Baryon-decuplet magnetic moments

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Abstract

We investigate the generalized Coleman–Glashow sum rules in the magnetic moments of the baryon decuplet in the chiral bag model. Due to the spin symmetries held both for the effective and naive NRQMs in the SU(3) flavor symmetric limit, the chiral bag model can be considered as an effective NRQM with meson cloud as in the baryon octet. In the multiquark structure with $U$-spin symmetry breaking, we predict the decuplet magnetic moments to be insensitive to the bag radius enough to suggest Cheshire catness.

1 Introduction

The chiral symmetry introduced in the framework of current algebra has been known to be a crucial symmetry of the strong interaction since it yields experimentally confirmed results such as the Goldberger–Treiman relation and the Adler–Weisberger sum rule [1] for pion–nucleon cross sections. The chiral and SU(3) flavor symmetry-breaking effect in the chiral bag model (CBM) [2] is induced by the different pseudoscalar meson masses and decay constants outside and the quark masses inside the bag. Especially the SU(3) flavor symmetry breaking originates from the strangeness degree of freedom in the baryonic structure, which has been significantly discussed after the European Muon Collaboration (EMC) experiment on deep-inelastic muon scattering [3] suggested a lingering question. The nontrivial strangeness in the EMC results has been interpreted in the context of CBM [4]. Moreover the role of the strangeness flavor has become one of the central issues in the kaon condensation [5] and their effects in the neutron star [6].

Also since Coleman and Glashow [7] predicted the magnetic moments of the baryon octet about thirty years ago, there has been a lot of progress in both the theoretical paradigm and experimental verification for the baryon magnetic moments.
In our previous paper [8] we have calculated the magnetic moments of the baryon octet in the SU(3) flavor CBM so that we could obtain the Coleman–Glashow sum rules [7] including the U-spin symmetry held up to the SU(3) flavor symmetric limit of the adjoint representation to suggest the possibility of a unification of the naive nonrelativistic quark model (NRQM) and the SU(3) CBM, which was proposed as the effective NRQM [9] with meson cloud around the quarks of the naive NRQM. Due to the Cheshire cat principle [10], the CBM has also been regarded as a model unifying the MIT bag and Skyrme models and gives model-independent relations insensitive to the bag radius.

The motivation of this paper is to generalize the conjecture of the effective NRQM used in the baryon-octet magnetic moments to the decuplet case so that one can investigate the spin symmetries and the generalized Coleman–Glashow sum rules. Moreover the new measurements of the magnetic moments of the decuplet baryons were recently reported for \( \mu_{\Lambda^+} \) [11] and \( \mu_{\Omega} \) [12] to yield a new avenue for understanding the hadronic structure.

On the other hand, in the naive NRQM one can start with the symmetric spin configuration in the ground state with \( \psi(\text{space}) \) so that the spin-\( \frac{3}{2} \) baryon-decuplet wave function goes with the symmetric flavor state to yield

\[
\psi(\text{baryon decuplet}) = \psi_s(\text{spin}) \psi_s(\text{flavor}).
\]  

(1)

The baryon-decuplet magnetic-moment spectra are then easily obtained by the linear sum of the three constituent-quark magnetic moments \( \mu_q = Q_q(m_q/m_N) \), where \( m_q (m_N) \) is the \( q \)-flavor quark (nucleon) mass,

\[
\begin{align*}
\mu_{\Delta^+} &= 3 \mu_d, \\
\mu_{\Delta^0} &= \mu_u + 2 \mu_d, \\
\mu_{\Delta^*} &= 2 \mu_u + \mu_d, \\
\mu_{\Delta^{**}} &= 3 \mu_u, \\
\mu_{\Xi^+} &= 2 \mu_d + \mu_s, \\
\mu_{\Xi^0} &= \mu_u + \mu_d + \mu_s, \\
\mu_{\Xi^{*0}} &= 2 \mu_u + \mu_s, \\
\mu_{\Omega^+} &= 3 \mu_s,
\end{align*}
\]

(2)

in units of nuclear magnetons \( (= e\hbar/2m_Nc) \). In the \( u \)- and \( s \)-channels the \( \Delta \) hyperon magnetic moments with the electromagnetic (EM) charge \( Q \) are given by

\[
\begin{align*}
\mu_{\Delta}^{(u)} &= (Q + 1) \left( \frac{2}{3} \frac{m_N}{m_u} \right), \\
\mu_{\Delta}^{(s)} &= 0.
\end{align*}
\]

(3)

With the above naive NRQM results in mind, we will show that the generalized Coleman–Glashow sum rules and the other model-independent relations in the baryon decuplet satisfied by the naive NRQM are also fulfilled by the CBM to support the effective NRQM conjecture.

