Particle Physics and Introduction to Field Theory

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Show that the classical soliton solution for
\[ U = \frac{\sigma^+ \sigma}{1 + e^2} \left[ (1 - \sigma^+ \sigma)^2 + e^2 \right] \]
is
\[ \sigma = \left( \frac{a}{1 + \sqrt{1 - a^2} \cosh y} \right)^{1/2} e^{-i\omega t} \]  
(7.112)
where \( a = (1 + e^2)(1 - \omega^2) \) and \( y = 2 \sqrt{1 - a^2} (x - \xi) \).

In both problems, for simplicity we set the scale so that \( g = 1 \)
and \( m = 1 \).

Problem 7.3. In the transformation \( x_i \rightarrow q_i = q_j(x_i) \) where the
subscripts can vary from 1 to \( n \), we define \( M \) to be the \( n \times n \) ma-
trix \( M = (M_{ij}) \equiv \left( \frac{\partial x_k}{\partial q_i} \frac{\partial x_k}{\partial q_j} \right) \), \( M^{-1} = (M^{-1}_{ij}) \) its inverse and
\[ |M| = g^2 \] its determinant. Show that
(i) \[ \frac{\partial}{\partial q_k} \left( \frac{1}{|M|} \frac{\partial M}{\partial q_k} \right) = M^{-1}_{ji} \frac{\partial M}{\partial q_k} = -2 \frac{\partial x_i}{\partial q_k} \frac{\partial}{\partial q_j} \left( \frac{\partial q_j}{\partial x_i} \right) \]
and
(ii) \[ \frac{\partial^2}{\partial x_i \partial x_l} = \frac{1}{g} \frac{\partial}{\partial q_i} \left( M^{-1}_{ji} \frac{\partial}{\partial q_j} \right) \]  
(7.113)

References
For quantization and especially topological solitons, there are some
excellent review articles:
S. Coleman, in New Phenomena in Subnuclear Physics, Part A, ed.

III. PARTICLE PHYSICS

ORDER-OF-MAGNITUDE ESTIMATIONS

From a phenomenological point of view, there are four distinct
interactions between particles, given in Table 1.

Table 1

<table>
<thead>
<tr>
<th>Interaction</th>
<th>Phenomenological Coupling Constant</th>
<th>Quanta of the Mediating Field</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong</td>
<td>( \sim 1 )</td>
<td>Color gluon</td>
</tr>
<tr>
<td>Electromagnetic</td>
<td>( \alpha \sim \frac{1}{137} )</td>
<td>Photon</td>
</tr>
<tr>
<td>Weak</td>
<td>( Gm_p^2 \sim 10^{-5} )</td>
<td>Intermediate boson</td>
</tr>
<tr>
<td>Gravitational</td>
<td>( \frac{G}{m_p^2} \sim 6 \times 10^{-39} )</td>
<td>Graviton</td>
</tr>
</tbody>
</table>

The four phenomenological classes of interactions, \( \alpha \) is the fine
structure constant, \( G \) is the Fermi constant for beta-decay, \( \frac{G}{m_p^2} \) is
Newton's gravitational constant and \( m_p \) is the mass of the proton;
all these constants are in the natural units. Among the quanta of the
mediating fields, only the photon has been detected experimentally.

Throughout our discussions we will concentrate only on the
strong, weak and electromagnetic interactions. Except for the quanta
of the mediating fields, all particles can be classified into two types.
Those that have strong interactions are called hadrons, all others are leptons. For example, $p$, $n$, $\Lambda$, $\pi$ are all hadrons; $e$, $\mu$, $\tau$ and various neutrinos are leptons. (Since the mass of $\tau$ is about 2 GeV, the word 'lepton', which means light particle, is something of a misnomer.) The detailed properties of these particles are given in a table at the end of this book.

