $N$ are given by

$$\langle i, x | \Delta | j, y \rangle = \left[ -\frac{\partial^2}{\partial x^2_\mu} \delta_{ij} + e^2 \xi^{-1}(\rho_R^2 + \rho_I^2)(\delta_{ij} - \hat{n}_i \hat{n}_j) \right] \delta^4(x-y),$$

(3.40)

where $\hat{n}$ is the unit vector parallel to $\rho_R$ and $\rho_I$, and

$$\langle i, x | N | j, y \rangle = \epsilon_{ijk} \frac{\partial}{\partial x_\mu} \left\{ [W_\mu(x)]_k \delta^4(x-y) \right\}$$

(3.41)

$$+ e\xi^{-1} \left[ (\rho_R \cdot \phi_R + \rho_I \cdot \phi_I) - (\rho_R)_i (\phi_R)_j - (\rho_I)_i (\phi_I)_j - (\delta_{ij} - \hat{n}_i \hat{n}_j) (\rho_R^2 + \rho_I^2) \right] \delta^4(x-y),$$

where $\epsilon_{ijk}$ is +1 or −1 if $(ijk)$ is an even or odd permutation of (123), and 0 otherwise. The last term in (3.38) is related to the so-called Fadeev-Popov ghost [38, 39]. For the matrix elements considered in this paper, this term makes no contribution.

In the following we give the various lowest order weak interaction transition elements that are relevant to the known hadrons. Only diagrams whose external lines are $p, n$ and $\lambda$ need be considered. These diagrams are listed in fig. 4.

The usual weak interaction of the known hadrons is given by the $W^\pm$-exchange diagram (i) and the $G^\pm$-exchange diagram (ii) in fig. 4. For on-mass-shell external lines, the sum of these two diagrams is independent of $\xi$. We find

$$i + (ii) = i 2^{-1/2} G_F(k^2 + m_W^2)^{-1} \left[ m^2_W / j^*_\mu j_\mu + s_s s^*_s \right],$$

(3.42)

where $k$ is the 4-momentum transfer,

$$G_F = (2\sqrt{2} m_W^2)^{-1} e^2 \sin^2 \beta,$$

$$j_\mu = i p^\dagger \gamma_4 \gamma_\mu (1 + \gamma_5) (n \cos \theta_c + \lambda \sin \theta_c),$$

$$j^*_\mu = i (n \cos \theta_c + \lambda \sin \theta_c)^\dagger \gamma_4 \gamma_\mu (1 + \gamma_5) p,$$

$$s_s = \cos \theta_c p^\dagger \gamma_4 [m_p (1 + \gamma_5) - m_n (1 - \gamma_5)] n + \sin \theta_c p^\dagger \gamma_4 [m_p (1 + \gamma_5) - m_\lambda (1 - \gamma_5)] \lambda,$$

(3.43)
and $s^\dagger_i$ is its Hermitian conjugate. These two diagrams violate $P$ and $C$ invariance, but conserve $CP$ and $T$; they also satisfy the usual $\Delta Y = 0$ and $\pm 1$ rule.

The $H^+$ exchange diagram (iii) in fig. 4 generates an additional weak interaction among the known hadrons. By using (3.29), we find

$$\text{(iii)} = i2^{-1/2}G_F(k^2 + m^2)\left(\rho_1 \rho_2 - i\rho_1 s^+ - is^+\right)(\rho_2^{-1} \rho_1 s^+ + is^+), \quad (3.44)$$

where

$$s^- = g_1 \rho_1 \rho_2^{-1}(\rho_1^2 + \rho_2^2)\exp(-i\alpha_p) p^\dagger \gamma_4 (1 + \gamma_5) (\cos \theta_c n + \sin \theta_c \lambda). \quad (3.45)$$

Thus, (iii) satisfies the $\Delta Y = 0$ and $\pm 1$ rule; it violates $P$, $C$ and $T$ invariance, but remains invariant under $CPT$.

