Moving Mirror Effects in Hadronic Reactions

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We present a theory of thermal distribution of hadrons on the basis of the moving mirror effects. A constant acceleration of walls of the "bag" due to the linear potential between quarks is responsible for the thermal gluon production. Applications to the $e^+e^-$ reaction are discussed.

§ 1. Introduction

Many particle physicists agree that a hadron is a spatially extended object ("bag") and a few quarks are within there. At relatively low energies, deformation, breaking, rejoining etc. of the bag accompanied by rearrangements of the quarks are predominant features of hadronic reactions. On the other hand, in the deep region, the interaction between quarks will be dominant. It is widely believed that the quark-quark interaction is mediated by the non-Abelian gauge fields. The quarks and the colored gauge fields are presumably confined within the bag.

In this paper, we accept such a picture of strong interactions and investigate the effects of the confinement by the bag. More precisely, we will study the effects of moving walls which imprison both the quarks and gluons.

Our conclusion is rather striking: The walls radiate gluons, which presumably transform into hadrons, according to the thermal law (Planck's distribution law) with the temperature \( T \approx 130 \, \text{MeV} \). This accurately explains the fireball aspect of multiple productions from the viewpoint of the confined quark picture.

Let us explain how is introduced such a thermodynamical concept like a temperature in the bag model of hadrons. We may idealize the walls of the bag as perfectly reflecting mirrors to the quarks and the gluons, which are almost free within the bag. Of course this may be an oversimplification. However our conclusion stated above will not change very much by possible elaborations to the simple-minded bag picture.

For definiteness, consider a quark-antiquark system within the bag. A pair of \( q \) and \( \bar{q} \) moves in opposite directions. In this situation one can imagine the wall as a cylinder with two lids, which are pushed by the quark outwards (Fig. 1). As is phenomenologically well known, the effective potential between quark and anti-quark is linear in distance when they are far apart. For simplicity we will employ the formula for the potential which gives a universal linear rising...
Regge trajectory:

\[ V(t) = \frac{1}{4\alpha'} t, \]  

(1.1)

where \( 2l \) is the distance between the \( q\bar{q} \) pair, and \( \alpha' \) is the universal Regge slope\(^{60,61} \) (\( \approx 1 \text{ GeV}^{-2} \)) (see Fig. 1). The acceleration of the quark (antiquark) due to the linear potential (1.1) is readily obtained as

\[ a = \frac{1}{m_q} \frac{\partial V}{\partial l} = \frac{1}{4m_q\alpha'}. \]  

(1.2)

which is independent of the distance \( l \). We can consider the acceleration \( a \) as that of each of lid which our energetic quark is pushing. Actually the lids decelerate our quarks. The world lines of the pair of lids are drawn in Fig. 2.

Now let us turn to the temperature and call attention to the beautiful results by the general relativists\(^{26} \) on the particle creations in curved space times. The thermal radiations seen by an accelerated observer, or due to an accelerated mirror have been studied by Polling,\(^{62} \) Davies\(^{63} \) and Unruh\(^{64} \) and are closely related to the Hawking effect\(^{65} \), the thermal radiation from black holes. They say that the constant acceleration, the origin of which we need not worry about, implies the thermal radiation with the effective temperature:

\[ kT = \frac{\hbar a}{2\pi c}. \]  

(1.3)

with \( k \) being the Boltzmann constant. In Eq. (1.3), we restore the dimension by explicitly writing the Planck constant and the light velocity to make sure that the temperature-acceleration connection is a quantum effect. We will show Eq. (1.3) explicitly in later in the case of \( q\bar{q} \) system which we are mainly
concerned. In our case what the moving lids radiate are gluons and quarks.

Although the temperature corresponding to the acceleration in everyday life is extremely low (10^{-6} K for 980 cm/sec), hadronic reactions give a considerably high temperature. We estimate

\[ E_T = 130 \text{ MeV} \quad (1.4) \]

with \( \alpha' = 1 \text{ GeV}^{-2} \) and \( m_s = 300 \text{ MeV} \). The obtained numerical value reminds us of the famous Hagedorn temperature\(^\text{11} \) and leads us to a natural speculation that all the seemingly mysterious thermodynamical features of hadronic reactions (e.g., \( p_T \) distribution\(^\text{12} \) in the low energy region) come from the moving mirror effects. From this point of view, the universality of the Hagedorn temperature is nothing more than that of the Regge slope or the universal linear potential.