In Section 2, the symmetry breaking in the CBM will be briefly discussed in terms of the physical operators in the adjoint representation and the baryon wave functions in the multiquark Fock space of the higher representation-mixing scheme.

In Section 3, we will apply the general formalism of the previous section to the baryon decuplet to explicitly investigate the generalized Coleman–Glashow sum rules in the magnetic moments and then introduce the \( U \)-spin symmetry-breaking effects in the multiquark structure so that one can predict the unknown experi-
ments as well as $\mu_{\Delta^{++}}$ and $\mu_Q$. Also in order to obtain the $I$-spin symmetry relations in the s-flavor components and the model-independent relations between the $u$- and $d$-flavor ones, we will evaluate the baryon magnetic moments in each flavor channel with the three flavor-valued EM currents.

2. Approaches to symmetry breaking

Now in order to introduce the broken chiral and SU(3) flavor symmetry in the CBM we start with the lagrangian of the form

$$\mathcal{L} = \mathcal{L}_{\text{CS}} + \mathcal{L}_{\text{CSB}} + \mathcal{L}_{\text{FSB}},$$

$$\mathcal{L}_{\text{CS}} = \bar{\psi}i\gamma^\mu\gamma^5\psi\partial_\mu - \frac{1}{2}\bar{\psi}U_3\psi\Delta_B + \left(-\frac{1}{2}f_\pi^2 \text{tr}(l^\mu l^\nu) + \frac{1}{32\epsilon^2} \text{tr}[l^\mu, l^\nu]^2 + \mathcal{L}_{\text{WZW}}\right)\overline{\Theta}_B,$$

$$\mathcal{L}_{\text{CSB}} = -\bar{\psi}M\psi\partial_\mu - \frac{1}{2}f_\pi^2 m_\pi^2 \text{tr}(U + U^\dagger - 2)\overline{\Theta}_B,$$

$$\mathcal{L}_{\text{FSB}} = \frac{1}{2}(f_\pi^2 m_K^2 - f_\pi^2 m_\rho^2) \text{tr}\left[(1 - \sqrt{3}\lambda_8)(U + U^\dagger - 2)\right]\overline{\Theta}_B - \frac{1}{12}(f_\pi^2 - f_\rho^2) \text{tr}\left[(1 - \sqrt{3}\lambda_8)(U\mu^\dagger U + l_\mu l^\mu U^\dagger)\right]\overline{\Theta}_B,$$

where one has the quark field $\psi$ with SU(3) flavor degrees of freedom inside the bag and the chiral field $U = \exp(i\lambda_a\tau_a/f_\pi) \in \text{SU}(3)$ described by the pseudoscalar meson fields $\tau_a$ ($a = 1, \ldots, 8$) outside the bag and the surface term playing the crucial role in the restoration of the chiral symmetry by coupling the pseudoscalar meson fields to the quarks on the bag boundary. Also Gell-Mann matrices $\lambda_a$ are normalized to satisfy $\lambda_a\lambda_b = \frac{1}{2}\delta_{ab} + (if_{abc} + d_{abc})\lambda_c$, and $\mathcal{L}_{\text{WZW}}$ stands for the topological Wess–Zumino–Witten (WZW) term and $l_\mu = U\partial_\mu U$. In the numerical calculation we will use the parameter fixing $\epsilon = 4.75$, $f_\pi = 93$ MeV and $f_K = 114$ MeV.

Here one notes that the chiral symmetry (CS) is broken by the quark masses $M = \text{diag}(m_u, m_d, m_s)$ and pion mass $m_\pi$ in $\mathcal{L}_{\text{CSB}}$ which is chosen such that it will vanish for $U = 1$, while the SU(3) flavor symmetry breaking (FSB) with $m_K/m_\rho \neq 1$ and $f_K/f_\rho \neq 1$ is included in $\mathcal{L}_{\text{FSB}}$.

