QUARKS AND LEPTONS*

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Abstract:

We review the physics of quarks and leptons within the framework of gauge theories for the weak and electromagnetic interactions. The Weinberg–Salam SU(2) × U(1) theory is used as a “reference point” but models based on larger gauge groups, especially SU(2)_L × SU(2)_R × U(1), are discussed. We distinguish among three “generations” of fundamental fermions: The first generation (e^+, e^-, u, d), the second generation (μ^+, μ^-, c, s) and the third generation (τ^+, τ^-, t, b). For each generation we discuss the classification of all fermions, the charged and neutral weak currents, possible right-handed currents, parity and CP-violation, fermion masses and Cabibbo-like angles and related problems. We review theoretical ideas as well as experimental evidence, emphasizing open theoretical problems and possible experimental tests. The possibility of unifying the weak, electromagnetic and strong interactions in a grand unification scheme is reviewed. The problems and their possible solutions are presented, generation by generation, but a brief subject-index (following the table of contents) enables the interested reader to follow any specific topic throughout the three generations.

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1. Introduction

1.1. Quark and lepton spectroscopy

At the time of this writing we have clear evidence for the existence of four different flavors of quarks and five flavors of leptons. There is indirect evidence for, at least, one additional quark and one additional lepton. In the last few years, various authors have entertained speculations involving even larger numbers of quarks and leptons. The proliferation of quarks and leptons has been sudden and largely unexpected. For ten years (1964–1974), the world of particle physics was more or less content with three quarks and four leptons. However, now we regard the number of fundamental fermions as yet another unknown parameter in our theoretical picture of the structure of matter.

With the proliferation of quarks and leptons, a new field of investigation has opened up. Suddenly, the old \( \mu-e \) puzzle is generalized to the much more complex problem of understanding the quark and lepton mass spectrum. The problem of computing the Cabibbo angle expands into the question of calculating several such angles in the quark and lepton sectors of the theory and a possible connection is established between these angles and \( CP \) violation. An important (but poorly understood) relation probably exists between the masses of the fundamental fermions and the Cabibbo angles. Several hints seem to connect quarks and leptons to each other and one wonders about the study of all fundamental fermions under the roof of a common “grand unification” gauge group of the strong, electromagnetic and weak interactions. Some of these problems have been with us in one form or another for several decades, but the theoretical and experimental developments of the last few years have provided a new dimension to each one of them.

Hadron spectroscopy is still very much alive. It is still interesting, as well as important, to classify the observed hadrons, study strong, weak and electromagnetic transitions between hadronic levels, understand the quark rules which govern hadronic collisions, and search for exotic hadrons. Theoretically, the study of the hadron as a system of confined quarks is one of the most important unsolved problems of physics. Quantum-Chromo-Dynamics (QCD) is the candidate theory for explaining quark confinement, and some of its features (such as asymptotic freedom) are tested in deep inelastic collisions and in processes which are forbidden by the Zweig–Iizuka rule. All of these topics relate to hadron spectroscopy.

At the same time, we begin to see glimpses of the next spectroscopy: the spectroscopy of quarks and leptons. Here we try to understand the mass pattern of the fundamental fermions (rather than the composite hadrons). We study weak transitions between fundamental fermions (sometimes by extracting the necessary information from weak hadronic transitions). We ask questions such as: How many types of quarks exist? How many leptons? What are their symmetry properties? What is the gauge group of the weak and electromagnetic interactions? To which representations of this gauge group do we assign the left-handed and the right-handed fundamental fermions? What are the properties of the gauge vector bosons and the Higgs scalar particles in such a theory? What are the experimental values of all Cabibbo-like angles? How can we calculate these angles? Can we incorporate the phenomenon of \( CP \)-violation into our gauge theory, and predict its observed magnitude?

The present review is devoted to the open problems of quark and lepton spectroscopy. Some of these problems are listed above while others will be raised throughout our discussion. Whenever the answers are known, we try to provide them. When they are not known (in most cases!) we outline some of the possible directions of attacking the problem. Whenever possible, we discuss experimental tests of the various ideas and models.
1.2. Theoretical framework: a “minimal” SU(2) × U(1) theory

We do not yet know which is the ultimate gauge algebra of the weak and electromagnetic interactions [1]. It is likely, but not certain, that the correct algebra includes SU(2) × U(1) as a sub-algebra (perhaps in a trivial way, i.e. the correct algebra is SU(2) × U(1)). We will devote some of our attention to models based on different gauge algebras. However, all the algebras considered by us include SU(2) × U(1). Consequently, we choose to use SU(2) × U(1) as the basic theoretical framework for our discussion [2].

Unless otherwise stated, we will therefore always assume that the weak and electromagnetic interactions are described by a renormalizable gauge theory based on SU(2) × U(1). The theory contains four gauge bosons W⁺, W⁻, Z° and γ, corresponding to the four generators of the algebra. The parameters of the theory include the fermion masses, the Cabibbo angles, the fine structure constant α, the W⁺-mass, and one additional parameter — the Weinberg angle θw, defined by the relation:

\[ Z_\mu = W_\mu^3 \cos \theta_w + B_\mu \sin \theta_w; \quad A_\mu = -W_\mu^3 \sin \theta_w + B_\mu \cos \theta_w. \quad (1.1) \]

Where \( Z_\mu, A_\mu, W_\mu^3, B_\mu \) are respectively, the vector fields of the neutral weak gauge boson, the photon, the third component of the SU(2) triplet of gauge fields, and the U(1) gauge field.

From time to time we will speculate on the properties of the Higgs scalar particles of the theory. However, unless otherwise stated, we will assume the simplest allowed Higgs structure. In the “standard” left-handed SU(2) × U(1) model, this involves four fields transforming like two SU(2) doublets. Three of these fields “convert” into the longitudinal modes of the vector fields when the W⁺, W⁻ acquire their masses, while one neutral scalar Higgs state remains as a physical particle. The simplest Higgs structure (as well as other patterns based only on SU(2) doublets) provide us with the well-known mass relation:

\[ M_Z = M_{W^\pm} / \cos \theta_w; \quad M_{W^\pm} = 37 \text{ GeV} / \sin \theta_w. \quad (1.2) \]

Whenever necessary, we will assume that the SU(3) gauge group of color is the underlying algebra of the interactions between quarks and gluons. This assumption is not crucial to most of our discussion, and it becomes relevant only when we discuss the possible unification of strong, weak and electromagnetic interactions.

We summarize: unless otherwise specified we assume throughout this review

(i) SU(2) × U(1) gauge theory of weak and electromagnetic interactions.
(ii) Simplest possible Higgs particle structure.
(iii) Color SU(3) gauge theory of quarks and gluons.

1.3. Outline of review

Our discussion does not follow the historical order of developments in the physics of quarks and leptons. Instead, we follow a path which is, pedagogically, more appealing to us and which could have been the historical order!

We introduce the fundamental fermions in three “generations”:

first generation: \( \nu_e, e^-, u, d; \)
second generation: \( \nu_\mu, \mu^-, c, s; \)
third generation: \( \nu_\tau, \tau^-, t, b. \)
For each generation we discuss all theoretical and experimental questions which arise, even if, historically, some of these questions may have emerged only after the discovery of a subsequent generation. Thus, the discussion of topics such as parity violation in Atomic physics or quark-lepton unification by an SU(5) or an SO(10) algebra precedes not only a discussion of the charmed quarks, but also the muon or the strange quark.

For each generation we study all known processes involving the fundamental fermions of that (and preceding) generation. At each stage we introduce the new physics ingredients which are necessary for a discussion of the appropriate generation, and review the combined pattern of quarks and leptons up to that point.

Due to our “quasi-historical” approach, certain topics are discussed several times in different sections. In order to enable the reader who is interested in one specific issue to find all references to it, we have included a brief subject index which appears immediately after the table of contents.

The detailed plan of the review is the following: sections 2, 3 and 4 are devoted, respectively, to the first, second and third generation of fermions. Section 5 mentions some open problems.

In section 2.1 we briefly review the properties of charged currents involving first-generation fermions. This is followed in section 2.2 by a discussion of neutral currents and their implications for the classification of right-handed fermions. We emphasize that neutral current experiments enable us to obtain information about the classification of a given fermion into a multiplet of the gauge group, even if other fermions in the same multiplet are not observed. Sections 2.3 and 2.4 are devoted, respectively, to the properties of the right-handed electron and the right-handed u and d quarks. The complete SU(2) × U(1) picture of left and right-handed first-generation fermions is then reviewed in section 2.5, reaching the tentative conclusion that the standard Weinberg–Salam model is extremely successful, except for the lack of observed violation of parity in atomic physics experiments. This last difficulty leads us to search for a somewhat larger gauge group which, however, duplicates the Weinberg–Salam model for charged current processes as well as neutrino-induced neutral currents. Such a model is the SU(2) × SU(2) × U(1) scheme which we discuss in detail in section 2.6.

In section 2.7 we turn to the mass spectrum of first-generation fermions. We observe that the spectrum is essentially well “understood”, at least qualitatively. (We realize that the pattern of the second generation turns this “understanding” into a totally unexplained mystery, but that happens only in section 3.10.) The possibility of relating quarks and leptons to each other is not treated in section 2.8, followed by a discussion of several unification schemes in sections 2.9 and 2.10. We distinguish between “simple unification” schemes which unify only the weak and electromagnetic interactions, using a simple (or “pseudosimple”) Lie algebra and “grand unification” schemes which relate the strong, weak and electromagnetic interactions. We show that all “simple unification” schemes must involve an SU(3) subalgebra. Among “grand unification” models we emphasize those based on SU(5) and SO(10). We conclude section 2 by a brief summary of the first-generation fermions.

Our discussion of the second generation of fermions begins in section 3.1, in which the left-handed fermions are classified and the Cabibbo angle is introduced. The possibility of right-handed doublets of quarks is discussed in sections 3.1, 3.2 and 3.3. Section 3.2 is devoted to arguments against the existence of a (c, d)R doublet. In section 3.3 we discuss experimental tests for the existence of a (c, s)R doublet. We conclude that, at present, there is no compelling evidence for or against such a doublet.

Flavor conserving neutral currents involving second-generation fermions are briefly discussed in section 3.4, followed by a general analysis of flavor conservation by neutral currents in section 3.5.
We review the theoretical conditions for “natural” flavor conservation, and conclude, in section 3.6, that there is strong evidence not only against $|\Delta S| = 1$ neutral currents, but also against $|\Delta C| = 1$ neutral currents. The $SU(2) \times U(1)$ classification of second-generation fermions is summarized in section 3.7.

Possible Cabibbo-like angles and mixing effects in the leptonic sector are introduced in section 3.8. Various mechanisms for transitions such as $\mu \rightarrow e \gamma$ are briefly discussed.

One of the most attractive features of some gauge models is the inclusion of a $CP$-violating interaction within the framework of the theory. At the level of the first two generations of fermions, $CP$-violation can be introduced only through the couplings of the Higgs mesons or through right-handed currents. All of these issues are analyzed in section 3.9, where we also conclude that the “standard” left-handed model can accommodate $CP$-violation only at the six-quark level. A brief discussion of the masses of second-generation fermions is given in section 3.10. Unfortunately, all we can do in this discussion is to expose our total ignorance and our lack of ability to compute either the fermion masses or the Cabibbo angle. While the main ingredients of the concept of “grand unification” are introduced already in section 2, we devote section 3.11 to grand unification schemes which relate the first and second generation fermions to each other. We emphasize the $E(7)$ model, but conclude that it leads to an unacceptable value of the Weinberg angle. Section 3.12 contains a brief summary of the second-generation fermions.

Our discussion of the third generation of fermions begins with a general introduction in section 4.1. We mention several possible patterns for the fifth and sixth quark, and conclude that the assumption that the third generation is similar to its two predecessors, is an attractive, but unproven, possibility. We then introduce the third-generation fermions one by one. Section 4.2 contains a discussion of the properties of the $\tau$-lepton with an emphasis on its decay modes. In section 4.3 we show that it is almost certain that the $\tau$-lepton is accompanied by its own neutral lepton. Whether this lepton is very light ($<300$ MeV) or heavy ($>2$ GeV), we do not know. There are many possible models for the leptonic sector including six or more leptons. We mention some of them in section 4.4. Section 4.5 is devoted to a general discussion of possible experimental and theoretical motivations for the introduction of the top and bottom (t and b) quarks. The “quarkonium” family of $b\bar{b}$ or $t\bar{t}$ mesons is discussed in section 4.6 with special emphasis on the properties of the recently discovered $T$ and $T'$ particles. We indicate that the $b\bar{b}$ assignment for $T$ is slightly favored over $t\bar{t}$.

Sections 4.7–4.10 contain a detailed discussion of the left-handed six-quark model. The model is introduced in section 4.7 where three Cabibbo-like angles are introduced and their numerical values are estimated. Sections 4.8 and 4.9 are devoted to a discussion of the weak production and weak decays of mesons including $b$-quarks and $t$-quarks, respectively. A detailed discussion of $CP$-violation in the $K^-K^+$ system and the electric dipole moment of the neutron is presented in section 4.10. In section 4.11 we briefly consider right-handed currents involving the six quarks of the first three generations. Using a variety of theoretical and experimental arguments, we conclude that within $SU(2) \times U(1)$, such currents are unlikely. Finally, we summarize the third-generation fermions in section 4.12.

Our final section is devoted to a brief overall summary and to the discussion of open experimental and theoretical problems. In section 5.1 we discuss our present overall view of the correct gauge theory of quarks and leptons. We consider $SU(2)_L \times SU(2)_R \times U(1)$ to be the most likely candidate. In section 5.2 we review several experimental issues which remain open. These include the trimuon events observed in neutrino scattering, the question of parity violation in neutral currents, the search for right-handed charged currents, the decays of the $\tau$ lepton, some open problems in the $\psi$-family...
and T-family and the question of neutrino masses. Our review concludes in section 5.3 with a short list of outstanding theoretical problems. Which is the correct gauge group? Do we have any theoretical guideline for the total number of quarks and leptons? How can we calculate the masses and angles which are generated by the spontaneous symmetry breaking mechanism? What is the connection between quarks and leptons? The answer to any of these profound questions will undoubtedly provide us with a significantly better understanding of the world of quarks and leptons.

2. First-generation fermions: $\nu_e$, $e^-$, $u$, $d$

2.1. Charged weak currents and the classification of left-handed fermions

The first generation of fermions includes two leptons ($\nu_e$ and $e^-$), and two quarks ($u$ and $d$). The charged currents represent the $\nu_e \leftrightarrow e^-$ transition and the $u \leftrightarrow d$ transition.

Information concerning $\nu_e \leftrightarrow e^-$ is obtainable from $\beta$-decay and $\mu$-decay:

$$\nu_e \rightarrow e^- + \bar{\nu}_e;$$

\hspace{1cm} (2.1)

$$\mu^- \rightarrow \nu_\mu + e^- + \bar{\nu}_e.$$ \hspace{1cm} (2.2)

In both cases all data are consistent with a pure left-handed $V$-$A$ leptonic charged current:

$$J^- \propto \bar{\nu}_e \gamma_\alpha (1 - \gamma_5) e^-.$$ \hspace{1cm} (2.3)

Small contributions from non-vectorial currents ($S, P, T$) are not completely excluded. However, in the absence of any good theoretical reason to consider such terms, we are probably safe in ignoring them. The contribution of right-handed $V + A$ currents is either absent or very small. Here we could envisage interesting theoretical models in which a right-handed $\bar{\nu}_e e^-$ term is actually small, but non-vanishing. Two examples of such schemes are:

(i) In an $SU(2)_L \times SU(2)_R \times U(1)$ gauge theory [3] (see section 2.6) we could have $M_{WR} \gg M_{WL}$, yielding right-handed charged-current effects which are usually suppressed by a factor: $(M_{WL}/M_{WR})^4$.

(ii) In an $SU(2) \times U(1)$ model, the right-handed electron could be in a doublet with a heavy neutral lepton $N_e$ while a right-handed $\nu_e$ is in a different SU(2) multiplet (assuming that it is not massless). We could then have a small Cabibbo-like angle $\epsilon$, mixing the right-handed $N_e$ and $\nu_e$. The right-handed $\nu_e \leftrightarrow e^-$ transition would then be suppressed by a factor $\sin^2 \epsilon$.

Both of these possibilities are not excluded. The most we could do within the framework of such models, would be to use the data in order to set an upper bound on $M_{WL}/M_{WR}$ or on $\sin \epsilon$. In both cases the bound will depend on other factors, such as the classification of $u$, $d$ and $\mu$.

Having mentioned these possibilities we will say no more about them, and assume that the left-handed $\nu_e$ and $e^-$ transform like a double of $SU(2) \times U(1)$:

$$\left( \begin{array}{c} \nu_e \\ e^- \end{array} \right) _L,$$ \hspace{1cm} (2.4)

while the right-handed electron $e_R^-$ is not in the same $SU(2) \times U(1)$ multiplet as $\nu_e$.

Note that we have not used here the information coming from the only measured purely leptonic process involving first generation leptons:

$$\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-.$$ \hspace{1cm} (2.5)
This process involves charged as well as neutral currents, and the present quality of the data enables us only to draw conclusions concerning the neutral currents, assuming that the charged currents are purely left-handed. We discuss $\bar{\nu}_e e^-$ elastic scattering in section 2.3.

The charged current involving u and d quarks cannot be studied directly. Like all quark currents, it can be determined only with the aid of several theoretical assumptions. Experimentally, $\beta$-decay data enable us to compute $G_A/G_V$ where $G_A$ and $G_V$ are defined by the phenomenological form of the hadronic current

$$J^- \propto \bar{p}\gamma_\alpha (G_V + G_A\gamma_5)n.$$  

The vector and axial-vector quark couplings are related to $G_A/G_V$ by the Adler–Weisberger relation [4]. Its success hints that the charged quark current is of the form:

$$J^- \propto \bar{u}\gamma_\alpha (1 - \gamma_5)d.$$  

(2.7)

This conclusion is supported by several other successes of the SU(2) × SU(2) algebra of currents [5]. These include several different sum rules for deep inelastic scattering, relations between deep inelastic ep and $\nu p$ scattering and several current algebra predictions. All of these support the charged quark current of eq. (2.7). However, small contributions from other terms cannot be excluded, and the two simple models outlined above in the leptonic case, could again yield small right-handed $u \leftrightarrow d$ transitions.

Barring such a situation, we assert that the left-handed u and d are in an SU(2) × U(1) doublet:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L,$$

(2.8)

while $u_R$ and $d_R$ are in two different SU(2) × U(1) multiplets.

The approximate equality ("universality") of the couplings of the $u \leftrightarrow d$ and $\nu e \leftrightarrow e^-$ transitions is a necessary consequence of the gauge theory. The deviation from universality is presumably due to impurities in the eigenstates of the weak SU(2) × U(1) multiplets, and the Cabibbo mixing of states explains this deviation (see section 3.1).

We conclude that the left-handed first-generation fermions fall, to a good approximation, into two SU(2) × U(1) doublets:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L; \begin{pmatrix} u \\ d \end{pmatrix}_L.$$  

(2.9)

We now turn to the classification of right-handed fermions.

### 2.2. Neutral weak currents and the classification of right-handed fermions

The SU(2) × U(1) classification of right-handed fermions could be studied, in principle, by observing right-handed charged current transitions involving $\nu e$, $e$, $u$, $d$, etc. No such transitions have been clearly observed. Within the SU(2) × U(1) theory this still leaves two possibilities:

(i) The right-handed fermions $\nu e_R$, $e_R$, $u_R$, $d_R$ belong to SU(2) × U(1) singlets. Consequently, they do not couple to $W^\pm$ and do not participate in right-handed charged current transitions.

(ii) Some or all of the right-handed fermions belong to SU(2) × U(1) doublets (or higher multiplets). The right-handed charged currents are not experimentally observed at a given energy because
the other fermions in the same multiplets are heavy, and cannot be produced at that energy.

At any given energy below the threshold for producing the alleged heavy right-handed fermions, it is not possible to distinguish between these two alternatives on the basis of charged current processes.

On the other hand, neutral current experiments do not necessitate the presence of the elusive heavy fermion (or fermions) which belong to the same representation. The weak neutral current in SU(2) \times U(1) has the form:

\[ J_{\text{neutral}} = J_3 - 2 \sin^2 \theta_W J_{\text{em}}, \quad (2.10) \]

where \( J_3 \) is the third component of the SU(2)-vector, and \( \theta_W \) is the Weinberg angle (eq. 1.1). Consequently, any neutral current measurement involving a fundamental fermion \( f \), provides us with a measurement of \( I_3(f) \). When the vector and axial vector neutral currents are separately determined, we may deduce the values of \( I_3(f_L) \) and \( I_3(f_R) \) for the left- and right-handed fermion \( f \). Note, however, that the Weinberg angle \( \theta_W \) has to be determined before we can translate neutral current data into statements about \( I_3(f_R) \).

In general, the determination of \( I_3(f_R) \) does not uniquely determine the SU(2) \times U(1) multiplet of the right-handed fermion (e.g. \( I_3 = 0 \) could correspond to \( f = 0 \) or \( f = 1 \), etc.). However, if we limit ourselves to SU(2) singlets and doublets, \( I_3(f_R) \), of course, determines the multiplet.

The crucial point here is the fact that we do not have to perform any high energy experiments in order to determine \( I_3(f_R) \), even if the other fermion in the same doublet is extremely heavy. Thus, if an atomic physics experiment or a nuclear reactor experiment tells us that \( I_3(e_R) = -\frac{1}{2} \), we know that a heavy right-handed neutral lepton with \( I_3 = \frac{1}{2} \) must exist. This is one of the most fascinating aspects of the properties of neutral currents in a gauge theory: a “table-top” atomic physics experiment can tell us about the existence of a new fundamental fermion with a mass of a few GeV!

In order to translate these principles into practical calculations, we now define the following left-handed and right-handed neutral current couplings [6] for each fermion \( f \):

\[ J_{\text{neutral}} \propto g_L^f \gamma_\alpha (1 - \gamma_5) f + g_R^f \gamma_\alpha (1 + \gamma_5) f. \quad (2.11) \]

In SU(2) \times U(1) we have:

\[ g_L^f = 2I_3(f_L) - 2 \sin^2 \theta_W Q(f), \quad (2.12) \]
\[ g_R^f = 2I_3(f_R) - 2 \sin^2 \theta_W Q(f), \quad (2.13) \]

where \( Q(f) \) is the electric charge of \( f \).

For the left-handed first-generation fermions we assume (section 2.1):

\[ I_3(\nu_e) = \frac{1}{2}; \quad I_3(e^-) = -\frac{1}{2}; \quad I_3(u_L) = \frac{1}{2}; \quad I_3(d_L) = -\frac{1}{2}. \quad (2.14) \]

There is no evidence for a right-handed \( \nu_e \). All known experiments involve only \( \nu_e \). Consequently, we can shed no light on the existence and assignment of \( \nu_e \). The quantities that we wish to study are: \( I_3(e_R), I_3(u_R) \) and \( I_3(d_R) \). For convenience, we introduce the notation:

\[ \alpha_f = 2I_3(f_R); \quad x = \sin^2 \theta_W. \quad (2.15) \]

Hence:

\[ g_L^e = -1 + 2x; \quad g_R^e = \alpha_e + 2x; \]
\[ g_L^u = 1 - \frac{4}{3}x; \quad g_R^u = \alpha_u - \frac{4}{3}x; \]
\[ g_L^d = -1 - \frac{2}{3}x; \quad g_R^d = \alpha_d + \frac{2}{3}x. \quad (2.16) \]
The next two subsections are devoted to a phenomenological discussion of the parameters \( \alpha_e, \alpha_u, \alpha_d \) and \( x \).