A few people\(^\text{10} \) including the author\(^\text{10} \) already speculated the possibility of the temperature-acceleration connection in hadronic reactions. In this paper we will develop such an idea at the fully quantitative level.

The present paper is organized as follows: In \( \S 2 \) we derive the thermal distribution for massless scalar particles, which mimics gluon distributions in the situation realized by the \( e^+e^- \) collision. In \( \S 3 \) implications of the particle production are examined in the \( e^+e^- \) reaction. The last section \( \S 4 \) is devoted to summary and discussion.

\section{2. Moving mirror effect}

In this section we derive the thermal distribution for a massless scalar particle which is confined by a pair of oppositely accelerated walls. The final result will not depend whether the particle under consideration is scalar or vector, so that we will study a scalar field for simplicity. The fermion case will be briefly discussed in \( \S 4 \). At the moment we ignore the transverse motion of the scalar particle and concentrate our attention on the longitudinal momentum distribution. The transverse momentum dependence of the particle distribution will be presented in the next section.

As far as the author knows, there has been no literature which explicitly calculated the thermal distribution in the situation we are interested in. In addition, the moving mirror effect is not too familiar to particle physicists. It seems worthwhile to derive the thermal distribution somewhat in detail.

\subsection{2.1. Kinematics}

We consider trajectories of a pair of lids (see Fig. 2):

\[ x_1 = x(t) = -\sqrt{t^2 + d^2} + x_0 \quad (x_0 > d) \]

\[ x_2 = - \bar{x}(t) \quad (2.1) \]

where \( d \) and \( x_0 \) are constants whose physical meanings are shortly explained. At

\[ t = -\sqrt{x_2^2 - d^2} \]

in the center-of-momentum:

until they collide.

Here we have used Eq. (2.2).

\subsection{2.2. Field equation}

Our massless scalar gauge field

\[ \phi \]

where we have imposed the condition at \( x = \pm \infty \).

The scalar field

\[ \phi \]

with the metric

\[ ds^2 = -dt^2 + \frac{dx_1^2 + dx_2^2}{t^2} \]
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\[ t = -\sqrt{x^2 - \beta^2}, \]

the \( q\bar{q} \) pair and thus the lids, which presumably run just ahead of the quarks, are created, say, by the \( e^+e^- \) annihilation. The initial energy of the quark (anti-quark) is given by

\[ E = \frac{x m_v}{b}, \]

in the centre-of-mass frame. Then the lids separate apart with a constant acceleration:

\[ a = \frac{d}{dt} \frac{v}{\sqrt{1 - v^2}} = \frac{1}{b^2}, \]

until they collide to each other at \( t = \sqrt{x^2 - \beta^2} \). The maximal stretch of the tube in Fig. 2 is \( 2x (x - b) \) at \( t = 0 \), which linearly grows as the initial energy \( E \) increases. The parameters of the trajectory (Eq. (2.1)) can be expressed by the universal Rogge slope \( a' \), the quark mass \( m_v \) and the initial energy of the quark:

\[ b = \frac{1}{a} = 4a' m_v, \]

\[ x_0 E = \frac{E}{m_v} = 4a' E. \]

Here we have used the arguments presented in the Introduction which leads to Eq. (2.2).

2.2 Field equation and boundary condition

Our massless scalar field \( \phi \) obeys the field equation:

\[ (\partial^2 - \beta \partial_x) \phi (x, t) = 0, \]

where we have ignored the other two space variables. We impose the boundary condition at \( x = \varepsilon (t) \) and \( x = -\varepsilon (t) \) as

\[ \phi (t, \varepsilon (t)) = \phi (t, -\varepsilon (t)) = 0. \]

