Elastic Form Factors of the Proton, Neutron and Deuteron

Michael Kohl

Laboratory for Nuclear Science and Bates Linear Accelerator Center, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

Abstract

In this paper an overview of past and future measurements of the elastic nucleon and deuteron electromagnetic form factors and a comparison with a selection of theoretical descriptions is provided.

Key words: Proton, deuteron, electromagnetic elastic form factor, Rosenbluth separation, polarization


1. Introduction

Elastic electromagnetic form factors of the proton, neutron and deuteron are fundamental quantities characterizing the distributions of charge and magnetization of the nucleon and the bound two-nucleon system. Significant advances in experiment and theory have been made over the last decade [1]. In particular, the use of spin degrees of freedom has led to unprecedented experimental precision in determinations of elastic form factors which has also triggered numerous theoretical advances.

The spatial distribution of charge and magnetization in the proton and neutron as spin-1/2 objects gives rise to two form factors, the electric (or charge) form factor $G_E$ and the magnetic form factor $G_M$. The deuteron as a spin-1 object has got one additional form factor, the charge quadrupole form factor $G_Q$ which is related to the deformation of the deuteron due to the small D-wave component of the ground state.

Precise mapping of the elastic form factors provides rigorous tests of nucleon and deuteron models. As strongly interacting many-body systems the nucleon and deuteron will ultimately be described by QCD solved on the lattice for which the results of current experiments will be used as benchmarks.

Recent precision measurements of form factor ratios using polarization methods have not only considerably reduced the uncertainties but have also revealed new phenomena.
such as the role of two-photon exchange at high four-momentum transfer squared $Q^2$ or subtle effects due to the meson cloud at low $Q^2$ and have hence significantly refined the theoretical understanding of the nucleon and deuteron. Most dramatic progress has been made in measurements of the neutron electric form factor during the last decade which was only poorly known until then.

The electric and magnetic form factors of the nucleon are also used as input for nuclear structure calculations and for the interpretation of parity violating electron scattering to extract the strange electromagnetic form factors.

In the following the experimental methods of measuring the elastic nucleon and deuteron form factors are briefly discussed, the existing experimental data on the proton, neutron and deuteron are presented and compared with various theories, and current and future experimental efforts are mentioned.

Fig. 1. World data of $G^p_E$ (left panel) and $G^p_M$ (right panel) from unpolarized measurements [4] using the Rosenbluth method, normalized to the dipole parameterization $G_D = (1 + Q^2/0.71)^{-2}$. Also shown are three theoretical curves [8–10].

Fig. 2. World data of the ratio $μ_p G^p_E/G^p_M$ from unpolarized measurements (black symbols) using the Rosenbluth method [4] and from double polarization experiments (colored symbols) [5–7]. Also shown are three theoretical curves [8–10].
In both panels three theoretical calculations [8–10] are compared to the data. Up to \( Q^2 \) value of \( 2 \) (GeV/c)\(^2\) the factor \( \tau \) corresponds to an exponential shape of the proton. The factor \( \tau \) is known as Rosenbluth formula [3]. A variation of the scattering angle at constant \( Q^2 \) allows to separate the Dirac and Pauli form factors \( F_1 \) and \( F_2 \). It is common to introduce the Sachs form factors \( G_E = F_1 - \tau F_2 \) and \( G_M = F_1 + F_2 \) which allow for a simple interpretation as spatial distributions of charge and magnetization in a special coordinate frame, the Breit frame. In the one-photon exchange approximation, the above form factors are experimentally accessible through the elastic scattering cross section

\[
\frac{d\sigma}{d\Omega} = A(Q^2) + B(Q^2) \tan^2 \frac{\theta}{2} = \frac{G_E^2(Q^2)}{(1 + \tau)} + \frac{\tau G_M^2(Q^2)}{1 + \tau} + 2\tau G_M^2(Q^2) \tan^2 \frac{\theta}{2}
\]

which is known as Rosenbluth formula [3]. A variation of the scattering angle at constant value of \( Q^2 \) allows to separate \( G_E^2 \) and \( G_M^2 \). The factor \( \tau = Q^2/(4m_N^2) \) (with \( m_N \) the nucleon mass) increases with \( Q^2 \) and eventually makes a separation of the two terms more and more difficult.

Figure 1 shows the existing data on separated electric and magnetic proton form factors from unpolarized measurements using the Rosenbluth method [4]. Up to \( Q^2 \approx 10 \) (GeV/c)\(^2\) both form factors are reasonably well described by the dipole parameterization \( G_D = (1 + Q^2/0.71)^{-2} \) which corresponds to an exponential shape of the proton. The unpolarized data indicated a scaling law of \( \mu p G_E^p / G_M^p = 1 \).

