Majorana neutrinos and their electromagnetic properties

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To help develop a picture of Majorana neutrinos, we study their electromagnetic properties. We show that CPT invariance forbids a Majorana neutrino from having a magnetic or electric dipole moment. Then, by considering the process $\gamma \rightarrow v\bar{\nu}$, we find the most general expression for the matrix element of the electromagnetic current of a Majorana neutrino. The result is verified in a way which leads us to explore the behavior under parity of such a particle. Next, we see how electromagnetic properties which follow from one-loop diagrams conform to our general results. Finally, we show how the striking electromagnetic differences between Majorana and Dirac neutrinos can become invisible as the neutrino mass goes to zero.

I. INTRODUCTION

A number of widely discussed recent theoretical models\textsuperscript{1} suggest that neutrinos are massive Majorana particles, identical to their antiparticles. Thus, it is of interest to develop a picture of the characteristics of a Majorana neutrino. Here we study its electromagnetic properties, and contrast them with those of a Dirac neutrino, which is distinct from its antiparticle. We begin by showing that CPT invariance forbids a Majorana neutrino from having either a magnetic or an electric dipole moment. Next, we question whether a physical Majorana neutrino state is indeed an eigenstate of charge conjugation $C$. Without assuming that it is, we derive in two ways the most general form for the matrix element

$$\langle \nu^M(p_f,s_f) | J^\text{EM}_\mu | \nu^M(p_i,s_i) \rangle,$$

where $J^\text{EM}_\mu$ is the electromagnetic current operator, and $\nu^M(p,s)$ is a Majorana neutrino of momentum $p$ and spin projection $s$. This matrix element contains only one form factor. We show that this fact follows very simply from the requirement that the final state in the crossed-channel process $\gamma \rightarrow \nu^M v^M$ be antisymmetric. The derivations of the electromagnetic matrix element reveal that a Majorana neutrino has very interesting parity properties, which we discuss. Next, noting that a Dirac neutrino has three more form factors than a Majorana neutrino, we examine how the extra form factors manage to vanish when the electromagnetic properties of a Majorana neutrino are calculated in $\text{SU}(2)\times\text{U}(1)$ to one-loop order. Lastly, we compare the electromagnetic interactions of a Majorana and a Dirac neutrino in the massless limit. We find that they conform to what seems to be a general rule: If all weak currents are left-handed, then the difference between a Majorana and a Dirac neutrino becomes invisible as the mass goes to zero. This occurs in spite of gross differences between these particles when the mass is not negligible.

II. STATIC ELECTROMAGNETIC PROPERTIES

It has been argued on various grounds, both in ancient papers and recent ones,\textsuperscript{2} that a Majorana neutrino cannot have a magnetic or electric dipole moment. It seems not to have been noticed, however, that this conclusion already follows trivially from the relatively weak assumption of CPT invariance. Suppose a Majorana neutrino has a magnetic dipole moment $\mu$ and electric dipole moment $d$. Then, when it is at rest, its interaction energy in a combination of static, uniform magnetic and electric fields is of the form $-\mu(\hat{s} \cdot \hat{B}) - d(\hat{s} \cdot \hat{E})$. Here $\hat{s}$ is, of course, the neutrino spin operator. Now, in the CPT-reflected state, the fields $\hat{B}$ and $\hat{E}$ are unchanged. However, the effect of CPT on a Majorana neutrino at rest is simply to reverse its spin (apart from a phase factor). Thus, the dipole interaction energy changes sign when we go to the CPT-reflected state, so if CPT invariance holds, $\mu$ and $d$ must vanish.\textsuperscript{3}
III. BEHAVIOR OF $\nu^M$ UNDER C

It might be thought that, alternatively, one could argue that $\mu$ and $d$ must vanish because of the $C$ properties of a Majorana neutrino. If such a neutrino is an eigenstate of $C$, then

$$\langle \nu^M | -\mu \tilde{s} \cdot \tilde{B} - d \tilde{s} \cdot \tilde{E} | \nu^M \rangle$$

$$= \langle \nu^M | C(-\mu \tilde{s} \cdot \tilde{B} - d \tilde{s} \cdot \tilde{E})C^{-1} | \nu^M \rangle$$

$$= \langle \nu^M | \mu \tilde{s} \cdot \tilde{B} + d \tilde{s} \cdot \tilde{E} | \nu^M \rangle,$$

since $\tilde{B}$ and $\tilde{E}$ are $C$-odd. Thus, $\mu = d = 0$.

