Coupling Constant Invariants in $\beta$-Decay

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Summary. — The principle recently used by Pauli (1) and Pursey (2) is extended and applied to single $\beta$-decay without lepton conservation, and to $\beta$-decay with non-vanishing neutrino mass. Identities are found between the coupling constant invariants for Pauli's (1) general interaction, and it is shown that these involve at most 35 independent real constants. Conditions are given for invariance under space inversion, time reversal, particle-antiparticle conjugation, and for four interesting special theories, including lepton conservation and the two-state neutrino theory. A brief discussion is given of how experimental knowledge of the invariants, in particular those occurring in single $\beta$-decay, can be used to check the various special cases.

1. — Introduction.

Recently, Pauli (1) and independently Pursey (2) used a little-known principle to show that $\beta$-decay processes can depend on the coupling constants only through certain combinations which are invariant under a transformation group generated by a group of unitary transformations of the theory. In this paper we are concerned primarily with the properties of these invariants, and what can be deduced from measurements of them, but we also find it necessary to state a more general form of the principle in order to be able to handle single $\beta$-decay in the case when leptons are not conserved. We also give a new application of the principle to derive some of the general features of Enz' (3) results on $\beta$-decay with non-zero neutrino mass.

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(1) W. Pauli: Nuovo Cimento, 6, 204 (1957).
We feel it may be helpful to say that in our opinion Sect. 2, 4 and 5 are of primarily theoretical interest while Sect. 3, 6 and 7 may be of interest also for the interpretation of experimental results.

2. - The fundamental principle.

Pauli's statement of the fundamental principle is as follows:

The S-matrix elements of any process are invariant (up to a phase) under any unitary transformation of the theory which leaves the initial and final states unaltered.

By leaving the initial and final states unaltered, we mean that the unitary operator of the transformation commutes with all the operators whose eigenvalues define the states; the state vectors may be multiplied by phase factors. If the relative phase of the initial and final state vectors is unaltered, the S-matrix element is strictly invariant, that is it is not even multiplied by a phase factor.

The principle as stated is adequate for the published applications, but is not sufficient for the discussion of single β-decay in the case when leptons are not conserved, for no final state of the system is invariant under the complete group of transformations used by Pauli. The difficulty is overcome by using the following generalization of the principle:

The average over given sets of initial and final states of the squared modulus of the S-matrix element for any process is invariant under any unitary transformation which leaves the sets of initial and final states invariant.

The wider applicability of this version is gained at the expense of the connection between the phase change of the S-matrix element and the change in relative phase of the initial and final states.

It is still possible to imagine situations in which applications of the principle similar to those already made seem impossible. For example, in single β-decay the principle would appear to be inapplicable if the plane of polarization of the neutrino were detected. However, the term plane of polarization of the neutrino has no observational meaning until an experiment has been described to measure it. This measurement must work through some neutrino interaction whose coupling constants will also be changed by the unitary transformations considered. The whole experiment, including both production and detection of the neutrino, is defined in terms of initial and final states or sets of states which are unaltered by the unitary transformations, and the predicted results depend on the coupling constants only through invariants built from those of both the production and the detection interactions. The non-invariant neutrino states are intermediate, are not directly observed, and are summed over in the course of calculation.
A similar argument applies to the problem of observing whether the emitted light neutral particle is a neutrino or an antineutrino if leptons are not conserved. This case has been discussed in more detail by Enz (3).

These remarks imply that the possibility of a natural and unambiguous experimental definition of, for example, the plane of polarization of the neutrino depends on special properties of the neutrino interactions. The same is true for the experimental definition of plane of polarization of electromagnetic radiation, or equivalently for the experimental distinction between $E$ and $H$. Suppose all particles carry both a normal electric charge and a magnetic (monopole) charge. If the ratio of magnetic to electric charges is the same for all particles, it is possible to redefine the fields and charges by a canonical transformation so that all the redefined magnetic charges vanish. In this case, we have the normal theory, with a natural definition of $E$ and $H$ which is experimentally unambiguous. However if the ratio of magnetic to electric charge is different for different particles, this is no longer so. An attempt to define $E$ and $H$ in the normal way will give results which depend on the test particle, and will be different for different particles.