### 2.3. The right-handed electron

The most direct information concerning the properties of the right-handed electron comes from neutrino-electron elastic scattering. Four such processes can be, in principle, measured:

\[
\begin{align*}
\bar{\nu}_e + e^- &\rightarrow \bar{\nu}_e + e^-, \\
\nu_e + e^- &\rightarrow \nu_e + e^-, \\
\bar{\nu}_\mu + e^- &\rightarrow \bar{\nu}_\mu + e^-, \\
\nu_\mu + e^- &\rightarrow \nu_\mu + e^-.
\end{align*}
\] (2.17, 2.18, 2.19, 2.20)

The processes (2.17), (2.18) involve charged currents as well as neutral currents, while (2.19), (2.20) are pure neutral current reactions (to lowest order). Assuming the standard V-A form for the charged currents and neglecting terms of order \( m_e/E_\nu \), we find:

\[
\begin{align*}
\sigma(\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-) &= K E_\nu \left[ (g_e^R)^2 + \frac{1}{3} (2 + g_e^L)^2 \right], \\
\sigma(\nu_e e^- \rightarrow \nu_e e^-) &= K E_\nu \left[ \frac{1}{3} (g_e^R)^2 + (2 + g_e^L)^2 \right], \\
\sigma(\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-) &= K E_\nu \left[ (g_\mu^R)^2 + \frac{1}{3} (g_\mu^L)^2 \right], \\
\sigma(\nu_\mu e^- \rightarrow \nu_\mu e^-) &= K E_\nu \left[ \frac{1}{3} (g_\mu^R)^2 + (g_\mu^L)^2 \right],
\end{align*}
\] (2.21, 2.22, 2.23, 2.24)

where

\[
K = G^2 m_e/2\pi.
\] (2.25)

Experimentally, the process (2.17) has been measured [7] using the \( \bar{\nu}_e \) flux coming out of a nuclear reactor. In that case, \( m_e/E_\nu \) is not negligible and additional experimental factors enter. The limits on \( g_e^R \) and \( g_e^L \) for two different intervals of the neutrino energy are shown in fig. 1.

The reaction (2.18) has not been measured. Reactions (2.19) and (2.20) were studied in the Gargamelle Bubble Chamber [8]. Three \( \bar{\nu}_\mu e \) events and no \( \nu_\mu e \) events were seen, yielding a measured rate for (2.19) and an upper limit for (2.20). The corresponding restrictions on \( g_\mu^L \) and \( g_\mu^R \) are shown in fig. 2. Additional data on the same reaction have been recently analyzed but not yet published by the Aachen-Padova collaboration [9]. Their recent data are consistent with fig. 2. The present data for the reactions (2.17)–(2.20) are summarized in fig. 3. We find that there are two overlap regions which are consistent with all data. Since eqs. (2.21)–(2.24) are invariant under \( g_e^R \leftrightarrow -g_e^R \), the two allowed regions are symmetric with respect to the \( g_e^L \) axis. The most probable values for \( g_e^L \) and \( g_e^R \) are

\[
\begin{align*}
g_e^L &= -0.5 \pm 0.2, \\
g_e^R &= \pm(0.5 \pm 0.2).
\end{align*}
\] (2.26, 2.27)

Since we know that

\[
g_e^L = -1 + 2x,
\] (2.28)
Fig. 1. The allowed range of $g_e^L$ and $g_e^R$ as determined from $\sigma_{el}(\nu_e e)$. The two shaded regions correspond to data from two different ranges of neutrino energy [7].

Fig. 2. The allowed range of $g_e^L$ and $g_e^R$ as determined from $\sigma_{el}(\nu_e e)$ and $\sigma_{el}(\bar{\nu}_e e)$ [8].

Fig. 3. The allowed range of $g_e^L$ and $g_e^R$, as determined from all available data on $\nu e$ scattering. The two diagonal lines represent a left-handed $SU(2) \times U(1)$ model ($g_L^L = g_R^R = 1$) and a vector $SU(2) \times U(1)$ model ($g_L^L = g_R^R$). In both cases $x = \sin^2 \theta_W \approx 0.25$. 
we determine $x$ to be:
\[ x = 0.25 \pm 0.1. \]  (2.29)

Consequently, the two solutions for $g_e^R$ yield the following values for $\alpha_e = 2I_e(e_R)$:

I. $\alpha_e = 0$; \hspace{1cm} $I_3(e_R) = 0$,

II. $\alpha_e = -1$; \hspace{1cm} $I_3(e_R) = -\frac{1}{2}$.

On the basis of the "world's data" on neutrino–electron scattering we find that the right-handed electron may be either in an SU(2) × U(1) singlet or in a doublet, while in both cases $x = \sin^2 \theta_W \approx 0.25$.

Note that solution I ($\alpha_e = 0$) corresponds to the "standard" Weinberg–Salam left-handed model. Solution II ($\alpha_e = -1$) leads to:
\[ g_e^L - g_e^R = 0 \]  (2.30)

and to a vanishing axial vector coupling of the electron to the neutral current. A new heavy neutral right-handed lepton is predicted, in this case, as the $I_3 = +\frac{1}{2}$ member of the doublet containing the right-handed electron.

Notice that both solutions happen to correspond to $(g_e^L)^2 = (g_e^R)^2$. Consequently, in both cases, $\sigma_{el}(\bar{\nu}_e e) = \sigma_{el}(\nu_e e)$. However, in the case of solution I ($\alpha_e = 0$) the ratio $\sigma(\bar{\nu}_e e)/\sigma(\nu_e e)$ is extremely sensitive to the value of $\sin^2 \theta_W$. The general expression is:
\[
\frac{\sigma_{el}(\bar{\nu}_e e)}{\sigma_{el}(\nu_e e)} = \frac{16x^2 + (1 - 4x) + 3\alpha(\alpha + 4x)}{16x^2 + 3(1 - 4x) + \alpha(\alpha + 4x)} \quad \text{for } \alpha = 0
\]
\[
\frac{16x^2 + (1 - 4x) + 3\alpha(\alpha + 4x)}{16x^2 + 3(1 - 4x) + \alpha(\alpha + 4x)} \quad \text{for } \alpha = -1.
\]  (2.31)

An accurate measurement of this ratio would provide, in this case, a good determination of $\theta_W$ (fig. 4). For solution II, $\sigma(\bar{\nu}_e e)/\sigma(\nu_e e) = 1$ regardless of the value of $\theta_W$. The unpublished data of ref. [9] seems to indicate that $\sigma_{el}(\bar{\nu}_e e) \neq \sigma_{el}(\nu_e e)$. It would be interesting to have better data on this issue.

An additional measurement of the neutral weak current of the electron is provided by the search for parity violation in atomic physics phenomena. Such parity violations can be due to products of vector and axial vector terms, where one term represents the electron neutral current while the other represents the nucleon (or u and d) neutral current.

An accurate measurement of this ratio would provide, in this case, a good determination of $\theta_W$ (fig. 4). For solution II, $\sigma(\bar{\nu}_e e)/\sigma(\nu_e e) = 1$ regardless of the value of $\theta_W$. The unpublished data of ref. [9] seems to indicate that $\sigma_{el}(\bar{\nu}_e e) \neq \sigma_{el}(\nu_e e)$. It would be interesting to have better data on this issue.
In general, the effects of atomic parity violation would be proportional to [10]:

\[ J^A_e J^V_q + J^V_e J^A_q, \]

where the indices \( A, V, e, q \) stand for axial vector, vector, electron and quark respectively. However, in heavy atoms the contribution of the \( J^A_q \) term is very small (proportional to the recoil of the nucleus), and the dominant term is given by:

\[ J^A_e J^V_q \propto (g^L_u - g^R_d) [(g^L_u + g^R_d)(2Z + N) + (g^L_d + g^R_d)(Z + 2N)]. \]  

(2.32)

Here \( Z \) and \( N \) are, respectively, the number of protons and neutrons in the atom under discussion.

The experiments carried out, so far [11], used bismuth atoms (\( Z = 83, N = 126 \)). Assuming \( x = 0.25 \), we get from eq. (2.16):

\[ g^L_u = 0.67; \quad g^L_d = -0.83; \quad g^R_u = \alpha_u - 0.33; \quad g^R_d = \alpha_d + 0.17. \]

(2.33)

Hence, the right-hand side of eq. (2.32) is:

\[ K = -(1 + \alpha_u)(-126 + 292\alpha_u + 335\alpha_d). \]

(2.34)

In the case of the standard Weinberg–Salam left-handed model (\( \alpha_e = \alpha_u = \alpha_d = 0 \)), we have \( K = 126 \). The latest experimental results together with the “standard” assumptions on the atomic wave functions, yield [11] (in the same units):

\[ K = -14 \pm 24 \text{ (Oxford)}; \quad K = +5 \pm 23 \text{ (Seattle)}. \]

The conversion of the directly measured rotation angle to a value of our \( K \)-parameters requires assumptions on the atomic wave-function. There are some indications of ambiguities in these calculations. The above values are obtained if we neglect all shielding corrections and configuration mixing effects. Such corrections may shrink the gap between theory and experiment. It is still possible that the discrepancy between the measured effects and the predictions of the Weinberg–Salam model is much smaller than suggested here. We will have to wait for more accurate experiments and a better understanding of the atomic physics factors, in order to be certain that the discrepancy is indeed substantial.

One intriguing possibility is, of course, that the correct value is \( K = 0 \). This could be due to the electron coupling, yielding:

\[ \alpha_e = 2I_3(e_R) = -1 \]

(2.35)

and preferring the solution II of fig. 3. It could also be due to the quark coupling, namely:

\[ -126 + 292\alpha_u + 335\alpha_d = 0. \]

(2.36)

This is an unlikely possibility, but we return to it briefly in section 2.4. A third possibility is that the weak neutral current actually conserves parity. Such a hypothesis, together with the \( \nu N \) and \( \bar{\nu} N \) scattering data, would force us outside \( SU(2) \times U(1) \), and necessitate more than one neutral weak boson. We return to this possibility in section 2.6.

We summarize our conclusions concerning the right-handed electron:

Within \( SU(2) \times U(1) \), neutrino–electron elastic scattering data yield \( \sin^2 \theta_w \approx 0.25 \) and allow the right-handed electron to be in an SU(2) singlet (\( \alpha_e = 0 \)) or doublet (\( \alpha_e = -1 \)). Atomic parity violation experiments indicate that the second possibility (\( e_R \) in a doublet) is more likely.
2.4. The right-handed u and d quarks

Information on the properties of right-handed u and d quarks can be obtained from several different processes. The most important among these are deep inelastic neutrino and antineutrino neutral-current processes.

The measured experimental quantities are:

\[ R_\nu = \frac{\sigma(\nu + N \to \nu + \text{anything})}{\sigma(\nu + N \to \mu^- + \text{anything})}, \quad (2.37) \]

\[ R_\bar{\nu} = \frac{\sigma(\bar{\nu} + N \to \bar{\nu} + \text{anything})}{\sigma(\bar{\nu} + N \to \mu^+ + \text{anything})}. \quad (2.38) \]

Experimentally, an average over the four available experiments [12], ignoring energy variation, gives:

\[ R_\nu = 0.28 \pm 0.04; \quad R_\bar{\nu} = 0.38 \pm 0.05. \]

All experiments are, more or less, consistent with these values.

If we assume that deep inelastic neutrino scattering can be described by the quark–parton model and if we temporarily neglect the contribution of the “ocean” of q\bar{q} pairs in the nucleon, we obtain:

\[ R_\nu = \frac{1}{4}(L + \frac{1}{4}R); \quad (2.39) \]

\[ R_\bar{\nu} = \frac{1}{4}(L + 3R). \quad (2.40) \]

where

\[ L = (g_u^L)^2 + (g_d^L)^2 = (1 - \frac{3}{4}x)^2 + (-1 + \frac{3}{4}x)^2, \quad (2.41) \]

\[ R = (g_u^R)^2 + (g_d^R)^2 = (\alpha_u - \frac{3}{4}x)^2 + (\alpha_d + \frac{3}{4}x)^2. \quad (2.42) \]

Experimentally, the quark–parton model is in reasonable agreement with experiment. However, the contribution of the “q “ocean” amounts to 10% or so of the cross section. In particular, the ratio:

\[ \sigma(\nu + N \to \mu^- + \text{anything})/\sigma(\bar{\nu} + N \to \mu^+ + \text{anything}) \]

is predicted to be \( \frac{1}{3} \) in the absence of “ocean” contributions. The measured values [13] are around 0.4–0.45, yielding:

\[ B = \int_0^1 \frac{[q(\xi) - \bar{q}(\xi)]d\xi}{[q(\xi) + \bar{q}(\xi)]d\xi} \approx 0.8, \quad (2.44) \]

where \( q(\xi), \bar{q}(\xi) \) are the usual momentum distribution functions for quarks and antiquarks in the nucleon and \( \xi \) is the momentum fraction carried by the quark. \( B = 1 \) corresponds to a vanishing “ocean” contribution.

In the presence of “ocean” contributions (but neglecting s\bar{s}, c\bar{c} pairs in the “ocean”), eqs. (2.39), (2.40) are modified as follows:

\[ R_\nu = \frac{1}{4}\left[ L + R\left(\frac{2 - B}{2 + B}\right)\right] \approx \frac{1}{4}(L + 0.42R); \quad (2.45) \]

\[ R_\bar{\nu} = \frac{1}{4}\left[ L + R\left(\frac{2 + B}{2 - B}\right)\right] \approx \frac{1}{4}(L + 2.3R). \quad (2.46) \]
It is easy to see that $R$ is not very sensitive to the values of $\alpha_u$ and $\alpha_d$. In fact, fig. 5 shows $R$ as a function of $x$ for all four combinations of $\alpha_u = 1, 0$ and $\alpha_d = 0, -1$ (using eq. (2.45) and assuming $B = 0.8$). In the experimentally interesting range ($R_p = 0.28 \pm 0.04$), all four curves yield similar values of $x$. We may therefore use the experimental value of $R$ in order to determine the approximate value of $x = \sin^2 \theta_W$, before we begin our discussion of $\alpha_u$ and $\alpha_d$. In all cases we obtain $0.2 < x < 0.5$. For $\alpha_d = 0, \alpha_u = 0$ we actually find

$$ x = 0.29 \pm 0.08. 
$$

In magnificent agreement with the value determined in section 2.3 from purely leptonic processes.

Having determined $x$, we may now use the measured value of $R_p$ in order to determine the allowed range for $\alpha_u, \alpha_d$ (using eqs. (2.46), (2.41), (2.42)). Fig. 6 shows the allowed values of $\alpha_u, \alpha_d$ for two representative $x$-values: $x = 0.25$ and $x = 0.35$. For all acceptable $x$-values, $\alpha_d = 1$ is excluded. We therefore conclude that the right-handed $d$-quark probably belongs to an $SU(2) \times U(1)$ singlet [6]. In the case of $x = 0.25$, $\alpha_u = 1$ is also excluded and the only allowed solution is $\alpha_d = \alpha_u = 0$, as required by the standard left-handed Weinberg--Salam model. However, for $x = 0.35, \alpha_u = 1$ is not ruled out. Consequently, we cannot completely reject the possibility that the right-handed $u$-quark is in an $SU(2) \times U(1)$ doublet. (Note that for $\alpha_u = 1, \alpha_d = 0$, fig. 5 actually gives $x = 0.32 \pm 0.05$.)

It might be interesting to see whether any of the allowed regions of $\alpha_u, \alpha_d$ would lead to vanishing parity violation in the atomic bismuth experiment. The condition for the vanishing of the quark vector neutral current is (eq. (2.32)):

$$ 292\alpha_u + 335\alpha_d = 332x + 43 = \begin{cases} 126 & (x = 0.25) \\ 159 & (x = 0.35). \end{cases} $
The only \( \alpha_u, \alpha_d \) values consistent with eq. (2.48) as well as with the observed value of \( R_p \) can be read off fig. 6.

They do not seem to correspond to a physically interesting model.

Elastic neutrino-nucleon and antineutrino nucleon experiments could serve as additional constraints [14] on \( \alpha_u, \alpha_d \) and \( x \). However, present data [15] cannot exclude any of the otherwise allowed possibilities. A substantial improvement in the elastic data (expected soon!) should provide additional information.

We conclude that, within the SU(2) \( \times U(1) \) theory, all data are consistent with \( x = \sin^2 \theta_w \approx 0.25 - 0.30 \) and the right-handed \( u \) and \( d \) quarks being in SU(2) singlets (\( \alpha_u = \alpha_d = 0 \)). The assignment of the right-handed \( u \) to a doublet (\( \alpha_u = 1 \)) is not completely ruled out, provided that \( x \) is somewhat larger. The smallness of parity violation effects in the bismuth experiment is probably not due to the hadronic vertex. It is presumably due to the electron vertex, implying \( \alpha_e = -1 \) (see section 2.3).

2.5. Summary: SU(2) \( \times U(1) \) classification of first-generation fermions

We now summarize the conclusions of the previous four sections, trying to remain within an SU(2) \( \times U(1) \) theory:

(i) the left-handed first-generation fermions belong to two doublets:

\[
\begin{pmatrix}
\nu_e \\
\bar{e}
\end{pmatrix}_L, \quad \begin{pmatrix}
\bar{u} \\
d
\end{pmatrix}_L.
\]

(ii) the Weinberg angle can be determined from elastic neutrino-electron scattering and, independently, from the deep inelastic neutral current process \( \nu + N \rightarrow \nu + \text{anything} \). In both cases, \( \sin^2 \theta_w \) can be approximately deduced without determining whether the right-handed \( e, u \) or \( d \) belong to singlets or doublets! We find:

\[
x = \sin^2 \theta_w = 0.25 \pm 0.1 \quad (\nu e \text{ elastic scattering}),
\]

\[
0.2 < \sin^2 \theta_w < 0.5 \quad (\nu + N \rightarrow \nu + \text{anything}).
\]

We can therefore assume that:

\[
0.2 < \sin^2 \theta_w < 0.35.
\]

(iii) If we accept the contradiction between the pure left-handed model and the atomic bismuth experiment, we must conclude that, within SU(2) \( \times U(1) \), the right-handed electron is probably in a doublet:

\[
\begin{pmatrix}
N_e \\
\bar{e}
\end{pmatrix}_R.
\]

The singlet assignment for \( e_R \) is consistent with all the elastic \( \nu-e \) data, but would lead to substantial parity violation in atoms.

(iv) The right-handed \( d \) quark is in an SU(2) \( \times U(1) \) singlet.

(v) The right-handed \( u \) quark is probably in a singlet, although the doublet assignment is not completely ruled out.
The most likely SU(2) × U(1) classification, consistent with all data, is:

\[
\begin{pmatrix}
\nu_e \\
e^-
\end{pmatrix}_{L} : \begin{pmatrix}
u_u \\
d
\end{pmatrix}_{L} : \begin{pmatrix}
N_e \\
e^-
\end{pmatrix}_{R} : (u)_R : (d)_R,
\]

(2.54)

where the only allowed modification is the assignment of \(u_R\) to a doublet.

Further experimental tests of these assignments and of the value of \(\sin^2 \theta_W\) will be provided by:

(a) Better data on all four \(\nu e\) elastic processes (eqs. (2.17)–(2.20)).

(b) Additional atomic-physics parity-violation experiments. In particular, the above assignment does not lead to a vanishing neutral axial-vector quark current. Consequently, it could lead to parity violation effects in light atoms, particularly in hydrogen!

(c) Elastic \(\nu N\) and \(\bar{\nu} N\) experiments could provide further tests for the classification of \(u_R\) and \(d_R\).

(d) Deep inelastic scattering of polarized electrons on nucleons provides another probe of the neutral weak current [16]. In this process, an asymmetry is expected from the above SU(2) × U(1) assignment (see also table 2 in section 2.6).

(e) A different type of asymmetry can be measured in the reaction \(e^+ + e^- \rightarrow \mu^+ + \mu^-\). The forward-backward asymmetry of the produced \(\mu^+ \mu^-\) pairs depends on the neutral axial vector current couplings of the electron and muon [17]. No asymmetry is expected in an SU(2) × U(1) model, if the right-handed electron belongs to a doublet.

On a phenomenological level, our analysis gave quite satisfactory results. We have a unique SU(2) × U(1) classification of all left-handed and right-handed first-generation fermions, with a unique value of the Weinberg angle. Several additional experimental tests are expected soon.

From the theoretical point of view, the resulting scheme is unsatisfactory: The symmetry between quarks and leptons is destroyed by the right-handed couplings; triangle anomalies are not cancelled; incorporation of the quarks and leptons in a larger unification scheme appears to be very difficult. Our theoretical prejudices tell us that we should be able to do better!

2.6. A possible extended gauge group: SU(2)_L × SU(2)_R × U(1)

In the previous subsection we have seen that the study of all available data involving first-generation fermions has yielded a unique consistent solution based on SU(2) × U(1). The solution treated electrons and quarks in a different way. It corresponded to a pure left-handed model in the quark sector and to a vector-like scheme in the lepton sector. While this may be the correct answer, our theoretical prejudices guide us in a different direction.

In order to pursue other possibilities we note that the standard Weinberg–Salam left-handed model is, so far, in very good agreement with all neutral current data involving neutrinos (namely: with all neutral current data except the atomic bismuth experiment). On the other hand, “all” neutral current data which do not involve neutrinos (namely: the atomic bismuth experiments) are consistent with parity conserving neutral currents.

Is it possible that all neutral currents conserve parity? The answer is yes! Within SU(2) × U(1), a parity conserving neutral current must give \(\sigma_{el}(\nu \bar{\nu}) = \sigma_{el}(\bar{\nu} \nu)\), \(\sigma(\nu + N \rightarrow \nu + \text{any}) = \sigma(\bar{\nu} + N \rightarrow \bar{\nu} + \text{any})\), etc. These results are inconsistent with the data. Hence, either parity is violated by neutral currents or SU(2) × U(1) should be enlarged. If SU(2) × U(1) is enlarged, parity conserving neutral currents could yield different \(\nu N\) and \(\bar{\nu} N\) elastic (or inelastic) cross sections. The neutral weak interaction could then conserve parity, but the initial neutrino (or antineutrino) beam would be left- (or right-) handed, thus introducing a nonsymmetric situation.
The challenge facing us would then be the following: Construct a theory in which all neutral
currents conserve parity, while all neutrino-related weak processes are described by the usual phenomenology of the Weinberg–Salam left-handed model. Such a theory should treat quarks and leptons
in the same way and should have only one Weinberg angle.