It seems, however, that this argument must be viewed with caution. The difficulty is that it is not obvious that a physical, dressed Majorana neutrino is indeed an eigenstate of $C$. To be sure, in free field theory a Majorana neutrino is defined to be an eigenstate of $C$. What is not obvious is that this property of $C$ self-conjugacy can be maintained once the $C$-violating weak interactions are turned on.

Consider, for example, the diagrams in Fig. 1, where the external Majorana neutrino is massive but relativistic, and has negative helicity. Due to its Majorana character, the incoming neutrino can produce both $l^- W^+$ and $l^+ W^-$ virtual states. However, if the charged-current weak interactions are purely left handed, the vertices in the $l^+ W^-$ diagram are severely depressed by helicity, compared to those in the $l^- W^+$ diagram. Thus, the $C$ mirror-image states $l^- W^+$ and $l^+ W^-$ occur with very unequal amplitudes in the dressed neutrino state. Consequently, that dressed state does not appear to be self-conjugate under $C$.

We shall assume that a photon can couple to a neutrino only by coupling, as in Figs. 2 and 3, to the charged particles in such virtual intermediate states as those in Fig. 1. Thus, if these same virtual states vitiate the $C$ self-conjugacy of Majorana neutrinos, that fact can hardly be disregarded when one is trying to understand the electromagnetic properties of these particles. Consequently, we shall derive the most general expression for the electromagnetic matrix element

$$\langle \nu^M(p_f, s_f) | J^E_{\mu} | \nu^M(p_i, s_i) \rangle$$

without assuming that $| \nu^M \rangle$ is an eigenstate of $C$.

We note in passing that if one takes the electromagnetic interaction of a Majorana neutrino to be given by the diagrams of Fig. 3, plus similar diagrams with the photon attached to the $W$ boson, one has taken $J^E_{\mu}$ to be $-ie\gamma_{\mu}j$, which is $C$-odd, plus a $C$-odd term for the $W$ boson. Thus, if the dressed $\nu^M$ were a $C$ eigenstate, these diagrams would give

$$\langle \nu^M | J^E_{\mu} | \nu^M \rangle = \langle \nu^M | C J^E_{\mu} C^{-1} | \nu^M \rangle$$

$$= -\langle \nu^M | J^E_{\mu} | \nu^M \rangle = 0.$$

However, explicit calculation shows that these diagrams do not yield a vanishing $\langle \nu^M | J^E_{\mu} | \nu^M \rangle$.

IV. THE ELECTROMAGNETIC CURRENT OF A MAJORANA NEUTRINO

We shall obtain the most general form for $\langle \nu^M | J^E_{\mu} | \nu^M \rangle$ in two ways. In the first of these we consider, not the $s$-channel process $\gamma + \nu \rightarrow \nu$ which this matrix element describes, but the related $t$-channel process $\gamma \rightarrow \nu + \bar{\nu}$. We do this because in the $t$ channel, the consequences of having neutrinos of Majorana character are particularly easy to see. (In the process $\gamma \rightarrow \nu + \bar{\nu}$ related through the crossing properties of local field theory to $\gamma + \nu \rightarrow \nu_f$, "$\nu_f$" is the $CPT$ (not $C$) conjugate of $\nu_i$, as one can show from the $CPT$ properties of fermion fields. Thus, in the Majorana case, the $t$-channel process involves the production of two identical particles, whether or not a Majorana neutrino is self-conjugate under $C$.)

For $\gamma \rightarrow \nu + \bar{\nu}$, the number of independent amplitudes is easy to determine. First, note that, even though the photon is off-shell, the conserved electromagnetic current to which it couples will only produce $\nu \bar{\nu}$ states with total angular momentum $J = 1$. States with $J = 0$ cannot be produced. For, if $| \phi_J(q) \rangle$ represents any system with $J = 0$ and momentum $q$, then

FIG. 1. Virtual components in the state of a Majorana neutrino $\nu^M$, whose subscript denotes its helicity. In the figure, $l$ is a charged lepton and $W$ the charged weak boson.
where $a$ is a form factor. Current conservation then requires that

$$q_{\mu} \langle \phi_{J=0}(q) | J_{\mu}^{EM} | 0 \rangle = a q_{\mu},$$

so that $a$ must vanish. Now, consider for a moment that Dirac case, in which the $t$-channel process is $\gamma \rightarrow \nu D^{\nu}$. In the nonrelativistic limit, the $v^D v^D$ pair can be produced in any one of four $J=1$ states: $3D_1$, $3P_1$, $3S_1$, and $1P_1$. Thus, a Dirac neutrino has four independent electromagnetic form factors. In the Majorana case, the t-channel process is $\gamma \rightarrow \nu M^{\nu} M$, with two identical fermions in the final state. Thus, this state must be antisymmetric. Of the four $J=1$ states just mentioned, only one, $3P_1$, meets this requirement. Therefore, a Majorana neutrino has only one electromagnetic form factor.

