EXPERIMENT ON THE SIGN AND MAGNITUDE OF THE $K_L^0-K_S^0$ MASS DIFFERENCE*

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On the basis of a definite result from a Kobzarev-Okun' experiment on coherent regeneration, and an optical model calculation, we present evidence that the $K_L^0$ is heavier than the $K_S^0$.

We are reporting here the results of an experiment designed to study the interference between the coherently regenerated $K_S^0$ from two regenerators of dissimilar material. This technique, originally suggested by Kobzarev and Okun' (henceforth referred to as the K-O method), measures the relative sign between the $K_L^0-K_S^0$ mass difference, $\delta$, and the regeneration phase difference, $\Delta \varphi$, of the two types of nuclei. Our experiment gives a statistically conclusive result as to the relative sign of $\delta$ and $\Delta \varphi$. In order to determine the sign of $\Delta \varphi$ we have performed an optical-model (OM) calculation making certain reasonable assumptions about the real parts of the OM potentials, as they are not well known in the energy range of interest. The results of this calculation are such as to make the $K_L^0$ heavier than the $K_S^0$. The strength of the evidence that the $K_L^0$ is heavier depends exclusively on the validity of the above mentioned assumptions.

The principle of the K-O method is the following: A $K_L^0$ beam is passed through two absorbers, $a$ (upstream) and $b$ (downstream), $X_a$ and $X_b$ cm thick and separated by $X_0$ cm. The amplitude for $K^0-\pi^+\pi^-$ in the forward direction, on the downstream side of absorber $b$, is given by the sum of three terms: $A_a^0$ and $A_b$ due to the coherent regeneration of $K_S^0$, and $A_{cp}$ due to decay mode $K_L^0-\pi^+\pi^-$. The number of decays is then given by

$$N_1 = |A_a^0 + A_b + A_{cp}|^2$$

$$= |1 - \exp(i\nu x_a^0)\exp(\nu(x_a^0 + x_b^0)) + \Re i\Delta \varphi[1 - \exp(i\nu x_b^0)] + (e/S)\exp[i(\eta - \varphi_a)]|^2, \quad (1)$$

where $\nu = \i \delta - \frac{1}{2}$, $\delta = K_L^0-K_S^0$ mass difference in units $h/\tau_S$ ($\tau_S = 0.91 \times 10^{-10}$ sec), $S = -i\Delta$ $\times N_a f_{21}(a)/\nu$, $\lambda = h/p$, $\Lambda = e\tau_S p/mc$ is the decay mean free path of $K_S^0$, $x_i = X_i/\Lambda$, $f_{21}(i(0)) = i f_{21}(i(0))\exp(\i \varphi_i)$ is the amplitude for regeneration by a single nucleus of element $i$, $R = N_b x_{21} f_{21}(b)/N_a f_{21}(a)$, $\Delta \varphi = \varphi_b - \varphi_a$, and $N_i$ is the atomic density.

The experiment was performed parasitically in a 30° neutral beam at the Brookhaven alternating gradient synchrotron (AGS) using a counter controlled spark chamber. A total of 120,000 spark-chamber photographs were taken using absorbers of carbon, 10.2 cm thick, and uranium, 6.25 cm thick, placed one behind the other and separated by a variable distance $X_0$. About two-thirds of these photographs, on which the reported results are based, were analyzed in a manner as described previously. The $K_L^0$ beam had a rather wide momen-
tum range; consequently, Eq. (1) was integrated over the momentum spectrum using a procedure similar to, but more approximate than, the one described earlier. The observed numbers of transmission-regenerated $K_S^0$ are given at various spacings $X_0$ in Fig. 1 together with some computed theoretical curves.