All of these requirements can be satisfied by a theory based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$ [18]. Various versions of such a theory were considered by several authors during the last
few years [19–22].

The $SU(2)_L \times SU(2)_R \times U(1)$ theory necessitates seven gauge bosons. The generators of $SU(2)_L$ and $SU(2)_R$ are denoted by $W^+_L$, $W^+_R$, $W^-_L$, $W^-_R$, $W^3_L$, $W^3_R$. The $U(1)$ generator is denoted by $B$. The $SU(2)_L$ and $SU(2)_R$ coupling constants are assumed to be equal, obeying a left-right symmetry. We
denote the $SU(2)$ coupling by $g$ and the $U(1)$ coupling by $g'$. The charged $W^+_L$ and $W^+_R$ bosons are chosen not to mix with each other. This can be easily arranged, e.g., if the Higgs mesons $\chi_L$ and $\chi_R$
belong to a $\left(\frac{1}{2}, 0\right)$ and $\left(0, \frac{1}{2}\right)$ multiplets of $SU(2)_L \times SU(2)_R$, with vacuum expectation values:

$$\langle \chi_L \rangle = \langle \chi_R \rangle = \left( \begin{array}{c} 0 \\ \lambda \end{array} \right).$$

In the absence of any other Higgs particles the mass matrix for $W^+_L$ and $W^+_R$ will be diagonal and:

$$M^2_{W^+_L} = M^2_{W^+_R} = \frac{1}{2} \lambda^2 g^2.$$  \hspace{1cm} (2.55)

The (mass)$^2$ matrix for the three neutral bosons $W^3_L$, $W^3_R$ and $B$ has the form [20]:

$$\begin{pmatrix}
  g^2 & 0 & -gg' \\
  0 & g^2 & -gg' \\
  -gg' & -gg' & 2g'^2
\end{pmatrix}.$$  \hspace{1cm} (2.56)

The eigenstates and eigenvalues are:

$$\Lambda = \sin \theta_w (W^3_L + W^3_R) + \sqrt{\cos 2 \theta_w} \ B; \hspace{2cm} M^2_\Lambda = 0,$$

$$Z_v = \frac{1}{\sqrt{2}} \cos 2 \theta_w (W^3_L + W^3_R) - \sqrt{2} \sin \theta_w B; \hspace{2cm} M^2_{Z_v} = \frac{1}{2} \lambda^2 (g^2 + 2g'^2),$$

$$Z_A = \frac{1}{\sqrt{2}} (W^3_L - W^3_R); \hspace{2cm} M^2_{Z_A} = \frac{1}{2} \lambda^2 g^2.$$  \hspace{1cm} (2.57)

The relations between $g$, $g'$ and $e$, $\theta_w$ are:

$$g = e/\sin \theta_w; \hspace{1cm} g' = e/\sqrt{\cos 2 \theta_w},$$  \hspace{1cm} (2.58)

where $e$ is the electromagnetic coupling constant and $\theta_w$ is a Weinberg-like angle. The masses of the
two neutral $Z$-bosons are related by:

$$M^2_{Z_A} = M^2_{Z_v} \cos 2 \theta_w.$$  \hspace{1cm} (2.59)

Notice that the left- and right-handed neutral bosons are mixed in a “maximal” way, creating a
pure vector and a pure axial-vector neutral boson $Z_v$ and $Z_A$.

All left-handed fermions are assigned to $\left(\frac{1}{2}, 0\right)$ representations. All right-handed fermions are in $\left(0, \frac{1}{2}\right)$ representations. The fermion masses (or, at least, mass differences) must be generated by Higgs
mesons belonging to the \((\frac{1}{2}, \frac{1}{2})\) multiplet. Such Higgs particles would also couple to the gauge Bosons and will spoil the simple \(W^\pm\) and \(Z\) eigenstates which emerged from our previous discussion. We assume that the vacuum expectation values of the \((\frac{1}{2}, \frac{1}{2})\) Higgs mesons are very small relative to \(\lambda\) and that, consequently, the physical \(W^\pm\) and \(Z\) states are not very different from the ones listed above.

At this point, we have to introduce into the theory some ingredient which would give us the usual left-handed charged currents and eliminate any presently observable right-handed charged currents. This can be done in two different ways:

(i) We may introduce yet another Higgs multiplet which will affect the masses of \(W^\pm_R\) without contributing to any vector boson mass. This could be done \([19, 20]\) by introducing Higgs fields \(\delta_L\) and \(\delta_R\) belonging to the \((1, 0)\) and \((0, 1)\) representations of \(SU(2)_L \times SU(2)_R\). These new Higgs fields do not contribute to the masses of the neutral vector bosons, but they may influence the charged-boson masses. We assume:

\[
\langle \delta_L \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad \langle \delta_R \rangle = \begin{pmatrix} 0 \\ \Lambda \end{pmatrix}.
\]  

If \(\Lambda \gg \lambda\), we have:

\[
M^2_{W^\pm_R} \gg M^2_{W^\pm_L}
\]  

while we still preserve the relations:

\[
M^2_{Z_A} = M^2_{W^\pm_L}; \quad M^2_{\nu_V} = M^2_{W^\pm_L}/\cos 2\theta_W.
\]

In such a scheme all right-handed charged current transitions are suppressed by, at least, factors of \(\lambda^2/\Lambda^2\). We may, for instance, assign \(u_R\) and \(d_R\) to the same \((0, \frac{1}{2})\) multiplet, leading to deviations of order \(\lambda^4/\Lambda^4\) from the usual \(V-A\) theory in \(\beta\)-decay. We can always choose \(\lambda/\Lambda\) to be sufficiently small so that the model is consistent with all present experiments. The set of Higgs particles which is introduced in this model suffers from an important theoretical drawback \([23]\): its pattern does not survive higher-order loop corrections. In this sense, the pattern of spontaneous breaking of parity invariance is not natural. Several ideas have been proposed in order to rectify this difficulty \([23]\), but the situation is still somewhat unclear.

(ii) A second option would be \([21]\) to keep \(M^2_{W^\pm_R} = M^2_{W^\pm_L}\) but exploit the orthogonality of \(W^\pm_L\) and \(W^\pm_R\). In that case we must assign heavy fermions to some of the right-handed doublets which contain \(c_R, u_R, d_R\). All charged current neutrino-initiated processes would follow the usual \(V-A\) theory, since the left-handed neutrinos in the beam would couple only to \(W^\pm_L\) which then couples only to \(u_L, d_L, \text{etc.}\). On the other hand, the successful \(V-A\) description of nucleon \(\beta\)-decay tells us that \(W^\pm_R\) does not contribute significantly and that at least \((u_R, d_R)\) or \((\bar{e}_R, e_R)\) cannot be in a \((0, \frac{1}{2})\) multiplet of \(SU(2)_L \times SU(2)_R \times U(1)\). Some heavy fermions are then needed in order to form doublets with \(c_R\) or \(u_R, d_R\).

In both versions of the theory, the weak neutral-current Lagrangian would be:

\[
\mathcal{L} = -\frac{g}{\sqrt{2}} \left[ J^3_A Z_A + \frac{1}{\sqrt{\cos 2\theta_W}} \left( J^3_{\nu} - \sin^2 \theta_W J^3_{em} \right) Z_{\nu} \right].
\]  

The effective Hamiltonian of the weak neutral current would then have the form:

\[
H_{weak} = \frac{g^2}{2} \left[ \frac{1}{M^2_{Z_A}} J_A J_A + \frac{1}{\cos 2\theta_W} \frac{M^2_{Z_{\nu}}}{M^2_{W^\pm_L}} J_{\nu} J_{\nu} \right] = \frac{g^2}{2M^2_{W^\pm_L}} (J_A J_A + J_{\nu} J_{\nu}).
\]
It is clear that the neutral-current interactions conserve parity. Furthermore, the equal coefficients of $J_A J_A$ and $J_V J_V$ guarantee that the left-handed component of the $\bar{\nu} \nu$ neutral current couples only to the left-handed component of the $\bar{\nu} e$ or $\bar{\nu} u$ or $\bar{\nu} d$ current. Consequently:

(i) All neutral current processes involving left-handed neutrinos obey the phenomenological description of the standard left-handed Weinberg–Salam model, yielding the same relations between $\sin^2 \theta_W$ and the measured experimental quantities. Thus, $\sin^2 \theta_W \approx 0.2 - 0.35$, as found in our analysis of sections 2.3 and 2.4.

(ii) All neutral current processes in which neutrinos do not participate, are manifestly parity conserving, yielding no parity violation in any atomic physics process and no asymmetry in deep inelastic scattering of polarized electrons on protons.

(iii) All charged current processes involving first-generation fermions obey the usual V-A theory.

The most remarkable property of the $\text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)$ model, is the identity between its predictions for neutrino-induced neutral-current processes and the predictions of the standard left-handed Weinberg–Salam theory. This identity is not accidental. In fact, it is a special case of a general theorem [22, 24] which states the following: Consider a gauge theory based on a group $G' \times G'' \times \text{U}(1)$ in which all Higgs particles are either $G'$-singlets or $G''$-singlets. In such a theory, all neutral current processes involving a neutral particle which is a $G''$-singlet, will be fully described by the $G' \times \text{U}(1)$ subgroup. The detailed proof can be found elsewhere [24], but the situation is intuitively clear: If a particle transforms as a $G''$-singlet, it couples only to the vector-Bosons and Higgs particles which are $G''$-singlets. In such a case, its interactions cannot possibly be influenced by the nature of $G'$. Hence, the left-handed neutrino which is an $\text{SU}(2)_R$ singlet, does not couple to $W^+_R, W^-_R, X_R$ or $\delta_R$. Its interactions are entirely described by $\text{SU}(2)_L \times \text{U}(1)$.

In table 2 we present a comparison of the predictions of four models:

(a) The standard left-handed Weinberg–Salam model (“standard model”).

(b) An $\text{SU}(2) \times \text{U}(1)$ vector model in which all fermions are in doublets and all neutral weak currents conserve parity (“vector model”).

(c) An $\text{SU}(2) \times \text{U}(1)$ model with the assignments of section 2.5: $e_R$ in doublet; $u_R, d_R$ in singlets (“asymmetric model”).

(d) The $\text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)$ model described in the present section (“left-right model”).

It is clear from the table that the standard model is in some difficulty with the atomic bismuth experiment and the vector model is ruled out by the neutral-current $\bar{\nu}/\nu$ ratio. The two other models (described respectively, in sections 2.5 and 2.6) are consistent with all present data, but differ in their predictions for the last three processes in the table [17, 22]. Any one of these processes could distinguish between these models. Our theoretical prejudices favor the $\text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)$ model.

2.7. The mass spectrum of first-generation fermions

The four first-generation fermions exhibit a very simple and reasonable mass spectrum. An old-fashioned, simple-minded, ignorant theorist might claim that all its qualitative features can be more or less “understood” by considering the different interactions involved:

(i) The $u$ and $d$ quarks are heavier than the $\nu_e$ and $e$ leptons. This is presumably due to the strong (QCD) interactions of quarks which somehow generate the quark–lepton mass differences.

(ii) The $\nu_e$–$e$ and the $u$–$d$ mass differences are of the same sign and roughly the same order of magnitude. Both are presumably due to electromagnetic differences, since the two fermions within
### Table 2

Some features of four possible gauge models

<table>
<thead>
<tr>
<th></th>
<th>&quot;Standard&quot;</th>
<th>&quot;Vector&quot;</th>
<th>&quot;Asymmetric&quot;</th>
<th>&quot;Left-right&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gauge group</strong></td>
<td>SU(2) × U(1)</td>
<td>SU(2) × U(1)</td>
<td>SU(2) × U(1)</td>
<td>SU(2)$_L$ × SU(2)$_R$ × U(1)</td>
</tr>
<tr>
<td><strong>Bosons</strong></td>
<td>$W^\pm, Z, \gamma$</td>
<td>$W^\pm, Z, \gamma$</td>
<td>$W^\pm, Z, \gamma$</td>
<td>$W^\pm_L, W^\pm_R, Z_V, Z_A, \gamma$</td>
</tr>
<tr>
<td><strong>Left-handed fermions</strong></td>
<td>$(\nu_e, u_L, d_L)$</td>
<td>$(\nu_e, u_L, d_L)$</td>
<td>$(\nu_e, u_L, d_L)$</td>
<td>$(\nu_e, u_L, d_L)$ in $(1, 0)$</td>
</tr>
<tr>
<td><strong>Right-handed fermions</strong></td>
<td>$(e^-)_R (u_R, d_R)$</td>
<td>$(N_e, b_R, d_R)$</td>
<td>$(N_e, b_R, d_R)$</td>
<td>$(N_e, b_R, d_R)$ in $(0, \frac{1}{2})$</td>
</tr>
<tr>
<td><strong>Minimal Higgs-system</strong></td>
<td>two doublets</td>
<td>doublets</td>
<td>doublets</td>
<td>$(\frac{1}{2}, \frac{1}{2})$ small VEV $(\frac{1}{2}, 0), (0, \frac{1}{2})$ medium VEV $(0, 1)$ large VEV $(\frac{1}{2}, 0), (0, \frac{1}{2})$ medium VEV</td>
</tr>
<tr>
<td><strong>Boson-mass relations</strong></td>
<td>$M_Z^2 (1 - x) = M_W^2$</td>
<td>$\frac{2}{3} Z^2 (1 - x) \leq M_W^2$</td>
<td>$M_Z^2 (1 - x) \leq M_W^2$</td>
<td>$M_{W_R}^2 &gt; M_{W_L}^2 = M_{Z_A}^2$ $M_{Z_V}^2 (1 - 2x) = M_{Z_A}^2$</td>
</tr>
<tr>
<td>$\sigma(\bar{\nu}<em>\mu e) / \sigma(\nu</em>\mu e)$</td>
<td>$1 - 4x + 16x^2$</td>
<td>$1$</td>
<td>$1$</td>
<td>Same as &quot;standard&quot;</td>
</tr>
<tr>
<td>$\sigma(N \to \bar{\nu} x)$ / $\sigma(N \to \nu x)$</td>
<td>$(2 - B)(1 - 2x) + \frac{86}{9} x^2$</td>
<td>$1$</td>
<td>Same as &quot;standard&quot;</td>
<td>Same as &quot;standard&quot;</td>
</tr>
<tr>
<td><strong>P-violation in heavy atoms</strong></td>
<td>yes (wrong?)</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td><strong>P-violation in hydrogen</strong></td>
<td>yes $\propto 1 - 4x$</td>
<td>no</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td><strong>Asymmetry in polarized ep scattering</strong></td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td><strong>Front-back asymmetry in $e^+ e^- \rightarrow \mu^- \mu^+$</strong></td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>
each doublet have identical weak and strong interactions. We have:

\[ m(e) - m(\nu_e) \approx 0.5 \text{ MeV}; \quad m(d) - m(u) \approx 2.5 \text{ MeV}. \]

We thus see that, naively speaking, the qualitative mass pattern is as simple as it could be, and all mass differences simply reflect the different interactions enjoyed by different fermions.

These simple remarks exhaust our understanding of the mass spectrum. From this point onward, we are only able to ask questions:

(a) Is the neutrino massless? If so, why? There must be some symmetry principle which prevents the neutrino from acquiring a mass. We do not know what it is. There may be a symmetry which forbids neutrino mass-terms to all orders, leading to a vanishing mass. Alternatively, it is conceivable that some symmetry prevents low order mass terms, and small high order terms are allowed. In that case a small but finite neutrino mass may be generated. The present limit [25]:

\[ m(\nu_e) < 60 \text{ eV} \]

still allows concoctions such as \( m(\nu_e) \approx \alpha^2 m(e) \). The (approximate?) masslessness of the neutrino is probably related to the left-right asymmetry observed in the weak charged currents, but we do not know of any satisfactory explanation of either phenomenon.

(b) What are the masses of u and d? The mass difference can be safely estimated. The masses themselves are either:

\[ m(u) \approx m(d) \approx 300 \text{ MeV}, \]

as obtained from “mechanical” mass formulae \( m(u) \approx \frac{1}{2} m(p) \) or:

\[ m(u) \approx m(d) \approx \text{few MeV}, \]

as obtained from calculations [26] of the symmetry breaking of chiral SU(2) × SU(2). It seems that in different contexts we must use different u and d masses [27]. This may mislead us when we try to relate u, d masses to c, s masses, etc.

(c) Isospin symmetry \( (m(u) \approx m(d)) \) appears natural when only first-generation fermions are considered. It presumably follows from

\[ e \ll g_{st}, \]

where \( e \) and \( g_{st} \) are the electromagnetic and strong (QCD) coupling constants, respectively. However, the peculiar pattern of second generation masses leads us to consider isospin symmetry as an exception rather than the rule. It is possible that isospin symmetry represents the overall smallness of the u and d masses, rather than the smallness of their mass difference [26].

(d) The alleged simplicity of the first-generation mass spectrum is valid only if no heavy fermions (such as \( N, b, t \)) appear in the same doublets with e, u, d. Among the models of table 2 only two have this feature: The standard model and the SU(2)\(_L\) × SU(2)\(_R\) × U(1) model with \( M_{w_R} \gg M_{w_L} \). In all other models, additional fermions play an important role.

(e) Finally, we must make the most obvious remark: If our “explanation” of first-generation masses is valid, the mechanism which generates the masses of the second and third generations must be completely different, and it involves totally new physics ingredients. Alternatively, if all fermion masses are generated in the same manner, the simplicity of the first-generation spectrum must be regarded as accidental. Both of these possibilities are unattractive. We return to discuss them in section 3.9, but we can offer no solution.
2.8. The quark-lepton connection: introduction

The building blocks of matter are quarks and leptons. Are they related to each other?

Six different arguments are usually mentioned as a motivation for considering a possible connection between quarks and leptons:

(i) To the extent probed by presently available momenta, quarks and leptons are “pointlike”, \( J = \frac{1}{2} \) fermions.

(ii) Quarks and leptons appear to respond to the weak interactions in an analogous way. Left-handed quarks and leptons are in \( SU(2) \times U(1) \) doublets. Right-handed quarks and leptons do not seem to participate in the observed charged currents. There is a general similarity between the observed spectra of quarks and leptons, generation by generation.

(iii) The quantization of electric charge in the quark sector is clearly related to that of the lepton sector. This is not guaranteed by \( SU(2) \times U(1) \). Within \( SU(2) \times U(1) \) there is no reason for any special relationship between, say, the electron charge and the proton charge (or between \( Q(e) \) and \( Q(u) \), etc.).

(iv) The Weinberg angle \( \theta_W \) is a free parameter in \( SU(2) \times U(1) \) or \( SU(2)_L \times SU(2)_R \times U(1) \). Hence, the weak and electromagnetic interactions are not truly unified in such models. In a true unification scheme we should be able to compute \( \theta_W \) from the symmetry properties of the unifying gauge group. Such a unification scheme may include the strong, weak and electromagnetic interactions and involve a quark-lepton connection.

(v) The strengths of the weak and electromagnetic interactions become comparable at \( s \gtrsim M_W^2 \).

The effective strong interaction coupling constant in asymptotically free QCD decreases logarithmically at high momenta. At sufficiently high energies, perhaps of order \( 10^{17} \text{ GeV} \) [28], the strong interactions presumably become comparable to the weak and electromagnetic ones. At such high energies, we may have an overall symmetry unifying all of these interactions, and relating the quarks to the leptons.

(vi) Empirically, the sum of electric charges of the first-generation fermions vanishes (counting the three quark-colors):

\[
Q(\nu_e) + Q(e^-) + 3Q(u) + 3Q(d) = 0.
\] (2.69)

A similar relation holds for second-generation fermions, etc. In a pure left-handed Weinberg–Salam model, this is actually a necessary condition for the elimination [29] of the triangle anomalies [30]. In other models, the anomalies are automatically cancelled, regardless of the sum of fermion charges. The fact remains, however, that the sum vanishes. One possible explanation for this may be the fact that in any model which contains the electric charge and which assigns quarks and leptons to the same multiplet of a simple group, the sum of quark and lepton charges must vanish. This may very well be the reason for the validity of eq. (2.69).

It is clear that a successful “grand unification scheme” of strong, weak and electromagnetic interactions would automatically account for all of the six items which we have just enumerated. However, we should perhaps study them separately, in order to see whether “grand unification” is indeed necessary for each one of them.

The first two points (i), (ii), are qualitative statements concerning the similarity between quarks and leptons. They do not impose any specific mathematical connection between them, although the similarity would perhaps be more natural in theories in which such a connection exists. In any event, even if we accept that points (i) and (ii) guide us towards a quark-lepton connection, there is no
The next two items (iii), (iv) clearly point at a “truly unifying” gauge group larger than $SU(2) \times U(1)$ or $SU(2)_L \times SU(2)_R \times U(1)$. The gauge group has to be either a simple group $G$ or a “pseudosimple” direct product of the form $G \times G$ with a discrete symmetry relating the coupling constant of the two $G$’s. It must unify the weak and electromagnetic interactions, but need not be related to the strong interactions. In such a situation, charge quantization becomes universal and the Weinberg angle is uniquely determined, without introducing quarks and leptons into the same multiplet. We refer to such a theory as “simple unification”.

Only the last two items (v), (vi), seem to require “grand unification” of strong, weak and electromagnetic interactions. In fact, the possible equality of strong and weak coupling constants actually sets the energy scale in which the symmetry limit of such a grand unification scheme may be realized. This energy scale ($\sim 10^{17}$ GeV) forces us to extrapolate our present ideas over many orders of magnitude. Most of the resulting symmetry relations are unlikely to be ever tested.

Summarizing our discussion, we find that we have:

(a) Qualitative motivation for a quark-lepton analogy (points (i), (ii)).

(b) Strong motivation for “simple unification” which, however, may leave quarks and leptons unrelated (points (iii), (iv)).

(c) Some motivation for “grand unification” (points (v), (vi)).

We must remember, however, that “grand unification” automatically achieves “simple unification” while the reverse is, of course, not true.

We now proceed to discuss the general features of “simple unification” and “grand unification”.

2.9 “Simple unification”

A “simple unification” scheme is based on a gauge group $G$ such that:

(i) $G$ is simple or pseudosimple (i.e. direct product of isomorphic groups, with equal coupling constants).

(ii) $G \supset SU(2) \times U(1)$.

(iii) $G$ commutes with $SU(3)_c$ and, therefore, does not connect quarks to leptons.

Any “simple unification” group would have additional weak gauge bosons, presumably heavier than $W^\pm$ and $Z$, but hopefully in the general mass range of $10^2-10^4$ GeV, well below the mass range required by “grand unification” (see section 2.10). The quantization of all electric charges follows a pattern which is determined by the gauge group, thus explaining the relation between quark and lepton charges. The Weinberg angle is determined by the group, and is given by [28]:

$$\sin^2 \theta_w = \frac{\sum I^2_3}{\sum |Q|^2},$$

(2.70)

where the summation is done over all quarks or all leptons in any representation of $G$. $I_3$ and $Q$ are the diagonal neutral generators obeying a relation of the form:

$$Q = I_3 + \sum \alpha_i G_i,$$

(2.71)
while the corresponding vector fields $A_\mu$ and $W_{3\mu}$ obey:

$$A_\mu = W_{3\mu} \sin \theta_W + \sum_j \beta_j W_j,$$

(2.72)

$G_j$ are all the diagonal generators of $G$ which are orthogonal to $I_3$; $W_j$ are their corresponding vector fields; $\alpha_j$ and $\beta_j$ are numerical coefficients.