The scalar field \( \phi \) with the boundary condition (2.7) is a mimic of the coloured gauge field which cannot escape outside the bag. Exploiting the two-dimensional feature of our problem, we perform the following conformal transformation: \( (t, x) \to (w, \phi) \):

\[ t = f (w - \phi), \]

\[ t + x = g (w + \phi), \]

with the metric

\[ ds^2 = dz^2 - dx^2 = f' (w - \phi) g' (w + \phi) (d\phi^2 - dt^2). \]
As is easily seen from Eq. (2-9), the field equation (2-6) does not change under the conformal transformation (2-8). We have
\[ \partial_s^2 - \partial_t^2 \phi = 0. \] (2.10)

Let us choose the functions \( f \) and \( g \) so that the boundary \( \lambda = -z(t) \) transforms to \( \lambda = 0 \) and \( x = \varepsilon(t) \) to \( \varepsilon = 1 \). Namely,
\[ t + \varepsilon(t) = f(w), \]
\[ t - \varepsilon(t) = g(w), \]
\[ t + \varepsilon(t) = f(w-1), \]
\[ t - \varepsilon(t) = g(w-1). \] (2.11)

Here \( \varepsilon(t) \) is explicitly given by Eq. (2.1). Introducing \( p = f^{-1} \), we can reduce Eq. (2.11) to
\[ p(t - \varepsilon(t)) = p(t + \varepsilon(t)) + 1, \]
\[ f(w) = g(w-1). \] (2.12)

Equation (2.12) is readily solved to give
\[ p(w) = \frac{1}{2u} \log \frac{\sinh \frac{\pi \varepsilon}{2}}{\sinh \frac{\pi}{2}}, \] (2.13)

where \( \eta \) is defined by
\[ \eta = \frac{a \sinh \eta}{a - \cosh \eta} \] (2.14)

Having prepared all the stuffs we need, we can easily obtain the normal modes of Eq. (2.6) with the boundary condition (2.7):
\[ \phi = \frac{1}{\sqrt{2\pi}} \sin \frac{1}{2} e^{-\omega t} \]
\[ = \frac{1}{\sqrt{2\pi}} \left( e^{-i\omega t} - e^{-i\omega t} \right) \]
\[ = \frac{1}{\sqrt{2\pi}} \left( e^{-i\omega t} - e^{-i\omega t} \right) \]
\[ = \frac{1}{\sqrt{2\pi}} \left( e^{-i\omega t} - e^{-i\omega t} \right) \] (2.15)

where \( p = f^{-1} \) is given by Eq. (2.13). In order to comply with the boundary condition at \( \lambda = 1 \), \( \omega \) is discrete and given by
\[ \omega = n\pi, \] (n: integer)

2.3. Quantum Field Theory

As our boson field is expanded in the form of a superposition of the modes of the field \( \phi \),

\[ \phi = \sum_{n=-\infty}^{\infty} a_n \phi_n, \]

The vacuum state is

\[ |0\rangle = \prod_{n=-\infty}^{\infty} |n\rangle. \]

Some generalizations related to \( \phi^\dagger \phi \)

The point is the superposition of such a mode.
2.3. Quantization

If the boundaries were at rest, appropriate normal modes would be

$$\phi^m = \frac{1}{2 \sqrt{\omega}} (e^{-i\omega t} - e^{-i\omega t}), \quad (2.16)$$

where $\omega = \pi \nu / L$ and $\nu = \omega L$. We take the box ($|x| < L/2$) very large so that our system of moving walls is contained in it. The standard quantization of the free field $\phi$ is

$$\hat{\phi} = \sum_{m} (a^m_\phi \phi^m + a^{\dagger m}_\phi \phi^{* m}), \quad (2.17)$$

$$[a^m_\phi, a^{\dagger n}_\phi] = \delta_{mn},$$

$$[a^m_\phi, c^p] = [a^{\dagger m}_\phi, c^p] = 0.$$  

(2.18)

The vacuum state $|0\rangle$ will be characterized by

$$a_{\phi}^{\dagger m}|0\rangle = 0. \quad (2.19)$$

As our boundaries actually move with a constant acceleration, $\phi$ should be expanded in the normal modes $\phi^m$ in Eq. (2.15). We write

$$\phi = \sum_{m} (a_\phi \phi^m + a^{\dagger}_\phi \phi^{* m}), \quad (2.20)$$

$$[a_\phi, a^{\dagger}_\phi] = \delta_\phi,$$

$$[a_\phi, a] = [a^{\dagger}_\phi, a] = 0. \quad (2.21)$$

We are now going into the most important part of this section. As shown in the Appendix, the annihilation operator $a_\phi$ and the creation operator $a^{\dagger}_\phi$ are related to $a^{\nu m}$ and $a^{\nu m \dagger}$ by the Bogoliubov transformation.