Using standard techniques, one easily finds that for a Dirac neutrino, the electromagnetic matrix element may be written in the form

$$\langle v^D(p_f,s_f) | J_{\mu}^{EM} | v^D(p_i,s_i) \rangle = i \bar{u}_f | F_D(q^2) \gamma_{\mu} + G_D(q^2)(q^2 \gamma_{\mu} - 2miq_{\mu}) \gamma_{5} + M_D(q^2) \sigma_{\mu v} q_{v} + E_D(q^2)i \sigma_{\mu v} q_{v} \gamma_{5} \rangle u_i.$$

(4.1)

Here $F_D$, $G_D$, $M_D$, and $E_D$ are form factors, $q = p_f - p_i$, and $m$ is the neutrino mass. Before one imposes the constraint that the neutrinos in the $t$ channel are identical, the analogous matrix element for a Majorana neutrino has exactly the same form. To see this, note that this matrix element comes from perturbation theoretic diagrams in which the incoming $\nu M$ is annihilated by a free Majorana field $\chi$, and the outgoing one is created by the corresponding $\bar{\chi}$, or else the other way around. Now, the momentum-space expansion of $\chi$ is

$$\chi = \sum_{\bar{\nu},s} \left( \frac{m}{E_{\bar{\nu}} - V} \right)^{1/2} \left[ a_{\bar{\nu},s} u_{\bar{\nu},s} e^{i p_{\bar{\nu}} \cdot x} + a_{\bar{\nu},s}^{\dagger} v_{\bar{\nu},s} e^{-i p_{\bar{\nu}} \cdot x} \right],$$

(4.2)

in an obvious notation. Thus, $\langle v^M | J_{\mu}^{EM} | v^M \rangle$ must have the form

$$\langle v^M(p_f,s_f) | J_{\mu}^{EM} | v^M(p_i,s_i) \rangle = i \bar{u}_f \Gamma_{\mu}^{M} u_i,$$

where $\Gamma_{\mu}^{M}$ is a combination of $\gamma$ matrices and momentum factors, and in the last step of Eq. (4.3) we have used the relation $v_{\bar{\nu},s} = \gamma_{2}u_{\nu,s}$. If we then find the most general expression for $\Gamma_{\mu}$ consistent with conservation of $J_{\mu}^{EM}$, we obtain for

$$\langle v^M(p_f,s_f) | J_{\mu}^{EM} | v^M(p_i,s_i) \rangle$$

a form identical to that of Eq. (4.1). Arguing similarly for the $t$-channel process, we find that

Using standard techniques, one easily finds that for a Dirac neutrino, the electromagnetic matrix element may be written in the form

$$\langle v^M(p_f,s_f) | J_{\mu}^{EM} | v^M(p_i,s_i) \rangle = i \bar{u}_f G_M(q^2) \gamma_{\mu} - 2miq_{\mu} \gamma_{5} u_i.$$

(4.5)
This is the most general expression for the matrix element of the electromagnetic current of a Majorana neutrino.

Note that Table I shows explicitly that in the t-channel matrix element, the \((q^2 \gamma_\mu - 2miq_\mu)\gamma_5\) term is antisymmetric in the nonrelativistic limit, while the others are not (the quantities \(\bar{T}\) and \(S\) are symmetric and antisymmetric, respectively). Of course, we need not have gone to the nonrelativistic limit to discover the symmetry properties of the different terms. The bracketed quantity in Eq. (4.4), \(\tilde{\Gamma}_\mu\), is symmetric under \((p_1, s_1)\leftrightarrow(p_2, s_2)\), so this interchange simply corresponds to \(i \bar{u}_1 \tilde{\Gamma}_\mu v_1 \leftrightarrow i \bar{u}_2 \tilde{\Gamma}_\mu v_2\). But

\[
i \bar{u}_1 \tilde{\Gamma}_\mu v_2 = -i \bar{u}_2 (C^{-1} \tilde{\Gamma}_\mu C)^T v_1 ,
\]

where \(C = \gamma_4 \gamma_2\). Hence, from the charge-conjugation properties of the various combinations of \(\gamma\) matrices, it follows immediately that under \((p_1, s_1)\leftrightarrow(p_2, s_2)\) all the candidate terms in Eq. (4.4) remain unchanged, except for the \((q^2 \gamma_\mu - 2miq_\mu)\gamma_5\) term, which undergoes the required change of sign.