The exchanges of $\phi_1^0$ and $\phi_2^0$ bosons lead to diagrams (iv), (v) and (vi) in fig. 4. By using (3.31) and (3.32), we find

$$\text{(iv)} = ix^\dagger(k^2 + m^2)^{-1}x,$$

$$\text{(v)} = i[x^\dagger(k^2 + m^2)^{-1}y + \text{h.c.}], \quad (3.46)$$

$$\text{(vi)} = iy^\dagger(k^2 + m^2)^{-1}y,$$

Fig. 4. Some second order weak interaction diagrams in the generalized Georgi-Glashow model (see (3.42), (3.44) and (3.46) for their values).
where $m_\phi$ is the $(2 \times 2)$ matrix given by (3.22)

$$
X = \begin{pmatrix}
g_R p^+ \gamma_4 (\cos \alpha_p - i \sin \alpha_p \gamma_5) p \\
g_1 p^+ \gamma_4 (\sin \alpha_p + i \cos \alpha_p \gamma_5) p 
\end{pmatrix},
$$

(3.47)

and

$$
y = \begin{pmatrix}
\rho_R^{-1} \sin^2 \beta (m_n^+ \gamma_4 n + m_\lambda \lambda^+ \gamma_4 \lambda) \\
0
\end{pmatrix}.
$$

(3.48)

Except for (vi), which is invariant under $C, P$ and $T$, the other two diagrams violate $P, T$ and $CP$ invariance, but remain invariant under $C$ and $CPT$; they all satisfy the $\Delta Y = 0$ rule.

**Remarks**

1. In the following, for order of magnitude estimations we shall assume

$$
m_H, m_\phi \sim O(m_w),
$$

and

$$
m_w \gg "quark" \text{ mass} \gg g_R \rho_R \text{ or } g_1 \rho_1,
$$

(3.49)

since $g_R \rho_R$ and $g_1 \rho_1$ represent some appropriate "quark" mass shifts. If we assume each of these inequalities carries a factor $\sim 10$, then to lowest order, the ratio of the $T$ violating amplitude to that of the usual $T$ conserving amplitude is typically

$$
\sim (g_1 \rho_1) m_p/m_H^2 \quad \text{or} \quad (g_1 \rho_1) (g_R \rho_R)/m_\phi^2,
$$

(3.50)

which is $\sim 10^{-3}$ or $10^{-4}$.

2. An interesting possibility is

$$
g_1 = 0 \quad \text{but} \quad g'_1 \neq 0.
$$

(3.51)

This condition implies that all the above lowest order weak interaction diagrams are $T$ conserving. $T$ violation occurs only in higher order diagrams, in which there is a nonzero probability of having a virtual "charmed" quark component in the known hadron state. Because of (3.17) and (3.18), (3.51) implies a "quark" mass relation

$$
m_p + m_q - 2m_n' \cos \beta = 0.
$$

(3.52)
3.4. $\epsilon$-parameter

In the usual analysis \[1\] of the $K^0 - \bar{K}^0$ system one defines a mass matrix $M$ and a decay matrix $\Gamma$, both Hermitians. The physical $K_L^o$ and $K_S^o$ states are eigenvectors of the equation

$$ (\Gamma + iM)|K^o\rangle = (\frac{1}{2}\gamma_j + m_j)|K^o\rangle, \quad (3.53) $$

where $j = L$ and $S$. Assuming $CPT$ invariance, the solutions $K_S^o$ and $K_L^o$ are given by (1.3) and (1.4), where to a good approximation, as discussed in section 1.1, the phase of the $\epsilon$-parameter is given by (1.5), and its magnitude is

$$ |\epsilon| \approx 2^{-3/2} \frac{|\text{Im} \langle \bar{K}^o | M | K^o \rangle|}{|\text{Re} \langle \bar{K}^o | M | K^o \rangle|} \quad (3.54) $$

If $T$ invariance were true, then $M$ would be real and $\epsilon$ should be zero.

In the present model, because of the lack of strong interaction, we can only compute the scattering matrix $S$ for

$$ n\bar{\lambda} = \lambda\bar{n}. \quad (3.55) $$

To simulate the $K$-meson, we define $|K^o\rangle$ and $|\bar{K}^o\rangle$ to be, respectively, the properly normalized free states $|n\bar{\lambda}\rangle$ and $|\lambda\bar{n}\rangle$, which have zero kinetic energy in their rest frame and a spin-parity $= 0$ — (in the convention that the relative parity between $n$ and $\lambda$ is $+1$). The rest energy of these states $K^o$ and $\bar{K}^o$ is therefore $(m_n + m_\lambda)$.