$$a_\phi = \sum_{m} (\alpha \phi m + \beta \phi m^\dagger) a^{\nu m},$$

$$a^{\dagger}_\phi = \sum_{m} (\alpha^{\dagger} \phi m - \beta^{\dagger} \phi m) a^{\nu m \dagger}. \quad (2.22)$$

Some generalities about the mode mixing are summarized in the Appendix. Using the completeness of $\phi^m \phi^m$ and $\phi^m \phi^m$ (see (A-3)), we have

$$\alpha (n, \omega) = i \int \phi^m \phi^m \phi^m dx,$$

$$\beta (n, \omega) = -i \int \phi^m \phi^m \phi^m dx. \quad (2.23)$$

The point is that the positive frequency mode $\phi^m$ referring to the "$v$-time" is a superposition of both the positive and negative frequency modes to the "$t$-time". Such a mode mixing brings about the Bogoliubov transformation (2.22). The
vacuum expectation value of the particle number confined between the accelerated walls is given by
\[
\langle 0 | a_x^+ a_x(0) \rangle = \sum_{a_0} \beta(a_0, u') \bar{\beta}^{*}(a_0, u'),
\]  
(2.24)
which is actually diagonal in the high energy limit \(E \to \infty\) as we shall subsequently show. Equation (2.24) implies that the accelerated walls create particles. We emphasize that our vacuum state \(|0\rangle\) is defined in the non-accelerated Minkowski frame: Eq. (2.19). In order to determine the precise form of the particle spectrum in our particular problem, let us concentrate on the calculation of \(\beta\).

2.4. Thermal distribution

We are going to evaluate the integral:
\[
\beta(a, u') = -i \int_{-\infty}^{\infty} dx \phi^2 \bar{\phi} a_x,
\]  
(2.25)
where \(\phi^2\) and \(\phi\) are given in Eqs. (2.15) and (2.19). As we shall explain in the next section, the moving mirror effects are most relevant to the reaction, \(e^+ e^-\)-hadrons in the case:
\[
z_x = \frac{E}{m_{e}} \gg |t| \gg |\bar{t}|,
\]  
(2.26)
In this limit we can ignore the interference terms of the left- and right-moving wave, which rapidly change in phase. We obtain
\[
\beta(a, u') \to \frac{-i}{4\pi \sqrt{|\omega|}} \int_{-\infty}^{\infty} dx (e^{iux} \bar{\beta}^{*}_x e^{-iux})
\]  
(2.27)
In the continuous limit of \(a'\)\(^{\dagger}\) (i.e., large normalization box limit), Eq. (2.27) becomes
\[
\beta(a, u') \to \frac{-i}{4\pi \sqrt{|\omega|}} \int_{-\infty}^{\infty} du e^{-iu} \bar{\beta}^{*}_x e^{-iu|x|},
\]  
(2.28)
\[p(x) = A \log \frac{z_x + z_{x'}}{z_x - z_{x'}}\]
It is straightforward to carry out the integration (2.28).

\(^{\dagger}\) For discrete values of \(a' = m_{n}\), the right-hand side of Eq. (2.28) turns out to be twice the rhs. of Eq. (2.28) if \(a = m_{n}\)-even and zero if \(a = m_{n}\)-odd, where \(a' = \mp X\). This is due to the reflection symmetry. In the continuous limit, we may take the average of the two cases to obtain (2.28).
\[ \beta(a, a') = \frac{i}{4 \sqrt{\alpha s}} \int_{-1}^{1} ds \exp\left(-i a' s \alpha \right) \tilde{\beta} \exp\left(-i s A \log \frac{1 + i s}{1 - i s} \right) \]

\[ = -\frac{1}{4 \sqrt{\alpha s}} \left[ \alpha s \tilde{A}_1 - \frac{1}{2} A \tilde{A}_1 - \frac{1}{2} A_1 \tilde{A} \right] \]

\[ I_1 = \int_{-1}^{1} ds (1 + s) \left( 1 - s \right) e^{-i s \alpha} e^{i s \alpha} \]

\[ I_2 = \int_{-1}^{1} ds (1 + s) \left( 1 - s \right) e^{-i s \alpha} e^{i s \alpha} \]

\[ I_3 = \int_{-1}^{1} ds (1 + s) \left( 1 - s \right) e^{-i s \alpha} e^{i s \alpha} \]