The electromagnetic matrix element of Eq. (4.5) is confirmed by finding the most general effective electromagnetic current \(J^\mu (\bar{X}, \chi)\) which can be constructed out of free Majorana fields \(\chi\) and \(\bar{X}\) and their derivatives up through second order, and taking the matrix element of this effective current between free Majorana neutrino states. The construction of \(J^\mu_{\text{eff}}\) does not assume \(C\) self-conjugacy of dressed Majorana neutrinos. Indeed, \(J^\mu_{\text{eff}}\) incorporates \(C\)-violating weak effects such as those in the diagrams of Fig. 3, replacing all but the external legs of such diagrams by a "black box." Since \(\chi^c\), the charge conjugate of the free Majorana field \(\chi\), is \(\chi\) itself, \(J^\mu_{\text{eff}} (\bar{X}, \chi)\) will necessarily be \(C\)-even, in contrast to the bare electromagnetic current, which is \(C\)-odd before it receives weak corrections.

Eliminating from \(J^\mu_{\text{eff}}\) such candidate terms as \(\bar{X} (\partial_\mu \chi) - (\partial_\mu \bar{X}) \chi\), which vanish because \(\chi^c = C\bar{X}^T = \chi\), we are left with

\[
J^\mu_{\text{eff}} = \partial_\mu [\bar{X} (a + b \gamma_5) \chi] + (c + d \Box^2) (\bar{X} \gamma_\mu \gamma_5 \chi) + [\bar{X} \sigma_{\mu \nu}(e + f \gamma_5)(\partial_\nu \chi) - (\partial_\nu \bar{X}) \sigma_{\mu \nu}(e + f \gamma_5) \partial_\nu \chi], \quad (4.6)
\]

where \(a, \ldots, f\) are constants. Current conservation then requires that

\[
\partial_\mu J^\mu_{\text{eff}} = \Box^2 [\bar{X} (a + b \gamma_5) \chi] + (c + d \Box^2) 2m (\bar{X} \gamma_\mu \gamma_5 \chi) + [2m \bar{X} \sigma_{\mu \nu}(e + f \gamma_5)(\partial_\nu \chi) - (\partial_\nu \bar{X}) \sigma_{\mu \nu}(e + f \gamma_5) \partial_\nu \chi] = 0 . \quad (4.7)
\]

This constraint demands that

\[a = b + 2md = c = e = f = 0 . \quad (4.8)\]

Thus,

\[
J^\mu_{\text{eff}} = -\frac{g}{m^2} [\Box^2 (\bar{X} \gamma_\mu \gamma_5 \chi) + 2m \partial_\mu (\bar{X} \gamma_\mu \gamma_5 \chi)] , \quad (4.9)
\]

where \(g\) is a dimensionless constant. Taking the matrix element of \(J^\mu_{\text{eff}}\) (0) between free \(\nu^M\) states, we obtain once again the result of Eq. (4.5), apart from the form factor. The latter corresponds to permissible additional powers of \(\Box^2\) operating on the entire current of Eq. (4.9).

V. PARITY OF MAJORANA NEUTRINOS

The structure of \(J^\mu_{\text{eff}}\), Eq. (4.9), calls attention to the interesting parity properties of Majorana neu-