In this chapter we shall illustrate how to estimate the order of magnitude of physical quantities, such as various particle sizes and cross sections. These estimations will employ very few input parameters, listed below:

- the fine structure constant $\alpha \approx \frac{1}{137}$,
- the Fermi constant $G \approx 10^{-5}/m_p^2$,
- the electron mass $m_e \approx 0.51\text{ MeV} \approx (4 \times 10^{-11}\text{ cm})^{-1}$,
- the proton mass $m_p \equiv 1800 m_e$,

and

- the pion mass $m_\pi \approx \frac{1}{7} m_p$.

The art of order-of-magnitude estimations is based on

(i) simple physical considerations,

and (ii) dimensional analysis,

as will be illustrated by the following examples.

### 8.1 Radius of the Hydrogen Atom

The hydrogen radius $r$ is determined by the orbit of the electron. Therefore, the momentum of the electron is $p \approx \frac{1}{r}$ and its kinetic energy is $\sim \frac{p^2}{2m_e}$. The electrostatic energy should be $\sim -\frac{\alpha}{r}$. The energy $E$ can then be estimated to be

$$ E \sim \frac{1}{2m_e} \left( \frac{1}{r} \right)^2 - \frac{\alpha}{r} \quad (8.2) $$

Its minimum is determined by

$$ \frac{dE}{dr} = 0 \quad (8.3) $$

which leads to

$$ r = \frac{1}{m_e \alpha} \quad (8.4) $$

By using the values of $\alpha$ and $m_e$ in (8.1) we see that

$$ r \approx 5 \times 10^{-9}\text{ cm} $$

which is the Bohr radius, now derived without solving any differential equations.

**Remarks.** In quantum electrodynamics there are three important lengths, differing from each other by powers of $\alpha$:

- Bohr radius $\frac{1}{m_e \alpha}$,
- Compton wavelength of $e$ $\frac{1}{m_e}$,
- classical radius of $e$ $\frac{\alpha}{m_e}$.

### 8.2 Hadron Size

According to Table 1, the strong interaction coupling constant is $\sim 1$ instead of $\alpha$. Therefore a glance at (8.5) tells us, for the hadrons, there is only one length, which can be taken as the Compton wavelength of the particle. Since the pion is the lowest-mass hadron, its Compton wavelength is therefore the largest. Because of strong interactions, pion clouds must exist in other hadrons such as $p$, \ldots
n, Λ, ... All these hadrons, including the pion, should be of size \( \sim \frac{1}{m_\pi} \) which according to (8.1) is \( \sim 10^{-13} \) cm = 1 fermi.

Therefore, one expects the charge radius \( r_p \) of the proton to be of the same order. Experimentally, one finds

\[
\begin{align*}
   r_p & \approx 0.81 \text{ fermi}, \\
   \sigma_{np} & \approx \sigma_{pp} \approx \sigma_{np} \approx \sigma_{np} . \tag{8.6}
\end{align*}
\]

consistent with the above simple estimation.

8.3 High-energy pp, πp and Kp Total Cross Sections

Because pp has strong interactions, we expect that at high energy the total cross section of a pp collision will be

\[
\sigma_{pp} \sim \pi r_p^2 . \tag{8.7}
\]

Since \( r_p \) is \( \sim 10^{-13} \) cm, we estimate

\[
\sigma_{pp} \sim 3 \times 10^{-26} \text{ cm}^2 \approx 30 \text{ mb} \tag{8.8}
\]

where 1 mb = \( 10^{-4} \) b and 1 b = 1 barn = \( 10^{-24} \) cm².