The \( I \)'s can be written in terms of the Whittaker function \( W_{\nu, \mu} \). That is,

\[ I_1 = \Gamma(1 - i a A) \Gamma(1 + i a A) M_{-i a, i a}(x), \]

\[ I_2 = \Gamma(-i a A) \Gamma(i a A) M_{i a, -i a}(x), \]

\[ I_3 = \Gamma(1 - i a A) \Gamma(i a A) M_{-i a, i a}(x), \]

where \( x = 2 \sqrt{\alpha s} \). (Note that \( -\pi/2 < \text{Arg} x < \pi/2 \) since \( \text{Im} s' \) is slightly negative.) Using the asymptotic form of \( M \) at \( |x| \to \infty \), we obtain

\[ \beta(a, a') = \frac{i}{2 \sqrt{\alpha s}} \Gamma(1 - i a A) e^{-i \alpha \log(2 \sqrt{\alpha s})} e^{-i \alpha s}. \]

Now that we have the transformation function \( \beta \) in Eq. (2.29), we can evaluate the particle number by using Eq. (2.24),

\[ \langle 0 | \hat{a}_\alpha^2 | 0 \rangle = \sum_{a, a'} \left\langle 0 \right| \hat{a}_\alpha^{a} \hat{a}_\alpha^{a'} \hat{a}_\alpha^{a'} \hat{a}_\alpha^{a} | 0 \rangle \]

\[ = \int \frac{d\omega'}{2\pi} \frac{1}{4 \sqrt{\alpha s}} \int \frac{d\omega}{2\pi} (2\pi a^2) \left( \frac{1}{4 \sqrt{\alpha s}} \right)^{\frac{1}{2}} e^{-i \alpha s} e^{-i \alpha s} \Gamma(1 - i a A) \Gamma(1 + i A A) \]

\[ = \frac{1}{e^{\alpha s} - 1} \delta(a - \tilde{a}). \]

Equation (2.30) reveals a striking feature of the radiation spectrum due to the accelerated mirrors. It is the Planck distribution! The frequency \( \omega \) measured by "w-time", however, does not have a direct physical meaning like an energy of a quantum. We would like to rewrite \( \omega \) in terms of the frequency measured by
the Minkowski time $t$ and at the one of the moving wall, say, at $s=0$. Writing
the energy and momentum in the Minkowski spacetime as $(e, p)$, we have
\[ \omega = \frac{\partial t}{\partial \tau} = \frac{\partial x}{\partial \tau} p. \]
For right-moving massless particles, this becomes
\[ \omega = f^r \left( \frac{r}{\partial x} \right) \epsilon = \frac{1}{\rho^r(x)} \epsilon, \tag{2-31} \]
due to Eq. (2-8). It is easy to show that at the wall $s=0$ Eq. (2-31) turns out to be
\[ \omega = \frac{k}{\sqrt{1-v^2}}, \quad \varepsilon = -\varepsilon, \tag{2-31'} \]
in the limit (2-26). We finally obtain the particle number spectrum
\[ n_r = \left( \frac{2}{\pi} \right)^{\frac{1}{2}} \frac{1}{2} \left( \frac{1-v^2}{1+\nu} \right) \epsilon \frac{1}{\rho^r(x)} \epsilon. \tag{2-32} \]
If we identify
\[ kT = \frac{1}{2 m}, \quad \frac{\alpha}{2z}, \tag{2-33} \]
(c.f., Eq. (2-4)), the distribution (2-32) is thermal with the correct Doppler shift. Equation (2-32) together with Eq. (2-33) is just what we have alluded to in the Introduction.