3. - Definition of invariants.

This section is primarily concerned with explaining our notation, which we try to keep as close to established usage as possible.

The coupling constants are defined by specifying the interaction hamiltonian density to be

$$H_{\text{int}} = \sum_j \bar{\psi}_p O_j \psi_p \bar{O}_j \left[ (C_j + C_j') \psi_p + (D_j + D_j') \gamma_5 \psi_p \right] + \text{h. c.} \quad (1)$$

Here $j$ ranges over the usual five interaction terms S, V, T, A and P; where we do not wish to be specific we may also distinguish different interactions by arabic numerals 1, ..., 5. The adjoint $\bar{\psi}$ and «charge» conjugate $\psi^c$ of a field $\psi$ are as defined by Pauli (4). The coupling constants $C_j$ and $C'_j$ are the same as those used by Lee and Yang (5) and most other authors; they differ from those used by Pauli (1). The coupling constants $D_j$ and $D'_j$ are defined in the way most convenient for the present paper, and also differ from the corresponding ones defined by Pauli (1). The relationship between Pauli's (1)

constants \( g_{1,s}, f_{1,s}, g_{11,s}, f_{11,s} \) and ours is as follows:

\[
\begin{align*}
g_{1,s} &= C^s_j, \\
f_{1,s} &= C^s_j, \\
g_{11,s} &= -D^s_j, \\
f_{11,s} &= D^s_j.
\end{align*}
\]

(2)

It is convenient to define two sets of two-component column matrices \( \xi_j \) and \( \eta_j \) by

\[
\xi_j = \begin{pmatrix} C_j + C^*_j \\ D_j + D^*_j \end{pmatrix}, \quad \eta_j = \begin{pmatrix} C_j - C^*_j \\ D_j - D^*_j \end{pmatrix}.
\]

(3)

We shall use \( \zeta \) or \( \bar{\zeta} \) when we do not wish to distinguish between a \( \xi \) and a \( \eta \). The usual Pauli spin matrices, operating in the space of the vectors \( \zeta \), will be denoted by \( \zeta_1, \zeta_2, \zeta_3 \) (collectively, \( \rho \)). By \( \zeta^+, \bar{\zeta} \) and \( \bar{\zeta}^+ \) we shall mean respectively the row matrix hermitian conjugate to \( \zeta \), the row matrix \( \zeta^T \rho \) where \( \zeta^T \) is the transpose of \( \zeta \), and the column matrix \( \rho^* \zeta^\dagger \) hermitian conjugate to \( \bar{\zeta} \).

For the invariance group we shall generally supplement Pauli's (1) four parameter unitary transformation of the neutrino field by a phase change of the electron field. This generates the coupling constant transformations

\[
\xi_j \rightarrow \xi'_j = \exp \left[ i(\varphi + \theta) \right] A \xi_j; \quad \eta_j \rightarrow \eta'_j = \exp \left[ i(\varphi - \theta) \right] A \eta_j;
\]

(4)

where \( \theta \) and \( \varphi \) are independent and \( A \) is a general \( 2 \times 2 \) unimodular unitary matrix. Pauli's transformation is given by \( \varphi = 0 \).

Apart from the phase factors, the transformations of the \( \zeta_j \) are identical with those of the spinor representation of the three-dimensional rotation group (4). It follows that the only quantities which are invariant up to a phase are products of factors such as \( \zeta_j^* \zeta_k \) and \( \bar{\zeta}_j \bar{\zeta}_k \).