<table>
<thead>
<tr>
<th>Term</th>
<th>Nonrelativistic limit</th>
<th>States produced</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\gamma_\mu)</td>
<td>(1 + \frac{\vec{p}^2}{12m^2}) (\bar{\eta}^T \bar{T} - \frac{\vec{p}^2}{6m^2} (3 \bar{\eta}^T \bar{\rho}^T \bar{\rho} - \bar{\eta}^T \bar{T}))</td>
<td>(3S^1, 3D_1)</td>
</tr>
<tr>
<td>((q^2 \gamma_\mu - 2miq_\mu)\gamma_5)</td>
<td>(\bar{\eta}^T \bar{T} \times \bar{\rho})</td>
<td>(3P_1)</td>
</tr>
<tr>
<td>(\sigma_{\mu \nu} q_\nu)</td>
<td>(1 - \frac{\vec{p}^2}{12m^2}) (\bar{\eta}^T \bar{T} + \frac{\vec{p}^2}{6m^2} (3 \bar{\eta}^T \bar{\rho}^T \bar{\rho} - \bar{\eta}^T \bar{T}))</td>
<td>(3S^1, 3D_1)</td>
</tr>
<tr>
<td>(\sigma_{\mu \nu} q_\nu \gamma_5)</td>
<td>(\bar{\eta}^T \bar{\rho} S)</td>
<td>(1P_1)</td>
</tr>
</tbody>
</table>
trinos. It is not obvious, but can be shown, that under parity $P$, a free Majorana field $\chi$ behaves just as does a Dirac field:

$$\chi'(\bar{x},t) \rightarrow \eta_P \gamma^4 \chi(-\bar{x},t) ,$$  

(5.1)

where $\eta_P$ is a phase factor. Thus, we can determine the parity of $J_\mu^{EM}$ by inspection, disregarding the fact that $\chi$ is not a Dirac field. We see that $\mathcal{J}_{\text{eff}}$ has even parity. However, from the $t$-channel analysis, we know that this operator, acting on the vacuum, produces a $v^{M}_{\nu} v^M$ state which in the non-relativistic limit is $3P_1$. Hence, we learn that

$$P \mid v^{M}_{\nu} v^M, 3P_1 \rangle = \pm i \mid v^{M}_{\nu} v^M, P_1 \rangle .$$  

(5.2)

That is, a state consisting of two identical Majorana neutrinos in a $P$ wave has positive parity. This result may seem somewhat strange, but it is true. It reflects the fact that the “intrinsic” parity of a single Majorana neutrino at rest is $\pm i$ (Ref. 7)

$$P \mid v^{M}_{\nu}(\bar{p}=0,s) \rangle = \pm i \mid v^{M}_{\nu}(p=0,s) \rangle .$$  

(5.3)

Thus, the intrinsic parity of two identical Majorana neutrinos is $(\pm i)^2$. This has measurable consequences, at least in principle. For example, in a world where parity is conserved, the decay $\Psi(2^+) \rightarrow v^{M}_{\nu} v^M$ of a particle $\Psi$ with $J^P=2^+$ would yield final states with odd values of orbital angular momentum. This fact would influence the angular distribution of the neutrinos.

To see why the intrinsic parity of a free Majorana neutrino is $\pm i$, consider the transformation law (5.1) for the free field. This law induces a related one for the charge-conjugate field $\chi^c$; namely,

$$\chi^c(\bar{x},t) \equiv \gamma^4 v^M \chi(\bar{x},t) \rightarrow -\eta_P \gamma^4 v^M (\bar{x},t) .$$  

(5.4)

Since $\chi^c(\bar{x},t)=\chi(\bar{x},t)$, we must require that the right-hand sides of (5.1) and (5.4) be equal. Thus, we must have $\eta_P = \pm i$. Insertion of the plane-wave expansion of $\chi$ in (5.1) then leads to the conclusion that

$$P \mid v^{M}(\bar{p},s) \rangle = \pm i \mid v^{M}(-\bar{p},s) \rangle .$$

(One finds that it also shows again that the properties of Majorana neutrinos under $P$ are not consistent unless $\eta_P = \pm i$.) The fixed value of the square of the intrinsic parity of a Majorana neutrino is in sharp contrast to the intrinsic parity of a Dirac particle, which is completely arbitrary.

VI. HOW LOOP DIAGRAMS YIELD ONLY ONE MAJORANA FORM FACTOR

The general expression for $\langle v^{M} \mid J_\mu^{EM} \mid v^{M} \rangle$, Eq. (4.5), shows that not only do the magnetic and electric dipole moments of a Majorana neutrino vanish, but the entire magnetic and electric dipole form factors $M_{\mu}(q^2)$ and $E_{\mu}(q^2)$ vanish as well. The “electric charge distribution” form factor $F_M(q^2)$ vanishes also.