As we shall see later in the discussion of the quark-parton model, a nucleon (proton or neutron) is made of three quarks while a meson (π or K) is made of two. Furthermore, at high energy we can treat these quarks as free particles, at least so far as total cross sections are concerned. Thus we expect

\[
\frac{\sigma_{\pi p}}{\sigma_{pp}} \approx \frac{\sigma_{K p}}{\sigma_{pp}} \approx \frac{2}{3}
\]

which is independent of the charge of π and K. By using (8.8), we can estimate, for the total cross sections,

\[
\begin{align*}
   \sigma_{\pi^+ p} & \approx \sigma_{\pi^- p} \approx 20 \text{ mb} , \\
   \sigma_{K^+ p} & \approx \sigma_{K^- p} \approx 20 \text{ mb} . \tag{8.9}
\end{align*}
\]

Likewise, we expect

\[
\sigma_{np} \approx \sigma_{pp} \approx \sigma_{np} \approx \sigma_{np} . \tag{8.10}
\]

The present high-energy experimental values are

\[
\sigma_{pp} \approx \sigma_{np} \approx \sigma_{np} \approx 45 \text{ mb} ,
\]

\[
\sigma_{\pi^\pm p} \approx 25 \text{ mb}
\]

and

\[
\sigma_{K^\pm p} \approx 20 \text{ mb} ,
\]

all consistent with the above estimations.

8.4 \( e^+ + e^- \rightarrow \mu^+ + \mu^- \)

We shall estimate the total cross section for this reaction at high energy. Since \( e^\pm \) and \( \mu^\pm \) are leptons, the strongest interaction between them is the electromagnetic. The lowest order diagram is given in Fig. 8.1 where the wavy line represents the virtual photon.

\[
\begin{align*}
   \mu^+ & \rightarrow \mu^- \\
   q & \rightarrow \mu^- \\
   e^+ & \rightarrow e^-
\end{align*}
\]

Fig. 8.1 Feynman diagram for \( e^+ + e^- \rightarrow \mu^+ + \mu^- \).

Let \( \sigma \) be the total cross section. In Fig. 8.1 each vertex carries a factor proportional to the electric charge; the Feynman amplitude is therefore proportional to the fine structure constant \( \alpha \). So far as the
total cross section is concerned, the only Lorentz-invariant variable in this problem is the square of the 4-momentum $q$ carried by the virtual photon. It is useful to visualize the process in the center-of-mass system (which is also the laboratory system if this is a standard colliding beam experiment), and to denote $s$ as the square of the center-of-mass energy $E_{\text{c.m.}}$. In our case, we have

$$s \equiv E_{\text{c.m.}}^2 = -q^2 = q_0^2 - \vec{q}^2.$$  \hspace{1cm} (8.11)

The total cross section must therefore be of the form

$$\sigma = \alpha^2 f(s, m_e, m_\mu)$$  \hspace{1cm} (8.12)

where $m_e$ and $m_\mu$ are respectively the masses of $e$ and $\mu$, and the function $f$ is to be determined. When $E_{\text{c.m.}}$ is much greater than either lepton mass, we may set $m_e = m_\mu = 0$ as an approximation. Now, $\alpha$ is dimensionless, and in the natural units the dimensions of $\sigma$ and $s$ are

$$[\sigma] = [L^2] \quad \text{and} \quad [s] = [L^{-2}],$$  \hspace{1cm} (8.13)

where $L$ denotes length. Thus, from dimensional analysis the function $f$ in (8.12) for large $s$ must be proportional to $s^{-1}$; i.e.,

$$\sigma \sim \frac{\alpha^2}{s}.$$  \hspace{1cm} (8.14)

The Feynman diagram in Fig. 8.1 can be readily evaluated, and the complete answer for $\sigma$ in the high-energy limit is

$$\sigma = \frac{4}{3} \pi \frac{e^2}{s}.$$  \hspace{1cm} (8.15)

[ See Problem 6.1. ] The point is that without any computation it is possible to give an estimation of the cross section, albeit without the factor $\frac{4}{3} \pi$.

From (8.14), one can readily estimate, e.g., that when $s \approx (1 \text{ GeV})^2$

$$\sigma(e^+e^- \rightarrow \mu^+\mu^-) \sim 4 \times 10^{-32} \text{ cm}^2 = 0.04 \mu\text{b},$$  \hspace{1cm} (8.16)

where $1 \mu\text{b} = 1 \text{ microbarn} = 10^{-6} \text{b}$.