The Noether scheme then enables the derivative terms in $\mathcal{L}_{\text{CS}}$ and $\mathcal{L}_{\text{FSB}}$ to yield the chiral symmetric EM currents $J^\mu_{\text{CS}}$ and $U$-spin symmetry-breaking currents $J^\mu_{\text{FSB}}$, respectively, so that one can have the conserved total EM currents

$$J^\mu = J^\mu_{\text{CS}} + J^\mu_{\text{FSB}},$$

$$J^\mu_{\text{CS}} = \bar{\psi}i\gamma^\mu\hat{Q}\psi\partial_\mu + \left(-\frac{1}{2}f_\pi^2 \text{tr}(\hat{Q}l^\mu) + \frac{i}{8\epsilon^2} \text{tr}[\hat{Q}, l^\mu][l^\nu, l^\nu] + U \leftrightarrow U^\dagger\right)\overline{\Theta}_B + \frac{N_c}{48\pi^2}\epsilon^{\mu\nu\alpha\beta} \text{tr}[\hat{Q}, l^\alpha l^\beta - U \leftrightarrow U^\dagger]\overline{\Theta}_B \quad (\epsilon^{0123} = 1),$$

$$J^\mu_{\text{FSB}} = -\frac{1}{12}i(f_\pi^2 - f_\rho^2) \text{tr}\left[(1 - \sqrt{3}\lambda_8)(U\hat{Q}l^\mu + l^\mu \hat{Q}U^\dagger) + U \leftrightarrow U^\dagger\right]\overline{\Theta}_B.$$

(5)
Here one should note that the mass terms in $\mathcal{L}_{\text{CSB}}$ and $\mathcal{L}_{\text{FSB}}$ yield no explicit contribution on the EM current operators.

Since the Euler equation with the meson fields in the nonlinear $\sigma$ model was analytically investigated [13] to obtain a specific classical solution for the meson fields whose isospin index points radially, $\pi'(r)/f_{\pi} = i\theta'(r)$, the so-called hedgehog solution, the spherically symmetric classical solution has been commonly used as a prototype ansatz in the literature of the skyrmion-related hadron physics. In the SU(3) CBM we suppose that the hedgehog classical solution in the meson phase $U_0 = \exp[i\lambda_i\theta(r)]$ ($i = 1, 2, 3$) is embedded in the SU(2) isospin subgroup of SU(3).1

Introducing the collective variable $\mathcal{A}(t) \in \text{SU}(3)$ and the hedgehog ansatz in the CBM one can generate the zero-mode with the slow rotation $U \rightarrow AU_0 At$ and $\psi \rightarrow \mathcal{A}\psi$ on the SU(3) group manifold so that the EM currents yield the magnetic-moment operators $\hat{\mu}^i = \hat{\mu}_{\text{CSB}}^i + \hat{\mu}_{\text{FSB}}^i$ where $\hat{\mu}_{\text{CSB}}^i = \hat{\mu}_{\text{FSB}}^i$ with

$$\hat{\mu}_{\text{CSB}}^i = -\mathcal{M} D^i_{ai} - \mathcal{N} \cdot \frac{1}{3} N_c d_{ipq} D^i_{ap} \hat{T}^R_i + \mathcal{N} \cdot \frac{1}{2} \sqrt{3} D^i_{ab} \hat{J}_a,$$

$$\hat{\mu}_{\text{FSB}}^i = -\mathcal{P} D^i_{ai} (1 - D^8_{88}) + \mathcal{Q} \cdot \frac{1}{2} \sqrt{3} d_{ipq} D^i_{ap} D^8_{8q},$$

where $\mathcal{M}$, $\mathcal{N}$, $\mathcal{N}'$, $\mathcal{P}$ and $\mathcal{Q}$ are the inertia parameters [8,14] calculable in the CBM and $D^8_{ab} = \frac{1}{2} \text{tr}(A^a A^b)$ are the Wigner $D$ functions in the adjoint representation of SU(3) and $\hat{T}_i = -\hat{T}^R_i$ ($i = 1, 2, 3$) and $\hat{T}_p$ ($p = 4, 5, 6, 7$) are the right SU(3) operators along the isospin and strangeness directions respectively.

