Test of CPT Symmetry through a Determination of the Difference in the Phases of $\eta_{00}$ and $\eta_{+-}$ in $K \rightarrow 2\pi$ Decays

M. Karlsson, G. D. Gollin, J. K. Okamitsu, and R. Tschirhart
Department of Physics, Princeton University, Princeton, New Jersey 08544

The Enrico Fermi Institute and Department of Physics, The University of Chicago, Chicago, Illinois 60637

E. Swallow
Department of Physics, Elmhurst College, Elmhurst, Illinois 60126
and Department of Physics, The University of Chicago, Chicago, Illinois 60637

G. J. Bock, R. Coleman, J. Enagonio, Y. B. Hsiung, K. Stanfield, R. Stefanski, and T. Yamanaka
Fermi National Laboratory, Batavia, Illinois 60510

P. Debu, B. Peyaud, R. Turlay, and B. Vallage
Département de Physique des Particules Elémentaires, Centre d’Etudes Nucléaires de Saclay,
F 91919 Gif-sur-Yvette CEDEX, France
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Data collected by the E731 experiment at Fermilab were used to search for CPT violation in $K^0 \rightarrow \pi\pi$ decays by measuring the difference $\Delta \phi$ between the phases of the CP-violating parameters $\eta_{00}$ and $\eta_{+-}$. Our result, $\Delta \phi = -0.3^\circ \pm 2.4^\circ \pm 1.2^\circ$, where the first error is statistical and the second systematic, is consistent with CPT symmetry.

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CPT symmetry is a natural consequence of local quantum field theory and guarantees the equality of masses and lifetimes of particles and antiparticles. It is conceivable, however, that a small violation of CPT symmetry could occur in extensions of quantum field theory (e.g., string theory); thus, it is imperative to check CPT symmetry wherever possible. The neutral-kaon system provides some of the most sensitive tests of CPT symmetry. The ratio of the decay amplitudes $\eta_{00}=\text{amp}(K_L^0 \rightarrow \pi\pi)/\text{amp}(K_S^0 \rightarrow \pi\pi)$ can be written as $\eta_{+-}=|\eta_{+-}|e^{i\phi_{+-}}=(\epsilon-\Delta+a)+\epsilon'$ for the $\pi^+\pi^-$ final state and $\eta_{00}=|\eta_{00}|e^{i\phi_{00}}=(\epsilon-\Delta+a)-2\epsilon'$ for the $\pi^0\pi^0$ final state (Fig. 1). Here $\epsilon$ is a measure of CP violation without CPT violation in $K^0-K^0$ mixing, and $\Delta$ is similarly a measure of CPT violation without T violation. Direct CP and CPT violations in the decay amplitudes are parametrized by $a=(A_0-A_\varphi)/(A_0+A_\varphi)$ (chosen to be real) and

$$\epsilon'=\frac{1}{\sqrt{2}} \frac{A_2-\bar{A}_2}{A_0+\bar{A}_0} e^{i(\delta_\varphi-\delta_\alpha)}, \tag{1}$$

where $A_\ell (A_\varphi)$ is the $2\pi$ decay amplitude of $K^0 (\bar{K}^0)$ to isospin-1 final state with corresponding phase shift $\delta_\ell$ from final-state interactions. CPT symmetry results in two constraints that can be tested by experiment: First, CPT symmetry implies that the phase of $2\eta_{+-}+\eta_{00}$ be within a few degrees of the “natural angle” $\phi_{\epsilon}=\text{arg}(\Gamma_S/2+i\Delta m)=43.7^\circ \pm 0.2^\circ$, where $\Gamma_S$ is the $K_S$ decay rate and $\Delta m=M_{K^0}-M_{K^0}$. Second, using the experimental value $\delta_\varphi-\delta_\alpha=-45^\circ \pm 10^\circ$ in Eq. (1), the CPT relation $A_\ell=A_\varphi^*$ leads to $\arg(\epsilon'/\epsilon)$. This, together with the current knowledge of $|\text{Re}(\epsilon'/\epsilon)|$, requires the phase difference $\Delta \phi=\phi_{00}-\phi_{+-}$ to be much less than $1^\circ$. With the accepted value $\phi_{+-}=44.6^\circ \pm 1.2^\circ$, CPT symmetry is not violated.