All quantities which are completely invariant under the transformations (4) must be products of factors of the four basic types

\[
\xi_j^* \xi_k, \quad \eta_j^* \eta_k, \quad \xi_j^+ \eta_k^+ \xi_m, \quad \xi_j^+ \eta_k^+ \bar{\eta}_l \bar{\xi}_m.
\]

(5)

These are not all independent: it is easy to show (most elegantly by using Fierz transformations for two-component spinors) that

\[
(\xi_j^+ \eta_k^+ \bar{\eta}_l \bar{\xi}_m) + (\xi_j^+ \eta_k \xi_m^+ \bar{\xi}_l) = (\xi_j^+ \xi_m^+ \eta_l \eta_k) .
\]

(6)

Further identities between the invariants (5) are obtained in Sect. 5.

If the group is restricted to Pauli's four parameter group, then quantities such as \( \bar{\eta}_l \xi_k \) are also invariant.

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(4) This remark is due to Professor Pauli.
For the purposes of comparison with experiment, it is convenient to introduce the notations

\[
K_{jk} = K_{jk}^* = \frac{1}{2}(\xi_k^+ \xi_j + \eta_k^+ \eta_j) = C_j^* C_k^* + C'_j C'_k^* + D'_j D'_k^* + D_j^* D_k^*,
\]

\[
L_{jk} = L_{jk}^* = \frac{1}{2}(\xi_k^+ \xi_j - \eta_k^+ \eta_j) = C'_j C_k^* + C_j C'_k^* + D'_j D_k^* + D_j D_k^*,
\]

\[
I_{jk}^* = I_{jk}^* = \frac{1}{2}i(\bar{\xi}_k \eta_j - \bar{\eta}_j \xi_k) = -C_j D_k + C'_j D_k + D_j C_k - D'_j C_k,
\]

\[
- J_{jk}^* = J_{jk}^* = \frac{1}{2}i(\bar{\xi}_k \bar{\eta}_j + \bar{\eta}_j \bar{\xi}_k) = -C_j D_k + C'_j D_k + D_j C_k - D'_j C_k.
\]

The invariants \(K_{jk}\) and \(L_{jk}\) and the quantities \(I_{jk}\) and \(J_{jk}\) (which are invariant if \(\varphi = 0\) in (4)) are identical with the quantities defined by Pauli (1).

\(K_{jk}\) and \(L_{jk}\) are the direct extension to the case of non-conservation of leptons of the quantities \(x_{jk}\) and \(\beta_{jk}\) introduced by Curtis and Lewis (7). From the general form of the fundamental principle it follows at once that any calculation for single \(\beta\)-decay with lepton conservation is extended to the case of lepton non-conservation by making the substitutions \(x_{jk} \rightarrow K_{jk}\), \(\beta_{jk} \rightarrow L_{jk}\).

4. - \(\beta\)-decay with non-zero neutrino mass.

If the neutrino mass is non-zero, the free neutrino hamiltonian density is not invariant under the unitary transformations which generate (4). If, however, we adopt the modified mass term

\[
\frac{1}{2}(\bar{\psi}_r, \psi, C \gamma_b) \rho^T (m + m') (\gamma_b \psi_r^c),
\]

then Pauli's unitary transformations generate linear transformations of the six mass parameters \(m\) and \(m'\). In (9), \(\rho\) is again the Pauli spin-matrix vector, now operating on the row and column matrices \((\bar{\psi}_r, \psi, C \gamma_b)\) and \((\gamma_b \psi_r^c, \gamma_b \psi_r^c)\); and \(m\) and \(m'\) are two vectors in the same three-dimensional space as the vector \(\rho\).

As the notation suggests, the transformations of \(m\) and \(m'\) generated by the unitary transformations are given by

\[
\begin{align*}
    m + m' &\rightarrow \exp [2i\theta] R(m + m'), \\
    m - m' &\rightarrow \exp [-2i\theta] R(m - m'),
\end{align*}
\]

where $\theta$ is as in (4), and $R$ is that rotation operator operating on the vectors $m$ and $m'$ which is represented by the matrix $A$ in (4) operating on the spinors $\zeta$.