8.5 $v + N \rightarrow \cdots$

Let $\sigma(vN)$ be the total cross section of this reaction summed over all final states. The initial $v$ can be either the $\mu$-neutrino $v_\mu$ or the $e$-neutrino $v_e$, and $N$ is the nucleon. As before, $s$ denotes the square of the center-of-mass energy. Since this is a weak process, its amplitude should be proportional to the Fermi constant $G$. Hence, $\sigma(vN)$ must be of the form

$$\sigma(vN) = G^2 f(s, m_N^2)$$  \hspace{1cm} (8.17)

where $m_N$ is the nucleon mass and the function $f$ is to be determined. For $s \gg m_N^2$, we can, as in the preceding example, set $m_N = 0$ as an approximation. Since the dimension of $G$ is

$$[G] = [L^2],$$  \hspace{1cm} (8.18)

by using (8.13) we see that in (8.17) the function $f$ must be proportional to $s$; therefore,

$$\sigma(vN) \sim G^2 s.$$  \hspace{1cm} (8.19)

The laboratory system in a typical high-energy neutrino experiment is one in which the initial nucleon is at rest. Let $E_v$ and $P_v$ be respectively the energy and the momentum of the neutrino in the laboratory system. We have

$$s = E_{\text{c.m.}}^2 = (E_v + m_N)^2 - P_v^2 = m_N(2E_v + m_N) \cong 2m_N E_v.$$  \hspace{1cm} (8.20)
Thus, (8.19) can also be written in terms of $E_v$:

$$\sigma(vN) \sim G^2 m_N E_v. \quad (8.21)$$

By using (8.1), we find at high energy

$$\sigma(vN) \sim 4 \times 10^{-38} \left(\frac{E_v}{m_N}\right) \text{ cm}^2. \quad (8.22)$$

The experimental result is

$$\sigma(vN) \approx 0.6 \times 10^{-38} \left(\frac{E_v}{m_N}\right) \text{ cm}^2. \quad (8.23)$$

Again, we can estimate the order of magnitude of this reaction without any computation. [Notice the huge difference between (8.8), (8.16) and (8.22) at a comparable energy.]

8.6 Compton Scattering

The lowest-order Feynman diagrams for Compton scattering

$$\gamma + e^\pm \rightarrow \gamma + e^\pm \quad (8.24)$$

are given in Fig. 8.2. By following the same reasoning which led to (8.12), we expect the total cross section $\sigma_{\text{Comp}}$ of this reaction to be of the form

$$\sigma_{\text{Comp}} = a^2 f(s, m_e) \quad (8.25)$$

where, as before, $s$ is the center-of-mass energy squared, and the function $f$ is to be determined.

At the nonrelativistic limit, $s \rightarrow m_e^2$ and therefore $f$ is a function which depends only on $m_e$. From dimensional considerations, we expect

$$\sigma_{\text{Comp}} \sim \frac{a^2}{m_e^2} \quad N, \ R. \quad (8.26)$$

where N.R. denotes the nonrelativistic limit. When $s \gg m_e^2$, we can neglect $m_e$ in (8.25), and that leads to, through dimensional analysis,

$$\sigma_{\text{Comp}} \sim \frac{a^2}{s} \quad E, \ R. \quad (8.27)$$

where E.R. denotes the extreme relativistic limit.

An accurate calculation of the Feynman diagrams in Fig. 8.2 gives

$$\sigma \sim \begin{cases} \frac{8}{3} \pi \left(\frac{a^2}{m_e^2}\right) & N, \ R. \\ 2\pi \frac{a^2}{s} \ln \frac{s}{m_e^2} & E, \ R. \end{cases} \quad (8.28)$$

Our estimation (8.26) differs from the accurate nonrelativistic formula (called the Thomson limit) only by a factor $\frac{8\pi}{3}$. However, in the extreme relativistic case, the estimation (8.27) misses an $s$-dependent factor $2\pi \ln \frac{s}{m_e^2}$. While this is a slowly varying function of $s$, the existence of such log terms has a general underlying reason, as we shall explain.