In addition to by the magnetic-moment operators $\hat{\mu}_{\text{FSB}}^i$ due to the symmetry-breaking kinetic term in the adjoint representation, the $U$-spin symmetry is also broken nonperturbatively by the mass terms in the higher-dimensional irreducible-representation (IR) channels where the CBM can be treated in the Yabu-Ando scheme [15] with the higher IR mixing in the baryon wave function to yield the multiquark structure [16] with the meson cloud inside the bag.

Now one can quantize the collective variable $\mathcal{A}(t)$ to obtain the hamiltonian

$$H = M + \frac{1}{2} \left( \frac{1}{\mathcal{F}_1} - \frac{1}{\mathcal{F}_2} \right) \hat{J}^2 + \frac{1}{2 \mathcal{F}_2} \left( h_{\text{SB}} - \frac{3}{4} \hat{Y}^2_R \right),$$

where $\mathcal{F}_1$ and $\mathcal{F}_2$ are the moments of inertia of the CBM along the isospin and the strangeness directions respectively and $\hat{J}^2$ is the Casimir operator in the SU($R$) group and $\hat{Y}_R$ is the right hypercharge operator to yield the WZW constraint $\hat{Y}_R|\text{phy} \rangle = +1|\text{phy} \rangle$. Here $h_{\text{SB}}$ is the representation-dependent part induced by the chiral and SU(3) flavor symmetry breaking with the strength $\omega$ [8],

$$h_{\text{SB}} = \hat{C}^2_2 + \frac{3}{2} \omega (1 - D^8_{88}),$$

1 The Fock space in the quark phase is described by the $N_c$ valence quarks in the $K^P = 0^+$ hedgehog ground state and the vacuum structure composed of quarks filling the negative-energy sea, where $K(K+1)$ and $P$ are the eigenvalues of the squared grand spin operator $K^2$ and the parity operator $P$ respectively.
where $\hat{C}^2$ is the Casimir operator in the SU$_3$ group. One can then directly diagonalize the hamiltonian $h_{SB}$ with the eigenstate $|B\rangle = \sum A C^B_A |B\rangle^A$, where $C^B_A$ is the representation-mixing coefficient and $|B\rangle^A = \Phi^A_B \otimes |\text{intrinsic}\rangle$. Here $\Phi^A_B$ is the collective wave function discussed above, and the intrinsic state degenerate to all the baryons is described by a Fock state of the quark operator and the classical meson configuration.

3. Generalized Coleman–Glashow sum rules

In order to obtain explicitly the baryon-decuplet magnetic moments in the adjoint representation of the CBM, we decompose the tensor product of the Wigner $D$ functions in the magnetic-moment operators $\hat{\mu}_{FB}^{(a)}$ of (6) into a sum of the single $D$ functions. The isovector and isoscalar pieces of the operator $\hat{\mu}_{FB}^{(a)}$ are then rewritten as

$$\begin{align*}
\hat{\mu}_{FB}^{(3)} &= \mathcal{D} \left( -\frac{4}{5} D_{3i}^8 + \frac{3}{10} D_{3i}^{27} \right) + \mathcal{C} \left( \frac{3}{10} D_{3i}^8 - \frac{3}{10} D_{3i}^{27} \right), \\
\hat{\mu}_{FB}^{(8)} &= \mathcal{D} \left( -\frac{2}{5} D_{8i}^8 + \frac{2}{20} D_{8i}^{27} \right) + \mathcal{C} \left( -\frac{3}{10} D_{8i}^8 - \frac{6}{20} D_{8i}^{27} \right),
\end{align*}$$

where one notes that the 1, 10 and 10 IRs do not occur in the decuplet baryons while 10 and 10 IRs appear together in the isovector channel of the baryon octet to conserve the hermiticity of the operator.