FIG. 1. Relationship among the CPT-violating parameters $a$, $\Delta$, and the directly measurable quantities $\eta_{+-}$, $\eta_{00}$. Since CPT symmetry predicts $\epsilon'$ to be parallel to $\epsilon$, a nonzero value of $\Delta \phi=\phi_{00}-\phi_{+-}$ also indicates CPT violation. The magnitudes of $a$, $\Delta$, and $\epsilon'$ are exaggerated for clarity.
however, the most recent published measurement\textsuperscript{9} of \(\phi_0 = 55.7^\circ \pm 5.8^\circ\) corresponds to a phase difference about 2 standard deviations away from the prediction of CPT symmetry;\textsuperscript{10} this has generated considerable interest and speculation.\textsuperscript{3}

\[
\frac{d^2I_{ex}}{dp dt} \propto F(P_k)e^{-\gamma(t)}[|p|^2e^{-i\tau_K} + |\eta|^2e^{-i\tau_\eta} + 2|p| |\eta|e^{-i\Delta m t + \phi_p - \phi_0}],
\]

where \(\tau_K, \tau_\eta\) is the \(K_S, K_L\) lifetime, \(\Delta m\) the mass difference between \(K_0\) and \(K_S, \rho = |\rho|e^{i\phi}\) the coherent regeneration amplitude, and \(F(P_k)\) the incident kaon momentum spectrum. The incident flux is normalized to the other vacuum beam whose \(\pi\pi\) rate is simply \(F(P_k) \times |\eta|^2e^{-i\tau_\eta}\). The factor \(e^{-\gamma(t)}\) accounts for absorption in the regenerated beam. As can be inferred from Eq. (2), the values extracted for \(\phi_+\) and \(\phi_0\) individually depend on \(\phi_+\) and \(\Delta m\), but the difference \(\Delta\phi = \phi_0 - \phi_+\) is insensitive to both parameters.

A description of the experimental setup can be found in Refs. 6 and 11, and only a brief account of the essential features is given here. The \(\pi^0\pi^0\) decays were detected with a lead-glass calorimeter having an energy resolution for photons of 2.5\% + 5\%/\sqrt{E} (\(E\) in GeV). The \(\pi^+\pi^-\) decays were reconstructed using a magnet and a sixteen-plane drift-chamber system with a typical momentum resolution of 1\%. All four decay modes, \(K_{S,L} \rightarrow \pi^+\pi^-\pi^0\pi^0\), were collected simultaneously and the regenerator alternated between the two beams. This minimizes sensitivity to differences in intensity and momentum spectrum between the beams as well as to reconstruction inefficiencies, dead-time effects, and calibration changes with time.

The phase-difference result presented here is based on the same data set that was recently used to determine \(\text{Re}(e'/e)\),\textsuperscript{6} and the event-reconstruction and background-subtraction techniques employed are the same as for that analysis. For neutral decays, however, the length of the decay region is extended downstream by 13 m to increase sensitivity to the phase in the interference term. Most of the phase information is provided by decays at the lower end of the energy range where a larger proper-time region can be sampled. For \(\pi^+\pi^-\) decays, the length of the decay region is 14 m (6.5 \(K_S\) lifetimes at 40 GeV), and for \(\pi^0\pi^0\) decays, it is 29 m (14 \(K_S\) lifetimes at 40 GeV). It is not possible to extend the decay region for the charged mode since it is hardware defined by a thin trigger hodoscope.