For hermiticity, $m$ must be real and $m'$ pure imaginary. If, in addition,

$$m \wedge m' = 0,$$

then it is possible to find a transformation (10) which makes all the mass parameters vanish except $m_3$; it is easily seen that (9) is then just the normal mass term for a particle of mass $m_3$.

The only invariants which can be built from the mass parameters alone are $m^2 - m'^2$ and $(m + m')(m - m')^2$. If (11) is satisfied then the second of these reduces to the square of the first.

For single $\beta$-decay, the general form of the fundamental principle shows that observable effects can depend only on invariants built from the coupling constants together with the neutrino mass parameters. The terms independent of the neutrino mass can depend only on $K_{jk}$ and $L_{jk}$ defined by (7). Terms linear in the mass can depend only on $(m + m') \cdot (\eta^+_j \rho \eta_k)$ and $(m - m') \cdot (\eta^+_j \rho \xi_k)$. For the standard representation in which the only non-vanishing mass parameter is $m_3 = m_\nu$, the only possible coefficients of the neutrino mass are $(\xi^+_j \rho \eta_k)$ and $(\eta^+_j \rho \xi_k)$. Half the sum of these gives the coefficient, found by Enz (4),

$$K'_{jk} = C^*_j C_k - C^*_k C_j - D^*_j D_k + D^*_k D_j,$$

while half the difference gives

$$L'_{jk} = C^*_j C_k - C^*_k C_j - D^*_j D_k + D^*_k D_j,$$

which can be expected to occur for processes which do not conserve parity.

For double $\beta$-decay without neutrinos, Pauli's form of the principle shows that terms in the $S$-matrix element linear in $m_\nu$ must be proportional to $(\eta^+_j \rho \eta_k)$ or $(\eta^+_j \rho \xi_k)$. (This is a case where the initial and final state vectors are strictly invariant, so the $S$-matrix element cannot suffer any phase change).

Suitable combinations of these invariants give the coefficients

$$I'_{jk} = C_j D_k + C^*_j D^*_k + D_j C_k + D^*_j C^*_k,$$

$$J'_{jk} = C_j D_k + C^*_j D^*_k + D_j C_k + D^*_j C^*_k.$$

$I'_{jk}$ and $J'_{jk}$ are just the coefficients found by Enz (4).

So long as we do not want to work to any higher order in perturbation theory, any additional powers of the neutrino mass must enter only through the invariant $m^2 - m'^2 = m_\nu$.

Henceforth we shall deal only with the case of zero neutrino mass.
5. Identities satisfied by the invariants.

Since the interaction involves 40 real coupling constants, and the observable combinations of them are invariant under a 5 parameter group, we expect the coupling constant invariants to involve at most 35 independent real constants. Obviously, many identities must exist among the invariants (4), and in (8) we have already found one set.

An obvious further set is

\begin{equation}
\begin{pmatrix}
(\xi_j^+ \eta_k^+ \xi_m^+ \eta_q^+ \xi_n^+ \eta_r^+ \xi_p) \\
(\xi_j^+ \eta_k^+ \xi_m^+ \eta_q^+ \xi_n^+ \eta_r^+ \xi_p)
\end{pmatrix} = (\xi_j^+ \eta_k^+ \xi_m^+ \eta_q^+ \xi_n^+ \eta_r^+ \xi_p),
\end{equation}

obtained by permuting scalar factors \( \eta^+ \xi \) and \( \tilde{\eta}^+ \xi \) between invariants.