8.7 Mass Singularity and High-energy Behavior

We first give the technical reason for the \( \ln \frac{s}{m_e^2} \) factor. Let us consider diagram (a) in Fig. 8.2. The virtual electron carries a 4-momentum

\[
q = p + k, \quad (8.29)
\]

where \( p \) and \( k \) are, respectively, the final 4-momenta of \( e \) and \( \gamma \). The components of \( p \) and \( k \) may be written as \( p = (\vec{p}, i p_0) \) and \( k = (\vec{k}, i k_0) \). Since the final \( e \) and \( \gamma \) are both on the mass shell, we have

\[
p^2 + m_e^2 = 0 \quad \text{and} \quad k^2 = 0,
\]

i.e.

\[
p_0 = \sqrt{p^2 + m_e^2} \quad \text{and} \quad k_0 = |k|. \quad (8.30)
\]

For definiteness, we consider the laboratory frame. In the E.R. limit, \( p_0 \) is \( \gg m_e \), and therefore

\[
p_0 \approx \sqrt{p^2 + m_e^2} \quad (8.31)
\]

In diagram (a) the electron propagator carries a denominator

\[
q^2 + m_e^2 = (p + k)^2 + m_e^2 \quad (8.32)
\]

which, because of (8.30), is equal to

\[
2 p \cdot k = 2 \vec{p} \cdot \vec{k} - 2 p_0 k_0. \quad (8.33)
\]

In the E.R. limit and for the nearly forward scattering case, we have

\[
q^2 + m_e^2 \approx -m_e^2 \frac{k_0}{p_0} - 2 \vec{p} \cdot k_0 (1 - \cos \theta)
\]

\[
\approx -k_0 \frac{m_e^2 + p_0^2 \theta^2}{p_0} \quad (8.33)
\]

where \( \theta \ll 1 \) is the angle between \( \vec{p} \) and \( \vec{k} \). Diagram (a) gives a contribution \( \sigma_a \) to the cross section that is inversely proportional to \( (q^2 + m_e^2)^2 \). For the region under consideration we expect the deviation of \( \sigma_a \) from the simple estimate (8.27) to be

\[
\frac{\sigma_a}{(a^2/s)} = \int \frac{\Theta^2 2 \pi \sin \theta \ d\theta}{(m_e^2 + p_0^2 \theta^2)^2} \cdot p_0^4
\]

\[
\sim 2 \pi \ln \left( \frac{p_0^2}{m_e^2} \right) \sim 2 \pi \ln \left( \frac{s}{m_e^2} \right) \quad (8.34)
\]

where \( 2 \pi \sin \theta \ d\theta \) is the solid angle, \( p_0^4 \) is there to make the whole expression dimensionless, and the \( \Theta^2 \) factor in the numerator is due to \( \gamma_5 \) invariance, as will be explained below. The new estimation (8.34) is in good agreement with the exact limit given by the Klein-Nishina formula \(^*\) given in (8.28).

The \( \gamma_5 \)-transformation

\[
\Psi \rightarrow \gamma_5 \Psi \quad (8.35)
\]

has been discussed in Section 3.8 in connection with the two component theory. For the electron field \( \Psi \), since \( m_e \neq 0 \), the mass term violates the \( \gamma_5 \) invariance. However, the electromagnetic interaction is invariant under the \( \gamma_5 \) transformation, as can be readily verified by substituting (8.35) into (6.7). In the E.R. limit, \( m_e \) can be neglected; therefore, by following the discussions given in Section 3.8 one sees that the helicity of the electron is unchanged through its electromagnetic interaction. Since the helicity of the photon is either \( +1 \) or \( -1 \), a helicity-conserving electron cannot emit or absorb a photon in the exact forward direction, as can be easily seen through angular-momentum conservation along the direction of motion. Thus, for zero \( m_e \) when the angle \( \theta \) between \( \vec{p} \) and \( \vec{k} \) is 0, the

Feynman amplitude must also be 0. Consequently, the matrix element for diagram (a) in Fig. 8.2 carries a factor proportional to \( \Theta \), and that explains the \( \Theta^2 \) term in (8.34). [For \( m_e \neq 0 \) but \( \Theta = 0 \), the amplitude is \( \propto m_e \); terms proportional to \( m_e \) do not lead to the mass singularity.]