§ 3. $e^+e^-$ reaction

In this section we shall discuss some implications of the moving mirror effects in the $e^+e^-\rightarrow$hadron reaction. Firstly the results of § 2 are directly applied to the longitudinal momentum distribution of soft particles relative to the jet axis. Then we would like to argue on the transverse momentum distribution.

3.1. Longitudinal momentum distribution

The linear potential assumed in Eq. (1-1) will be valid when the distance between $q$ and $\bar{q}$ are relatively large, say, $t=1 \sim 1$ fm. At a relatively short distance, the Coulomb-like gluon exchange force will be more important.

In the calculation of the particle number, we have ignored the pressure which would be caused by multiple collisions of the produced gluons onto the walls. There may be thermal quark-antiquark pair productions due to the moving lids of the bag. And hence there is a chance that one of the thermal quarks (anti-

The condition (2-26) shows that a single accelerated case $|t| \gg b$. The one of the other side. We will estimate the quarks. We refer to Ref. 21). Presumably from the bag picture; we assume outside the bag. We assume the distribution without the assumption of order $\alpha P^{-2}$ which implies that the part
\[ \frac{1}{x} + \frac{dE}{dx} \]
Here $\nu = T/\sqrt{T^2 + \nu^2}$ left lid from which to reach $x_1$ at $t=x_1$ (e.g., geometrical optics gives
\[ \frac{1}{1+\nu} \]
By integrating over $x$ to get the longitudinal momentum
\[ N._{dx} \]}
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$q_{\bar{q}}$ shoots the locomotive anti-quark (quark) to produce hard mesons after the time travel $t\sim a_0 - bE/m_{P}$. Hence it will be easy to use our formula with the limitation $|r| \ll c_0$.

Combining the two inequalities together we see that the moving mirror effect is relevant if

$$z_{0} \gg |r| \gg b.$$  \hspace{1cm} (2.26)

As we have shown in the previous section, the particle number produced by the bag obeys the thermal law:

$$n_{L} = \exp \left( \frac{c}{kT} \frac{1 - v^2}{1 + v^2} - 1 \right).$$  \hspace{1cm} (2.27)

The condition (2.26) can be seen in other ways. It is not difficult to show that a single accelerated mirror gives rise to the thermal distribution (2.27) in the case $|r| \gg b$. The condition $z_{0} \gg |r|$ guarantees that we can safely ignore the mirror of the other side. We will use the same sort of argument in §3.2.

Of course there will be ordinary bremsstrahlungs of gluons from the accelerated quarks. We will not discuss the particle production of this origin (see, e.g., Ref. 11)). Presumably there are two sources of gluon productions: thermal radiations from the bag and bremsstrahlungs from the quarks. As we have argued in the Introduction, we presume that the produced gluons will eventually transform into hadrons (mainly into mesons). Following Bjorken’s picture, we assume the gluons in some ways change into hadrons and then escape outside the bag. We may consider the gluon distribution (2.27) as the meson distribution with a great error. Incidentally, the “mean free path” of the quark is of order $m_{P} \sim E$ which also accomodates Bjorken’s picture. Equation (3.26) implies that the particle number at $t = q$ from the left lid in the interval $[x_{0}, x_{0} + d x_{0}] \times [t, t + d t]$ is given by

$$\left[ \exp \left( \frac{c}{kT} \frac{1 - v^2}{1 + v^2} - 1 \right) \right] d x_{0} d t.$$  \hspace{1cm} (3.1)

Here $v = \gamma / \sqrt{1 - \beta^2}$ is the velocity of the left lid from which our particle is emitted to reach $x_{0}$ at $t = t_{0}$ (see Fig. 5). Elementary geometrical optics give us

$$\sqrt{1 - \beta^2} = \frac{x_{0} + x_{0} - t_{0}}{b}.$$  \hspace{1cm} (3.2)

By integrating over the position $x_{0}$, we have the longitudinal momentum distribution:

$$N_{d t} = 2 \int_{x_{0}}^{x_{0} + d x_{0}} \exp \left( \frac{c}{kT} (x_{0} + x_{0} - t_{0}) - 1 \right) d x_{0}.$$  \hspace{1cm} (3.3)