The $S$-matrix can be decomposed into a sum of a $T = +1$ part $S_+$ and a $T = -1$ part $S_-$ where

$$ S_+ = \frac{1}{2} [S + TS^T T^{-1}], \quad S_- = \frac{1}{2} [S - TS^T T^{-1}], \quad (3.56) $$

and $T$ is the usual antiunitary time reversal operator. The $\epsilon$ parameter in our model is then given by

$$ |\epsilon| \approx 2^{-3/2} \frac{|\langle n\bar{\lambda} | S_+ | \lambda\bar{n}\rangle|}{|\langle n\bar{\lambda} | S_- | \lambda\bar{n}\rangle|} \quad (3.57) $$

at zero kinetic energy.

The lowest order $T$ conserving diagrams for the $\Delta Y = \pm 2$ process (3.55) are fourth order ones via exchanges of two $W^\pm$:

$$ n\bar{\lambda} = W^+ W^- = \lambda\bar{n}, \quad (3.58) $$

where, for convenience of notation, $W^\pm$ represents either $W^\pm$ or $G^\pm$. These diagrams have been calculated by Lee, Primack and Treiman [28], and the resulting $S_+$ is given by

$$ S_+ \approx -i (24\pi^2)^{-1} G_p^2 \sin^2 \theta_c \cos^2 \theta_c \left[ 3(\Delta m_p + \Delta m_q)^2 - 5(\Delta m_p - \Delta m_q)^2 \right] $$

$$ \times [\lambda^+ \gamma_4 \gamma_\mu (1 + \gamma_5) n] [\lambda^+ \gamma_4 \gamma_\mu (1 + \gamma_5) n], \quad (3.59) $$
where
\[ \Delta m_p = m_{p'} - m_p \quad \text{and} \quad \Delta m_q = m_{q'} - m_q. \] (3.60)

The lowest order \( T \) violating diagrams for the same process are also fourth order ones, but via exchanges of \( W^\pm \) and \( H^\pm \):
\[ n\bar{\lambda} = W^2 H^\pm = \lambda\bar{n}. \] (3.61)

These diagrams are given in fig. 5. It is convenient to assume (3.49), and to consider
\[ \Delta m_p, \quad \Delta m_q, \quad (\Delta g_R)\rho_R \equiv (g'_R - g_R)\rho_R, \quad \text{and} \]
\[ (\Delta g'_R)\rho'_R \equiv (g'_R - g_R)\rho'_R \quad \text{to be of comparable magnitude.} \] (3.62)

Furthermore, we shall regard \( \rho'_R \) to \( \rho_R \) to be of the same order of magnitude, and also \( \Delta g_1, \Delta g_R, (g'_1 + g_1) \) and \( (g'_R + g_R) \) to be of the same order of magnitude (although \( g'_1 \) may be much larger than \( g_1 \), leaving the possibility (3.51) open). Such an estimate is consistent with the view that all

Fig. 5. Diagrams for the \( T \) violating scattering \( S \) of the \( \Delta Y = \pm 2 \) reaction \( n_a n_b \rightarrow \lambda c \lambda d \). Each wavy line can be either \( W^\pm \) or \( G^\pm \), and each solid internal line can be either \( p^\pm \), or \( p'^\pm \), or \( q^\pm \), or \( q'^\pm \). Thus, each diagram refers to 16 different assignments, making a total of 128 possibilities.
these quark-like hadron masses are \( \sim \) a few GeV, all their mass differences and mass shifts (due to the Goldstone-Higgs mechanism) are of an order of magnitude smaller, while all integer-spin boson masses (except the photon's) are an order of magnitude bigger. With this in mind, it is only necessary to evaluate \( S_- \) for the reaction (3.61) to order \((\Delta g_1)(\Delta m_i)\), neglecting \((g'_i + g_i)(\Delta m_i)(\Delta m_j)\) where \( i \) and \( j \) can be \( p \) or \( q \).