Acting on the magnetic-moment operator $\hat{\mu}^i$ the baryon-decuplet wave function $\Phi^i_B = \sqrt{\text{dim}(\lambda)} D_{ab}^\lambda$ with the quantum numbers $a = (Y; I, I_3)$ ($Y$: hypercharge, $I$: isospin) and $b = (Y_R; J, -J_3)$ ($Y_R$: right hypercharge, $J$: spin) and $\lambda$ the dimension of the representation, one can formulate hyperfine structure in the adjoint representation,

$$\begin{align*}
\mu_{\Delta^+} &= -\frac{1}{16} \mathcal{C} - \frac{1}{4} \left( N - \frac{1}{2} \sqrt{\frac{1}{3}} N' \right) - \frac{1}{7} \mathcal{D} + \frac{1}{7} \mathcal{C}, \\
\mu_{\Delta^0} &= \frac{1}{21} \mathcal{C} + \frac{11}{168} \mathcal{C}, \\
\mu_{\Delta^+} &= \frac{1}{16} \mathcal{C} + \frac{1}{4} \left( N - \frac{1}{2} \sqrt{\frac{1}{3}} N' \right) + \frac{5}{21} \mathcal{D} + \frac{1}{84} \mathcal{C}, \\
\mu_{\Delta^+} &= \frac{1}{8} \mathcal{C} + \frac{1}{2} \left( N - \frac{1}{2} \sqrt{\frac{1}{3}} N' \right) + \frac{3}{7} \mathcal{D} - \frac{3}{56} \mathcal{C}, \\
\mu_{\Sigma^+} &= -\frac{1}{16} \mathcal{C} - \frac{1}{4} \left( N - \frac{1}{2} \sqrt{\frac{1}{3}} N' \right) - \frac{12}{84} \mathcal{D} + \frac{13}{168} \mathcal{C}, \\
\mu_{\Sigma^0} &= \frac{1}{84} \mathcal{D} - \frac{1}{84} \mathcal{C}, \\
\mu_{\Sigma^+} &= \frac{1}{16} \mathcal{C} + \frac{1}{4} \left( N - \frac{1}{2} \sqrt{\frac{1}{3}} N' \right) + \frac{19}{84} \mathcal{D} - \frac{17}{168} \mathcal{C}, \\
\mu_{\Sigma^+} &= -\frac{1}{16} \mathcal{C} - \frac{1}{4} \left( N - \frac{1}{2} \sqrt{\frac{1}{3}} N' \right) - \frac{11}{42} \mathcal{D} + \frac{1}{84} \mathcal{C}, \\
\mu_{\Sigma^0} &= -\frac{1}{42} \mathcal{D} - \frac{17}{168} \mathcal{C}, \\
\mu_{\Omega^-} &= -\frac{1}{16} \mathcal{C} - \frac{1}{4} \left( N - \frac{1}{2} \sqrt{\frac{1}{3}} N' \right) - \frac{9}{28} \mathcal{D} - \frac{1}{56} \mathcal{C}.
\end{align*}$$
Now we can derive the generalized Coleman–Glashow sum rules which will be shown to be shared by the naive NRQM and CBM. In the SU(3) flavor symmetric limit with the chiral symmetry-breaking masses $m_u = m_d = m_s$, $m_K = m_{\pi}$ and decay constants $f_K = f_{\pi}$, the above magnetic moments of the decuplet baryons with the EM charge $Q$ are simply given by [17]

$$\mu_B = Q \left[ \frac{1}{16} \mathcal{M} + \frac{1}{4} \left( \mathcal{N} - \frac{1}{2} \sqrt{\frac{3}{3}} \mathcal{N}' \right) \right].$$

Here one notes that one has the similar SU(3) flavor symmetry relations $\mu_B = Q(m_N/m_u)$ in the naive NRQM without the chiral symmetry, which is preserved in the adjoint representation of the CBM as in (11) since the chiral symmetry-breaking masses do not yield any contribution to the EM currents.