The 2\(\pi^0\) mass distribution for the regenerated beam has a small (0.04\%) background which is dominated by 3\(\pi^0\) decays from the \(K_S\) beam transmitted through the regenerator. A 3\(\pi^0\) decay can fake a 2\(\pi^0\) signal when two out of the six photons are lost either by escaping the detector or by merging with other photons in the lead-glass calorimeter. Figure 2 shows the vertex distributions for signal and backgrounds in the regenerated beam. For the vacuum beam, the level of 3\(\pi^0\) background is higher (0.99\%); its vertex distribution has the same shape as that of the regenerated beam. Another source of background arises from incoherently produced kaons that emerge with nonzero scattering angle from the regenerator. This amounts to 2.7\% in the regenerated beam, and its vertex distribution is also shown in Fig. 2. The corresponding background level for the vacuum beam is 2.8\%. There is also a background due to nuclear interactions at the regenerator \((z = 123\text{ m})\) and at the 2-mm-thick trigger hodoscope \((z = 138\text{ m})\). In the charged mode, the background in the vacuum beam (0.32\%) is dominated by \(K_{S,3}\) decays, whereas the background in the regenerated beam (0.13\%) comes from incoherent regeneration.

In order to extract \(\phi_+ - \phi_p\) and \(\phi_0 - \phi_p\) from the shapes of the vertex distributions in the regenerated beam, the acceptance as a function of decay vertex must be well understood. Accurate knowledge of the accep-
tance is essential for the determination of $\text{Re}(\epsilon'/\epsilon)$, and is discussed in detail in Ref. 6. The fit is done by comparing the background-subtracted and acceptance-corrected $z$ distributions with the predicted rate $I_{\epsilon_0}$ of Eq. (2), constrained by the total number of events in the vacuum beam. In the fit for $\phi_+ -$ and $\phi_{00}$, the kaon parameters $\tau_5$, $\tau_L$, and $\Delta m$ are fixed to their world-average values. The parameter $\text{Re}(\epsilon'/\epsilon)$, which is a measure of the difference in magnitude between $\eta_+ -$ and $\eta_{00}$, is allowed to float in the fit. The assumptions made for the regeneration amplitude are the following: (a) The difference in the forward-scattering amplitudes between $K^0$ and $\bar{K}^0$ has a power-law dependence on the kaon momentum; namely, $|\langle f - \bar{f} \rangle/k| \propto P_k^{-\alpha}$; (b) the phase of $\langle f - \bar{f} \rangle/k$ is given by the analyticity condition $\text{arg}(\langle f - \bar{f} \rangle/k) = -(2-a)/2$. The absorption factor $e^{-\chi}$ is measured to better than 1% of itself from $K_L \rightarrow \pi^+ \pi^- \pi^0$ and $K_L \rightarrow \pi^0 \pi^0 \pi^0$ decays in the two beams. This uncertainty has a negligible effect on $\Delta \phi$.

The result of the fit is $\phi_+_0 = 47.7^\circ \pm 2.0^\circ$, $\phi_{00} = 47.4^\circ \pm 1.4^\circ$, and $\Delta \phi = \phi_{00} - \phi_+ = -0.3^\circ \pm 2.4^\circ$. The errors are statistical and $\chi^2 = 316$ for 340 degrees of freedom. Figure 3 shows the quality of the fit to the data for both modes. As a check of our understanding of the acceptance, we have also fitted for $\tau_5$ and $\Delta m$, and we have obtained $\tau_5 = (0.8882 \pm 0.0030) \times 10^{-10}$ s (charged mode), $\tau_5 = (0.8929 \pm 0.0033) \times 10^{-10}$ s (neutral mode), $\Delta m = (0.5377 \pm 0.0098) \times 10^{-10}$ h s$^{-1}$ (both modes combined), where the errors are statistical only. These results are in good agreement with the corresponding world averages.

The systematic errors on $\Delta \phi$ come from various sources. When the parameters $\tau_5$ and $\Delta m$ are varied by 1 standard deviation around their world-average values, the value of $\Delta \phi$ changes by 0.2$^\circ$ for $\tau_5$ and 0.1$^\circ$ for $\Delta m$. Similarly the value of $\Delta \phi$ changes by $+0.8^\circ$ for a change of $+10^{-3}$ on $\text{Re}(\epsilon'/\epsilon)$. The acceptance was carefully studied using high-statistics modes ($10^7 K_{S3}$ events for charged mode and $6 \times 10^6 3\pi^0$ for neutral mode). The remaining uncertainty in the acceptance corresponds to an error in $\Delta \phi$ of 0.9$^\circ$. The uncertainty in the absolute energy calibration (0.1%) is determined from the sharp upstream edge of the decay distribution in the regenerated beam. The resulting error in $\Delta \phi$ is 0.6$^\circ$. The subtraction of the $3\pi^0$ and incoherent backgrounds in neutral mode contributes an error of 0.3$^\circ$. Accidental overlaps in the detector have a negligible effect on the result. The combined systematic error on $\Delta \phi$ is thus 1.2$^\circ$. Excluding the regeneration phase uncertainty, the systematic error on $\phi_+ -$ is 0.9$^\circ$ (dominated by acceptance uncertainty) and the systematic error on $\phi_{00}$ is 0.8$^\circ$ (dominated by absolute energy calibration).