The evaluation of the \( T \) violating scattering matrix element is straightforward though somewhat tedious. The resulting \( S_- \) is given in the Appendix. It leads to (at small values of external 4-momenta)

\[
|e| = 2^{-3/2} \left( \frac{(\Delta g_1)\rho_1(\Delta m_p - \Delta m_q)(\rho_1^2 + \rho_R^2)(m_1^2 - m_\lambda^2)}{3(\Delta m_p + \Delta m_q)^2 - 5(\Delta m_p - \Delta m_q)^2}\rho_R^2 m_H^2 \right) .
\]  

(3.63)

To have an understanding of this expression, we may consider the special case \( m'_0 = m_0 \); this implies that the mass differences \( \Delta m_p \) and \( \Delta m_q \) are entirely due to the Goldstone-Higgs mechanism. By using (3.17) and neglecting higher order corrections, we find

\[
\Delta m_p = -\Delta m_q = (\Delta g_R)\rho_R .
\]

(3.64)

Consequently, (3.63) reduces to

\[
|e| = \rho_R^{-3} \rho_1(\rho_1^2 + \rho_R^2) \left( \frac{\Delta g_1}{\Delta g_R} \right) \frac{m_1^2 - m_\lambda^2}{20\sqrt{2}m_H^2} .
\]

(3.65)

If we set \( \rho_1 \approx \rho_R \), \( \Delta g_1 \approx \Delta g_R \), then from the observed value \( |e| \approx 2 \times 10^{-3} \) it follows that

\[
m_H^2 \approx 25\sqrt{2}|m_1^2 - m_\lambda^2| .
\]

(3.66)

Thus, if \( m_1 \) and \( m_\lambda \) are in the range of \( \sim 5 \) GeV, their difference of the order of 1 GeV, then \( m_H \) is \( \sim 20 \) GeV, quite consistent with our general order of magnitude considerations.

We emphasize that since \( e \) depends on the difference \( \Delta g_1 \), one has the attractive possibility of setting \( g_1 = 0 \), and therefore \( g'_i = \Delta g_i \) while leaving \( e \) unchanged. In this case, as already discussed in sections 2.3 and 3.3, the milliweak character of \( T \) violation resides only in the “charmed” hadrons. In all known \( \Delta Y = 0 \) and \( \pm 1 \) weak processes, one can certainly neglect the contributions of “charmed” hadrons. If we consider only the lowest order diagrams for these weak processes, then all amplitudes are \( T \) conserving, and that resembles a superweak theory. In particular, in the \( 2\pi \) decay of \( K_L^0 \) and \( K_S^0 \), we have

\[
e' = 0 .
\]

(3.67)

where \( e' \) is defined in (1.10). As mentioned before at the end of section 2.3, to find the difference between such a theory and a \textit{bona fide} superweak theory, one must consider other higher order diagrams, such as those for the electric dipole moment (see eq. (3.73) below).
3.5. Electric dipole moment

In this model, it is quite simple to calculate the electric dipole moment of the quark-like hadrons n, λ, etc. However, without properly taking into account the strong interaction, one cannot determine with any certainty the electric dipole moment of the physical baryon. For definiteness, let us adopt (3.35); we consider the physical neutron N to be a composite \((Z^o n\bar{\lambda})\). As an approximation, we may expect the electric dipole moment of N to be linearly related to those of its constituents

\[ D_E(N) \approx f_n D_E(n) + f_{\bar{\lambda}} D_E(\bar{\lambda}), \tag{3.68} \]

where \(f_n\) and \(f_{\bar{\lambda}}\) are constants, assumed to satisfy \(|f_n| + |f_{\bar{\lambda}}| \leq 1\). \(D_E(n)\) and \(D_E(\bar{\lambda})\) are respectively the electric dipole moment of n and \(\bar{\lambda}\). (The electric dipole moment of \(Z^o\) is by assumption zero.) If in this composite state the spin of N is completely correlated with that of \(Z^o\) (e.g., \(n\bar{\lambda}\) is in a singlet state), then we have \(f_n = f_{\bar{\lambda}} = 0\), and consequently

\[ D_E(N) \approx 0. \tag{3.69} \]

If on the other hand the spin of N is completely correlated with that of n, or \(\bar{\lambda}\), then we expect \(D_E(N) \approx D_E(n)\), or \(D_E(\bar{\lambda})\).