In this SU(3) flavor symmetric limit of the CBM and naive NRQM, one can also obtain the $U$-spin symmetry relations

$$\mu_\Delta = \mu_\Sigma^* = \mu_\Xi^* = \mu_\Omega^*,$$
$$\mu_{\Delta^-} = \mu_{\Sigma^*} = \mu_{\Xi^*} = \mu_{\Omega^*},$$
$$\mu_{\Delta^0} = \mu_{\Sigma^0} = \mu_{\Xi^0},$$
$$\mu_{\Delta^+} = \mu_{\Sigma^+},$$

and the other generalized Coleman–Glashow sum rules,

$$\mu_{\Sigma^0} = \frac{1}{2} (\mu_{\Sigma^*} + \mu_{\Sigma^0}),$$
$$\mu_{\Delta^+} + \mu_{\Delta^0} = \mu_{\Delta^+} + \mu_{\Delta^0},$$
$$\mu_{\sum B \in \text{decuplet}} \mu_B = 0,$$

since the CBM has the degenerate $d$- and $s$-flavor charges in the EM charge operator $\hat{Q}$ of the EM currents (5) and the $d$-quark and $s$-quark have the same magnetic moments in the naive NRQM in this limit. Moreover, in the more general SU(3) flavor symmetry-broken case with $m_u = m_d \neq m_s$, $m_{\pi} \neq m_K$ and $f_{\pi} \neq f_K$, the first two sum rules in (13) are satisfied in the CBM and naive NRQM since they are derived in the same strangeness sector.

Next one can obtain the $\Delta$ hyperon magnetic moments in the $u$- and $s$-flavor channels of the CBM similar to (3) in the naive NRQM,

$$\mu_\Delta^{(u)} = \frac{7}{24} \mathcal{M} - \frac{1}{6} \left( \mathcal{N} - \frac{1}{2} \sqrt{\frac{1}{3}} \mathcal{N}' \right) - \frac{2}{21} \mathcal{P} + \frac{2}{21} \mathcal{L},$$
$$\mu_\Delta^{(u)} = \frac{1}{3} \mathcal{M} + \frac{2}{63} \mathcal{P} + \frac{11}{282} \mathcal{L},$$
$$\mu_\Delta^{(u)} = \frac{3}{8} \mathcal{M} + \frac{1}{6} \left( \mathcal{N} - \frac{1}{2} \sqrt{\frac{1}{3}} \mathcal{N}' \right) + \frac{10}{63} \mathcal{P} + \frac{1}{126} \mathcal{L},$$
$$\mu_\Delta^{(u)} = \frac{5}{12} \mathcal{M} + \frac{1}{3} \left( \mathcal{N} - \frac{1}{2} \sqrt{\frac{1}{3}} \mathcal{N}' \right) + \frac{5}{28} \mathcal{P} - \frac{1}{18} \mathcal{L},$$
$$\mu_\Delta^{(s)} = -\frac{7}{48} \mathcal{M} + \frac{1}{12} \left( \mathcal{N} - \frac{1}{2} \sqrt{\frac{1}{3}} \mathcal{N}' \right) + \frac{2}{21} \mathcal{P} + \frac{5}{158} \mathcal{L},$$

where we have used the $q$-flavor EM currents $J^{(q)}$ given by replacing the EM charge operator $\hat{Q}$ with the $q$-flavor EM charge operator $\hat{Q}_q$ in (5).
Table 1

<table>
<thead>
<tr>
<th>Flavor States</th>
<th>(a₁, a₂, a₃, a₄)</th>
<th>(b₁, b₂, b₃, b₄)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ⁻</td>
<td>(7, -1, -12, 24)</td>
<td>(-7, 1, 12, 15)</td>
</tr>
<tr>
<td>Δ⁰</td>
<td>(8, 0, 4, 13)</td>
<td>(-7, 1, 12, 15)</td>
</tr>
<tr>
<td>Δ⁺</td>
<td>(9, 1, 20, 2)</td>
<td>(-7, 1, 12, 15)</td>
</tr>
<tr>
<td>Δ⁺⁺</td>
<td>(10, 2, 36, -9)</td>
<td>(-7, 1, 12, 15)</td>
</tr>
<tr>
<td>Σ⁻⁻</td>
<td>(7, -1, -17, 13)</td>
<td>(-8, 0, 1, -4)</td>
</tr>
<tr>
<td>Σ⁺⁺⁺</td>
<td>(8, 0, 1, -2)</td>
<td>(-8, 0, 1, -4)</td>
</tr>
<tr>
<td>Σ⁺⁺⁺⁺</td>
<td>(9, 1, 19, -17)</td>
<td>(-8, 0, 1, -4)</td>
</tr>
<tr>
<td>Σ⁺⁺⁺⁺⁺</td>
<td>(7, -1, -22, 2)</td>
<td>(-9, -1, -12, -15)</td>
</tr>
<tr>
<td>Σ⁺⁺⁺⁺⁺⁺</td>
<td>(8, 0, -2, -17)</td>
<td>(-9, -1, -12, -15)</td>
</tr>
<tr>
<td>Σ⁺⁺⁺⁺⁺⁺⁺</td>
<td>(7, -1, -27, -9)</td>
<td>(-10, -2, -27, -18)</td>
</tr>
</tbody>
</table>