Fig. 3. A glass emitted at $(T, X)$ travels to the point $(n, x_{0})$.
where the particles from the right hit also are added. In the limit $x_w \gg |t_4|, h$, Eq. (3.3) becomes

$$N_4 dt = \frac{2dt}{2\pi} \sqrt{\sin^2 (\pi s - 1)} \cdot d\epsilon .$$

(5-6)

Since $x_w \rightarrow 0$ as $|t_4| \gg h$, the integral is formally divergent. If we take account of the transverse momenta and the mass of mesons, a simple kinematics shows

$$x_w = \epsilon \left( \frac{m_t^2 - t_4}{t_4} \right), \quad (h \gg m_t)$$

(5-5)

where $m_t^2 = p_t^2 + m_t^2$ is the so-called transverse mass squared. Hence

$$N_4 dt \leq \frac{2dt}{2\pi} \int \frac{dx}{\sin^2 \epsilon - 1} \cdot (\epsilon \rightarrow \infty).$$

(5-6)

Optimal choice of $t_4$, $t_4 = (h/2m_t)$, saturates the bound (3-6) and gives a flat distribution in the rapidity space. In order to get a precise form of the distribution, a detailed study of the transformation of gluons into hadrons has to be done.

3.2. Transverse momentum distribution

The reader may perhaps be uneasy about the previous paragraph. In the derivation of the thermal distribution we have heavily relied on the two-dimensional peculiarities, e.g., the conformal invariance of the wave equation. And hence he may even doubt the arguments on the transverse momentum. However, if we admit the fact that in the high energy limit $x_w \rightarrow \infty$ the two walls can be dealt separately and if we can simply add the contributions of the two, we can resort to the results by Fulling and Unruh. They actually studied the thermal productions of massive particles in three-dimensional space seen by an accelerated observer. Their results can easily be translated into single mirror problem and give the thermal distribution:

$$\left[ \exp \left( \frac{-\epsilon \cdot p_t}{\sqrt{\sin^2 (\pi s - 1) + \epsilon}} \right) \right]^{-1},$$

(3.7)

where $p_t$ is the velocity of the mirror which emits a particle with an energy and momentum $(E = \sqrt{p_t^2 + p_t^2 + m_t^2}, p_t, p_t)$. Equation (3.7) is a direct generalization of Eq. (2.32) to the three-dimensional case. Equation (3.7) straightforwardly leads to Eq. (3.4) with $x_w$ given by Eq. (3.5). Equation (3-6) strongly suggests that the transverse momentum distribution is roughly given by

$$\propto p_t \exp \left( \frac{-m_t}{130 \text{ MeV}} \right) dp_t .$$

(3-8)

This result accommodates the experimental data very well. Again much work has to be done.

We have shown for hadronic reactions at the walls of quarks. The acceleration of the Regge slope and the theory of quarks is not well understood today.

Under some condition of secondaries,

$$\text{Our discussion transform into a},$$

but this is not more definite.

It will not be $pp \rightarrow X$. Expect the viewpoint of this.

As is recently seen, (as it should be quark distribution). The author of on the quantity.

We expect...
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If we take account of the kinematics shown \hspace{1cm} (3.5)

Hence \hspace{1cm} (3.6)

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Equation (3.6) is roughly given by

Again much work

\begin{align*}
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\end{align*}

has to be done to get the precise form of the distribution.

§ 4. Summary and discussion

We have shown that the accelerated mirror effects are quite relevant in the
hadronic reaction especially in the $e^+e^-$ reactions. Gluons are thermally produced
at the walls of the bag which are pushed by energetic but permanently confined
quarks. The standard linear potential between quarks implies a constant ac-
celeration of the walls. The acceleration gives rise to a temperature according to
the formula:

\begin{equation}
kT = \frac{\hbar}{2mc} \times \text{acceleration},
\end{equation}

in the theory of quantization in curved spacetimes. The standard values of the
Regge slope and the light quark mass give

\begin{equation}
kT \approx 130 \text{ MeV}.
\end{equation}

Under some conditions, which we have not yet fully clarified, the rapidity distribu-
tion of secondaries is flat and the transverse momentum distribution is of the form:

\begin{equation}
p_z \exp \left( - \frac{m_{\pi^+}}{130 \text{ MeV}} \right) dp_z.
\end{equation}

Our discussion is not cut at the point how the thermally created gluons transform into mesons and eventually escape outside the bag. Hopefully, if we have a good model of this phenomenon, we can calculate the distribution function more definitely.