Now, for Dirac neutrinos, the electromagnetic form factors have been calculated in SU(2)$_L \times U(1)$ in terms of one-loop diagrams such as that in Fig. 2. This and subsequent figures in which a photon is attached to a charged lepton line, the additional diagram in which the photon is attached to the $W$ boson line will always be understood. Also, we imagine that the calculations we shall discuss are carried out in a gauge in which diagrams such as that of Fig. 2, but with the $W$ replaced by an unphysical charged Higgs particle, or by $W$ and charged-Higgs-boson lines meeting at a photon $W$-Higgs-boson vertex, do not contribute. It is well known that the loop diagrams yield a nonvanishing magnetic moment for a Dirac neutrino, but suppose the external neutrino is replaced by a Majorana one. How is it that these diagrams will now give vanishing values, as they must, for the magnetic moment and, indeed, for all the form factors which are forbidden to a Majorana neutrino? The answer follows from Fig. 3. This reminds us that when the incoming neutrino is a $v^M$, there will be an extra diagram in which the neutrino produces an $l^+ W^-$ intermediate state, rather than an $l^- W^+$ one. The term in the weak Lagrangian which was active at the initial (final) vertex of the original diagram will be active at the final (initial) vertex of the new one. Figure 3 indicates at each vertex the interaction term that is active there, but in the extra diagram peculiar to $v^M$, this interaction is written in terms of charge-

FIG. 2. One-loop diagram for the electromagnetic interaction of a Dirac neutrino $v^D$ with initial momentum and spin projection values $i$ and final ones $f$. The term in the weak Lagrangian which is active at each vertex is written next to it.
neutrino and a Majorana neutrino. That is, the four states which normally comprise a Dirac neutrino decouple into two disconnected pairs. The pair consisting of the left-handed Dirac neutrino and its right-handed so-called antineutrino can equivalently be regarded as the left- and right-handed states of a Majorana neutrino. The other pair need not even exist. Once the mass is nonzero, however, then no matter how small it is, a Dirac and a Majorana neutrino are different: the former involves four distinct states, connected by Lorentz transformations and CPT invariance, while the latter involves only two. Nevertheless, it appears that for practical purposes the massless limit is still a smooth one. Indeed, it seems that this limit can be described by a “practical Majorana-Dirac confusion theorem”: Assume that all weak currents are left-handed. Assume further that experiments on a given neutrino are always done with one of two incoming states—a state of negative helicity, “v_−,” or its positive-helicity CPT conjugate, “v_+.” Then, as the neutrino mass goes to zero, it gradually becomes impossible to tell experimentally whether v_− and v_+ are actually v_D and v_i, two of the four states of a Dirac neutrino (v_D, v_D, v_D, v_D), or v_D and v_D, the two states of a Majorana neutrino (v_M, v_M).

This “theorem” applies, in particular, to the behavior which has been found\(^\text{11}\) for neutrino neutral-current weak interactions. We shall show explicitly that it also applies to the electromagnetic interactions of neutrinos, even though, as we have seen, these interactions distinguish sharply between a Majorana particle and a Dirac one when the mass is not small. We shall also compare the electromagnetic interactions of Majorana and Dirac neutrinos in the massless limit when right-handed currents are present.

VII. THE MASSLESS LIMIT

For massless neutrinos in a world where all weak currents are left-handed, there is no distinction between a two-component Dirac (i.e., Weyl)
Assume first that there are no right-handed currents. Then the electromagnetic interactions of a Dirac neutrino are described by diagrams such as those of Fig. 4. The initial and final vertices are always weak vertices involving either a charged or neutral left-handed current. Now, suppose the neutrino mass $m$ is very small compared to its momentum $|\mathbf{p}|$. Then, due to the handedness of the final vertex, the helicity-flipping transition $\nu^d_--\nu^d_+$ is highly suppressed compared to the helicity-preserving one $\nu^d_-\nu^d_-$, as Fig. 4 indicates. From Eq. (4.1), this means that the helicity-flipping form factors $M_D(q^2)$ and $E_D(q^2)$ must go to zero with $m$, and that the helicity-flipping dipole terms in Eq. (4.1) may be neglected when $m \ll |\mathbf{p}|$. In addition, the quantity multiplying $G_D$ in Eq. (4.1) obviously simplifies to $\gamma_\mu\gamma_5$ when $m \ll |\mathbf{p}|$, so that in this limit, the surviving, helicity-conserving electromagnetic matrix element obeys

$$
\langle \nu^d_- | J^\mu_{\text{EM}} | \nu^d_- \rangle \rightarrow i\mathbf{u} f(-)[F_D\gamma_\mu + G_D q^2 \gamma_\mu\gamma_5]u_i(-)
$$

$$
\simeq (F_D + G_D q^2) i\mathbf{u} f(-)\gamma_\mu u_i(-) .
$$

(7.1)

(7.2)

Here $u_i(-)$ and $u_f(-)$ are spinors for initial and final states of negative helicity, and in obtaining Eq. (7.2) we have used the relation $\gamma_5 u_i(-) \simeq u_i(-)$, which holds when $m \ll |\mathbf{p}|$.