An important consequence of any “simple unification” scheme are the conditions:

$$\sum_{\text{quarks}} Q_i = 0; \quad \sum_{\text{leptons}} Q_i = 0,$$

(2.73)

for all quarks or all leptons in any given representation of $G$. Starting with the $u, d$ quarks with charges $\frac{2}{3}, -\frac{1}{3}$ we immediately conclude that any “simple unification” theory must include, as a subgroup, an SU(3) gauge group of the weak and electromagnetic interactions. Moreover, if we restrict our attention to models in which all quark charges are $\frac{2}{3}$ or $-\frac{1}{3}$ (excluding $\frac{1}{6}, -\frac{1}{6}$, etc.) we find that the quarks must always come in SU(3)-triplets. Each triplet will then include an SU(2) doublet ($Q = \frac{2}{3}, -\frac{1}{3}$) and SU(2) singlet ($Q = -\frac{1}{2}$). The full “simple unification” group may be larger than SU(3) but it must contain an SU(3) subgroup in such a way as to guarantee that all quarks are in triplets of that SU(3) subgroup [31]. This can be achieved with $G = SU(6), SU(3) \times SU(3), SU(3) \times SU(3) \times SU(3), \ldots$ The full scheme would then be based on:

$$G \supset SU(3) \supset SU(2) \times U(1).$$

(2.74)

It is easy to see that all such theories will have several common properties:

(i) We must have at least four (possibly many more) gauge bosons beyond those of SU(2) × U(1). Their masses are likely to be, at least, several hundred GeV.

(ii) The first generation of quarks must be supplemented by, at least, one additional $Q = -\frac{1}{2}$ quark.

(iii) Leptons and quarks must belong to inequivalent representations. Leptons belong, at least, to SU(3) octets.

(iv) We must either have positively charged leptons, or have leptons and antileptons in the same SU(3) multiplet, leading to lepton number nonconservation.

(v) The Weinberg angle is determined. If the operator $I_3$ in eqs. (2.70), (2.71), (2.72) is identified with the third generator of an SU(2) subgroup of SU(3), we have:

$$\sin^2 \theta_w = \frac{\lambda}{2},$$

(2.75)

in clear contrast with experiment. However, if $G$ is sufficiently large, we may identify $I_3$ in several different ways within $G$. Thus, if $G = SU(3)_L \times SU(3)_R$ and if $I_3$ of eqs. (2.70), (2.71), (2.72) is a generator of SU(3)$_L$, we may obtain [32]:

$$\sin^2 \theta_w = \frac{\lambda}{8},$$

(2.76)

which is perhaps not completely ruled out by experiment (see sections 2.3, 2.4, 2.5). We do not know how to construct a reasonable “simple unification” scheme with a value of $\sin^2 \theta_w$ smaller than $\frac{\lambda}{3}$.

(vi) Any value of the Weinberg angle which is determined by $G$ will not be substantially modified by renormalization effects, once we move away from the mass region of the heavy vector bosons. Thus, the theoretical value of $\theta_w$ can be directly confronted with experiment. Furthermore, the
A grand unification scheme [33], [34] is based on a group G* such that:

(i) G* is simple or pseudosimple,
(ii) G* ⊇ SU(2) × U(1) × SU(3)_{color}.

The gauge bosons of G* include the twelve gauge bosons of SU(2) × U(1) × SU(3)_{c} (W^±, Z, γ and eight gluons) as well as many additional bosons, some of which must transform nontrivially under both SU(2) and SU(3)_{c}.

The “grand unification” group G* may or may not include a “simple unification” subgroup G such that G commutes with SU(3)_{c} and:

\[ G^* \supset G \times SU(3)_{c} \supset SU(2) \times U(1) \times SU(3)_{c}. \] (2.77)

If it does, the entire discussion of section 2.9 applies to G*, including the determination of \( \theta_W \), the requirements concerning an SU(3) subgroup and the restrictions on the quark and lepton spectrum. However, it is entirely possible that G* does not contain any “simple unification” subgroup. In that case, all results of the previous subsection are inapplicable to G*.

All grand unification schemes have several important common features:

(i) The mass scale in which the grand unification scheme achieves its symmetric limit can be estimated by studying the momentum dependence of the “running” coupling constant in asymptotically free QCD. Various estimates [28], [35] range between \( 10^{15} \) and \( 10^{19} \) GeV. At least some of the gauge bosons which generate G* presumably acquire masses of that order, in the process of spontaneous symmetry breaking.

(ii) G* must contain a set of gauge bosons [33], [34] which carry the quantum number of both SU(2)_{weak} and SU(3)_{color}. The minimal set of such bosons transform as a (2, 3) multiplet of SU(2) × SU(3). Together with the conjugate states in a (2, 3−) multiplet, we must have, at least, twelve such bosons. These bosons convert a quark state into a lepton state and vice versa. They are sometimes referred to as “leptoquarks”. They must be extremely heavy and they may be “confined” if all color-carrying states are “confined”.

(iii) Quarks and leptons may be assigned to the same irreducible representation of G*. This immediately means that baryon number and lepton number cannot commute with all generators of
Thus, baryon number and lepton number, if they are to be conserved, must be identified within \( G^* \). Any exactly conserved quantum number within a gauge theory corresponds to a massless gauge boson. Consequently, if baryon and/or lepton number are exactly conserved, we must have at least one additional neutral massless gauge boson, in addition to the photon and the (confined?) gluons. Such a massless boson does not seem to exist in nature. \textit{We therefore conclude that baryon number and/or lepton number cannot be exactly conserved in a grand unification scheme} [33, 34]. This leads to the prediction that the proton is not stable. Its decay modes depend on the specific theory, but they may be e.g.:

\[
p \rightarrow e^+ + \pi^0 \quad \text{or} \quad p \rightarrow \nu + \bar{\nu} + \pi^+.
\]

In order to preserve agreement between the observed lower limit of the proton's lifetime (~\(10^{30}\) years) and a given grand unification scheme, we may need to postulate that the masses of the gauge bosons which mediate processes such as \( p \rightarrow e^+ + \pi^0 \) are somewhere between \(10^{15}\) and \(10^{19}\) GeV, depending on the model. This is consistent with estimates of the energy scale of grand unification, but it is certainly far removed from any present or future experimental studies.

(iv) The Weinberg angle is, again, determined by the following expression (see section 2.9, eqs. (2.70), (2.71), (2.72)):

\[
\sin^2 \theta_w = \frac{\sum P_{3i}^2}{\sum Q_i^2}
\]

where now the summation is over all quarks and leptons in the same representation of \( G^* \). In general, the value of \( \theta_w \) obtained in such a way will be valid only at the grand unification mass (\(10^{15} - 10^{19}\) GeV). In order to estimate \( \theta_w \) at currently available energies, we have to compute the renormalization corrections to \( \theta_w \). This cannot be done in a reliable way, and it involves a wild extrapolation. Crude estimates based on the procedure of Georgi, Quinn and Weinberg [28] indicate that \( \sin^2 \theta_w \) may move from 0.375 at the grand unification mass to 0.2–0.3 or so at present energies [35]. However, such a substantial renormalization correction appears only when \( G^* \) does not contain a "simple unification" subgroup \( G \). If such a subgroup \( G \) exists, \( \theta_w \) is determined by \( G \) and its value cannot be changed in the process of spontaneous symmetry breaking from \( G^* \) to \( G \).

Note that a complete list of all fermions in one representation of \( G^* \) determines the symmetry value of \( \sin^2 \theta_w \). Hence, any grand unification scheme based on the first generation fermions \( \nu_e, e, u, d \) with no additions, would give:

\[
\sin^2 \theta_w = \frac{3}{8} [\text{if } I_3(f_R) = 0]; \quad \sin^2 \theta_w = \frac{3}{4} [\text{if } I_3(f_R) = I_3(f_L)].
\]

We now briefly mention several specific grand unification schemes which are of interest:

(i) \textit{The SU(5) model [36]}. The minimal grand unification scheme is based on SU(5). This is the smallest simple algebra containing SU(2) × U(1) × SU(3), and the only one which has the minimal number of leptoquarks (twelve). The left-handed first-generation fermions and antifermions are assigned to a reducible 10 + \( \overline{5} \) representation:

\[
\overline{5} \supset (2, 1) + (1, 3) = (\nu_e, e^-)_L + (\bar{d})_L,
\]

\[
10 \supset (2, 3) + (1, \overline{3}) + (1, 1) = (u, d)_L + (\bar{u})_L + (e^+_L).
\]

The subsequent SU(2) × U(1) classification is, of course, that of the standard left-handed Weinberg–Salam model (which disagrees with the atomic physics parity violation experiments; see sections 2.3,
The neutrino must be massless. The Weinberg angle is $\sin^2 \theta_W = \frac{3}{8}$, but it may be renormalized down to around 0.2 (see [35]). The leptoquark masses must be, at least, of the order of $10^{14}$ GeV and the Higgs particles must also be extremely heavy in order to protect the present limit on the proton lifetime. In SU(5), not only baryon and lepton number, but even fermion number, is not conserved. The theory has no anomalies.

(ii) The SO(10) model [34], [35]: A more attractive scheme is based on SO(10). This is the smallest simple group which contains SU(2)$_L \times$ SU(2)$_R \times$ U(1) $\times$ SU(3)$_c$ and is therefore the natural grand unification scheme for the SU(2)$_L \times$ SU(2)$_R \times$ U(1) theory of weak and electromagnetic interactions (see section 2.6). The left-handed first-generation fermions and antifermions are assigned to the 16-dimensional spinor representation of SO(10). We may consider several different symmetry breaking chains:

(a) $\text{SO}(10) \supset \text{SU}(5) \supset \text{SU}(2)_L \times \text{U}(1) \times \text{SU}(3)$.

Here the 16-dimensional representation of SO(10) has the following SU(5) decomposition:

$16 \supset 10 + \bar{5} + 1$. \hspace{1cm} (2.83)

This provides us with the feature of including all left-handed fermions and antifermions of one generation in one irreducible representation (unlike the SU(5) case). The additional singlet is the left-handed antineutrino which could have a mass.

(b) $\text{SO}(10) \supset \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1) \times \text{SU}(3)_c$.

In this chain the 16-dimensional representation has the following content:

$16 \supset (2, 1, 3) + (2, 1, 1) + (1, 2, \bar{3}) + (1, 2, 1)$. \hspace{1cm} (2.84)

The four terms in eq. (2.84) correspond, respectively, to $(u, d)_L; (\nu_e, e^-)_L; (\bar{u}, \bar{d})_L; (\bar{\nu}_e, e^+)_L$.

(c) $\text{SO}(10) \supset \text{SU}(2)_L \times \text{SU}(2)_R \times \text{SU}(4)_c$.

Here the 16-dimensional multiplet decomposes into:

$16 \supset (2, 1, 4) + (1, 2, \bar{4})$. \hspace{1cm} (2.85)

SU(4)$_c$ is the generalized color group, including lepton number as a “fourth color”. This was advocated a long time ago by Pati and Salam [33] who pioneered the study of grand unification schemes.

The different symmetry breaking chains of SO(10) can be summarized by the diagram:

![Diagram of SO(10) symmetry breaking chains]

The SO(10) scheme predicts $\sin^2 \theta_W = \frac{3}{8}$. The renormalization correction may modify this value to about 0.25—0.3. Chanowitz, Ellis and Gaillard have actually argued [35] that the renormalization corrections to $\theta_W$ in SO(10) are smaller than those in SU(5), yielding a more satisfactory value of $\theta_W$. 
Here again, leptoquarks must be extremely heavy, fermion number is not conserved and the theory is anomaly-free.

(iii) Gursey and collaborators [37] have proposed models based on the exceptional groups E(6) and E(7). We return to these models in section 3.11. At this point we remark, however, that all such models include a "simple unification" subgroup and are therefore subject to our discussion in section 2.8. In particular, the E(7) scheme predicts $\sin^2 \theta_w = \frac{3}{4}$, a value which is not substantially modified by renormalization, and is inconsistent with experiment.

We believe that the most attractive grand unification scheme for first-generation fermions is the SO(10) model. It provides a reasonable value of $\theta_w$, "explains" why the sum of quark and lepton charges equals zero, provides a natural classification of the observed quarks and leptons and reduces naturally to $SU(2)_L \times SU(2)_R \times U(1)$. It suffers from the common unpleasantness of all grand unification schemes: Extremely heavy gauge bosons, baryon and lepton number violations, and a grand unification mass which necessitates extrapolations over 15–20 orders of magnitude.

3. Second-generation fermions: $\nu_\mu, \mu^-, c, s$

3.1. Charged weak currents and the classification of second-generation fermions

The second generation of fermions includes two leptons ($\nu_\mu$ and $\mu^-$) and two quarks ($c$ and $s$). The muon was the first fermion of that generation to be discovered. When it was identified as a lepton, the central problems of the second generation of fermions were already clearly stated: Why do these fermions exist? What distinguishes them from first-generation fermions? First we had the $\mu^-$, duplicating all properties of $e^-$, except its mass. Later, $\nu_\mu$ was found, presumably duplicating $\nu_e$. The strange and charmed quarks seem to duplicate the strong, weak and electromagnetic properties of the down and up quarks, respectively (again except for their masses). The central mystery of the
second generation, namely the reason for its existence, remains unsolved, forty years after the discovery of the muon. More on that in section 3.10.

The charged weak currents of the second generation involve the $\nu_\mu \leftrightarrow \mu^-$ and the $c \leftrightarrow s$ transitions. However, we now have, for the first time, a possibility of "generation mixing". There is nothing, a priori, to stop the mixing of $e^-$ and $\mu^-$, $\nu_e$ and $\nu_\mu$, $u$ and $c$, $d$ and $s$. Every such pair contains two particles with identical values of all the conserved quantum numbers of the non-strong interactions. Consequently, if the usual particle labels define the "physical" mass eigenstates, the SU(2) × U(1) representations may contain mixtures of such states.

In the leptonic case we will temporarily assume $m(\nu_e) = m(\nu_\mu)$ (experimentally: $m(\nu_e) < 60$ eV, $m(\nu_\mu) < 650$ keV [25]). In that case, the mass does not select specific eigenstates $\nu_e$ and $\nu_\mu$. We can then define the left-handed $\nu_e$ and $\nu_\mu$ to be the SU(2) × U(1) companions of the left-handed $e^-$ and $\mu^-$, respectively, and no further mixing is allowed among left-handed leptons. We return to this question in detail in section 3.8 and consider the case $m(\nu_e) \neq m(\nu_\mu)$. Until then, we assume equal neutrino masses and no mixing of left-handed leptons.

The decay

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

is consistent with a pure V-A transition. Hence, the $\mu^- \leftrightarrow \nu_\mu$ transition presumably involves only left-handed leptons. The relevant charged current is:

$$J^-_\alpha \propto \bar{\nu}_\mu \gamma_\alpha (1 - \gamma_5) \mu^-$$

and the left-handed $\nu_\mu$, $\mu$ are in an SU(2) × U(1) doublet:

$$\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L$$

(3.3)

The quark case is, of course, different. We define $u$, $d$, $c$, $s$ to be the mass eigenstates (also the strong interaction eigenstates). If we then postulate that all four left-handed quarks are in SU(2) × U(1) doublets, we may choose either $u$ and $c$ or $d$ and $s$ to define the two SU(2) × U(1) doublets. We then have the following equivalent SU(2) × U(1) assignments of the four left-handed quarks of the first two generations:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L$$

or

$$\begin{pmatrix} u' \\ d \end{pmatrix}_L, \begin{pmatrix} c' \\ s \end{pmatrix}_L,$$

(3.4)

where

$$d' = d \cos \theta + s \sin \theta \quad u' = u \cos \theta - c \sin \theta$$

$$s' = -d \sin \theta + s \cos \theta \quad c' = u \sin \theta + c \cos \theta$$

(3.5)

and $\theta$ is the Cabibbo angle. For $\theta \neq 0$ we now have not only $u \leftrightarrow d$, $c \leftrightarrow s$ charged current transitions, but also $u \leftrightarrow s$, $c \leftrightarrow d$ transitions of strength $G \sin \theta$. We will consider every one of these four transitions separately. The two transitions involving u-quarks are well studied:

(i) $u \leftrightarrow d$. We have already discussed this transition in section 2.1 and noted that it is consistent with being a pure left-handed transition. We are now in a position to use the strength of the $u \leftrightarrow d$ transition in order to determine $\theta$. The properties of the gauge theory dictate equal couplings for the
charged currents:
\[ \bar{\nu}_\mu \gamma_\alpha (1 - \gamma_5) \mu^- \quad \text{and} \quad \bar{u}_\gamma (1 - \gamma_5) d'. \] (3.6)

Hence, the strength of the u ⇆ d transition should be given by \( G_\mu \cos \theta \) where \( G_\mu \) is the coefficient of the \( \nu_\mu \leftrightarrow \mu \) term. Experimentally, the vector couplings of nucleon \( \beta \)-decay and \( \mu \)-decay yield [38]:
\[ \cos^2 \theta = 0.948 \pm 0.004. \] (3.7)

(ii) \( u \leftrightarrow s \). The strangeness changing charged current can be studied in semi-leptonic hyperon decays and \( K \)-decays such as \( K^- \leftrightarrow \mu^- \bar{\nu}_\mu, K \rightarrow \pi \nu, \Lambda \rightarrow \pi e^+ \bar{\nu}_e, \Sigma^- \rightarrow \Lambda e^- \bar{\nu}_e \), etc. All of these transitions are consistent with a \( V-A \) current:
\[ \bar{u}_\gamma (1 - \gamma_5) s, \] (3.8)

with strength \( G \sin \theta \) where [38]:
\[ \sin \theta = 0.229 \pm 0.003. \] (3.9)

While there is no evidence for any right-handed \( u \leftrightarrow s \) transition, its existence (with a strength smaller than \( G \sin \theta \)) cannot be excluded (see also our discussion in section 2.1). The value of the Cabibbo angle as determined from the \( |\Delta S| = 1 \) charged current is in extremely good agreement with the \( \beta \)-decay determination of \( \cos^2 \theta \). Several recent discussions of the separate determination of \( \cos \theta \) and \( \sin \theta \) yield [38]:
\[ 1 - (\sin^2 \theta + \cos^2 \theta) \lesssim 0.004. \] (3.10)

The calculation of Sirlin [39] actually leads to an even more stringent limit:
\[ 1 - (\sin^2 \theta + \cos^2 \theta) \lesssim 0.001. \] (3.11)

This "maximal allowed deviation from a four-quark Cabibbo theory" will play an important role in our discussion of the left-handed six quark model (section 4.7).

The \( V \) and \( A \) structure of transitions involving the charmed quark (\( c \leftrightarrow d, c \leftrightarrow s \)) has not been directly studied, so far. We have, however, indirect considerations which enable us to determine the properties of these transitions. The next two sections are devoted, respectively, to the \( c \overleftarrow{d} \) and \( c \overleftarrow{s} \) charged currents.

3.2. The case against a right-handed \( \overleftarrow{c}d \) charged current

The \( c \leftrightarrow d \) charged-current transition can be directly studied in the decay process:
\[ c \rightarrow d + e^+ + \nu_e, \] (3.12)

or the production process:
\[ \nu_\mu + d \rightarrow \mu^- + c. \] (3.13)

We have four different indications which favor a predominantly left-handed \( c \leftrightarrow d \) transitions:

(i) \( D \)-meson decays. The charmed \( D \)-mesons should decay through a \( c \leftrightarrow s \) or \( c \leftrightarrow d \) transition. The \( c \leftrightarrow s \) left-handed transition is of strength \( G \cos \theta \) while the \( c \leftrightarrow d \) left-handed coupling is \( G \sin \theta \). Consequently, if all \( c \)-decays are left-handed, we expect:
\[ \frac{\Gamma(c \rightarrow d + e^+ + \nu_e)}{\Gamma(c \rightarrow s + e^+ + \nu_e)} \approx \tan^2 \theta \approx 0.05. \] (3.14)
On the other hand, if we have a $c \leftrightarrow d$ right-handed transition of strength $G$ we would expect:

$$\Gamma(c \rightarrow d + e^+ + \nu_e) \approx \Gamma(c \rightarrow s + e^+ + \nu_e).$$

(3.15)

Experimentally, one should measure the ratio:

$$\frac{\Gamma(D \rightarrow e^+ + \nu_e + \overline{K} + \text{anything})}{\Gamma(D \rightarrow e^+ + \nu_e + \text{anything})}.$$

(3.16)

This ratio would be around 95% for the standard left-handed model, but around 50% for a model with a full-strength right-handed $c \leftrightarrow d$ transition. Semileptonic D-decays have been observed both in $e^+e^-$ collisions and in neutrino reactions. The fraction of decays containing K-mesons seems to be substantial and probably larger than 50%. However, the semileptonic data [40] are not yet sufficiently precise in order to definitely distinguish between a branching ratio of 50% and 95%. A somewhat less definitive prediction involves the overall (semileptonic and nonleptonic) ratio:

$$\frac{\Gamma(D \rightarrow \overline{K} + \text{anything})}{\Gamma(D \rightarrow \text{anything})}.$$

(3.17)

Here, again, the ratio would be either 95% or 50%, but in order to obtain this prediction we have to assume that we have no special enhancement or suppression mechanisms of specific nonleptonic channels. Such effects exist, of course, in nonleptonic K-decays and hyperon decays, where $\Delta I = \frac{1}{2}$ amplitudes are strongly enhanced, leading to a small semileptonic branching ratio. We believe, however, that such enhancement effects are not very prominent in D-decays in view of the "reasonable" semileptonic branching ratio [41]:

$$\frac{\Gamma(D \rightarrow \text{semileptonic})}{\Gamma(D \rightarrow \text{all})} \approx 0.2.$$

(3.18)

We therefore consider the ratio (3.17) to be a reasonable test for the existence of right-handed $c \leftrightarrow d$ transitions. Experimentally, it seems that most D-decays do contain K-mesons and that the 95% estimate is probably preferred [42].

Finally, we have an upper limit [43] on:

$$\frac{\Gamma(D^0 \rightarrow \pi^+ + \pi^-)}{\Gamma(D^0 \rightarrow K^+ + \pi^+)} \leq 0.07.$$

(3.19)

This, again, indicates the absence of right-handed $\bar{c}d$ terms which might yield:

$$\Gamma(D^0 \rightarrow \pi^+ + \pi^-) \approx \Gamma(D^0 \rightarrow K^- + \pi^+).$$

(3.20)

We summarize: The percentage of K-meson events in two-body nonleptonic D-decays is definitely above 90%. It is almost certainly around 90% in overall D-decays, and it is probably around the same value in semileptonic decays. All of these indicate, with varying degrees of experimental and theoretical certainty, that there is no substantial $c \leftrightarrow d$ right-handed transition.

