ON EFFECTS OF D°–D° MIXING AT SPEAR

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We discuss the channels in which D°–D° mixing effects might be observed at SPEAR.

The general theoretical industry of the last two years on the properties of charmed particles [1] has been crowned by the announcement [2] of a candidate for the particle D° and its antiparticle in the decay D° → K⁻π⁺ at SPEAR. Clearly, e⁺e⁻ colliding beams are a prime laboratory for the study of the decays of these particles.

Previously the most direct signal for the possible weak decay of charmed particles was the presence of μ⁺μ⁻ events [3–5] in ν and v̄ beams and e⁺μ⁻ events [6, 7], often produced with strange particles, in ν beams. In connection with this, a number of μ⁻μ⁺ events in ν beams [4], at about 1/10 of the level of the μ⁻μ⁺ events, have been of considerable interest. Two possible mechanisms considered for these μ⁻μ⁺ events have been the associated production of charmed particles by the charm-conserving part of the weak current, or the effect of D°–D° mixing before decay [8, 9]. D°–D° mixing is expected in the GIM model [10] to be highly suppressed [8], but clearly this matter requires empirical investigation. Other models involving heavy quarks or a charm-changing neutral current [11] are capable of producing very large D°–D° mixing. Here we analyse the phenomenological effect of D°–D° mixing in e⁺e⁻ colliding beams [12]. The appropriate ratio for analysis is

\[ r = \frac{N(K^+K^+) + N(K^-K^-)}{N(K^+K^-)}, \]  

where each kaon observed is taken to be the only strange particle in the decay of a charmed particle. We shall see how r varies with the type of state produced by the virtual photon.

Assuming CP conservation, the mass eigenstates are

\[ |D_+⟩ = \frac{1}{\sqrt{2}}(|D^0⟩ + |\bar{D}^0⟩), \quad |D_-⟩ = \frac{1}{\sqrt{2}}(|D^0⟩ - |\bar{D}^0⟩), \]  

with masses and inverse lifetimes \( m_+ \) and \( \lambda_+ \), \( m_- \) and \( \lambda_- \), respectively. A produced \(|D^0⟩\), created at time \( t=0 \), evolves as

\[ a(t)|D^0⟩ + b(t)|\bar{D}^0⟩ + \sum_i c_i(t)|X_i^-⟩ + \sum_i d_i(t)|X_i^+⟩ + ..., \]  

where \(|X_i^-⟩\) is a decayed state of \(|D^0⟩\) of energy \( E_i \) containing a K⁻ together with non-strange particles, and \(|X_i^+⟩\) is the state obtained by acting on \(|X_i^-⟩\) with the operator CP, and so contains a K⁺. From the equations of motion we learn that

\[ a(t) = \frac{1}{2} \left[ \exp (-im_+t/2) + \exp (-im_-t - \lambda_-t/2) \right], \]  

while \( c_i(t), d_i(t) \) obey the equations

\[ \frac{d}{dt} c_i(t) = \alpha_i a(t) - iE_i c_i(t), \]  

and

\[ \frac{d}{dt} d_i(t) = \alpha_i b(t) - iE_i d_i(t), \]  

which have the solutions

\[ c_i(t) \{d_i(t)\} = \frac{\alpha_i}{2} \left[ \exp (-im_+t - \lambda_+t/2) - \exp (-im_-t - \lambda_-t/2) \right] \]  

\[ + \left\{-\exp (-im_-t - \lambda_-t/2) - \exp (-im_+t - \lambda_+t/2) \right\}, \]  

\[ \alpha_i \] is the partial rate for \(|D^0⟩\) to decay to the state \(|X_i^-⟩\).

Asymptotically, a produced \(|D^0⟩\) evolves to the state

\[ \sum_i c_i|X_i^-⟩ + \sum_i d_i|X_i^+⟩, \]  

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where
\[ c_i |d_i\rangle = \frac{\alpha_i}{2} \frac{1}{\lambda_+ /2 + i(m_+ - E_i)} \]
\[ + \left( -\right) \frac{1}{\lambda_- /2 + i(m_- - E_i)} \exp(-iE_it). \]

By CP conservation, a produced $|D^0\rangle$ evolves to the state
\[ \sum_i c_i |X_i^+\rangle + \sum_j d_j |X_j^-\rangle. \]

We first consider the production of a neutral weakly decaying D together with a charged weakly decaying charmed particle, and other particles. In this case the mixing is the simple case already analysed [8], as only the uncharged D will mix, and we obtain
\[ r = \frac{\delta m^2 + (\delta \lambda)^2 / 4}{2 \lambda^2 + \delta m^2 - (\delta \lambda)^2 / 4} \equiv r_0, \]
where
\[ \delta m = |m_+ - m_-|, \delta \lambda = |\lambda_+ - \lambda_-|, \lambda = \frac{1}{2} (\lambda_+ + \lambda_-). \]

We next consider the production of the $D^0\bar{D}^0$ system. Two particular cases arise. If the $D^0\bar{D}^0$ is produced together with charged particles, it will often be possible to identify which particle was produced as a $D^0$, even in the ideally mixed case. For example, it might be possible to identify the reaction
\[ e^+ e^- \rightarrow D^{+\ast} D^{\ast\ast} \rightarrow \pi^- D^{0}. \]

For the subset of events for which this is possible, we see that the amplitudes for the states $|D^0\rangle |\bar{D}^0\rangle$ to evolve to the states $|X_i^+\rangle |X_i^+\rangle, |X_i^+\rangle |X_j^-\rangle, |X_i^-\rangle |X_j^+\rangle$ and $|X_i^-\rangle |X_j^-\rangle$ are $c_i c_j, c_i d_j, d_i c_j$ and $d_i d_j$ respectively. Since there is no interference, we easily obtain
\[ r = \frac{2r_0}{1 + r_0^2}, \]
where $r_0$ is the result for $r$ in eq. (10).

Finally we consider the case where the $D^0\bar{D}^0$ system is produced exclusively, or only together with other neutral $C = +1$ particles such as $\pi^0, \eta$ and $\eta'$. In that case the produced $D^0\bar{D}^0$ state has $C = -1$, in the one-photon approximation, and so is $|D^0\rangle |\bar{D}^0\rangle - |\bar{D}^0\rangle |D^0\rangle$.

The amplitude to find the states $|X_i^+\rangle |X_j^-\rangle$ and $|X_i^-\rangle |X_j^+\rangle$ are now $(c_i d_j - d_i c_j)$ and $(c_i c_j - d_i d_j)$ respectively.

By calculation we obtain
\[ r = \frac{\int |c_i d_j - d_i c_j|^2 dE_i dE_j}{\int |c_i c_j - d_i d_j|^2 dE_i dE_j} = r_0, \]
the same ratio as obtained in (10) rather than approximately twice as much, as in (13). This difference is due to the obvious interference effect.

We have ignored the production of $K^-K^-$ states which can occur at about the 0.2% level due to the sin$^4\theta_c$ decay of a charmed particle in a $\Delta C = -\Delta S$ mode. Mixing at that level would best be analysed in terms of specific final states, as in ref. [8].

Clearly, if the mixing ratio $r_0$ is not very small, as might be indicated if mixing is the correct explanation of the Fermilab $\mu^-\mu^-$ events, this effect should be clearly visible at SPEAR. If it is seen it will be a strong signal for the necessity of corrections to the GIM model of the weak interactions. Any breaking of the result calculated in eq. (14), for the case of neutral particle production only, would presumably signal CP violation in the $D^0\bar{D}^0$ system.

References