For a Majorana neutrino, Eq. (4.5) shows that as $m \rightarrow 0$, the electromagnetic matrix element reduces to the helicity-conserving $\gamma_\mu\gamma_5$ term, regardless of the nature of the weak currents. In particular,

$$
\langle \nu^M_- | J^\mu_{\text{EM}} | \nu^M_- \rangle \rightarrow G_M q^2 i\mathbf{u} f(-)\gamma_\mu u_i(-) .
$$

(7.3)

Thus, if

$$
\langle \nu^M_- | J^\mu_{\text{EM}} | \nu^M_- \rangle = G_M q^2 i\mathbf{u} f(-)\gamma_\mu u_i(-) .
$$

(7.4)

when $m \rightarrow 0$, then not only do the electromagnetic interactions of $\nu^d_-$ and $\nu^d_+$ both become helicity preserving in this limit, but they become completely indistinguishable. [Of course, the interactions of $\nu^d_+$ and $\nu^d_+$ also become indistinguishable if Eq. (7.4) holds.] Note that indistinguishability requires that the Majorana and Dirac form factors be related by Eq. (7.4) for any given set of (left-handed) weak interactions. Otherwise, we could determine the weak interactions through some experiments which do not settle the Majorana-Dirac issue, then calculate both the Majorana and Dirac form factors from the measured weak interactions, then measure $\langle \nu^- | J^\mu_{\text{EM}} | \nu^- \rangle$, and see whether it is described by $G_M q^2$, or $F_D + G_D q^2$.

At the one-loop level, it is easy to show that relation (7.4) is indeed satisfied. In fact, from Fig. 3 we see that in the extra diagram peculiar to $\nu^M$, the weak currents are effectively right-handed. Hence, when $m \rightarrow 0$, the contribution of this diagram to the transition $\nu^M \rightarrow \nu^M$ becomes negligible compared to that of the diagram common to $\nu^M$ and $\nu^d$, so that

$$
\langle \nu^M_- | J^\mu_{\text{EM}} | \nu^M_- \rangle \rightarrow (F_D | J^\mu_{\text{EM}} | D^-) .
$$

(7.5)

Going to a bit more detail, let us first note that when there are no right-handed currents,

$$
F_D = G_D q^2
$$

(7.6)

when $m \rightarrow 0$. To see this, observe from Fig. 4 that $\langle \nu^d_+ | J^\mu_{\text{EM}} | \nu^d_+ \rangle$ must vanish with $m$. Through the analog of Eq. (7.1) for $\langle \nu^d_+ | J^\mu_{\text{EM}} | \nu^d_+ \rangle$, this implies Eq. (7.6). Now, suppose that when $m \ll |\mathbf{p}|$, the one-loop diagram of Fig. 2 yields a matrix element $\langle \nu^d_+ | J^\mu_{\text{EM}} | \nu^d_+ \rangle$ of the form required by Eq. (7.1). Then, according to Eq. (6.2), the sum of diagrams in Fig. 3 will yield

$$
G_M q^2 = F_D + G_D q^2
$$

are now diagrams which lead to helicity-flipping transitions $\nu^d_- \rightarrow \nu^d_+$ whose amplitudes remain finite when $m \rightarrow 0$. The dipole form factors $M_D$ and $E_D$, and in particular the magnetic and electric dipole moments of $\nu^d$, need no longer vanish with $m$.

Thus, even when $m = 0$, a Dirac neutrino is now quite distinct from a Majorana neutrino. Through helicity flipping in an external $\mathbf{B}$ or $\mathbf{E}$ field, $\nu^d_-$
FIG. 5. A helicity-flipping transition involving the action of a right-handed current of strength \( r \). The amplitude for this transition does not vanish when \( m \to 0 \).

and \( \bar{\nu}^D + \) can be converted, respectively, into \( \nu^D + \) and \( \bar{\nu}^D - \), the other two particles of the Dirac foursome. The latter two particles cannot be identical to \( \bar{\nu}^D + \) and \( \nu^D + \). If they were, we would have a neutrino involving only two states, and could prove through one of the approaches in Sec. IV that its electromagnetic matrix element is given by Eq. (4.5) and does not allow any helicity flipping when \( m = 0 \).