The final result is then $\Delta \phi = \phi_{00} - \phi_+ = -0.3^\circ \pm 2.4^\circ$ (stat) $\pm 1.2^\circ$ (syst). This result is consistent with zero and thus with the prediction of $\text{CPT}$ symmetry. With the world average of $\phi_+ -$ our measurement of $\Delta \phi$ leads to a value for $\text{arg}(2\eta_+ + \eta_{00})$ of $44.5^\circ \pm 1.5^\circ$, which is well within 1 standard deviation of the natural angle $\phi_+ = 43.7^\circ$. Our measurement of $\phi_+ -$ is consistent with the world average. This supports the assumptions made for the regeneration amplitude in the fit. In conclusion, our measurement shows no indication of $\text{CPT}$ violation. Further confirmation is provided by a recent experiment which finds $\Delta \phi = 0.2^\circ \pm 2.9^\circ$.

The limit on $\Delta \phi$ also leads to a limit on the component of $\epsilon'$ perpendicular to $\epsilon$ using the relation $\text{Im}(\epsilon'/\epsilon) = -\Delta \phi/3$ which can be easily seen from Fig. 1. With the measured value of $\Delta \phi$, we obtain a 95%-confidence limit on $\text{Im}(\epsilon'/\epsilon)$ without any assumption of $\text{CPT}$ invariance: $-0.03 < \text{Im}(\epsilon'/\epsilon) < 0.03$.

![Figure 3](image-url)

**FIG. 3.** The vertex distributions in the regenerated beam after background subtractions and acceptance corrections for the (a) neutral and (b) charged modes. The solid curves are the result of the fit described in the text. The momentum cuts on the kaon energy are (a) [40,50] GeV and (b) [30,40] GeV.
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(a) Current address: Department of Physics, Loomis Laboratory of Physics, University of Illinois, 1110 West Green Street, Urbana, IL 61801.
(b) Current address: Princeton Combustion Laboratories, 4275 U.S. Highway 1, Monmouth Junction, NJ 08852.
(c) Current address: Fermi National Laboratory, Batavia, IL 60510.
(d) Current address: Stanford Linear Accelerator Center, P.O. Box 4349, Stanford, CA 94309.

4The relative phase of $K_0$ and $\bar{K}_0$ is chosen such that $\arg A_0 = \arg A_0$ (Wu-Yang phase convention).
10The discrepancy between $\arg (2\pi + \eta_\text{std})$ and $\phi$ is also about 2$\sigma$ if it is assumed that the $\Delta S = \Delta Q$ rule holds and that there is no anomalously large CP or CPT violations in $3\pi^0$ and semileptonic modes; Barmin et al., Ref. 3.
12In the determination of $\text{Re}(e'/e)$ in Ref. 6, the phase difference was fixed at zero. The value obtained for $\text{Re}(e'/e)$ in this analysis is consistent with that given in Ref. 6.
14The regeneration amplitude $p$ is related to $(f - f)/(k) = \text{Re}(e'/e)$ in $\Delta S = \Delta Q$ by $p = \pi N L g (f - f)/(k)$, where $N$ is the density of scatterers, $L$ is the length of the regenerator, and $g = (1 - e^{-x})/x$ is a geometrical factor with $x = (1 - i\delta m T_3)/\Lambda_3$ with $\Lambda_3$ the $K_3$ decay length. For a thin regenerator $g$ is unity, and $|g - 1|$ is always less than 0.2 in our case.