A third set may be found by noting that at most two of the \( \zeta \)'s can be linearly independent. It follows at once that

\begin{equation}
\begin{vmatrix}
\zeta_j^+ \eta_m^+ \\
\zeta_k^+ \eta_n^+ \\
\zeta_l^+ \eta_p^+
\end{vmatrix} = 0.
\end{equation}

If we make suitable choices for the \( \zeta \)'s, expand the determinant, and use (6), we find

\begin{equation}
\begin{pmatrix}
(\xi_j^+ \eta_k^+ \xi_m^+ \eta_q^+ \xi_n^+ \eta_r^+ \xi_p) \\
(\xi_j^+ \eta_k^+ \xi_m^+ \eta_q^+ \xi_n^+ \eta_r^+ \xi_p)
\end{pmatrix} = (\xi_j^+ \eta_k^+ \xi_m^+ \eta_q^+ \xi_n^+ \eta_r^+ \xi_p) + (\xi_j^+ \eta_k^+ \xi_m^+ \eta_q^+ \xi_n^+ \eta_r^+ \xi_p) - (\xi_j^+ \eta_k^+ \xi_m^+ \eta_q^+ \xi_n^+ \eta_r^+ \xi_p),
\end{equation}

and the analogous set

\begin{equation}
\begin{pmatrix}
(\xi_j^+ \eta_k^+ \xi_m^+ \eta_q^+ \xi_n^+ \eta_r^+ \xi_p) \\
(\xi_j^+ \eta_k^+ \xi_m^+ \eta_q^+ \xi_n^+ \eta_r^+ \xi_p)
\end{pmatrix} = (\xi_j^+ \eta_k^+ \xi_m^+ \eta_q^+ \xi_n^+ \eta_r^+ \xi_p) + (\xi_j^+ \eta_k^+ \xi_m^+ \eta_q^+ \xi_n^+ \eta_r^+ \xi_p) - (\xi_j^+ \eta_k^+ \xi_m^+ \eta_q^+ \xi_n^+ \eta_r^+ \xi_p),
\end{equation}

and

\begin{equation}
\begin{pmatrix}
(\xi_j^+ \eta_k^+ \xi_m^+ \eta_q^+ \xi_n^+ \eta_r^+ \xi_p) \\
(\xi_j^+ \eta_k^+ \xi_m^+ \eta_q^+ \xi_n^+ \eta_r^+ \xi_p)
\end{pmatrix} = (\xi_j^+ \eta_k^+ \xi_m^+ \eta_q^+ \xi_n^+ \eta_r^+ \xi_p) + (\xi_j^+ \eta_k^+ \xi_m^+ \eta_q^+ \xi_n^+ \eta_r^+ \xi_p) - (\xi_j^+ \eta_k^+ \xi_m^+ \eta_q^+ \xi_n^+ \eta_r^+ \xi_p).
We demonstrate that this now gives a complete set of identities by showing how to express all the invariants in terms of 35 real constants.

Suppose that $\xi_i^+, \eta_i^+$, $\xi_i^+, \eta_i$ and $\xi_i^+, \eta_i^+$, $\eta_i \xi_i$, are known and are all non-zero: these are all real, so that, by (6), they involve only three independent real constants. Now suppose $\xi_i^+ \eta_j^+$, $\eta_i^+ \eta_j^+$, $\xi_i^+ \eta_j^+ \eta_i \xi_j$ and $\xi_i^+ \eta_j^+ \eta_i^+ \xi_j$ are also known for $j$ running over the remaining four interactions: these involve 32 real constants, making 35 altogether. By use of (6) and (18) it is now possible to find $(\xi_j^+ \eta_i \xi_j^+ \eta_i)$ and $(\xi_j^+ \eta_i \eta_i^+ \eta_i)$, in the first place when two of the indices are 1, and then for arbitrary indices. Either (17) or (18) then gives $(\xi_j^+ \xi_j)$ and $(\eta_i \eta_i)$.

This is the least favourable case. If, for example, it is found that one of $\eta_i \eta_i \xi_1$, and $\xi_i \eta_i \xi_1$ is zero, then the above solution is not possible, but a more efficient one using at most 34 real constants (including the three $\xi_i^+ \xi_i$, $\eta_i^+ \eta_i$, $\xi_i^+ \eta_i \eta_i^+ \xi_i$) can be found.