The diagrams for the electric dipole moment of n are given in fig. 6. We find

\[ D_E(n) = -(4\sqrt{2} \pi^2)^{-1} eG_F(m_H \rho_R)^{-2} \rho_1 (\rho_1^2 + \rho_R^2) m_n \]

\[ \times [m_p g_1 \cos^2 \theta_c \cos \alpha_p F(n, p) + m_p g_1' \sin^2 \theta_c \cos \alpha_p' F(n, p')] \]

\[ + m_q g_1 \cos^2 \theta_c \cos \alpha_q F(n, q) + m_q g_1' \sin^2 \theta_c \cos \alpha_q' F(n, q')] \tag{3.70} \]

where

\[ F(n, p) = - \frac{m_H^2}{2m_n^2} \ln \frac{m_H^2}{m_p^2} + \frac{m_H^2}{2m_n^2} R^{-1}(m_H^2 - m_p^2 + m_n^2) \ln \frac{m_H^2 + m_p^2 - m_n^2 + R}{m_H^2 + m_p^2 - m_n^2 - R}, \tag{3.71} \]

and

\[ R = [(m_H + m_p)^2 - m_n^2]^{1/2} [(m_H - m_p)^2 - m_n^2]^{1/2}. \]

For \(m_H \gg m_n\) and \(m_p\), the function \(F\) becomes

\[ F(n, p) = \ln (m_H^2/m_p^2) - 1 + O[(\ln (m_H^2/m_p^2) m_H^{-2}(m_H^2 - m_p^2)] \tag{3.72} \]

Similarly, one finds the electric moment of λ to be given by the same expression (3.70), provided \(n\) is replaced by \(\lambda\) and \(\theta_c\) by \(\frac{1}{2} \pi - \theta_c\).

It is of interest to examine the special case \(g_1 = 0\). As noted before, in this case all lowest order
Fig. 6. Diagrams for the electric dipole moment of the quark-like hadron $n^a$. The solid internal line can be either $p^+$, or $p^{-}$, or $q^+$, or $q^{-}$. Thus, each diagram represents four possibilities.

weak processes among known hadrons are $T$ conserving. Nevertheless, the electric dipole moment may be quite different from that of bona fide superweak theory. For order of magnitude estimations, we may set, in addition to $g_1 = 0$, also $m_p' \approx m_q', \rho_1 \approx \rho_R$ and the angles $\alpha_p, \alpha_q$ much less than 1. Equation (3.70) becomes

$$D_E(n) \approx -(\sqrt{2} \pi^2)^{-1} e E \cdot m_n \cdot m_p \cdot m_{H}^{-2} (g'_1 \rho_1) \sin^2 \theta_c \cdot F(n, p).$$

(3.73)

If we assume $m_{H}$ to be an order of magnitude larger than $m_n$ or $m_p$, and the mass shift $g'_1 \rho_1$ to be $\sim$, or less than, 1 GeV, then the product $m_n \cdot m_p \cdot m_{H}^{-2} (g'_1 \rho_1)$ is $\approx 10^{-2}$ GeV; therefore, the magnitude of $D_E(n)$ becomes $\leq e \times 10^{-23}$ cm. It is reasonable to expect the electric dipole moment $D_E(N)$ of the physical neutron to be also to be of a similar order of magnitude, retaining the milliweak character of the theory. Of course, as shown in (3.69), $D_E(N)$ may also be much smaller, depending on the spin correlation between N and its constituents.

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Appendix

In this appendix we calculate the $T$ violating amplitude for the scattering

$$n_a + n_b \rightarrow \lambda_c + \lambda_d,$$

(A.1)

where $a, b, c, d$ denote the appropriate 4-momenta of the initial and final particles. The diagrams are given in fig. 5. Let us introduce

$$\sigma_{\mu} = (a + b)_{\mu}, \quad s = -\sigma_{\mu}^2,$$

$$\tau_{\mu} = (a - d)_{\mu}, \quad \text{and} \quad t = -\tau_{\mu}^2.$$

(A.2)
In evaluating these diagrams we shall assume $\Delta m_p$, $\Delta m_q$ small, and keep only the first order terms in $\Delta m_p$ and $\Delta m_q$, but neglect all quadratic and higher order terms in these mass differences. The coupling constants $g_i$ and $g'_i$ are assumed to be small, and therefore all higher order terms in $g_i$ and $g'_i$ will also be neglected. In addition, we assume $m_w$ and $m_H$ to be much bigger than the masses of the quark-like hadrons.