2. Proton

The pioneering measurements of elastic form factors were done more than 50 years ago at SLAC [2]. It was discovered that the finite size of the nucleon plays an important role in the description of the proton and deuteron form factors. Up until the 1990’s all experiments were essentially unpolarized. The elastic form factor is generally defined as the matrix element of the electromagnetic current operator, which reads in the case of a spin-1/2 target

\[
\langle N(P')|J_{EM}^\mu(0)|N(P)\rangle = \bar{u}(P') \left[ \gamma^\mu F_1(Q^2) + i\sigma^{\mu\nu} \frac{q_\nu}{2M} F_2(Q^2) \right] u(P) \tag{1}
\]

with the Dirac and Pauli form factors \( F_1 \) and \( F_2 \). It is common to introduce the Sachs form factors \( G_E = F_1 - \tau F_2 \) and \( G_M = F_1 + F_2 \) which allow for a simple interpretation as spatial distributions of charge and magnetization in a special coordinate frame, the Breit frame. In the one-photon exchange approximation, the above form factors are experimentally accessible through the elastic scattering cross section

\[
\frac{d\sigma}{d\Omega} = A(Q^2) + B(Q^2) \tan^2 \frac{\theta}{2} = \frac{G_E^2(Q^2)}{(1 + \tau)} + \frac{\tau G_M^2(Q^2)}{1 + \tau} + 2\tau G_M^2(Q^2) \tan^2 \frac{\theta}{2}
\]

which is known as Rosenbluth formula [3]. A variation of the scattering angle at constant value of \( Q^2 \) allows to separate \( G_E^2 \) and \( G_M^2 \). The factor \( \tau = Q^2/(4m_N^2) \) (with \( m_N \) the nucleon mass) increases with \( Q^2 \) and eventually makes a separation of the two terms more and more difficult.

Figure 1 shows the existing data on separated electric and magnetic proton form factors from unpolarized measurements using the Rosenbluth method [4]. Up to \( Q^2 \approx 10 \) (GeV/c)\(^2\) both form factors are reasonably well described by the dipole parameterization \( G_D = (1 + Q^2/0.71)^{-2} \) which corresponds to an exponential shape of the proton. The unpolarized data indicated a scaling law of \( \mu p G_E^p / G_M^p = 1 \).
The development of polarized beams, targets and recoil polarimeters in the 1990’s enabled access to the form factor ratio $G_E/G_M$ through a spin correlation in double polarization experiments. The interference of $G_E$ and $G_M$ yields an asymmetry $\bar{A}$

$$-\sigma_0 P_e \bar{P}_N \cdot \bar{A} = P_e P_N \left[ \sqrt{2} \epsilon (1 - \epsilon) G_E G_M \sin \theta^* \cos \phi^* + \tau \sqrt{1 - \epsilon^2} G_M^2 \cos \theta^* \right], \quad (3)$$

where the scalar product of $\bar{P}_N$ and $\bar{A}$ can either be interpreted as that of recoil polarization and analyzing power in a polarization transfer experiment, or as target polarization and beam-target asymmetry in a polarized-target experiment. The double asymmetry requires a longitudinally polarized electron beam ($P_e$). The angles $\theta^*$ and $\phi^*$ denote the spin orientation of the recoil or target nucleon relative to the momentum transfer direction, $\sigma_0$ is the unpolarized elastic cross section, and $\epsilon = [1 + 2(1 + \tau) \tan^2(\theta/2)]^{-1}$ is the virtual photon polarization.

The world data of the proton form factor ratio $\mu_p G_E^p/G_M^p$ from double polarization experiments [5–7] is shown in Fig. 2 along with that obtained from Rosenbluth-separated form factors. Starting at $Q^2 > 2$ (GeV/c)$^2$ the polarization data are monotonically decreasing, dramatically different from the unpolarized data which followed the scaling law. The data in Figs. 1, 2 are compared to three theoretical calculations based on dispersion theory [8], generalized parton distributions [9] and a cloudy bag model with relativistic constituent quarks [10]. All three models are able do reproduce the main features of the form factor distributions while differing at a more detailed level.

The discrepancy of the unpolarized and polarized data is most likely explained by the effect of two-photon exchange [11]. Additional current and future experiments will address
this controversy with Rosenbluth-type measurements [12], $\epsilon$- (or angular) dependence of the form factor ratio [13], $e^+p/e^-p$ cross section asymmetry [14–16], and single-spin asymmetries [17]. Future recoil polarization experiments at Jefferson Lab will extend the $Q^2$ range up to 9 (GeV/c)$^2$ with a new recoil polarimeter [18] and up to 14 (GeV/c)$^2$ after the 12 GeV upgrade [19]. Also at low $Q^2$, new unpolarized and polarized measurements will yield additional precision data [20,21] to investigate the observed structure beyond the dipole form that might arise from the effect of the meson cloud.