(ii) **Charmed particle production by neutrinos.** Events of the type:

$$\nu_\mu + N \rightarrow \mu^- + \mu^+ + \text{anything}$$

(3.21)

are presumably mostly due to the production of a charged particle followed by its semileptonic decay. The average $\xi$-value (momentum fraction) of the $\mu^-\mu^+$ events in $\nu N$ scattering is significantly larger than the corresponding $\xi$-value for $\bar{\nu} N$ [44]. The absolute rate of $\mu^-\mu^+$ events is larger in $\nu N$ than in $\bar{\nu} N$ [44]. Both of these facts indicate that most of the $\mu^-\mu^+$ events in $\nu N$ scattering occur when the charmed quark is produced off a valence quark:

$$\nu_\mu + d \rightarrow \mu^- + c.$$

(3.22)
By studying the $\gamma$-distribution of these events we may then determine whether they are produced via a left-handed $d \leftrightarrow c$ transition (yielding a constant $\gamma$-dependence) or a right-handed transition (yielding a $(1 - \gamma)^2$ dependence). Furthermore, the production rate would be much higher in the case of a right-handed transition.

The production rate of $\mu^-\mu^+$ events in $\nu N$ scattering is [44] somewhat less than 1%, consistent with a production fraction of the order of $\sin^2 \theta \approx 0.05$ times a decay branching ratio of the order of 10%. Should we have a right-handed $d \leftrightarrow c$ transition of strength $G$, approximately 25% of all $\nu N$ events would contain charmed particles and the rate of $\mu^-\mu^+$ events would be larger than 2%. This seems somewhat large when compared with the observed $\mu^-\mu^+$ rate.

The $\gamma$-distribution of $\mu^-\mu^+$ events in $\nu N$ collisions is consistent with a pure left-handed current and inconsistent with a dominant right-handed $c \leftrightarrow d$ transition [44]. We, therefore, conclude that the neutrino data favors a left-handed $c \leftrightarrow d$ transition.

(iii) $K^0 - K^0_L$ mass difference. The mass of the charmed quark was originally estimated [45], within the standard left-handed model of Glashow, Iliopoulos and Maiani (GIM) [46] from the observed $K^0 - K^0_L$ mass difference. The estimate is based on the diagram of fig. 7a. One obtains:

$$\Delta M = K(m_c^2 - m_u^2) \cos^2 \theta \sin^2 \theta,$$

where the constant $K$ involves factors (of $G, M_w$, etc.) which are irrelevant to the present argument. Inserting the experimental values of $\Delta M$ and $\theta$ one obtains the successful estimate [45]:

$$m_c \approx 1.5 \text{ GeV}.$$  

If we now have a right-handed $c \leftrightarrow d$ coupling of strength $G$, the expression for $\Delta M$ acquires a contribution (fig. 7b):

$$\Delta M = K(m_c^2 - m_u^2) \cos^2 \theta \cdot 2(\ln(M_w/m_c) - 1).$$

where $K$ is the same factor as in eq. (3.23). It is easy to see that the latter contribution to $\Delta M$ is larger than the first one by a factor of 100. Consequently, the estimate of $m_c$, based on a right-handed $c \leftrightarrow d$ coupling and the experimental value of $\Delta M$, yields the totally unacceptable value $m_c \approx 150 \text{ MeV}$. This argues very strongly against such a right-handed current [47].

(iv) Nonleptonic K-decays. Golowich and Holstein [48] have presented another interesting argument against a right-handed $c \leftrightarrow d$ transition. They studied the transformation properties of the weak Hamiltonian $H_w$ for nonleptonic K-decays, under chiral SU(2) × SU(2). In a V-A theory, the
$\Delta I = \frac{1}{2}, \frac{3}{2}$ pieces of $H_w$ transform like the $(\frac{1}{2}, 0), (\frac{3}{2}, 0)$ representations of $SU(2) \times SU(2)$. Consequently:

$$[H_w, Q + Q_s] = 0 \quad \text{and} \quad [H_w, Q] = -[H_w, Q_s], \quad (3.26)$$

where $Q, Q_s$ are, respectively, the vector and axial vector charges. In a theory with a substantial $V + A$ current connecting $c \leftrightarrow d$, the $\Delta I = \frac{1}{2}$ piece of $H_w$ will contain a term:

$$\bar{s}_\gamma(1 - \gamma_5) c \cdot \bar{s}_\gamma(1 + \gamma_5) d. \quad (3.27)$$

Such a term belongs to the $(0, \frac{1}{2})$ representation of $SU(2) \times SU(2)$ and obeys:

$$[H_w^{\frac{1}{2}}, Q - Q_s] = 0 \quad \text{and} \quad [H_w^{\frac{1}{2}}, Q_s] = [H_w^{\frac{1}{2}}, Q]. \quad (3.28)$$

If the $\Delta I = \frac{1}{2}$ piece of $H_w$ is dominated by this term we have the following situation:

(a) In the standard left-handed model:

$$[H_w^{\frac{3}{2}}, Q_s] = -[H_w^{\frac{3}{2}}, Q]; [H_w^{\frac{3}{2}}, Q_s] = -[H_w^{\frac{1}{2}}, Q]. \quad (3.29)$$

(b) In a model with a right-handed $c \leftrightarrow d$ current:

$$[H_w^{\frac{3}{2}}, Q_s] = -[H_w^{\frac{3}{2}}, Q]; [H_w^{\frac{1}{2}}, Q_s] = [H_w^{\frac{1}{2}}, Q]. \quad (3.30)$$

We can then distinguish between the two models by measuring the relative sign of the $I = \frac{1}{2}$ and $I = \frac{3}{2}$ amplitudes in $K$-decays. These amplitudes are related by PCAC to the corresponding $[H_w, Q_s]$ commutators, and their measured values clearly favor the case (a), again ruling out a substantial $c \leftrightarrow d$ right-handed term [48].

The four arguments listed above are more than sufficient for excluding a right-handed SU(2) $\times U(1)$ doublet of the form:

$$\begin{pmatrix} c \\ d \end{pmatrix}. \quad (3.31)$$

None of these arguments can, at present, exclude a right-handed $c \leftrightarrow d$ transition of strength $G \sin \theta$, namely, a comparable amount of left and right-handed $c \leftrightarrow d$ couplings. Such a possibility can be excluded only by much better data on the production of charmed mesons by neutrinos, or by a detailed analysis of decays such as:

$$D^+ \rightarrow \mu^+ + \nu; D \rightarrow \rho + e^+ + \nu_e. \quad (3.32)$$

The last two arguments (concerning the $K^0_L - K^0_S$ mass difference and nonleptonic decays) are important because they lead to a general result: In an SU(2) $\times U(1)$ model, no heavy quark can have a left-handed coupling of order $G$ to the s-quark (or d-quark) and at the same time a right-handed coupling of order $G$ to the d-quark (or s-quark). This is a significant constraint on models involving third generation quarks.

3.3. Is there a right-handed $\bar{c}s$ charged current?

The left-handed $c$ and $s$ quarks are presumably connected by a charged current of strength $G \cos \theta$, leading to predominant strange particle decays of charmed mesons (see section 3.2). Do we also have right-handed $c \leftrightarrow s$ transitions of strength $G$?
The phenomenological analysis of this problem is more difficult than the corresponding discussion of right-handed c \rightarrow d terms in section 3.2. A right-handed (c, d)_{R} doublet would lead to an almost pure V + A coupling for c \leftrightarrow d (because of the sin \theta suppression of the left-handed coupling). On the other hand, a (c, s)_{R} doublet would yield approximately equal amounts of V + A and V - A couplings, leading to an almost pure vector interaction. While in the c \rightarrow d case it was sufficient to exclude a dominant V + A term, here we try to exclude an equal amount of V + A and V - A.

At present, there is no convincing experimental or theoretical indication against a right-handed c \leftrightarrow s term. There are, however, five items that we would like to discuss in this connection:

(i) D \rightarrow K^{*}\nu_{e}. The decay D \rightarrow K^{*}e^{+}\nu_{e} is, presumably, the dominant semileptonic decay of the D-meson. It is also the simplest decay which could proceed by both V and A currents. A detailed measurement of the electron momentum spectrum in this decay could distinguish between V - A, V + A or pure V [49]. The present data [50] (which represents D \rightarrow e^{+} + anything, without K*-detection) is consistent with a pure V - A transition. However, pure V cannot yet be excluded.

We must emphasize that tests concerning D decays (or \Lambda_{c} and F^{+} decays) probe the effective V, A structure of the meson weak current rather than the "bare" weak couplings of the c and s quarks. The connection could, in principle, be made using methods similar to those of Adler and Weisberger [4], but the application of such methods for the \bar{c}s current is not straightforward.

(ii) \bar{p} + N \rightarrow \mu^{+} + \mu^{-} + anything. A somewhat more direct test of the c \leftrightarrow s current involves the production of \mu^{+}\mu^{-} pairs in \bar{p}N reactions. Assuming that such pairs emerge from the production and decay of charmed particles, the dominant mechanism will be:

\[\bar{p} + \bar{\Xi} \rightarrow \mu^{+} + \bar{\Xi} \rightarrow \mu^{-} + \bar{s} + \bar{\nu}_{\mu}\]  

where the struck \bar{s} belongs to the q\bar{q} "ocean" in the nuclear target. The observed rate of antineutrino \mu^{+}\mu^{-} events, as well as their \xi distribution, are consistent with this assumption [44]. We may then use the \nu\nu distribution in order to determine the V, A structure of the production mechanism. For a pure V - A model the struck \bar{s} and the produced \bar{\Xi} will be right-handed, yielding a constant \nu\nu dependence. A pure V + A (not wanted by anyone) would produce a \nu\nu dependence. A pure vector (representing approximately equal left-handed and right-handed c \leftrightarrow s terms) would give:

\[d\sigma/d\nu \approx 1 + (1 - \nu)^{2}\]  

The present data [44] is consistent with a constant, is inconsistent with \nu\nu, but probably cannot exclude \nu\nu. Better data on \bar{p}-production of \mu^{+}\mu^{-} pairs is likely to be the best way of settling this issue.

(iii) Charmed baryon decays. The decay of charmed baryons into strange baryons provides us with several measurable quantities which depend on the V, A structure of the c \leftrightarrow s current [51]. Of particular interest is the reaction \nu_{\mu} + n \rightarrow \mu^{-} + \Lambda_{c}^{+} followed by the decay \Lambda_{c}^{+} \rightarrow \Lambda + e^{+} + \nu_{e}. The \Lambda-polarization as well as the asymmetry in the angular distribution of the \Lambda-particles, depend on the c \leftrightarrow s current and on the polarization of the decaying \Lambda_{c}^{+}. The latter is determined by the properties of the c \leftrightarrow d current which is presumably understood (see section 3.2).

(iv) F^{+}decays. Certain rare decays of the F^{+}-meson provide a relatively clean test of the V, A structure of the c \leftrightarrow s transition [52]. The decay F^{+} \rightarrow \mu^{+}\nu can proceed only via an axial vector coupling. It is forbidden if the \bar{c}s current is pure vector and is suppressed by a factor sin^{4} \theta if we have a right-handed doublet (c, s)_{R}. Similarly, the decay F^{+} \rightarrow (3\pi)^{+} is strongly suppressed in the presence of a (c, s)_{R} doublet. Both of these tests will be decisive [52], but they involve extremely rare decay modes of the F^{+}. 
Fig. 8. Possible contributions to (a) strangeness changing and (b) charm changing nonleptonic decays. The diagrams represent a weak Hamiltonian which is a "single-quark" operator, guaranteeing a $\Delta I = \frac{1}{2}$ rule. The shaded "blob" represents gluon exchanges.

(v) $\Delta I = \frac{1}{2}$ enhancement in nonleptonic strange particle decays. It has been known for a long time that $\Delta I = \frac{1}{2}$ matrix elements of the effective weak Hamiltonian for nonleptonic decays of strange particles are strongly enhanced relative to $\Delta I = \frac{3}{2}$ nonleptonic matrix elements and relative to semileptonic transitions. There is some circumstantial evidence that the enhancement comes from "single quark operators" such as the one shown in fig. 8a. Such terms contribute only to the $\Delta I = \frac{1}{2}$ nonleptonic decay. The case in which both couplings in fig. 8a are left-handed or right-handed, contributes to the renormalization of the quark wave-function rather than to the nonleptonic decay matrix element. The right-left coupling not only contributes, but is proportional to the mass of the intermediate charmed quark and is therefore presumably enhanced over other terms. Since we excluded a substantial right-handed $c \leftrightarrow d$ coupling, the most logical possibility [53] would be to have a right-handed $c \leftrightarrow s$ coupling of order $G$ followed by a left-handed $c \leftrightarrow d$ coupling of strength $G \sin \theta$.

Whether the strength of such a term is sufficient for explaining the $\Delta I = \frac{1}{2}$ enhancement we do not know, but it is certainly an interesting possibility. One immediate corollary of this mechanism is the prediction that nonleptonic charmed particle decays will not have such an enhancement because the intermediate quark mass ($m_q$) is much smaller (fig. 8b). Consequently, semileptonic branching ratios for $D, F$ and the charmed baryons would be substantially larger than the corresponding branching ratios for $K_S^0, \Lambda, \Sigma$ and $\Xi$. Experimentally, this is verified in the case of D-decays [41]. It is interesting to note that in the SU(2)$_L \times$ SU(2)$_R \times$ U(1) model discussed in section 2.6, such a mechanism cannot operate. In such a model $W_R^\pm$ and $W_L^\pm$ are orthogonal eigenstates, and we cannot have a right-left term in fig. 8.

Our overall conclusion is that there is no direct experimental evidence for or against a $(c, s)_R$ doublet. However, several tests involving the decays $D \rightarrow K^* \nu_e, \Lambda^+ \rightarrow \Lambda \nu_e, F^+ \rightarrow \mu^+ \nu_\mu, F^+ \rightarrow 3\pi$, as well as the production of $\mu^+\mu^-$ pairs by antineutrinos, may help us to resolve this question. The only theoretical motivation for a $(c, s)_R$ doublet comes from a speculation involving the $\Delta I = \frac{1}{2}$ enhancement in nonleptonic $|\Delta S| = 1$ decays. On the other hand, we will see in section 3.7 that there are indirect theoretical arguments against a $(c, s)_R$ doublet.

3.4. Flavor conserving neutral currents

Flavor conserving neutral currents involving second-generation fermions are, experimentally, extremely elusive. An exception is, of course, the neutral current involving $\nu_\mu$ which is measured in $\nu_\mu e^-\nu_\mu$ and $\nu_\mu N$ scattering, and which was already discussed in detail in sections 2.3, 2.4. The neutral current of the muon will hopefully be probed when the asymmetry in $e^+e^- \rightarrow \mu^+\mu^-$ is detected (see section 2.6 and table 2).
Flavor conserving neutral currents involving $s$ and $c$ quarks are more difficult to detect. It is unlikely that the contribution of the $s\bar{s}$ or $c\bar{c}$ pairs in the $q\bar{q}$ "ocean" will ever be isolated in $\nu + N \rightarrow \nu +$ anything. The only other method of detecting such currents would be through the decays $\phi \rightarrow \nu \bar{\nu}$, $\psi \rightarrow \nu \bar{\nu}$. We can express such decays in terms of the neutral current couplings:

$$
\begin{align*}
&g_c^L = 1 - \frac{3}{2}x; \quad g_c^R = \alpha_c - \frac{3}{2}x \\
&g_s^L = 1 + \frac{3}{2}x; \quad g_s^R = \alpha_s + \frac{3}{2}x
\end{align*}
$$

(3.35)

defined in analogy with eq. (2.16) in section 2.2. The $\psi \rightarrow \nu \bar{\nu}$ decay branching ratio is given by [54]:

$$
\frac{\Gamma(\psi \rightarrow \nu_\mu \bar{\nu}_\mu)}{\Gamma(\psi \rightarrow \mu^+ \mu^-)} = \frac{9G^2 M^4_\psi}{256 \pi^2 \alpha^2} (1 + \alpha_c - \frac{3}{2}x)^2 = 8 \times 10^{-7} (1 + \alpha_c - \frac{3}{2}x)^2.
$$

(3.36)

The total decay rate of $\psi$ to all types of massless neutrinos is obtained by multiplying eq. (3.36) by the number of different types of neutrinos. For $\alpha_c = 0$, $x = 0.25$ and three types of neutrinos ($\nu_e$, $\nu_\mu$, $\nu_\tau$) we get:

$$
\Gamma(\psi \rightarrow \nu \bar{\nu}) = 1.5 \times 10^{-3} \text{ eV.}
$$

(3.37)

The $\psi \rightarrow \nu \bar{\nu}$ decay rate can, in principle, be measured by studying the process:

$$
e^+ + e^- \rightarrow \psi' \rightarrow \pi^+ + \pi^- + \text{missing mass}
$$

where the missing mass is $m_\psi$ and no hadrons or photons are emitted from the decay of $\psi$. It is unlikely, however, that such a measurement would achieve the required sensitivity in the foreseeable future.

It is interesting that the decay $\psi \rightarrow \nu \bar{\nu}$ is forbidden in the standard left-handed model, when $\sin^2 \theta_W$ assumes its "grand unification" value of $\frac{3}{8}$. (Note, however, that renormalization effects would shift $\sin^2 \theta_W$ away from this value; see section 2.10.) In a vector SU(2) $\times$ U(1) theory we have $\alpha_c = 1$, but the "grand unification" value is $\sin^2 \theta_W = \frac{3}{8}$, again leading to a forbidden $\psi \rightarrow \nu \bar{\nu}$ decay. This peculiar coincidence reflects the fact that the vector weak coupling of $I_3 = +\frac{1}{2}$ quarks (u, c, etc.) vanishes in most "grand unification" schemes, including all the schemes mentioned in this review (SU(5), SO(10), E(7), etc.). There is an interesting analogy here between the vanishing vector electromagnetic coupling of $I_3 = +\frac{1}{2}$ leptons (neutrinos) and the vanishing of the vector weak coupling of $I_3 = +\frac{1}{2}$ quarks. One can prove that these two facts actually follow from each other in a large class of grand unification schemes, which includes all interesting models [55]. We do not know how important or profound this observation is.

The decay $\phi \rightarrow \nu \bar{\nu}$ is even more difficult to detect than $\psi \rightarrow \nu \bar{\nu}$, and it is practically hopeless.

We therefore conclude that a direct study of flavor conserving neutral currents involving $s$ or $c$ quarks is unlikely in the foreseeable future.

### 3.5. "Natural" conservation of flavor by neutral weak currents

When a generator of a Lie algebra acts on a component of a given irreducible representation, it transforms it into another component of the same irreducible representation. In a gauge theory, the vector boson fields generate the gauge algebra. Consequently, they can only connect states belonging to the same representation. If two different flavors of quarks with equal electric charges belong to two different representations of the gauge algebra, the neutral-current generators cannot connect them.
In the standard left-handed model we can always select u and c or s and d to be the unmixed states in SU(2) X U(1) doublets (see section 3.1). We can do so, regardless of the specific value of the Cabibbo angle $\theta$. Hence, in such a theory, there will be no strangeness changing (sxd) or charm changing ($\bar{c}u$) neutral currents. This is, of course, the famous GIM mechanism [46], which originally led to the prediction of charm. The conservation of flavor by the neutral current is "natural" in the sense that it occurs for any value of $\theta$ and does not require a specific value for any mass or angle parameter.

Our ability to assign u and c or s and d as pure SU(2) X U(1) states, regardless of the value of $\theta$, follows from the assumption that the equal-charge quarks were assigned to identical SU(2) X U(1) multiplets. Consequently, any linear combination of them would also entirely reside in such a multiplet. Had we assigned, say, u and c into a doublet and a singlet, respectively, the SU(2) X U(1) eigenstates would have been uniquely determined by the value of $\theta$ and we would have no freedom in rearranging u and c to belong to two different SU(2) X U(1) representations. In such a case, charm-changing neutral currents would exist, and their strength would depend on the value of the u-c mixing angle.

The general conditions for "natural" flavor conservation have been studied by several authors [56]. Within SU(2) X U(1) we can define neutral-current coupling matrices $G_L$ and $G_R$, generalizing our definitions of the couplings $g_L$ and $g_R$ in section 2.2. We then have the matrix relation:

$$G_L = I_L^3 - 2 \sin^2 \theta W Q, \quad G_R = I_R^3 - 2 \sin^2 \theta W Q.$$  \hspace{1cm} (3.38)

If we use the basis of the "physical" (mass eigenstates) quarks, flavor conservation means that $G_L$ and $G_R$ must be diagonal. However, if we consider all quark flavors with a common charge $Q$, "natural" flavor conservation means that $G_L$ and $G_R$ be diagonal for any set of values of the (generalized) Cabibbo angles. This can happen only if $G_L$ and $G_R$ are multiples of the unit matrix for all quarks of the same charge. Hence, all equal-charge quarks must have the same $I_L^3$-values and the same $I_R^3$-values.

If we further want to exclude effective flavor changing neutral currents of order $G_\alpha$ we have to consider single loop contributions to the effective coupling. The effective matrix element to order $G_\alpha$ may include all neutral second order operators, namely:

$$(I_L^3)^2, \quad (I_R^3)^2, \quad (I_L^3)^2, \quad (I_R^3)^2.$$  

Hence, the required condition is that all equal-charge quarks have identical values of $I_L$ and identical values of $I_R$. The leading term in the effective neutral weak interaction will then be of order $G_\alpha (m_\nu^2/M_W^2)$, consistent with the observed magnitude of the $K_L^0 - K_S^0$ mass difference and the $K_L^0 \rightarrow \mu^+\mu^-$ decay [56].

Additional constraints are required if we want to prevent effective flavor-changing neutral currents due to the exchange of neutral Higgs particles.

An important theoretical question is whether we must demand that any gauge theory obeys "natural" flavor conservation. The success of the GIM mechanism is impressive, and the smallness of charm changing neutral currents is gradually being established. It would be extremely ugly if these would be mere accidents. However, even if we adopt the point of view that there is nothing accidental about flavor conservation, we still have two options:

(i) Adhere to the conditions listed above. This would practically limit us to SU(2), U(1) or their products, as the candidate algebras of weak and electromagnetic interactions. All larger groups such as SU(3), and all "simple unification" schemes are excluded [35].
(ii) We might use a larger gauge group, but impose "natural" flavor conservation by introducing additional discrete symmetries. Such a mechanism exists, for instance, in the SU(3) × U(1) model [57].

Our own prejudice is that "natural" flavor conservation should be a property of the correct gauge theory, and that the introduction of a special discrete symmetry for the sole purpose of enforcing it, is somewhat artificial. We do not feel that imposing "natural" flavor conservation by the neutral Higgs particles is necessarily a mandatory requirement.