It is obviously not practicable to check the identities (6), (15), (17) and (18) experimentally, but they may prove useful in deducing the values of invariants which it is impracticable to measure directly.

6. - Conditions for invariance and for other special cases.

The conditions for invariance under space inversion ($P$), time reversal ($T$) and particle-antiparticle conjugation ($C$) which were given by Pursey (2) must be amplified before they are valid for the interaction (1). The generalized conditions are

**$P$ invariance:**

\[ L_{jk} = 0 , \]

and either \[ I_{jk} = 0 \]

or \[ J_{jk} = 0 \]

(all $j, k$).

**$T$ invariance:**

\[ K_{jk}, L_{jk}, I_{jk}^*, I_{lm}^*, J_{jk}^*, J_{lm}^*, L_{jk}, J_{lm}^* \]

all real

(all $j, k, l, m$).

**$C$ invariance:**

\[ K_{jk}, L_{jk}, I_{lm}^*, J_{jk}, J_{lm}^*, L_{jk}, J_{lm}^* \]

all real,

all pure imaginary

(all $j, k, l, m$).

These conditions can be obtained by showing that if they are satisfied then (1) can be transformed by (5) into a form which is trivially invariant, and vice versa. For $P$ invariance, there are two inequivalent trivially invariant...
possibilities, namely $C_j = D'_j = 0$ (all $j$) and $C'_j = D_j = 0$ (all $j$), which cannot be transformed one into the other by (4). Only the second case is consistent with a parity conserving two-state neutrino theory.

We shall now consider the conditions for a number of interesting special cases.

**Case A.** This is defined by

$$
\tilde{\xi}_j \xi_k = \tilde{\eta}_j \eta_k = 0 , \quad (\text{all } j, k),
$$

which is equivalent to

$$
\left\{ \begin{array}{l}
K_{jk}K_{lm} + L_{jk}L_{lm} = K_{jm}K_{lk} + L_{jm}L_{lk}, \\
K_{jk}L_{lm} + L_{jk}K_{lm} = K_{jm}L_{lk} + L_{jm}K_{lk},
\end{array} \right. \quad (\text{all } j, k, l, m),
$$

where $K_{jk}$ and $L_{jk}$ are the invariants measured in single $\beta$-decay defined by (7) above. The importance of this theory is that there exist simple relations involving only those invariants occurring in single $\beta$-decay. A special case is when the $D$ and $D'$ coupling constants vanish.

**Case B.** In this case, the light neutral particle emitted in $\beta$-decay, whether it is a neutrino or antineutrino, is always circularly polarized in the same sense. The condition for this is

$$
\text{either } \xi_j = 0, \quad (\text{all } j), \quad \text{or } \eta_j = 0, \quad (\text{all } j),
$$

or equivalently

$$
\left\{ \begin{array}{l}
\text{either } L_{jk} = K_{jk}, \\
\text{or } L_{jk} = -K_{jk},
\end{array} \right. \quad (\text{all } j, k).
$$

This is a particular and rather exceptional case of a theory conserving leptons.

**Case C: Lepton conservation.** By lepton conservation we mean the impossibility of double $\beta$-decay without neutrinos. The condition is

$$
\tilde{\xi}_j \eta_k = 0 , \quad (\text{all } j, k),
$$

or

$$
I_{jk} = J_{jk} = 0 , \quad (\text{all } j, k).
$$

With the exception of the special Case B, lepton conservation also implies Case A.
Case D: Two state neutrino. In writing down the interaction (1) we have allowed for a four state neutrino theory. The most direct way of specializing to a two-state theory is to use the version due to Weyl (8) and other recent workers (9) rather than that of Majorana (10). The relationship between the two versions has been fully discussed by several authors (11).