In addition, when there are right-handed currents, the helicity-preserving matrix elements \( \langle \nu^M | J^E_\mu | \nu^M \rangle \) and \( \langle \nu^D | J^E_\mu | \nu^D \rangle \) do not become equal, for a given set of weak interactions, as \( m \to 0 \). When the currents acting in the first diagram of Fig. 3 include right-handed pieces, those acting in the second diagram include effectively left-handed ones. Thus, the latter diagram, which is unique to \( \nu^M + \), makes a nonvanishing contribution to \( \langle \nu^M | J^E_\mu | \nu^M \rangle \) when \( m = 0 \). Hence, Eq. (7.5) fails, as does, of course, the relation

\[
G_M q^2 = F_D + G_D q^2,
\]

VIII. SUMMARY

While a Dirac neutrino has four electromagnetic form factors, a Majorana neutrino has only one. This result is obtained very easily by considering the \( t \)-channel process \( \gamma \to \nu^M \bar{\nu}^M \), and requiring that the two-neutrino final state obey Fermi statistics. The same procedure shows that it is an essentially axial-vector form factor which survives in the Majorana case. The electric-charge-distribution form factor, and the magnetic and electric dipole form factors, all vanish. We saw explicitly how these form factors vanish when they are calculated from one-loop diagrams. We also observed that the magnetic and electric dipole moments are already forbidden by CPT invariance.

The electromagnetic matrix element

\[
\langle \nu^M | J^E_\mu | \nu^M \rangle
\]

derived by the \( t \)-channel analysis is confirmed by constructing an effective electromagnetic current out of free Majorana fields. Neither of these derivations assumes that a physical Majorana neutrino is an eigenstate of \( C \). Taken together, they point to the surprising fact that the intrinsic parity of a pair of identical free Majorana neutrinos is \(-1\).

In spite of the very different electromagnetic properties of Dirac and Majorana neutrinos, the electromagnetic interactions of \( \langle \nu^D_\mu, \nu^D_\mu \rangle \) and \( \langle \nu^M_\mu, \nu^M_\mu \rangle \) smoothly become helicity-preserving and indistinguishable as \( m \to 0 \) if there are no right-handed weak currents. This strengthens our expectation that all differences between Dirac and Majorana behavior disappear with the neutrino mass if right-handed currents are absent. On the other hand, if such currents are present, then the electromagnetic interactions of \( \langle \nu^D_\mu, \nu^D_\mu \rangle \) and \( \langle \nu^M_\mu, \nu^M_\mu \rangle \) are very different, even when \( m = 0 \).

Note added in proof. A new analysis of electromagnetic properties and decays of Dirac and Majorana neutrinos by R. Shrock has now appeared [Stony Brook Report No. ITP-SB-82-2 (unpublished)]. This analysis focuses on topics complementary to those treated here, and confirms our result for \( \langle \nu^M | J^E_\mu | \nu^M \rangle \), Eq. (4.5).

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4Throughout this paper we shall omit the normalization factor \([m/E_V|m/E_V]|^{1/2}\) from expressions such as the right-hand side of Eq. (4.1).

5After this work was mostly completed, we received a paper by J. Nieves [Phys. Rev. D (to be published)], in which another derivation of the electromagnetic matrix element \(\langle \psi^M | J_{EM}^M | \psi^M \rangle\) is presented. The result of that derivation agrees with our Eq. (4.5). Also, an effective photon-Majorana neutrino interaction, expressed in a two-component formalism and involving one form factor, has been given by Schechter and Valle (Ref. 2). Now, our \(t\)-channel analysis shows that there can only be one form factor. Thus, the effective interaction of Schechter and Valle must be equivalent to our \(J_{EM}^{\text{eff}}\), Eq. (4.9), and J. Schechter (private communication) has explicitly shown that it is equivalent.

6The author thanks L. N. Chang for a very helpful conversation in this connection.


10A related analysis has been given by J. Schechter and J. Valle, Ref. 2. We thank J. Schechter for bringing this work to our attention.


12The form factor \(M_M(q^2)\) calculated to one-loop order in Ref. 8 explicitly exhibits this behavior.

13Relations between the Majorana and Dirac form factors for the electromagnetic transition between two different neutrinos \(\nu_1\) and \(\nu_2\) have been given by J. Nieves, Ref. 5.

14See Ref. 8, and, for a specific example, Bég et al. (Ref. 9).