3.6. Experimental evidence against |ΔS| = 1 and |ΔC| = 1 neutral currents

The smallness of |ΔS| = 1 neutral weak interactions has been known for many years. The two outstanding examples are, of course [25]:

\[ m(K^0_L) - m(K^0_S) \approx 3.5 \times 10^{-6} \text{ eV} \]  
\[ (K^0_L \rightarrow \mu^+ \mu^-)/(K^0_L \rightarrow \text{all}) \approx 10^{-8}. \]

Both of these results are significantly below the expected values from an effective neutral interaction of order G. Both are consistent with estimates of order \( G\alpha(m_c^2/M^{2}_{W}) \), and are smaller than flavor-conserving neutral weak processes by seven orders of magnitude. It is these experimental observations which led to the prediction of charm, and they form the basis to our belief in the GIM mechanism [46].

Now that charmed particles have been discovered, we are immediately led to ask whether charm changing neutral currents are also absent. There are two simple methods for searching for such currents, and both have yielded negative results:

(i) \( \bar{D}^0 - D^0 \) mixing. A charm changing neutral interaction could produce \( \bar{D}^0 - D^0 \) mixing through a diagram such as fig. 9. The amount of \( \bar{D}^0 - D^0 \) mixing depends on the relative strength of \( D^0 \) decay and the \( D^0 \rightarrow \bar{D}^0 \) transition. If the \( D^0 \rightarrow \bar{D}^0 \) transition is of order G we would expect complete \( D^0 - \bar{D}^0 \) mixing. If charm changing neutral currents are absent, we expect almost no mixing [58].

The dominant decay modes of \( D^0 \) include \( K \) particles. Most \( D^0 \) decays include \( K \)'s. In the case of \( D^0 - \bar{D}^0 \) mixing we would expect a \( K \) with the "wrong" strangeness to be produced in \( D^0 \)-decay. In particular, one may study the process:

\[ e^+ + e^- \rightarrow D^0 + K^\pm + \text{anything} \]
\[ \rightarrow K^\pm \pi^\pm. \]

In this process, the two detected charged \( K \)-mesons will have opposite charges in the absence of \( D^0 - \bar{D}^0 \) mixing (neglecting effects proportional to \( \tan^4 \theta \approx 0.002 \)). In the case of complete \( D^0 - \bar{D}^0 \)}
mixing there will be no correlation between the charges of the two K-mesons. It is convenient to define:

\[ \epsilon = \frac{(N_{\text{opposite}} - N_{\text{same}})}{(N_{\text{opposite}} + N_{\text{same}})} \]

where \( N_{\text{opposite}} \), \( N_{\text{same}} \) are, respectively, the number of events showing K-meson pairs with opposite charges and the same charges. For complete mixing: \( \epsilon = 0 \). In the absence of mixing: \( \epsilon = 1 \). The preliminary experimental determination gave [59]:

\[ \epsilon = 0.76 \pm 0.17. \]

Thus, complete mixing is excluded. The existence of a \( |\Delta C| = 1 \) neutral current of order \( G \) is extremely unlikely, and the leading \( |\Delta C| = 1 \) term may well be of the same order of magnitude as the \( |\Delta S| = 1 \) neutral interaction. Note that a \( \Delta C = 1 \) neutral current which is much weaker than a “normal” weak current could still produce complete \( D^0 - \bar{D}^0 \) mixing [58]. Hence, the absence of complete mixing sets a very stringent limit!

(ii) \( \nu + u \rightarrow \nu + c \). The production of charmed particles by a neutral-current neutrino interaction can be identified by detecting a “wrong sign” lepton. The detected process would be:

\[ \nu_\mu + N \rightarrow \mu^+ + \text{anything} \ (\text{no } \mu^-) \]

or

\[ \nu_\mu + N \rightarrow e^+ + \text{anything} \ (\text{no } \mu^-). \]

In both cases, such a final state would emerge from:

\[ \nu + u \rightarrow \nu + c \]

\[ \begin{cases} 
\mu^+ + \text{anything} \\
\ e^+ + \text{anything.} 
\end{cases} \]

One should remember, of course, that “wrong sign” leptons may come from \( \bar{\nu}_\mu \) or \( \bar{\nu}_e \) contamination in the neutrino beam.

The best available limit on reaction (3.45) is given by the Brookhaven–Columbia bubble chamber experiment [60]:

\[ \frac{\sigma(\nu + N \rightarrow \nu + \text{charm})}{\sigma(\nu + N \rightarrow \mu^- + \text{anything})} \cdot \frac{\Gamma(\text{charm} \rightarrow e^+ + \text{anything})}{\Gamma(\text{charm} \rightarrow \text{anything})} \leq 5 \times 10^{-4}. \]

This, again, completely excludes \( |\Delta C| = 1 \) neutral currents of order \( G \). The limit provided by eq. (3.46) is, however, less significant than the one obtained from eq. (3.42).

There are other, more difficult, methods of studying \( |\Delta C| = 1 \) neutral currents, such as looking for a charm threshold in \( R_\nu \) (as defined in eq. (2.37)). No evidence for any such effects was found, so far.

We may thus safely conclude that no \( |\Delta C| = 1 \) neutral current exists to lowest order. The GIM mechanism is probably operative, giving further support to the notion of “natural” flavor conservation by neutral currents (section 3.5).

3.7. Summary: \( \text{SU}(2) \times U(1) \) classification of second-generation fermions

The discussion of the previous sections leads us to the following conclusions concerning the \( \text{SU}(2) \times U(1) \) classification of second generation fermions:
(i) The left-handed fermions are in doublets:

\[
\begin{pmatrix}
\nu^c_L \\
\mu^- \\
\end{pmatrix}_L = \begin{pmatrix}
c \\
\ell \\
\end{pmatrix}_L
\]

where \( s' \) is the Cabibbo-rotated s quark.

(ii) There is no evidence for any charged right-handed current involving a second-generation fermion. There is substantial evidence against right-handed currents with an effective coupling of order \( G \), connecting \( \nu_\mu \leftrightarrow \mu^- \), \( u \leftrightarrow s \), \( c \leftrightarrow d \). A \( c \leftrightarrow s \) right-handed doublet cannot be excluded, at present. Note, however, that the "natural" conservation of strangeness and charm by neutral currents implies that \( d_R \) and \( s_R \) on one hand, and \( u_R \) and \( c_R \) on the other hand, must have identical \( I \) and \( I_3 \) values. Consequently, if \( (c, s)_R \) form a doublet, \( u_R \) and \( d_R \) must be in doublets, contrary to our analysis in section 2.4. Such a possibility is unlikely.

Within the SU(2) \( \times \) U(1) model, all eight fermions of the first and second generation must be in left-handed doublets. At the same time, all possible right-handed doublets are excluded except:

\[
\begin{pmatrix}
c \\
\ell \\
\end{pmatrix}_R
\]

and even this is unlikely, in view of our last argument.

There is no evidence against the suggestion that the weak and electromagnetic properties of the second-generation fermions are identical to those of the first generation. If this hypothesis is correct, we would be left, again, with only one possible SU(2) \( \times \) U(1) classification, duplicating the classification of the first generation (see section 2.5). Alternatively, we would extend the gauge group into SU(2)_L \( \times \) SU(2)_R \( \times \) U(1) and have left-handed doublets in the \( (1/2, 0) \) representation and right-handed doublets in the \( (0, 1/2) \) (see section 2.6). The present data concerning second-generation fermions (except for \( \nu_\mu \)) sheds no light on this question and adds no information concerning \( \sin^2 \theta_w \).

\section{Flavor conservation and Cabibbo-like angles in the leptonic sector}

The "generation mixing" introduced by the Cabibbo angle is due to the fact that the "mass eigenquarks" do not coincide with the SU(2) \( \times \) U(1) eigenquarks. The transformation between the two sets of eigenquarks is given by the Cabibbo rotation. In the leptonic case, a similar situation would occur if we could uniquely define "mass eigenleptons". We now consider several cases:

(i) \( m(\nu_e) = m(\nu_\mu) \) in a left-handed model. In this case (including, of course, the possibility of \( m(\nu_e) = m(\nu_\mu) = 0 \)) the "mass eigenleptons" can be arbitrarily chosen. The Cabibbo rotation becomes meaningless, and we actually define \( \nu_e \) and \( \nu_\mu \) to be the SU(2) \( \times \) U(1) left-handed partners of \( e^- \) and \( \mu^- \), respectively. In that case, all couplings between fermions and gauge bosons will automatically conserve \( \mu \)-number and \( e \)-number. If all right-handed leptons are in singlets and if we temporarily ignore the Higgs mesons, \( \mu \)-number would be conserved to all orders of the weak interactions, without having to postulate it as a special symmetry. A sufficiently complex set of Higgs mesons would then be the only agent which could lead to \( \mu \)-number nonconservation in a left-handed SU(2) \( \times \) U(1) model [61]. Typical one loop and two loop diagrams contributing to \( \mu \rightarrow e \gamma \) in such a theory are shown in fig. 10. The two-loop contribution is probably dominant and one finds [61]:
Fig. 10. Typical (a) one-loop and (b) two-loop contributions to $\mu^- \rightarrow e^- \gamma$ in a model involving a non-minimal Higgs structure [61].

The two-loop diagram is probably dominant.

This is comparable with the present experimental upper limit of $3.6 \times 10^{-9}$ [62]. Note, however, that with the minimal set of Higgs particles no such contributions exist and $\mu \leftrightarrow e\gamma$ is forbidden.

(ii) $m(\nu_e) \neq m(\nu_\mu)$ in a left-handed model. If the two neutrinos have different masses, the "mass eigenleptons" will, in general, be different from the SU(2) $\times$ U(1) eigenleptons. In that case, the entire Cabibbo formalism is reproduced. The left-handed doublets will be:

$$\left(\begin{array}{c}
\nu'_e \\
\nu'_\mu
\end{array}\right) = \left(\begin{array}{c}
\nu_e \\
-\nu_\mu
\end{array}\right) \cos \phi + \left(\begin{array}{c}
\nu_e \\
\nu_\mu
\end{array}\right) \sin \phi,$$

where

$$\nu'_e = \nu_e \cos \phi + \nu_\mu \sin \phi, \quad \nu'_\mu = -\nu_e \sin \phi + \nu_\mu \cos \phi.$$ (3.51)

$\nu_e, \nu_\mu$ are the "mass-eigenleptons" and $\phi$ is a leptonic Cabibbo angle. We continue to assume that all right-handed leptons are singlets. The mixing of neutrinos will allow $\mu$-number violating transitions, through diagrams such as fig. 11. However, the rate for $\mu^- \rightarrow e^- + \gamma$ in such a model is hopelessly small [63]:

$$\frac{(\mu^- \rightarrow e^- + \gamma)}{(\mu^- \rightarrow e^- + \nu_e + \nu_\mu)} = O\left(\frac{\alpha}{\pi} \frac{\Delta m^2_{\mu e}}{M^2_W} \sin 2\phi\right).$$ (3.52)

For the present upper limit on $m(\nu_\mu)$ we obtain a ridiculously small branching ratio of less than $10^{-19}$. It is clear that any effect which is proportional to $\Delta m^2_{\mu e}$ is unlikely to be detected in this case! The $\nu_e - \nu_\mu$ mixing leads to other effects such as "neutrino oscillations" [64]. This would happen in the following way: Neutrinos emitted in $\pi^+ \rightarrow \mu^+ + \nu_e$ decay would be the $\nu'_\mu$ mixture of $\nu_e$ and $\nu_\mu$. The resulting neutrino beam will be a time-dependent mixture of the two mass eigenstates $\nu_e$ and $\nu_\mu$. The probability of finding the SU(2) $\times$ U(1) eigenlepton $\nu'_e$ then depends on the time evolution of the beam. Consequently, the ratio between produced $e^+$ and $\mu^+$ will vary along the beam line. Such effects can be, in principle, detected in neutrino experiments.

Fig. 11. A mechanism for the reaction $\mu^- \rightarrow e^- \gamma$ in a model involving Cabibbo-like mixing in the leptonic sector.
(iii) \(\mu\)-number violation in a right-handed model. We now consider a model (see section 2.5) in which both the left-handed and the right-handed \(e^-\) and \(\mu^-\) are in SU(2) \(\times\) U(1) doublets:

\[
\begin{pmatrix}
\nu_e^L & \nu_\mu^L & N_e^L & N_\mu^L \\
e^-_L & \mu^-_L & e^-_R & \mu^-_R
\end{pmatrix}.
\]

(3.53)

For simplicity, \(\nu_e\) and \(\nu_\mu\) may be massless. \(N_e^L\) and \(N_\mu^L\) are neutral SU(2) \(\times\) U(1) eigenleptons representing a mixture of "mass eigenleptons" \(N_e\) and \(N_\mu\) [63]:

\[
N_e^L = N_e \cos \chi + N_\mu \sin \chi; \quad N_\mu^L = -N_e \sin \chi + N_\mu \cos \chi.
\]

(3.54)

The mechanism which violates \(\mu\)-number conservation is, in this case, identical to the one described above in the case \(m(\nu_\mu) \neq m(\nu_e)\). The theoretical expression for the \(\mu \to e + \gamma\) branching ratio is similar:

\[
\frac{\left(\frac{\alpha}{\pi} \frac{m(N_\mu)^2 - m(N_e)^2}{M_W^2} \sin 2\chi\right)}{\left(\frac{\alpha}{\pi} \frac{m(N_\mu)^2 - m(N_e)^2}{M_W^2} \sin 2\chi\right)}.
\]

(3.55)

However, it is now entirely possible that \(m(N) = \text{few GeV}\). Consequently, we may obtain \(\mu \to e + \gamma\) branching ratios around \(10^{-8} - 10^{-10}\), comparable with present experimental upper limits [62]. The left-handed \(N\)-leptons (about which we assumed nothing, so far) may also mix with the neutrino, leading to additional contributions to \(\mu \to e + \gamma\) [65].

There are, of course, many other variations of the SU(2) \(\times\) U(1) theory, leading to different forms of \(\mu\)-number violation. The common feature of all such theories is the fact that in the absence of Cabibbo mixing, \(\mu\)-number is automatically conserved (except for the Higgs couplings). Consequently, all \(\mu\)-number violations are proportional to the mixing parameters as well as to the (mass)\(^2\) differences of the mixed states. In order to obtain a nontrivial effect, these masses have to be of the order of a GeV or so, implying the existence of heavy neutral right-handed leptons.

In an SU(2)\(_L\) \(\times\) SU(2)\(_R\) \(\times\) U(1) theory, the \(\mu\)-number violation effects discussed here are basically unchanged. The only minor differences involve (i) replacing \(M_w\) by \(M_{w_L}\) and \(M_{w_R}\) according to the coupling involved and (ii) elimination of contributions from right-left diagrams in which the two W-couplings to the leptons are of opposite handedness.

3.9. CP-violation in a gauge theory

The charged weak current of the standard Weinberg–Salam left-handed model can be written as:

\[
J = (\bar{u}c)\gamma_\mu(1 - \gamma_5)A^\dagger \begin{pmatrix} d \\ s \end{pmatrix}
\]

(3.56)

where \(A\) is a unitary 2 \(\times\) 2 matrix. In principle, such a unitary matrix can be fully parametrized in terms of four real parameters. However, three of these parameters can be "absorbed" into the definitions of the quark states, u, d, c, s. In other words, we can redefine u as \(ue^{i\phi}\) without suffering any observable consequences. Four quark states can absorb only three phase parameters -- one for each quark except for one overall phase. We therefore remain with an \(A\)-matrix which is fully determined by one real parameter. \(A\) is then necessarily an orthogonal matrix and the single parameter can be chosen as the Cabibbo angle \(\theta\):

\[
A = \begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}.
\]

(3.57)
All matrix elements of weak currents involving the four quarks and the four vector gauge particles \((W^+, W^-, Z^0, \gamma)\) will be relatively real in such a theory. Consequently, \(CP\) is necessarily conserved in a gauge theory based on the left-handed model of first and second-generation quarks.

One might suggest that the interaction responsible for \(CP\) violation is not an integral part of the gauge theory of weak and electromagnetic interactions. In that case, all the fundamental questions which were solved by the introduction of gauge theories must be reopened. It is not clear, for instance, that a gauge theory with an external \(CP\)-violating piece remains renormalizable, etc.

It would be much more attractive to be able to account for \(CP\)-violations within the framework of the gauge theory, in a fashion that preserves all the beautiful features of the theory. This could be achieved if the matrix elements of the weak current would contain complex phases which cannot be eliminated by redefining the physical states, and which are, therefore, experimentally observable.

There are, at least, three ways to achieve this within the framework of the SU(2) \(\times\) U(1) gauge theory (but not within the simplest four-quark version of the standard left-handed model). We now discuss them briefly:

(i) \(CP\)-violation and Higgs mesons. We may remain with only four quarks and only left-handed currents but introduce the complex phase parameter into the interactions of the Higgs particles [66]. The “minimal” set of Higgs particles in the Weinberg–Salam model includes four scalar states. Three of them are “eaten up” by the three massive vector gauge bosons and one remains as a physical particle. The couplings of this particle cannot induce \(CP\)-violation. It is clear, however, that we can also introduce a larger set of Higgs particles. Such an assumption is neither elegant nor necessary, but it is perfectly consistent with all the requirement of the theory. Weinberg [66] has pointed out that the (otherwise ugly) possibility of increasing the number of Higgs particles, enables us to introduce an arbitrary relative phase parameter between the interaction of different Higgs particles. Such a phase will produce \(CP\)-violation through the interference of diagrams involving different virtual Higgs particles. This is the only way of incorporating \(CP\)-violation into the standard left-handed model without introducing new quarks or new currents. The attractive feature of this idea is the natural smallness of the \(CP\)-violating amplitude, relative to other weak amplitudes. The resulting \(CP\)-violating amplitude successfully “imitates” the predictions of the superweak theory, in accordance with experiment. The unattractive feature is the explicit dependence on the properties of the Higgs particles, and the required non-minimal set of such particles.

(ii) \(CP\)-violation and right-handed currents. If we do not appeal to the Higgs particles, we may still introduce \(CP\)-violation at the level of second generation quarks. This can be done by the introduction of right-handed weak currents and in particular the \(c \leftrightarrow s\) right-handed transition discussed in section 3.3. The idea is simple: the \(2 \times 2\) matrix \(A\) which we have discussed above, has “lost” three of its arbitrary parameters through a redefinition of the quark states \(u, d, c, s\). If, however, some of the same quarks also participate in a right-handed current, we do not have any more freedom to absorb the phase parameters of the additional current into the redefined quark states. In other words, a relative phase between the left-handed transitions and the right-handed matrix elements cannot be, in general, eliminated. This method of violating \(CP\) was first suggested by Mohapatra [67] several years ago, and was later discussed by other authors [68]. It also leads to predictions which imitate the “superweak” results for the \(K^0 – \overline{K}^0\) system, but the typical strength of the \(CP\)-violating interaction has to be postulated and cannot be derived.

(iii) \(CP\)-violation and the number of quarks. It is clear that the simplest way to produce \(CP\)-violation would be to increase the number of quarks [69]. The argument for \(CP\)-conservation in the case of the left-handed four-quark model started with four (i.e. \(2 \times 2\)) parameters of a unitary matrix and
continued with three (i.e. $4 - 1$) "absorbed" phases. Generalizing the argument to $n$ doublets of left-handed quarks would give an $n \times n$ unitary matrix $A$ with $n^2$ real parameters. The total number of quarks is $2n$, allowing us to "absorb" $(2n - 1)$ phases into the definitions of the quark states. We remain with $n^2 - (2n - 1)$ real parameters, while an $n \times n$ orthogonal matrix allows only $\frac{1}{2}n(n - 1)$ parameters. We are therefore left with an $n \times n$ unitary matrix containing:

$$\frac{1}{2}n(n - 1) \text{ real rotation angles (generalized Cabibbo angles)}$$

$$\frac{1}{2}(n - 1)(n - 2) \text{ phase parameters (CP violating phases).}$$

For one generation of quarks ($n = 1$) we have no angles and no phases. For two generations, we obtain one Cabibbo angle and no phases. For three generations, we obtain three angles and one phase parameter, yielding CP-violating effects within a purely left-handed SU$(2) \times U(1)$ gauge theory [69]. We discuss this possibility in detail in sections 4.7 and 4.10.

We do not have a strong prejudice for or against any of the above mechanisms for CP-violation. It is important to realize that almost any extension of the four-quark left-handed model leads to CP violation. At the present time the extension to six quarks appears to be most attractive, due to reasons which are unrelated to CP-violation (see section 4.5). We therefore prefer the six-quark version of CP-violation over the options of introducing right-handed $c \leftrightarrow s$ currents or complicated Higgs spectra.

3.10. The mass spectrum of second-generation fermions and the Cabibbo angle

We have seen, so far, that the second-generation fermions behave in all respects like their first-generation predecessors. Their weak, electromagnetic and strong interaction patterns are identical. The only exception is the mass spectrum, which was simple and qualitatively "understandable" for the first-generation fermions, but is totally puzzling in the case of the second generation.

There is no explanation whatsoever for the $\mu$-e, c-u and s-d mass differences. While the $\nu_e$-e and u-d differences may well be of electromagnetic origin, the $\nu_\mu$-u and c-s differences are larger by 2--3 orders of magnitude. The $\nu_e$-e and the u-d differences are of essentially comparable order of magnitude and equal signs. The $\nu_\mu$-u and c-s differences are of different orders of magnitude and opposite signs. Except for the presence of the Cabibbo angle which is likely to enter into various mass calculations, we see no reason for the totally different mass patterns of the two generations.

On the level of naive qualitative speculations, the simplest "explanation" could have been the following: There is a new interaction (beyond the usual weak, electromagnetic and QCD interactions) which distinguishes between fermions of the two generations. First-generation fermions do not participate in this interaction (presumably having vanishing values of some new charge which couples to it). Second-generation fermions respond to the "new interaction" and largely acquire their masses through it. Thus, the alleged "new interaction" is responsible for the $\mu$-e mass difference as well as for the mass values of the c and s quarks. Such a speculation is not, apriori, unattractive. After all, something must eventually distinguish between the muon and the electron!

The real difficulty of our naive "new interaction" is, of course, the incredibly accurate measurement [70] of the muon magnetic moment, which agrees with the QED prediction to within 25 parts per $10^9$ in $g$ (or 25 parts per million in $g - 2$). Our "new interaction" must be strong enough to account for the muon mass, but weak enough to contribute to $(g - 2)_\mu$ below the present experimental uncertainty, and below the contribution of the ordinary weak interactions.
An example for such an artificial interaction (which, however, teaches us nothing new) is the coupling of the Higgs particles to the fermions. This coupling is, by definition, proportional to the fermion mass. It distinguishes between \( \mu \) and \( e, c \) and \( u, \) etc. At the same time, the Higgs-particle contribution to \( g - 2 \) is well below the present level of experiments [71]. The Higgs couplings teach us nothing new, since the fermion masses are introduced into them “by hand”. Whether a more meaningful “new interaction” exists we do not know, but it is attractive to assume that in some approximation, all first-generation fermions are massless, while second generation fermions acquire their mass through a mechanism which distinguishes between the two generations.