\[ \mu_B^{(u)} = \frac{Q_d}{Q_u} \mu_B^{(u)} \, , \quad \mu_B^{(s)} = \mu_B^{(s)} \],

(15)

with \( \bar{B} \) the isospin conjugate baryon in the isomultiplets of the baryon.

In general all the baryon-decuplet magnetic moments in the CBM (see Table 1) and naive NRQM fulfill the model-independent relations in the \( u- \) and \( d- \) flavor components, and the \( I- \) spin symmetry of the isomultiplets with the same strangeness in the \( s- \) flavor channel.

In order to induce the meson-cloud contributions inside the bag to the decuplet baryons with \( Y_R = 1 \) and \( J = \frac{3}{2} \), we now consider the minimal multiquark Fock space \( qqq + qqqqq \) whose possible SU(3) representations are constrained by the Clebsch–Gordan series \( 10 \otimes 27 \otimes 35 \) [8] so that the representation-mixing coefficients can be fixed by solving the eigenvalue problem of the \( 3 \times 3 \) hamiltonian matrix in (7). Since in the multiquark scheme the baryon-decuplet wave functions act nonperturbatively on the magnetic-moment operators consisting of both the quark and meson phase inertia parameters, one could have the meson-cloud \( \bar{q} q \) content, referring to all the possible flavor combinations to construct the pseudoscalar mesons, inside the bag through the channel of \( qqqqq \) multiquark Fock space where the two-body operator effect [9] discussed in the SU(2) CBM was proposed to be induced by the composite operators \( \psi i \gamma_5 \lambda_a \psi \) (\( a = 1, \ldots, 8 \)) in the SU(3) flavor sector [8].

Fig. 1 and Table 2 show that in the multiquark structure of the CBM with the chiral and SU(3) flavor symmetry breaking, the baryon-decuplet magnetic moments are predicted for the range of the bag radius up to 1.0 fm, together with those of the naive NRQM and the known experimental data. In particular we obtain \( \mu_{\Delta^{++}}^{\text{CBM}} = (1.01-1.37) \mu_p \) comparable to the experimental value \( \mu_{\Delta^{++}}^{\text{exp}} = (1.62 \pm 0.18) \mu_p \) [11] and the naive NRQM prediction \( \mu_{\Delta^{++}}^{\text{naive}} = 2 \mu_p \). For \( \mu_{\bar{p}} \), the CBM seems to predict well the value \( -(1.45-1.72) \) n.m., consistent with the experimental data \( -(1.94 \pm 0.17 \pm 0.14) \) n.m. [12] and the naive NRQM prediction \(-1.83 \) n.m. since
the $\mu_\Omega^-$ could be mainly achieved from the strange quark and kaon whose masses are kept in our massless profile approximation $(m_u + m_d)/m_s \approx m_\Xi^0/m_\Omega^0 = 0$. Also one can easily see in Fig. 1 that the full symmetry-breaking effects induce the

![Graph](image)

Table 2
The SU(3) CBM results in the $U$-spin symmetry-broken case compared with the naive NRQM and the experimental data