From (8.34) one observes that in the E.R. limit the cross section has a logarithmic singularity (called mass singularity) when the electron mass \( m_e \to 0 \). The origin of the mass singularity is associated with the fact that (8.33) becomes 0 when \( m_e = \Theta = 0 \); i.e., the relevant propagator becomes infinite, which in turn means the virtual particle is approaching its mass shell. There is a simple, but general, reason for this: If we have a number of zero-mass particles moving in the same direction, their total energy \( E = \sum E_a \) and the magnitude of their total 3-momentum \( \vec{p} = \sum \vec{p}_a \) become equal

\[
\sum_a E_a = \left| \sum \vec{p}_a \right| \quad (8.36)
\]

where the subscript \( a \) denotes the \( a \)th particle. Thus, when \( m_e \to 0 \), conservation of energy follows from the conservation of momentum whenever the momenta of \( e \) and \( \gamma \) are all parallel; i.e., in

\[ e = e + \gamma \]

all particles can be on-mass-shell in this special limit.

A general theorem can be established which states that, for any process

\[
i \to f \quad (8.37)
\]

although the Feynman diagrams have mass singularities, the square of the S-matrix element

\[
\sum_{\{i\},\{f\}} \left| \langle f \mid S \mid i \rangle \right|^2 \quad (8.38)
\]

summed over the sets \( \{i\} \) and \( \{f\} \) which respectively contain all states that are degenerate (within an energy width \( \epsilon \neq 0 \), which can be arbitrarily chosen) with \( i \) and \( f \), does not. Expression (8.38) depends on the width \( \epsilon \) but has a finite limit when \( m_e \to 0 \) to every order in the perturbation expansion. This theorem is valid not only in quantum electrodynamics, but also in other field theories such as quantum chromodynamics. The proof is quite elementary, and is given in Chapter 23.

8.8 \( e^+e^- \) Pair Production by High-energy Photons

Let us consider the reaction

\[
y + Z \to e^+ + e^- + Z \quad (8.39)
\]

where \( Z \) denotes a heavy nucleus of charge \( Z \) (in units of positron charge). The lowest-order diagrams are given in Fig. 8.3 in which the wavy lines represent real or virtual photons. In order to establish the cross section of (8.39) we first consider the reaction of pair production by two photons

\[ \gamma \to e^+ + e^- \]

Fig. 8.3. Lowest-order Feynman diagrams for (8.39).
\[ \gamma + \gamma \rightarrow e^+ + e^- \]  \hspace{1cm} (8.40)

whose diagram is given in Fig. 8.4. By following the same argument,

\[ \gamma \rightarrow e^+ \]

\[ \gamma \rightarrow e^- \]

Fig. 8.4. Lowest-order diagram for \( \gamma + \gamma \rightarrow e^+ + e^- \).

which leads to (8.27), we may estimate at high energy

\[ \sigma (\gamma \gamma \rightarrow e^+ e^-) \sim \frac{\alpha^2}{s} \]  \hspace{1cm} (8.41)

Next we compare the difference between reactions (8.39) and (8.40). For simplicity the nucleus is assumed to be extremely heavy, at rest in the laboratory system. The electrostatic potential generated by the nucleus at distance \( r \) is

\[ \frac{Ze}{r} \]  \hspace{1cm} (8.42)

Thus, in the laboratory frame, the 4-momentum carried by the virtual photon in Fig. 8.3 is \( q = (q', 0) \); i.e., the fourth component of \( q \) is 0. The distribution of \( q' \) is given by the Fourier transformation of (8.42), and therefore it is proportional to

\[ \frac{1}{q'^2} \]  \hspace{1cm} (8.43)

The process \( \gamma + Z \rightarrow e^+ + e^- + Z \) can then be viewed as a particular case of \( \gamma + \gamma \rightarrow e^+ + e^- \) in which one of the \( \gamma \)'s is virtual with a momentum distribution given by (8.43) in the laboratory frame.