The assumption of zero “bare mass” for the first-generation fermions leads to an interesting speculation concerning the value of the Cabibbo angle [72]. Assuming that prior to the Cabibbo mixing:

\[
m_u^0 = m_d^0 = 0,
\]

we may obtain the following mass matrix for the \( u \) and \( c \) quarks:

\[
\begin{pmatrix}
0 & m_{uc} \\
m_{cu}^* & m_c^0
\end{pmatrix}.
\]

(3.59)

The unitary matrix needed to diagonalize this mass matrix can be defined by a rotation angle \( \theta_1 \) which can be expressed in terms of the final mass eigenvalues \( m_u \) and \( m_c \):

\[
\tan 2\theta_1 = 2\sqrt{m_u m_c} / (m_c - m_u).
\]

(3.60)

A similar angle, \( \theta_2 \), “rotates” the “bare” \( d_0 \) and \( s_0 \) states into the “physical” \( d \) and \( s \):

\[
\tan 2\theta_2 = 2\sqrt{m_d m_s} / (m_s - m_d).
\]

(3.61)

The Cabibbo angle is, of course, given by:

\[
\theta = \theta_2 - \theta_1 = \frac{1}{2} \left\{ \arctan \frac{2\sqrt{m_d m_s}}{m_s - m_d} - \arctan \frac{2\sqrt{m_u m_c}}{m_c - m_u} \right\}.
\]

(3.62)

Assuming:

\[
m_u \approx 4 \text{ MeV}; m_d \approx 7 \text{ MeV}; m_s \approx 150 \text{ MeV}; m_c \approx 1500 \text{ MeV}
\]

(3.63)

we find:

\[
\theta \approx 9^\circ.
\]

Compared with the “experimental” value of 13° (see section 3.1). The agreement is not bad, in view of the crude assumptions used here.

This calculation of the Cabibbo angle ignores several important effects and involves some unjustified assumptions. We do not consider it to be a solution to the problem of understanding the relation between fermion masses and the Cabibbo angle. We have mentioned it here only as an illustration of one possible way of attacking the problem. Other interesting attempts have been suggested by different authors [73].
3.11. Grand unification and the second generation of fermions

We have indicated in sections 2.9 and 2.10 that all the motivations for a quark–lepton connection or for a grand unification of strong, electromagnetic and weak interactions already exist at the level of first-generation fermions. The second-generation fermions do not add to these motivations. Models such as the SU(5) and SO(10) schemes (see section 2.10) relate only quarks and leptons of the same generation. Consequently, each generation of fermions is assigned to a separate representation of the grand unification algebra. In the case of SO(10), the left-handed fermions $\nu_{\mu}, \mu, \tau$ (and the left-handed antifermions) form a 16-dimensional multiplet, analogous to the one formed by $\nu_e, e, u, d$. In SU(5), the second-generation fermions form an additional set of $10 + 5$ multiplets. Such grand unification schemes shed no light on the distinction between different generations. In fact, it is clear that in such models the Cabibbo angle (or angles) which represents “generation mixing” cannot be understood within the framework of the grand unification group. It is also apparent that the mass differences between first- and second-generation fermions are not due, in such models, to any of the interactions which are represented by the gauge bosons of the grand unification algebra.

There are more ambitious models which try to assign all fermions (of both generations) to one large multiplet of a grand unification scheme. The most interesting model in this category is the E(7) model [37].

In E(7), all fundamental fermions are in the “spinor” 56-dimensional representation. E(7) has a maximal subgroup SU(6) $\times$ SU(3). SU(6) is the flavor gauge algebra and SU(3) is the usual color gauge group. The 56-multiplet of E(7) decomposes into the following SU(6) $\times$ SU(3) multiplets:

$$56 \supset (6, 3) + (\bar{6}, \bar{3}) + (20, 1).$$

Hence, the fundamental fermions are:

(i) Six quarks (color triplets)

(ii) Six antiquarks (color antitriplets)

(iii) Twenty leptons and antileptons (color singlets).

Since the electric charge is an SU(6) generator, we conclude that, in this model, SU(6) is a “simple unification” group (see section 2.9). It includes an SU(3) subgroup and dictates the charges of the six quarks to be $\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}$. Hence, the four quarks of the first two generations are supplemented by two additional $Q = -\frac{1}{3}$ quarks. The Weinberg angle is predicted, at the SU(6) level, to obey:

$$\sin^2 \theta_w = \frac{3}{4}$$

in clear conflict with experiment. There is no reason to expect the SU(6) gauge bosons to have very high masses. Consequently, the renormalization effects on $\sin^2 \theta_w$ should not be substantial (see sections 2.9, 2.10).

The leptons and antileptons fall into a 20-dimensional representation of SU(6). Lepton number is not conserved. There are four negatively charged leptons ($e^-, \mu^-, \tau^-$ and one more), four positively charged leptons (their antiparticles) and twelve neutral leptons. Of the four negatively charged leptons, two must be in SU(2) $\times$ U(1) triplets, and two are in doublets.

The E(7) model represents a bold attempt to unite all fermions in one multiplet. Unlike SU(5) or SO(10) it makes a definite prediction about the total number of quarks and leptons. It predicts an unequal number of $Q = \frac{1}{3}$ and $Q = -\frac{1}{3}$ quarks. All of these predictions can be tested. The main difficulty of the model is, however, its unacceptable value for $\sin^2 \theta_w$. 

Any model (including E(7)) which introduces fermions of different generations into the same multiplet of a grand unification gauge group must violate natural flavor conservation by neutral currents. If the s and d quarks belong to the same representation of such an algebra, it must have a generator corresponding to an $\bar{s}d$ neutral current. Such a current can be made small by various means, such as assuming a very large mass for the corresponding vector boson. However, the absence of $|\Delta S| = 1$ and $|\Delta C| = 1$ neutral currents must be introduced explicitly into the model, and it does not follow “naturally”.

We believe that the two major difficulties mentioned above (i.e. $\sin^2 \theta_W$ and the absence of natural flavor conservation) provide us with strong arguments against such models. However, their explicit predictions concerning the existence and number of various quark- and lepton-flavors will serve as the most conclusive test.

### 3.12. Summary: second-generation fermions

The second generation of fermions raises only one new fundamental question: Why do these fermions exist, and what distinguishes them from their predecessors?

All weak interactions of second-generation fermions are, so far, consistent with being identical to those of the first generation. There is no indication for right-handed charged currents. All left-handed fermions seem to fit in SU(2) $\times$ U(1) doublets. There is no evidence against the hypothesis that the classification of second-generation fermions is identical to that of the first generation.

The mass pattern of the second generation fermions is completely mysterious and the presence of “$\Delta$-generation mixing” by the Cabibbo angle is the only obvious connection between the two generations. It is very likely that the Cabibbo angle (or angles) and the fermion masses are intimately related.

The natural conservation of all flavors by the neutral currents emerges as an attractive hypothesis, and it provides an important constraint for model building.

### 4. Third-generation fermions: $\nu$, $\tau$, t, b

#### 4.1. Introducing the third generation of fermions

Unlike the first two generations of fermions, the third generation is not yet established. We have very strong evidence for the existence of the $\tau^-$-lepton, some evidence for the fifth quark (probably b), indirect evidence for a neutral lepton related to $\tau$ and no concrete evidence for a sixth quark.

We do not know of any profound reason for the existence of the third generation of fermions (or for the existence of the second generation, for that matter). The discovery of the $\tau^-$-lepton (like the discovery of the $\mu^-$-lepton) was both surprising and “unwanted” by any convincing theory.

While the first two generations are clearly similar to each other in many respects, the pattern of the third generation is not yet established. There are several possibilities. Some of them are the following:

(i) The third generation of fermions is analogous to the first two generations. It includes two leptons ($\nu_\tau$ and $\tau^-$) and two quarks (t and b). The left-handed leptons and quarks belong to SU(2) $\times$ U(1) doublets with possible Cabibbo-like mixing with earlier generations [74].

(ii) There is no third generation. The fifth and sixth left-handed quarks are associated, respectively, with the first and second generations. Both have $Q = -\frac{1}{3}$ and they are denoted by b and h. Each
generation of quarks forms an SU(3)-triplet. The t-quark does not exist. The additional leptons join \( \nu_e, e, \mu \) in an SU(3) octet. Such a situation occurs in practically all "simple-unification" schemes (see section 2.9), such as SU(3) \( \times \) SU(3), E(6), E(7), etc. [31, 32, 37].

(iii) The \( \tau \)-lepton belongs to a third generation which is similar to the first two, but each generation of left-handed leptons contains three leptons: \( (\nu_e, e^-, E^-); (\nu_\mu, \mu^-, M^-); (\nu_\tau, \tau^-, T^-) \). The fifth and sixth left-handed quarks belong to the first two generations which contain (each) a triplet of left-handed quarks. This is the case in the SU(3) \( \times \) U(1) model [57] invoked to account for the trimuon events observed in neutrino processes (see section 5.2).

There are other possibilities which we will not discuss here. All of them can be tested experimentally by studying the production and decay properties of the new leptons and new quarks.

As a reference point and a convenient theoretical framework, we will use the possibility (i), namely: a third generation of fermions which is as similar as possible to the first two generations. Whenever possible, we will discuss experimental tests of this hypothesis.

4.2. The \( \tau \)-lepton: experimental facts and open problems

The first indications for the existence of the \( \tau \)-particle were announced [75] in July 1975. In the two years since that time, many of its properties were elucidated. It is now clear that, barring the possibility of a completely new type of particle, the \( \tau \) appears to be a lepton. Its known properties are briefly summarized here:

(i) The mass of \( \tau^- \) is around 1.8 GeV [76, 77]. The \( \tau \) has now been observed in \( \psi ' (3.68) \) decays and the possibility that \( \tau \) is a charmed particle is completely ruled out.

(ii) The energy dependence of the production cross section \( e^+ + e^- \rightarrow \tau^+ + \tau^- \) is consistent with the expression computed for a point-like \( J = \frac{1}{2} \) particle [78]. Better data are required in order to fully establish this point. The energy dependence for \( \tau^+ \tau^- \) production is different from the energy dependence for DD production.

(iii) The decays:

\[
\begin{align*}
\tau^- &\rightarrow e^- + \nu + \bar{\nu}, \\
\tau^- &\rightarrow \mu^- + \nu + \bar{\nu}
\end{align*}
\]

are observed at a branching ratio of 15%–20% (each) [79]. This is consistent with theoretical expectations [80]. We have deliberately avoided any labels for the neutrinos, and we return to this question in section 4.3.

(iv) The momentum spectra of the electrons and muons produced in \( \tau \)-decay are consistent with the prediction of a pure V-A theory [78]. However, other possibilities such as various combinations of V – A and V + A are not yet excluded. The momentum spectrum of electrons in \( \tau \)-decay is completely different (much "harder") than the one observed in D-decay.

(v) The \( \rho^- \nu \) and \( A_1 \bar{\nu} \) decay modes have been observed with the following branching ratios [81, 82]:

\[
\begin{align*}
B(\tau^- \rightarrow \rho^- + \nu) &\approx 0.24 \pm 0.09, \\
B(\tau^- \rightarrow A_1 + \nu) &\approx 0.11 \pm 0.07
\end{align*}
\]
These values are consistent with theoretical expectations [80]. The decay: $\tau^\rightarrow \pi^- + \nu$ has not been observed [81] and there is some doubt as to whether it occurs at the expected branching ratio of 7–10%. We hope that this point will soon be clarified experimentally.

(vi) Assuming that a new neutral lepton $\nu_\tau$ is emitted in all $\tau$-decays, the following upper limits for its mass are obtained [82]:

$$m(\nu_\tau) < 300 \text{ MeV} \quad \text{(from } \tau^\rightarrow A_1^- + \nu_\tau), \quad (4.5)$$

$$m(\nu_\tau) < 550 \text{ MeV} \quad \text{(from } \tau^\rightarrow e^- + \bar{\nu}_e + \nu_\tau). \quad (4.6)$$

(vii) The following upper limits have been obtained for $\tau$-decays in which e-number, $\mu$-number or $\tau$-number are not conserved [83]:

$$B(\tau^\rightarrow e^- + \gamma) < 2.6\%, \quad (4.7)$$

$$B(\tau^\rightarrow \mu^- + \gamma) < 1.3\%, \quad (4.8)$$

$$B(\tau^\rightarrow \ell^- + \ell^- + \ell^+) < 0.6\% \quad \text{($\ell^\pm \equiv e^\pm \text{ or } \mu^\pm$).} \quad (4.9)$$

(viii) All properties of $\tau$ are consistent with the hypothesis that it is a sequential lepton, namely — a lepton carrying its own conserved (or, effectively conserved; see section 3.8) quantum number [78].

We will therefore assume that $\tau$ is a lepton which belongs to a new SU(2) × U(1) multiplet, and is not associated with e or $\mu$.

4.3. Does $\tau^-$ have its own neutral lepton?

The simplest pattern for the coupling of the $\tau$-lepton to the charged weak current duplicates the couplings of the electron and muon. We assume that $\tau^-$ and a new neutral lepton $\nu_\tau$ form a left-handed SU(2) × U(1) doublet, forming a charged weak current of the form:

$$\bar{\nu}_\tau \gamma_\alpha (1 - \gamma_5) \tau^- . \quad (4.10)$$

There is no evidence against this simple hypothesis. However, it is interesting to study whether the existence of a new neutral lepton is indeed a necessary consequence of the present experimental information on the $\tau$-particle [84].

We could avoid the new neutral lepton by assigning the left-handed $\tau^-$ to an SU(2) × U(1) singlet. In the absence of Cabibbo-like mixing between $\tau^-$, $e^-$ and $\mu^-$, a singlet $\tau^-$ would be stable. This is, of course, unacceptable. However, there is no a priori argument against $\tau^-$-$\mu$ or $\tau^-$-$e$ mixing. When such mixing is present we would have two doublets and one singlet:

$$\begin{pmatrix} \nu_e \\ e' \end{pmatrix}_L , \quad \begin{pmatrix} \nu_\mu \\ \mu' \end{pmatrix}_L , \quad \begin{pmatrix} \tau' \end{pmatrix}_L , \quad (4.11)$$

where the "weak eigenleptons" $e'$, $\mu'$, $\tau'$ are given by:

$$\begin{pmatrix} e' \\ \mu' \\ \tau' \end{pmatrix} = B \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} , \quad (4.12)$$

and $B$ is a unitary $3 \times 3$ matrix. The decay of $\tau^-$ will then proceed through the $\tau^-$-component resid-
ing in the same doublet with \( \nu_e \) or \( \nu_\mu \). The leptonic decays of \( \tau^- \) would then involve an "ordinary" neutrino (\( \nu_e \) or \( \nu_\mu \)) and an "ordinary" antineutrino (\( \bar{\nu}_e \) or \( \bar{\nu}_\mu \)). Horn and Ross [84] have studied this possibility and showed that the following partial widths emerge for purely leptonic \( \tau^- \) decays:

\[
\Gamma(\tau^- \rightarrow \mu^- \nu_\mu \bar{\nu}_e) = K(1 + 4x + 2x^2),
\]

\[
\Gamma(\tau^- \rightarrow \mu^- \nu_e \bar{\nu}_e) = 4K,
\]

\[
\Gamma(\tau^- \rightarrow e^- \nu_\mu \bar{\nu}_e) = 4K,
\]

where \( K \) is a factor which depends on the mixing angles of the matrix \( B \). A similar set of expressions with a different factor \( K' \), is obtained by replacing \( \mu \leftrightarrow e, \nu_\mu \leftrightarrow \nu_e \). From all of these relations, we obtain the following interesting formula:

\[
\frac{\Gamma(\tau^- \rightarrow e^- \nu_e \bar{\nu}_e)}{\Gamma(\tau^- \rightarrow \mu^- \nu_\mu \bar{\nu}_e)} = 1 - 4x + 2x^2 \approx 0.4.
\]

From all of these relations, we obtain the following interesting formula:

\[
\Gamma(\tau^- \rightarrow \ell^- \nu_\ell \bar{\nu}_e) = 1 - 4x + 2x^2 \approx 0.4
\]

where \( \Gamma(\tau^- \rightarrow \ell^- \nu_\ell \bar{\nu}_e) \) represents the sum of all possible combinations for \( \ell^\pm \equiv e^\pm \) or \( \mu^\pm \), and \( \Gamma(\tau^- \rightarrow \mu^- \nu_\mu \bar{\nu}_e) \) represents a sum over all types of neutrinos. We have used \( x \sim 0.25 - 0.3 \). The observed branching ratio [79] for \( \tau^- \rightarrow \mu^- \nu_\mu \bar{\nu}_e \) then leads to the prediction:

\[
\Gamma(\tau^- \rightarrow \ell^- \nu_\ell \bar{\nu}_e) \approx 6\% - 8\%.
\]

in gross disagreement with the much smaller experimental upper limit [83]:

\[
\Gamma(\tau^- \rightarrow \ell^- \nu_\ell \bar{\nu}_e) < 0.6\%.
\]

We therefore conclude that the "concocted" model which attempts to avoid a new neutral lepton, fails [84]. Thus, we have indirect evidence that such a lepton exists, and is probably associated with \( \tau^- \) in the same left-handed doublet:

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\tau^-
\end{pmatrix}_L
\]

We still have two possibilities concerning the mass of \( \nu_\tau \):

(i) \( m(\nu_\tau) < m(\tau^-) \). In this case \( \nu_\tau \) is emitted in all \( \tau^- \) decays and the experimental limit [82]:

\[
m(\nu_\tau) < 300 \text{ MeV}
\]

is valid.

(ii) \( m(\nu_\tau) > m(\tau^-) \). In this case we expect Cabibbo-like mixing among the three left-handed doublets: (\( \nu_e, e^- \)); (\( \nu_\mu, \mu^- \)); (\( \nu_\tau, \tau^- \)). The \( \tau^- \) could decay through such mixing effects without emitting a \( \nu_\tau \). The mass of \( \nu_\tau \) could easily be of the order of 10 GeV [85]. An amusing consequence of this scheme is the transition \( \mu^- \rightarrow e^- + \gamma \) which proceeds through the Cabibbo-like mixing of the leptonic doublets. The branching ratio for \( \mu^- \rightarrow e^- + \gamma \) is then proportional (see section 3.8) to \( |m(\nu_\tau)|^2 \) and is of the order of \( 10^{-9} \) for \( m(\nu_\tau) \approx 20 \text{ GeV} \). We have no reason to believe that \( m(\nu_\tau) > m(\tau^-) \) and we consider it to be a somewhat ugly (but not yet excluded) possibility.

We summarize: we have indirect evidence for the existence of a neutral lepton associated with \( \tau^- \). It is probably lighter than 300 MeV, although it may still be heavier than \( \tau^- \).
4.4. Models with six or more leptons

Assuming that $\nu_\tau$ exists, we have at least six leptons. There is a wide variety of models based on six or more leptons which are not inconsistent with the available data. We briefly list here several such models. Our list is far from exhaustive.

(i) **Left-handed six-lepton SU(2) $\times$ U(1) model.** This is the simplest scheme involving three left-handed doublets:

\[
\left( \begin{array}{c} \nu_e \\ e^-_L \\ \nu_\mu \\ \mu^-_L \\ \nu_\tau \\ \tau^-_L \end{array} \right).
\]

(4.22)

All right-handed leptons are in SU(2) $\times$ U(1) singlets. This classification contradicts the atomic physics parity violation experiment (see sections 2.3, 2.5). Each of the three neutral leptons $\nu_e, \nu_\mu$ and $\nu_\tau$ may or may not be massless. If one or more neutral leptons has a mass, Cabibbo-like mixing is very likely, leading to decays such as $\mu^- \to e^- + \gamma, \tau^- \to \mu^- + \gamma, \tau^- \to e^- + \gamma$. The rates for these decays are proportional to the mixing angles and to the neutrino mass differences (see eq. (3.52), section 3.8).

(ii) **Vector-like leptonic SU(2) $\times$ U(1) model.** The only SU(2) $\times$ U(1) classification of the right-handed electron, which is consistent with all data (sections 2.3, 2.5), is in a doublet. Extending this conclusion to all other leptons we end up with a six-lepton vector-like model, having the three left-handed doublets of eq. (4.22) and three right-handed doublets:

\[
\left( \begin{array}{c} N_e \\ e^-_R \\ N_\mu \\ \mu^-_R \\ N_\tau \\ \tau^-_R \end{array} \right).
\]

(4.23)

Here, at least some of the neutral leptons must be massive in order to avoid conflict with the V-A features of $\mu$-decay, $\beta$-decay and hyperon decays. It is clear that in such a model $m(N_e) \neq m(N_\mu) \neq m(N_\tau)$. Consequently, a $3 \times 3$ matrix of Cabibbo-like angles leads to elaborate mixing effects, as well as to violations of c-number, $\mu$-number and $\tau$-number conservation. It is also possible that some of the right-handed neutral leptons are related to some of the left-handed ones. For instance, if $\nu_\tau$ is heavy (see section 4.3), it is entirely possible that $\nu_\tau \equiv N_e$ or $\nu_\tau \equiv N_\mu$. A six-lepton model with massive neutral leptons of different masses will, in principle, exhibit $CP$-violation effects analogous to those expected in a six-quark model (see sections 3.9 and 4.10).

(iii) **More complicated SU(2) $\times$ U(1) models.** More complex SU(2) $\times$ U(1) models may involve larger number of leptons, or the assignment of some right-handed leptons to doublets and others to singlets. Some other schemes propose SU(2) triplets of the type:

\[
\left( \begin{array}{c} 1^+ \\ \nu_e \\ e^- \end{array} \right).
\]

(4.24)

and yet others involve doubly charged leptons. We see no reason to adopt any of these models and will not discuss them any further.

(iv) **SU(2)$_L \times$ SU(2)$_R \times$ U(1).** In section 2.6 we discussed the attractive possibility of an SU(2)$_L \times$ SU(2)$_R \times$ U(1) gauge algebra. The full lepton classification of such a model presumably includes the three left-handed doublets of eq. (4.22) in three $(\frac{1}{2}, 0)$ representations of SU(2)$_L \times$ SU(2)$_R \times$
U(1). The corresponding right-handed leptons are in three \( (0, \frac{1}{2}) \) representations. Note, however, that in the version in which \( M_{W_R} \gg M_{W_L} \) (see section 2.6) it is entirely possible that the right-handed neutral companion of the electron (or, respectively, \( \mu \) or \( \tau \)) is \( \nu_e \) (or, respectively, \( \nu_\mu \) or \( \nu_\tau \)). In such a case all \( \nu \) particles have non-vanishing masses. The dominance of left-handed couplings in \( \mu \)-decay and \( \beta \)-decay is then guaranteed by the asymmetry of the \( W^\pm \) masses rather than by an asymmetric lepton classification.

There is only one piece of experimental data which might motivate us to consider complicated leptonic models, involving eight or more leptons and large gauge groups: the trimuon events observed in neutrino reactions. We return to this issue in section 5.2.

4.5. Do we need the \( t \) and \( b \) quarks?

There is no fundamental theoretical argument which tells us that the number of different types of quarks must be larger than four. However, there are several phenomenological and theoretical considerations which may already necessitate additional quarks. Some of these considerations are:

(i) The observations of the new \( \mu^+\mu^- \) resonance \( \Upsilon(9.5) \) \([86]\) and its possible excited state \( \Upsilon'(10.1) \) \([87]\) are most easily interpreted as bound states of a new type of quark. We discuss the implications of the \( \Upsilon \)-particles in section 4.6. At this point we only state that their existence is, at present, the strongest indication for the presence of a fifth quark.