3. Neutron

Figure 3 shows the world data collection of the neutron form factors $G^n_E$ (left panel) and $G^n_M$ (right panel). Before polarization degrees of freedom were available the only possible extraction of $G^n_E$ was from $A(Q^2)$ in elastic electron-deuteron scattering which appeared to be largely model-dependent [22,23], indicated by the shaded band in Fig. 3. This difficulty has been overcome in recent years with the advent of polarized beams, targets and polarimeters which minimize both the systematic errors and the model dependency [24]. In absence of a free neutron target, quasielastic scattering from $^2$H and $^3$He is employed. The slope of $G^n_E$ at $Q^2 = 0$ is constrained by the neutron square radius [25]. A new and much more precise picture of the $G^n_E$ distribution has emerged which is also in good agreement with $G^n_E$ extracted from the available data of the deuteron quadrupole form factor $G_Q$ [26]. The results from additional measurements with polarized $^3$He at higher momentum transfer up to $Q^2 = 3.5$ (GeV/c)$^2$ at Jefferson Lab [27] and up to $Q^2 = 1.5$ (GeV/c)$^2$ at Mainz [28] are awaited, while another experiment at Jefferson Lab to measure $G^n_E$ at $Q^2 = 4.3$ (GeV/c)$^2$ via recoil polarimetry is approved [29].

The magnetic form factor $G^n_M$ has been measured traditionally by the ratio of quasielastic $(e,e'n)/(e,e'p)$ cross sections [30–33]. The requirement to know the neutron detection efficiency has most likely been the cause for the significant scatter between experiments. A more recent trend has been set by the data of [31,32] which agree very well with $G^n_M$ extracted from double polarized inclusive electron scattering off polarized $^3$He [34] and vector-polarized $^2$H [35]. At higher $Q^2$ the preliminary unpolarized data from CLAS at Jefferson Lab [30] is significantly lower than [31,32] in the overlap region. The three theoretical models [8–10] show some variations, predominantly for $G^n_M$. 

Fig. 5. Left panel: available world data of $T_{20}$ in comparison with various theoretical calculations. Right panel: Reduced $T_{20R}$ normalized to the leading dependence $Q^2Q_d$. 

M. Kohl / Nuclear Physics A 805 (2008) 361c–368c
4. Deuteron

The elastic response functions $A$ and $B$ of the deuteron and the tensor analyzing power $T_{20}$ are related to the three form factors $G_C$, $G_Q$ and $G_M$,

\[
A(Q^2) = G_C^2(Q^2) + \frac{8}{9} \eta^2 G_Q^2 + \frac{2}{3} \eta G_M^2(Q^2), \quad B(Q^2) = \frac{4}{3} \eta(1 + \eta)G_M^2(Q^2)
\]

\[
T_{20} = -\frac{1}{\sqrt{2S}} \left[ \frac{8}{3} \eta G_C G_Q + \frac{8}{9} \eta^2 G_Q^2 + \frac{1}{3} \eta \left(1 + 2(1 + \eta) \tan^2 \frac{\theta}{2}\right) G_M^2 \right],
\]

with $\eta = Q^2/(4m_D^2)$ and $m_D$ the deuteron mass. Measurements of $A$ and $B$ by means of a Rosenbluth separation are only sufficient to separate $G_M$ and a combination of $G_C$ and $G_Q$, while another independent measurement of a polarization observable such as $T_{20}$ is required to disentangle $G_C$ and $G_Q$. The tensor analyzing power $T_{20}$ can be measured with elastic scattering from a tensor-polarized deuteron, and is equivalent to the induced tensor polarization of an elastically recoiling deuteron.

Figure 4 shows a collection of the world data on separated structure functions $A$ and $B$, respectively, in comparison with various deuteron model predictions [46]. The data in the bottom panels are normalized to a recent parameterization of the world data [37]. While there is a discrepancy of about two standard deviations between the two recent measurements of $A$ at Jefferson Lab [38,39], a larger and long-standing discrepancy at low $Q^2$ between data from Mainz [40] and Saclay [23] of several standard deviations is awaited to be resolved by new measurements made at Jefferson Lab [41]. The diffractive minimum of $B$ found at SLAC [42] still needs to be confirmed in a future experiment at Jefferson Lab [43]. The available data of the tensor analyzing power $T_{20}$ [44,45] is displayed in Fig. 5. The resulting separated form factors $G_C$ and $G_Q$ are shown in Fig. 6. The recent preliminary results from BLAST at MIT-Bates [45] significantly improve the precision for the separated form factors $G_C$ and $G_Q$ and confirm the node of $G_C$ at $Q^2 = 4.19$ fm$^{-2}$.

This work has been supported by the U.S. Department of Energy.
References


[16] Proposal for BLAST @ DORIS/DESY, R. Milner, private communication.