Accordingly, in reaction (8.39) the 3-momenta \( p_+ \) and \( p_- \) of \( e^+ \) and \( e^- \) are independent; therefore, the final-state phase space differs from that of (8.40) by an additional factor

\[ \frac{d^3 p^+}{p^0} \sim p^2 \sim s \]  \hspace{1cm} (8.44)

where \( p_0 = (p^2 + m_e^2)^{1/2} \) and \( p \) can be either \( p_+ \) or \( p_- \). By multiplying (8.44) and (8.41) we see that the s-factors cancel, and therefore at high energy the cross section \( \sigma_{\text{pair}} \) for reaction (8.39) is approximately independent of \( s \). In Fig. 8.3, each Feynman diagram has three vertices, one is proportional to \( Ze \) and the other two to \( e \).

Thus, \( \sigma_{\text{pair}} \) is of the form

\[ \sigma_{\text{pair}} \sim Z^2 \alpha^3 f(m_e) \]

where the function \( f \) can be determined through dimensional analysis.

We then obtain the estimation

\[ \sigma_{\text{pair}} \sim \frac{Z^2 \alpha^3}{m_e^2} \]  \hspace{1cm} (8.45)

So far, we have only made the crudest estimation, ignoring completely the possibility of mass singularity. By following arguments similar to that given between (8.29) and (8.34), we can derive a better high-energy estimation

\[ \sigma_{\text{pair}} \sim \frac{Z^2 \alpha^3}{m_e^2} \ln \frac{E_\gamma}{m_e} \]  \hspace{1cm} (8.46)

where \( E_\gamma \) is the laboratory energy of the initial \( \gamma \) in reaction (8.39). The exact limit given by the Bethe-Heitler formula* is

\[ \sigma_{\text{pair}} = \frac{28}{9} \frac{Z^2 \alpha^3}{m_e^2} \ln \frac{E_\gamma}{m_e} \]  \hspace{1cm} (8.47)

consistent with the above estimation.

Equation (8.47) is valid for a point nucleus without screening. If one has complete screening, then the log factor in (8.47) should be replaced by a constant \( \approx \ln(183 Z^{-2/3}) \). Unlike \( \sigma_{\text{Comp}}/\sigma_{\text{pair}} \) does not decrease with increasing energy. This has an important experimental consequence; it implies that in matter a high-energy photon has a finite, nonzero, almost energy-independent mean free path, which is, e.g., about 460 meters in air (1 atm. pressure and 0° C), 13 cm in aluminum and 7 mm in lead.

IIA. PARTICLE PHYSICS: SYMMETRY

Chapter 9

GENERAL DISCUSSION

Since the beginning of physics, symmetry considerations have provided us with an extremely powerful and useful tool in our effort to understand nature. Gradually they have become the backbone of our theoretical formulation of physical laws. In this chapter we shall review these symmetry operations and examine their foundation. Such an examination is useful, especially in view of the various symmetries that have been discovered during the past quarter century.

There are four main groups of symmetries, or broken symmetries, that are found to be of importance in physics:

2. Continuous space-time symmetries, such as translation, rotation, acceleration, etc.
3. Discrete symmetries, such as space inversion, time reversal, particle-antiparticle conjugation, etc.
4. Unitary symmetries, which include
   \( U_1 \)-symmetries such as those related to conservation of charge, baryon number, lepton number, etc.,
   \( SU_2 \) (isospin)-symmetry,
   \( SU_3 \) (color)-symmetry,
   and \( SU_n \) (flavor)-symmetry.