The conditions for a two state theory are

\begin{equation}
\xi_j^* \eta_k = 0, \quad \xi_j^* \xi_k = \eta_j^* \eta_k = 0, \quad \text{(all } j, k)\end{equation}

or alternatively

\begin{equation}
\begin{align*}
I_{jk}^* I_{lm} &= \frac{1}{2} (K_{mk} K_{lj} - L_{mk} L_{lj} + L_{ik} K_{mj} - L_{ik} L_{mj}), \\
J_{jk}^* J_{lm} &= \frac{1}{2} (K_{mk} K_{lj} - L_{mk} L_{lj} - K_{ik} K_{mj} + L_{ik} L_{mj}), \\
I_{jk}^* J_{lm} &= \frac{1}{2} (K_{mk} L_{lj} - L_{mk} K_{lj} - K_{ik} L_{mj} + L_{ik} K_{mj}),
\end{align*}
\end{equation}

together with (20). This does not imply lepton conservation nor does it imply the polarized neutrino theory of Case B (1). Case A is always implied by a two-state theory. Except for a two-state theory which also satisfies B, the conditions (19) or (20) defining Case A are redundant in the definition of Case D.

Case A and B taken together imply both Cases C and D, and vice versa: this is the two-state theory envisaged by Landau, Lee, Yang and Salam (7). Cases A and C together imply the possibility of choosing a representation in which \( D = D' = 0, \) (all \( j \)).

7. – Conclusion.

In this Section, we shall try very briefly to relate our considerations to experimental results, especially on single \( \beta \)-decay, which we hope will in time become available.

The most interesting theoretical questions are whether leptons are conserved and whether a two-state theory is adequate. Direct checks of these involve difficult experiments on double \( \beta \)-decay or of the type performed by Davis (12). However, it is possible to get some indirect evidence from single \( \beta \)-decay alone, by testing the conditions (20) and (22) for the special Cases A and B. If both conditions are satisfied, then leptons are conserved and a two

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state theory is valid. If (22) is satisfied but not (20), then leptons are conserved but a two state theory is impossible. If (20) is satisfied but not (22), then further tests of lepton conservation and the two-state theory, involving the Davis type of experiment, will be necessary. If neither (20) nor (22) is satisfied, then leptons can not be conserved, and also the two-state theory is impossible.

If Case $A$ is true but not Case $B$, we are faced with the problem of deciding whether leptons are conserved, or whether a two state theory is valid. A complete check on these points appears impossibly difficult, but we notice that for a two state theory the result of any experiment on neutrino capture (of a type equivalent to a double $\beta$-decay experiment with no neutrinos) can be predicted from information obtained from single $\beta$-decay and used in (26). It may in time be practicable to check the total probability for a positive result in the Davis ($^{12}$) experiment in this way.

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**Note Added in proof.**

Since this paper was written, we have learnt of a paper by G. LüDERS (to be published in Nuovo Cimento) which covers similar ground. We wish to thank Dr. LüDERS for sending us a prepublication copy of his paper, and for a valuable correspondence.

RIASSUNTO (*)

Il principio recentemente applicato da PAULI (1) e PURSEY (2) si estende e si applica al singolo decadimento $\beta$ senza conservazione dei leptoni e al decadimento $\beta$ con massa del neutrino non tendente a zero. Si trovano identità tra gli invarianti della costante d'accompagnamento per l'interazione generale di Pauli, e si dimostra che detti invarianti contengono al massimo 35 costanti reali indipendenti. Si danno le condizioni per l'invarianza rispetto all'inversione dello spazio, del tempo, della coniugazione particella-antiparticella e per quattro interessanti teorie speciali, includenti la conservazione dei leptoni e la teoria del neutrino a due stati. Si discute brevemente come la conoscenza sperimentale degli invarianti, in particolare di quelli che si presentano nel singolo decadimento $\beta$, possa essere usata per controllare i casi speciali.

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