Let $L_{\alpha}$ be the effective Lagrangian density for the $T$ violating part of the $\alpha$th diagram in fig. 5. For the first two diagrams, we find

$$L_1 + L_2 = iK \cdot f(s)(\Delta m_p - \Delta m_q)$$

$$\times \left\{ m_n [\lambda^+_c \gamma_4 \gamma_\nu (1 + \gamma_5) n_a \cdot \lambda^+_d \gamma_4 \gamma_\nu (1 - \gamma_5) n_b + \lambda^+_c \gamma_4 \gamma_\nu (1 - \gamma_5) n_a \cdot \lambda^+_d \gamma_4 \gamma_\nu (1 + \gamma_5) n_b] - \lambda^+_c \gamma_4 \gamma_\nu (1 + \gamma_5) n_a \cdot \lambda^+_d \gamma_4 \gamma_\nu (1 + \gamma_5) n_b + \lambda^+_c \gamma_4 \gamma_\nu (1 + \gamma_5) n_a \cdot \lambda^+_d \gamma_4 \gamma_\nu (1 + \gamma_5) n_b] \right\}, \quad (A.3)$$

where

$$K = (24\pi^2)^{-1} (m_H \rho^2_R)^{-2} \rho_1 (\rho_1 + \rho_2^2) (G_F \sin \theta_c \cos \theta_c)^2 (g'_1 - g'_i), \quad (A.4)$$

$$f(s) = s^{-2} 3 m_p m_q \left\{ (m_p^2 - m_q^2) \ln (m_p^2/m_q^2) - 2s + [s(m_p^2 + m_q^2) - (m_p^2 - m_q^2)^2] \right\} \times [R(s)]^{-1} \ln \frac{m_p^2 + m_q^2 - s + R(s)}{m_p^2 + m_q^2 - s - R(s)} , \quad (A.5)$$

$$R(s) = [s - (m_p + m_q)^2]^{1/2} [s - (m_p - m_q)^2]^{1/2}, \quad (A.6)$$

the initial and final Dirac spinors are denoted by $n_a$, $n_b$. $\lambda_c$ and $\lambda_d$ respectively, and $\varphi = -i\gamma_\mu \sigma_\mu$.

At $s = 0$ and $m_p = m_q$, the function $f(s)$ is 1.

The sum of the effective Lagrangian densities for the third and fourth diagrams is given by

$$L_3 + L_4 = iK \cdot [f_p(t) \Delta m_p - f_q(t) \Delta m_q]$$

$$\times \left\{ m_n [-\lambda^+_c \gamma_4 \gamma_\nu (1 - \gamma_5) n_a \cdot \lambda^+_d \gamma_4 \gamma_\nu (1 + \gamma_5) n_b + \lambda^+_c \gamma_4 \gamma_\nu (1 + \gamma_5) n_a \cdot \lambda^+_d \gamma_4 \gamma_\nu (1 - \gamma_5) n_b]$$

$$+ m_n [-\lambda^+_c \gamma_4 \gamma_\nu (1 + \gamma_5) n_a \cdot \lambda^+_d \gamma_4 \gamma_\nu (1 + \gamma_5) n_b + \lambda^+_c \gamma_4 \gamma_\nu (1 + \gamma_5) n_a \cdot \lambda^+_d \gamma_4 \gamma_\nu (1 + \gamma_5) n_b] \right\}, \quad (A.7)$$

where

$$f_p(t) = t^{-2} 6 m_p^2 \left\{ -1 + 2 m_p^2 [R_p(t)]^{-1} \ln \frac{R_p(t) - t}{R_p(t) + t} \right\}, \quad (A.8)$$

$$R_p(t) = [t(t - 4 m_p^2)]^{-1}, \quad (A.9)$$
The function \( f_p(t) \) is given by the same expression (A.8) but with the subscript \( p \) replaced by \( q \). At \( t = 0 \), the function \( f_p(t) \) is 1. The other four diagrams in fig. 5 are the same as the first four provided one interchanges the final states \( \lambda_c \) with \( \lambda_d \).

For the \( K^0 \rightarrow \bar{K}^0 \) transition, we have to consider the crossed reaction

\[
n + \bar{\lambda} = \lambda + \bar{n}, \tag{A.10}
\]

at zero kinetic energy in their rest frame. Therefore, \( \sigma_n = 0 \). In fig. 5, only diagrams 3, 4, 7 and 8 contribute. For simplicity, we set \( f_p(t) \) and \( f_q(t) \) equal to their value at \( t = 0 \); the result leads to eq. (3.63).

References

A. Salam and J.C. Ward, Phys. Letters 13 (1964) 168;