<table>
<thead>
<tr>
<th>$R$</th>
<th>$\mu_{\Delta^-}$</th>
<th>$\mu_{\Delta^0}$</th>
<th>$\mu_{\Delta^+}$</th>
<th>$\mu_{\Delta^{++}}$</th>
<th>$\mu_{\Sigma^-}$</th>
<th>$\mu_{\Sigma^0}$</th>
<th>$\mu_{\Sigma^+}$</th>
<th>$\mu_{\Xi^-}$</th>
<th>$\mu_{\Xi^0}$</th>
<th>$\mu_{\Omega^-}$</th>
</tr>
</thead>
<tbody>
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<td>-1.97</td>
<td>-0.38</td>
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<td>2.81</td>
<td>-1.88</td>
<td>-0.17</td>
<td>1.64</td>
<td>-1.72</td>
<td>0.14</td>
<td>-1.45</td>
</tr>
<tr>
<td>0.10</td>
<td>-2.03</td>
<td>-0.40</td>
<td>1.23</td>
<td>2.87</td>
<td>-1.94</td>
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<td>1.70</td>
<td>-1.77</td>
<td>0.17</td>
<td>-1.47</td>
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For the experimental data $\mu_{\Xi^0} = 4.52 \pm 0.50$ and $\mu_{\Omega^-} = -1.94 \pm 0.17 \pm 0.14$ we have referred to [11] and [12], respectively.
magnetic moments of the baryon decuplet to pull the $U$-spin symmetric predictions back to the experimental data. Here it is interesting to note from the vertical lines that the neutral baryons respect the Cheshire catness stronger than the charged ones do and the SU(3) CBM could be considered to be a candidate unifying the MIT bag and Skyrme models with predictions good enough to be insensitive to the bag radius.

Also the $s$-flavor magnetic moments $\mu_s^{(s)}$ given in Table 3 show us the stronger Cheshire catness than in $\mu_B$ and the fairly good consistency with the naive NRQM. Here one notes that the CBM prediction has a nonvanishing non-strangeness contribution to $\mu_\Omega$ as shown in Tables 1 and 3, with (30–40)% discrepancy with that of the naive NRQM only with the strangeness component. Moreover, the relations (15) are fulfilled even in the multiquark structure where the SU(3) flavor symmetry-breaking masses $m_u = m_d \neq m_s$, $m_\pi \neq m_K$ and decay constants $f_\pi \neq f_K$ do not affect the relations (15) in the $u$- and $d$-flavor channel without any strangeness and in the $s$-flavor channel with the same strangeness.

4. Conclusions

In the context of the naive NRQM and the SU(3) CBM in the adjoint representation with the SU(3) flavor symmetric limit, we have formulated model-independent relations of the baryon-decuplet magnetic moments, including the generalized Coleman–Glashow sum rules and $U$- and $I$-spin symmetries, all of which suggest the possibility of the CBM as a good candidate to unify the previous models such as the naive NRQM, Skyrme and MIT bag models. Furthermore, together with the above model-independent relations shared by two models, the SU(3) CBM numerical predictions propose in the baryon decuplet the effective NRQM [8] which has the meson cloud located both in the quark and meson phases around the quarks of the naive NRQM.
From the full CBM lagrangian in the adjoint representation where $\hat{\mathcal{L}}_{\text{CSB}}^{(a)}$ vanishes, we have derived the explicit magnetic-moment operators to evaluate the magnetic moments of the decuplet baryons with the SU(3) flavor symmetry breaking. To include the missing symmetry-breaking mass effect, we have introduced the higher representation mixing in the baryon-decuplet wave functions in the minimal multiquark structure where the symmetry-breaking mass could be treated nonperturbatively to yield the magnetic-moment spectrum with all the contributions from the chiral and SU(3) flavor symmetry-breaking terms.

As shown in Table 2 we have obtained the theoretical prediction of the CBM with the bag radius up to 1.0 fm, $\mu_{\Delta^+} = 2.81 - 3.81$ n.m. and $\mu_{\Omega} = -(1.45 - 1.72)$ n.m., which are around 25% and 15% in agreement with the experimental data, respectively. For comparison with the naive NRQM, the other baryon-decuplet magnetic moments in the CBM are also predicted with the Cheshire cat property in Fig. 1, and using the flavor projection operators [8] we have singled out the strangeness components in these magnetic-moment predictions. Here one could figure out that the meson-cloud contributions originated from the pseudoscalar quark bilinear composite operators $\bar{\psi} \gamma_5 \lambda_a \psi \sim \pi_a$ and the strange quarks inside the bag, additional to the massive kaons outside the bag.

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References