It will not be difficult to extend our theory to other hadronic reactions, e.g., $pp \rightarrow eX$. Experimental evidence\textsuperscript{26} in the small $p_z$ region seem to support our viewpoint of thermal distributions due to the moving bag.

In this paper we have been mainly concerned with the gluon distributions, As is recently shown\textsuperscript{26}, the quark and anti-quark distributions obey the Fermi law (as it should be!) in the accelerated wall problem. The implications to the soft quark distribution will be investigated elsewhere.

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Appendix

We recapture general formulae of the Bogoliubov transformation. For details
we refer the reader to Ref. 10.

A scalar field $\phi$ is expanded in two different ways:

$$\phi = \sum_{\sigma} a_{\sigma}^\dagger u_{\sigma}^0 + \sum_{\sigma^*} a_{\sigma^*}^\dagger u_{\sigma^*}^0,$$

$$+ \sum_{\sigma} (a_{\sigma} u_{\sigma}^0 + a_{\sigma}^* u_{\sigma}^0),$$  \hspace{1cm} (A-1)

where $u_{\sigma}^0$ and $u_{\sigma^*}^0$ are positive frequency modes for "$\sigma$-time" and for "$\sigma^*\text{-time}"$, respectively. Suppose we expand $\phi$ in terms of $u_{\sigma}^0$:

$$\phi_t = \sum_{\sigma} (a_{\sigma} u_{\sigma}^0 + a_{\sigma}^* u_{\sigma}^0),$$  \hspace{1cm} (A-2)

then we have

$$\phi_0 = \langle \phi_0, u_{\sigma}^0 \rangle,$$

$$\beta_{\sigma}^0 = \langle\phi^0, u_{\sigma}^0 u_{\sigma}^*\rangle.$$  \hspace{1cm} (A-3)

with

$$\langle A, B \rangle = \int A^T \overline{B} d\Sigma,$$  \hspace{1cm} (A-4)

where $d\Sigma$ is an appropriate Cauchy surface. It is easy to show that

$$\sum_{\sigma} (a_{\sigma}^* a_{\sigma} - \beta_{\sigma}^0 a_{\sigma}) = j_\mu,$$

$$\sum_{\sigma}(-a_{\sigma}^* a_{\sigma} + \beta_{\sigma}^0 a_{\sigma}) = 0.$$  \hspace{1cm} (A-5)

Combination of Eqs. (A-1), (A-2) and (A-5) gives

$$a_{\sigma} = \sum_{\sigma} (a_{\sigma}^* a_{\sigma} - \beta_{\sigma}^0 a_{\sigma}^*).$$  \hspace{1cm} (A-6)

Equation (A-5) guarantees the consistency of the conjugation relations:

$$[a_{\sigma}, a_{\sigma'}^*] = [a_{\sigma}^*, a_{\sigma'}] = \delta_{\sigma \sigma'},$$

$$[a_{\sigma}, a_{\sigma'}] = [a_{\sigma}^*, a_{\sigma'}^*] = 0.$$  \hspace{1cm} (A-7)

The vacuum state $|0\rangle$ will be naturally defined in the ordinary Minkowski space time. We impose

$$a_{\sigma}^\dagger |0\rangle = 0 \text{ for all } \sigma.$$  \hspace{1cm} (A-8)

By using Eqs. (A-6) and (A-8), we obtain the vacuum expectation value of the particle number:

$$\langle 0 | a_{\sigma}^\dagger a_{\sigma} | 0 \rangle = \sum_{\sigma} \beta_{\sigma}^0.$$  \hspace{1cm} (A-9)
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References

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(A-1) and for "w-time",

(A-2)

(A-3)

(A-4)

(A-5)

(A-6)

(A-7)

(A-8)

(A-9)

Notes

(A-10)