If the decay mode $K_L^0 \rightarrow \pi^+ + \pi^-$ is ignored, which is justified in this case because the ratio $\epsilon/S$ is relatively small, then there are three important parameters in Eq. (1), namely $\delta$, $R$, and $\Delta \varphi$. These parameters either have never been measured (like $\Delta \varphi$), or have been measured with an accuracy (like $\delta$ and $R$) which is not high enough to justify the neglect of the existing uncertainties. Therefore, we considered it necessary at first to treat all three parameters as free and determine them by making a least-squares fit. In this way we obtained for the best values $|\delta| = 0.35 \pm 0.15$, $R = 0.143 \pm 0.015$, $|\Delta \varphi| = 0.70 \pm 0.25$, and that $\delta$ and $\Delta \varphi$ have opposite signs. The errors quoted here are statistical only. Several causes of systematic errors were considered in detail, and because of their presence, the errors stated above should be augmented. Accordingly, our best estimates become $|\delta| = 0.35 \pm 0.15$, $|\Delta \varphi| = 0.70 \pm 0.25$, and $R = 0.143 \pm 0.015$. These results are consistent with independent measurements of $\delta$ and $R$ and our OM calculation discussed below. Our previously published measurement for $\delta$ using an iron regenerator has been corrected and is now $|\delta| = 0.72 \pm 0.15$. Two measurements of $\delta$ which were performed differ by almost twice their combined standard deviation. The relatively large difference might be due to some undetected systematic errors or simply a statistical fluctuation. The weighted average from both measurements is $|\delta| = 0.53 \pm 0.11$, which is in good agreement with a combined world average of $\delta = 0.60 \pm 0.06$.

Combining the above results with independent measurements of $|\delta|$ and $R$ and with the OM calculation described below, it seems reasonable to restrict (with an estimated 99% confidence level) the ranges of $|\delta|$ to $0.4-0.7$, $R$ to $0.12-0.16$, and $|\Delta \varphi|$ to $0.25-1.0$. In Table I are given the results of a $\chi^2$ analysis for several sets of $\delta$, $R$, and $\Delta \varphi$ so chosen as to allow one to make a crude interpolation within the ranges considered. The first line corresponds to the point where $\chi^2$ is at the minimum. From the last two columns of Table I, it can easily be seen that the betting odds (likelihood ratios) of $\delta$ and $\Delta \varphi$ having the same sign against their having the opposite sign are always smaller than $10^{-2}:1$ if $\delta$, $R$, and $\Delta \varphi$ are within the above defined ranges. A detailed investigation of systematic errors showed that the betting odds change only insignificantly when these errors are taken into account.

We have investigated the effects of the decay mode $K_L^0 \rightarrow \pi^+ + \pi^-$ and have found that they are small. The correction to $|\delta|$ is always $\lesssim 0.04$; to $\Delta \varphi$, $\lesssim 0.1$; and to $R$, $\lesssim 0.015$. The conclusion on the relative sign of $\delta$ and $\Delta \varphi$ is

![Figure 1](image)

**Figure 1.** Computed regeneration curves and the experimental points for the case of performed K-O experiment using a combination of uranium and carbon absorbers. Each pair of curves was calculated ($A_{2\pi}$ neglected) by making a least-squares fit to all 11 points simultaneously, i.e., with the same normalizing constant. The values of parameters used and $\chi^2$ obtained for each pair of curves is shown.

| $|\delta|$ | $|\Delta \varphi|$ | $R$ | $\chi^2_{-}$ | $\chi^2_{+}$ |
|------|------|------|------|------|
| 0.35 | 0.7  | 0.143| 6.1  | 78.6 |
| 0.55 | 0.5  | 0.14 | 13.0 | 137  |
| 0.40 | 0.25 | 0.12 | 35.8 | 90   |
| 0.40 | 0.25 | 0.12 | 9.2  | 37.6 |
| 0.40 | 1.0  | 0.12 | 12.4 | 216  |
| 0.40 | 1.0  | 0.16 | 25.3 | 150  |
| 0.70 | 0.25 | 0.12 | 75   | 163  |
| 0.70 | 0.25 | 0.16 | 23.8 | 98.6 |
| 0.70 | 1.0  | 0.12 | 35.4 | 302  |
| 0.70 | 1.0  | 0.16 | 48.4 | 271  |
unchanged, since the betting odds again change only insignificantly.

Because no experimental information on the sign of $\Delta \varphi$ exists in the momentum region of interest to us, we tried to compute it by performing an OM calculation. In this calculation the nuclear density function in uranium was assumed to be given by a Fermi distribution with a nuclear radius of 1.15A$^{1/3}$ F and a fall-off parameter of 0.57 F. In carbon the nucleon density distribution was assumed to be the same as the nucleon charge density distribution determined by the electron scattering measurements. The imaginary parts, $W$ and $\bar{W}$, of the $K^0$ and $\bar{K}^0$ potentials were computed from the experimentally determined values of the total $K$-nucleon cross sections. Little is known about $V$ and $\bar{V}$, the real parts of the OM potentials, in this energy range; nevertheless, it is reasonable to assume that $|V/W| < 1$ and $|\bar{V}/\bar{W}| < 1$. The second assumption is justified, since the $\bar{K}^0$-nucleon interaction is strongly absorptive in the 1- to 2-BeV/c region. Under these assumptions the OM calculation shows that $0 < |\Delta \varphi| < 0.35$ and $0.125 < R < 0.145$, which is in agreement with our measurements; also it shows that the sign of $\Delta \varphi$ depends crucially on whether the $K^0$ (or $K^+$, if we assume charge independence) potential is attractive\(^7\) or repulsive. It has been shown that at 515 and 700 MeV/c the $V/W$ ratio is +1.43 and +0.77, respectively. From the measurements of $K^+p$ elastic cross section, it follows (if it is assumed that $K^+p$ behavior is the same as $K^+\bar{p}$) that in the momentum range between 920 and 1970 MeV/c, the absolute value of the ratio $V/W$ is fairly constant and consistent with the value $|V/W| = 0.44 \pm 0.20$.\(^8\) Our own investigation of the energy dependence of $\Delta \varphi$ is also consistent with the above statement, which is tantamount to saying that $\Delta \varphi$ does not change sign in the momentum interval of interest.

If one more reasonable assumption is made, viz. that $V$ does not change sign in the range between 700 and 900 MeV/c, then it follows that $V/W$ is positive, thereby implying that the uranium-carbon phase difference is negative.

Combining then the conclusive result of our K-O experiment with a negative value for $\Delta \varphi$, as obtained on the basis of the above assumptions, it follows that the $K_L^+$ is heavier than the $K_S^0$. This result is in agreement with three other experiments\(^9\) which have also reported evidence for a heavier $K_L^0$.

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\(^7\)U. Camerini, D. Cline, J. B. English, W. Fischbein, W. F. Fry, J. A. Galdos, R. L. Huntman, R. H. March, and R. Stark, Phys. Rev. 150, 1148 (1966). When the results listed in that paper are combined with those presented here and those given by M. Bott-Bodenhagen et al., Phys. Letters 20, 212 (1966), a combined world average becomes $16l=0.54 \pm 0.04$.

\(^8\)We measured the ratio of regeneration amplitudes in an auxiliary experiment by comparing numbers of transmission-regenerated $K_S^0$ produced in the absorbers of carbon and uranium each 12.6 cm thick. We quoted previously a preliminary result, $R=0.152 \pm 0.010$, which was based on about one-fourth of data taken.

\(^9\)The systematic errors considered in detail were (a) the separation of transmission-regenerated $K_S^0$ from background, (b) the effects of changes in the assumed shapes for the transmission peak, (c) the statistical effects of experimental points with small numbers of counts, (d) the effects of the approximate integration procedure over the incident $K_L^0$ momentum, and (e) the possible momentum dependence of $R$ and $\Delta \varphi$ investigated by dividing data into three momentum intervals and performing the least-squares fits independent-
We have made some additional corrections to the previously published number (see Ref. 3) for the mass difference which was $|\delta| = 0.82 \pm 0.12$. The first correction actually influences the preliminary value for the nuclear mean free path $\mu$ measured in a separate attenuation experiment. A systematic effect was taken into account and more data were analyzed. The average value for $\mu$ changed from the reported $14.1 \pm 1.3$ to $13.0 \pm 1.0$ cm. The second correction had to do with a detailed geometry of the regenerator. Applying both corrections together we obtain from the same data that $|\delta| = 0.72 \pm 0.15$. The increased error appears as a consequence of strongly correlated errors between $\delta$ and $\mu$.


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Erratum


Results similar to ours have been previously obtained by K. Itabashi, Phys. Rev. 136, B221 (1964); D. Ballin, Nuovo Cimento 38, 1342 (1965); and S. P. Rosen, S. Pakvasa, and E. C. G. Sudarshan, Phys. Rev. 146, 1118 (1966). The SU(3) techniques used by these authors are quite different from those used by us, and they employ an effective Lagrangian method rather than considering the physical decay amplitude as we have done. This perhaps obscures the kinematical symmetry-breaking effect mentioned in our Letter. We should also like to emphasize that our result follows only if the vector and axial-vector Cabibbo angles are equal. We should like to thank S. P. Rosen, H. Schnitzer, G. Guralnik, and S. Pakvasa for bringing this previous work to our attention.