(ii) The first two generations of fermions exhibit a certain degree of quark-lepton symmetry or, at least, analogy. The existence of the \( \tau \)-lepton and its probable neutral partner \( \nu_\tau \), implies that a new doublet of quarks will be needed in order to preserve the same symmetry \([74]\).

(iii) In a pure left-handed \( SU(2) \times U(1) \) model, the requirement of an equal number of left-handed quark doublets and lepton doublets is not just a question of aesthetics or a pleasant analogy. The divergent parts of the triangle anomaly diagrams \([30]\) are removed \([29, 74]\) only if the \( (\nu_\tau, \tau^-) \) doublet is supplemented by a \( (t, b) \) left-handed doublet.

(iv) We have seen in section 3.9 that in a pure left-handed \( SU(2) \times U(1) \) model, \( CP \)-violation is naturally introduced at the level of six quarks, while at the four-quark level it can be only induced through the couplings of a sufficiently complicated set of Higgs particles. While there are several different ways of incorporating \( CP \)-violation into a gauge theory of the weak interactions, the simplest and most direct appears to be the introduction of six (or more) quarks \([69]\). We discuss \( CP \)-violation in the six-quark model in section 4.10.

(v) In section 2.6 we have discussed a left-right symmetric \( SU(2)_L \times SU(2)_R \times U(1) \) model. We mentioned two variations of the model: one \([19, 20]\) in which \( M_{W_L} \ll M_{W_R} \) and one \([21]\) in which \( M_{W_L} = M_{W_R} \) but the right-handed doublet always connect light fermions to heavy fermions. In this second version, the right-handed partners of the \( u \) and \( d \) quarks must be new quarks, presumably \( b \) and \( t \), respectively. Note that this requirement is not related to the "\( \rho \)-anomaly". No \( \rho \)-anomaly is expected in such a model, because of the orthogonality of \( W_L^\pm \) and \( W_R^\pm \). However, the fifth and sixth quarks are necessary in the second variation of the \( SU(2)_L \times SU(2)_R \times U(1) \) model.

Except for the existence and interpretation of the \( \Upsilon \) particles, none of the above arguments is entirely convincing. However, their accumulated weight should certainly lead us to study seriously the various features of the proposed \( t \) and \( b \) quarks.
4.6. New mesons with b or t quarks: the T-family, B mesons and T mesons

One of the simplest methods of discovering a new type of quark q is, of course, the observation of its bound q̅q system. The successful description of the ψ-family in terms of the simple “charmonium” model [88] encourages us to speculate on the features of similar systems involving heavier quarks such as b or t. The “standard” work on this subject is the analysis of Eichten and Gottfried [89], which predicts the level structure of fig. 12 for a q̅q system, as a function of the mass of the heavy quark. Eichten and Gottfried used the same potential which proved successful in the case of charmonium and showed that with such a potential the energy differences between the levels shrink as the mass of the quark increases. Furthermore, the number of levels below the threshold for the production of pairs of new mesons, increases as a function of the quark mass.

The observation of the T particle in the reaction [86]

\[ p + \text{nucleus} \rightarrow T + \text{anything} \rightarrow \mu^+ + \mu^- \] (4.25)

immediately led to its interpretation as a bound state of a new heavy quark and antiquark with a quark mass around 4–5 GeV. For such a mass value, the Eichten–Gottfried analysis leads to the following predictions [89]:

(i) \( m(T') - m(T) \approx 420 \text{ MeV} \). (4.26)

(ii) From the mass difference quoted in (i), it follows that the decay

\[ T' \rightarrow T + \pi + \pi \] (4.27)

is not a dominant decay mode of T'. Consequently, the branching ratios:

\[ B' = \frac{\Gamma(T' \rightarrow \mu^+\mu^-)}{\Gamma(T' \rightarrow \text{all})} \quad \text{and} \quad B = \frac{\Gamma(T \rightarrow \mu^+\mu^-)}{\Gamma(T \rightarrow \text{all})} \] (4.28)

are not very different. Assuming that T and T' are b̅b or t̅t bound states, we obtain a rough estimate [90]:

![Fig. 12. Relative positions of the levels of “quarkonium” as a function of the quark mass [89].](image-url)
\[ B'/B \approx 0.6 \quad \text{(for a b-quark; } Q = -\frac{1}{3}) \]  
(4.29)

\[ B'/B \approx 0.4 \quad \text{(for a t-quark; } Q = \frac{2}{3}) \]  
(4.30)

(iii) A third level, \( \mathcal{T}'' \) is predicted below the "continuum", with

\[ m(\mathcal{T}'') - m(\mathcal{T}') \approx 330 \text{ MeV} \]  
(4.31)

and

\[ B'' = \frac{\Gamma(\mathcal{T}'' \rightarrow \mu^+\mu^-)/\Gamma(\mathcal{T}'' \rightarrow \text{all})}{\Gamma(\mathcal{T} \rightarrow \text{all})} \approx 0.7 \quad \text{(for b or t).} \]  
(4.32)

(iv) In a hadronic production process, such as \( p + p \rightarrow \mathcal{T} + \text{anything} \), the expected production rates of \( \mathcal{T}, \mathcal{T}' \) and \( \mathcal{T}'' \) are comparable to each other. The mass differences are of the order of 5\%, and the full mass dependence of the production cross-section is expected to yield effects of the order of, at most, factor of two or three [90]. Since both the production cross-section and the \( \mu^+\mu^- \) branching ratios for \( \mathcal{T}, \mathcal{T}' \) and \( \mathcal{T}'' \) are predicted to be more or less of the same order of magnitude, we expect to see all three resonances as \( \mu^+\mu^- \) bumps in pp scattering. We denote:

\[ N(\mathcal{T}) = \sigma(p + p \rightarrow \mathcal{T} + \text{anything}) \cdot \frac{\Gamma(\mathcal{T} \rightarrow \mu^+\mu^-)/\Gamma(\mathcal{T} \rightarrow \text{all})}{\Gamma(\mathcal{T} \rightarrow \text{all})}. \]  
(4.33)

A typical estimate then yields [90]:

\[ N(\mathcal{T}) \cdot N(\mathcal{T}') \cdot N(\mathcal{T}'') \approx 1:0.3:0.15 \quad \text{(for b-quark; } Q = -\frac{1}{3}) \]  
(4.34)

\[ N(\mathcal{T}) \cdot N(\mathcal{T}') \cdot N(\mathcal{T}'') \approx 1:0.17:0.09 \quad \text{(for t-quark; } Q = \frac{2}{3}). \]  
(4.35)

Note that all estimates for the branching ratios \( B \) and the resonance signals \( N \) depend on the smallness of the decay \( \mathcal{T}' \rightarrow \mathcal{T} + \pi + \pi \). This, in turn, is sensitive to the \( \mathcal{T}' - \mathcal{T} \) mass difference which is predicted to be 420 MeV.

Experimentally, a double peak structure is observed [87] in the 10 GeV region (fig. 13). However, the mass difference between the two peaks appears to be closer to 600 MeV, and definitely larger than the predicted 420 MeV. This immediately implies that the branching ratio

\[ B' = \frac{\Gamma(\mathcal{T}' \rightarrow \mu^+\mu^-)/\Gamma(\mathcal{T}' \rightarrow \text{all})}{\Gamma(\mathcal{T} \rightarrow \text{all})} \]  
(4.36)

Fig. 13. The measured cross section \( d^2\sigma/dm dy y = 0 \) for \( p + p \rightarrow \mathcal{T} + \text{anything} \) times the branching ratio \( B = \Gamma(\mathcal{T} \rightarrow \mu^+\mu^-)/\Gamma(\mathcal{T} \rightarrow \text{all}) \), after background subtraction. The two peaks presumably correspond to \( \mathcal{T} \) and \( \mathcal{T}' \) [87].
is significantly smaller than the predicted value (eqs. (4.29), (4.30)). Consequently, the resonance signal \( N(T') \) should be smaller than predicted by eqs. (4.34), (4.35).

Experimentally [87]:

\[
N(T')/N(T) \approx \frac{1}{3}
\]  

(4.37)

consistent with eq. (4.33), but inconsistent with our entire line of reasoning (because eq. (4.33) was based on a 420 MeV \( T' - T \) mass difference while the data give a 600 MeV difference!)

This puzzling situation may or may not be settled in the near future. However, we still feel fairly confident that the \( T \) and \( T' \) represent bound states of (at least one) new heavy quark.

Assuming that we have here only one new quark, is it the bottom quark (b) or the top quark (t)?

(i) Various estimates of the absolute production cross-section favor the b-hypothesis [90], [91].

However, we cannot regard these estimates as conclusive since there are still some important open questions concerning the mechanism of \( \psi \) production in hadronic reactions.

(ii) The relative size of the \( T \) and \( T' \) signals also tends to favor the b-hypothesis. We have seen in eqs. (4.34) and (4.35) that \( N(T')/N(T) \) is expected to be larger for b than for t. If anything, it is too large.

(iii) It is likely that a reliable picture of the \( T \)-family will emerge only from the observation of these particles in \( e^+e^- \) collisions. The production cross-sections for:

\[
e^+ + e^- \rightarrow T, T', T'', \text{etc.}
\]

as well as the widths for radiative decays between \( C = -1 \) and \( C = +1 \) states depend on \( Q^2 \), the squared charge of the new quark. This should enable us to determine whether we are facing a b-quark or a t-quark.

Note that the properties of "toponium" or "bottomonium" are independent of the weak interaction properties of the new quarks. On the other hand, most of the properties of low-lying mesons containing one new quark are sensitive to the gauge theory assignment of the left- and right-handed b-quark (or t-quark). The most notable exception to this statement are the charges of the new mesons. For instance, the lowest-lying mesons including b-quarks presumably correspond to the isospin doublet:

\[
B^0 \equiv (b\bar{d}); \quad B^- \equiv (b\bar{u}),
\]

while the lowest-lying mesons with a t-quark are:

\[
T^+ \equiv (t\bar{d}); \quad T^0 \equiv (t\bar{u}).
\]

The observation of new charged mesons and their decay modes to charmed or strange mesons could determine whether we have a b or a t quark. The \( B^- \) and \( T^+ \) would decay into charm = +1 or strangeness = -1 systems while \( T^- (= T^+) \) and \( B^+ (= B^-) \) would decay into charm = -1 or strangeness = +1 systems.

4.7. The left-handed six-quark model

We now proceed to introduce the left-handed SU(2) \( \times \) U(1) six-quark model [69, 92], including the left-handed doublets

\[
\begin{pmatrix}
u \\ d
\end{pmatrix}_L, \quad \begin{pmatrix}c \\ s
\end{pmatrix}_L, \quad \begin{pmatrix}t \\ b
\end{pmatrix}_L.
\]  

(4.38)
with possible Cabibbo-like mixing, and the right-handed singlets:

\[(u)_R, (d)_R, (c)_R, (s)_R, (t)_R, (b)_R.\]  

(4.39)

While our discussion centers on the SU(2) × U(1) model, we note that an SU(2)_L × SU(2)_R × U(1) model based on the same six quarks (section 2.6) would lead to the same predictions for all charged current and flavor-changing neutral current processes. In such a model the left- and right-handed quarks are in the \((\frac{1}{2}, 0)\) and \((0, \frac{1}{2})\) doublets, respectively.

The charged quark current of the SU(2) × U(1) model has the form:

\[J^- = (\bar{u} \gamma_\mu (1 - \gamma_5) A (d))_R (s),\]  

(4.40)

where \(A\) is a unitary 3 × 3 matrix. The most general form of the matrix \(A\) can be represented by [69]:

\[A = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 + s_2 s_3 e^{i\delta} & c_2 c_3 e^{i\delta} \\ s_1 s_2 & c_2 s_3 e^{i\delta} & c_1 s_2 + s_2 c_3 e^{i\delta} \end{pmatrix},\]  

(4.41)

where:

\[c_j = \cos \theta_j; \quad s_j = \sin \theta_j; \quad j = 1, 2, 3.\]

The three angles \(\theta_1, \theta_2, \theta_3\), are Cabibbo-like angles, representing mixing of the three left-handed doublets. The phase angle \(\delta\) introduces CP-violation (see sections 3.9, 4.10).

What can we say about the values of the parameters \(\theta_1, \theta_2, \theta_3, \delta\)? The angle \(\theta_1\) is the original Cabibbo angle. The value of \(\cos \theta_1\) is directly measured in nucleon \(\beta\)-decay and is found to be

\[\cos \theta_1 = 0.974 \pm 0.002,\]  

(4.42)

\[\theta_1 = (13.2 \pm 0.5)^\circ.\]  

(4.43)

The combination \(\sin \theta_1 \cos \theta_3\) appears in all strangeness changing semileptonic decays. It corresponds to the \(\sin \theta\) factor in ordinary Cabibbo theory. We find:

\[\sin \theta_1 \cos \theta_3 = 0.229 \pm 0.003,\]  

(4.44)

yielding:

\[\cos \theta_3 \geq 0.96,\]  

(4.45)

or:

\[\theta_3 \leq 16^\circ.\]  

(4.46)

We do not have a direct determination of \(\theta_2\). However, we can set a bound on \(\theta_2\) using a theoretical estimate of the \(K_S^0 - K_L^0\) mass difference [90]. We have already mentioned (in section 3.2) that a successful estimate of the mass of the charmed quark can be obtained by considering the contribution of fig. 7a to the \(K_S^0 - K_L^0\) mass difference. In a model involving a t-quark in addition to the c-quark, such an estimate should presumably remain valid. It now relates the \(K_S^0 - K_L^0\) mass difference to a certain function of \(m_c, m_t\) and \(\theta_2\). The usual expression (eq. 3.23):
\[ \Delta M \approx K m_c^2 \cos^2 \theta \sin^2 \theta \]
is replaced by [90]:

\[
\Delta M = K s_c^2 c_c^2 c_\theta^2 \left[ c_c^2 m_c^2 + s_c^2 m_c^2 + \frac{2 s_c^2 c_c^2 m_t^2 m_c}{m_t^2 - m_c^2} \ln \left( \frac{m_t^2}{m_c^2} \right) \right] \tag{4.47}
\]

where \( K \) is, again, a factor depending on parameters such as \( G \) and \( M_W \) but not on the quark masses or Cabibbo angles.

It is clear that for very large values of \( m_t \), eq. (4.47) implies that \( s_c^2 \ll 1 \). Crudely speaking, we must have:

\[
\tan^2 \theta_2 < m_c/m_t. \tag{4.48}
\]

If \( m_t \) is somewhere in the 5–20 GeV range, we obtain:

\[
\theta_2 < 30^\circ. \tag{4.49}
\]

For larger values of \( m_t \) we clearly obtain lower values of \( \theta_2 \).

We cannot establish an upper bound on the phase angle \( \delta \). We will see in section 4.10 that we can only state that \( \sin \delta \neq 0 \) and is larger than \( 5 \times 10^{-3} \).

We conclude that all \( \theta \)-angles in the Cabibbo matrix \( A \) are small:

\[
\theta_1 \approx 13^\circ; \quad \theta_2 < 30^\circ; \quad \theta_3 < 16^\circ. \tag{4.50}
\]

The three doublets:

\[
\begin{pmatrix} u \\ d \end{pmatrix}_L, \quad \begin{pmatrix} c \\ s \end{pmatrix}_L, \quad \begin{pmatrix} t \\ b \end{pmatrix}_L \tag{4.51}
\]

are mixed by relatively little amounts, and the main weak transitions will normally connect \( u \) to \( d, c \) to \( s, t \) to \( b \). Transitions proportional to \( \sin \theta_i \) \((i = 1, 2, 3)\) become important only when the main transitions are kinematically forbidden, as is the case in strange particle decays and, possibly in bottom-quark decays (see section 4.8).

### 4.8. Weak production and weak decays of b-quarks in the left-handed six-quark model

Having determined the bounds on the mixing angles \( \theta_1, \theta_2 \) and \( \theta_3 \), we can now proceed to discuss the production of b-quarks in neutrino reactions and the weak decays of mesons containing b-quarks.

The simplest and most likely mechanism for b-production is the reaction:

\[
\bar{\nu} + u \rightarrow \mu^+ + b. \tag{4.52}
\]

The production rate is proportional to \( \sin^2 \theta_1 \cdot \sin^2 \theta_3 \). Using the known value of \( \theta_1 \) and the bound on \( \theta_3 \) (eq. (4.50)) we get:

\[
\sigma(\bar{\nu} + N \rightarrow \mu^+ + b + \text{anything})/\sigma(\bar{\nu} + N \rightarrow \mu^+ + \text{anything}) < 0.5\%. \tag{4.53}
\]

Another mechanism for b-production in \( \bar{\nu}N \) scattering involves the c-quarks of the “qQ-ocean” in the target nucleon. The cross-section for:

\[
\bar{\nu} + c \rightarrow \mu^+ + b \tag{4.54}
\]
includes the coupling strength of the $c \leftrightarrow b$ transition, which is likely to be significantly larger than the corresponding $u \leftrightarrow b$ transition. However, the production cross section is strongly suppressed by the small parameter which represents the probability of finding a $c\bar{c}$ pair in the nucleon. If the bound $\theta_3 < 16^\circ$ is more or less saturated, the process (4.52) is expected to dominate. However, if $\theta_3 \ll 16^\circ$, production of a $c$-quark may become the largest source of $b$-quarks. In that case, the production rate will probably be significantly smaller than the 0.5% of eq. (4.53).

The production of $b$-quarks in neutrino (rather than antineutrino) reactions proceeds only via the "$q\bar{q}$-ocean", and the expected rate is substantially smaller than in $\nu N$ reaction.

Assuming that $m_b < m_t$, the dominant weak decay modes of the $b$-quark are:

$$b \rightarrow c + e^- + \bar{\nu}_e; \quad b \rightarrow c + \mu^- + \bar{\nu}_\mu; \quad b \rightarrow c + d + \bar{u}. \quad (4.55)$$

All important decay modes will include charmed particles [92]. Assuming that the semileptonic branching ratio

$$\Gamma(b \rightarrow c + \mu^- + \bar{\nu}_\mu)/\Gamma(b \rightarrow \text{all})$$

is of the order of 10%, we conclude that for every event of the type:

$$\bar{\nu} + u \rightarrow \mu^+ + b$$

we will have:

(i) 0.1 events with a $\mu^+\mu^-$ pair

(ii) 0.1 events with a $\mu^+\mu^+$ pair

(iii) 0.01 events with a $\mu^+\mu^-\mu^+$ triplet.

If $\theta_3 \approx \theta_1$, the overall rate of "unusual" multimuon events due to $b$-production would be:

$$\sigma(\bar{\nu} + N \rightarrow \mu^+ + \mu^+ + \text{anything})/\sigma(\bar{\nu} + N \rightarrow \mu^+ + \text{anything}) \approx 5 \times 10^{-4} \quad (4.58)$$

$$\sigma(\bar{\nu} + N \rightarrow \mu^+ + \mu^- + \mu^+ + \text{anything})/\sigma(\bar{\nu} + N \rightarrow \mu^+ + \text{anything}) \approx 5 \times 10^{-5}. \quad (4.59)$$

If $\theta_3 < \theta_1$, the corresponding rates are, of course, lower. At the same time, trimuon events and equal-sign dimuons may come from other sources such as production of $c\bar{c}$ pairs.

If $m_b > m_t$, the dominant weak decay modes of the $b$ quark are:

$$b \rightarrow t + e^- + \bar{\nu}_e; \quad b \rightarrow t + \mu^- + \bar{\nu}_\mu; \quad b \rightarrow t + d + \bar{u}. \quad (4.60)$$

In that case, $t$-decays resemble $c$-decays (see section 4.9), and the decays $b \rightarrow t$ will have similar characteristics to the decays $b \rightarrow c$ mentioned above. Consequently, the general features of the experimental signature of $b$-decays are independent of whether $m_b < m_t$ or $m_t < m_b$.

The lowest lying mesons containing $b$-quarks are expected to be:

$$B^0 \equiv (b\bar{d}); \quad B^- \equiv (b\bar{u}).$$

Their leading decay modes are, presumably:

$$B^0 \rightarrow D^+ + e^- + \bar{\nu}_e; \quad D^+ + \mu^- + \bar{\nu}_\mu; \quad D + \text{pions} \quad (4.61)$$

$$B^- \rightarrow D^0 + e^- + \bar{\nu}_e; \quad D^0 + \mu^- + \bar{\nu}_\mu; \quad D + \text{pions}. \quad (4.62)$$

If the angles $\theta_2, \theta_3$ are sufficiently small, the lifetime of the $B^0$ meson may be relatively long. In that case, $B^0 - \bar{B}^0$ mixing effects may be substantially larger than $D^0 - \bar{D}^0$ mixing and $CP$-violating effects may be stronger than they are in the $K^0 - \bar{K}^0$ system [90, 93].
If we identify the $T$ as a $b\bar{b}$ state, the mass of the $B^0$ and $B^-$ mesons is expected to be approximately 5 GeV. The presently available neutrino beams should enable us to produce such mesons, and with improved studies of multimuon events in $\bar{\nu}N$ scattering, we may find further evidence for the existence of the $b$-quark.

Needless to say, in the left-handed SU(2) $\times$ U(1) model no "$y$-anomaly" is expected in $\bar{\nu}N$ scattering. In the SU(2)$_L$ $\times$ SU(2)$_R$ $\times$ U(1) model with a $(u, b)_R$ doublet in the $(0, \frac{1}{2})$ representation (see section 2.6) we do not expect right-handed $b$-production in anti-neutrino experiments, because of the orthogonality of $W^\pm_L$ (which couples to the $\nu_\mu$ or $\bar{\nu}_\mu$ beam) and $W^\pm_R$ (which couples to the right-handed $b$). In such a model, the experimental situation of $b$-production in neutrino processes is identical to that of the left-handed SU(2) $\times$ U(1) model discussed in this section. However, any degree of mixing between $W^\pm_L$ and $W^\pm_R$ would change this conclusion and would allow for the production of right-handed $b$-quarks.

4.9. Weak production and weak decays of $t$-quarks in the left-handed six-quark model

We first consider the case $m_t < m_b$. In this case direct production of $t$-quarks in neutrino reactions will mostly proceed by converting a "valence" $d$-quark into a $t$-quark:

$$\nu_\mu + d \rightarrow \mu^- + t.$$  \hspace{1cm} \hspace{0.5cm} (4.63)

A second possible mechanism would utilize the strange quark of the "$q\bar{q}$-ocean":

$$\nu_\mu + s \rightarrow \mu^- + t,$$

$$\bar{\nu}_\mu + \bar{s} \rightarrow \mu^+ + \bar{t}.$$ \hspace{0.5cm} (4.64)

The production of a $d$-quark is favored by the large "valence" to "ocean" probability ratio. The production of an $s$-quark enjoys a larger "Cabibbo-factor". Both of these mechanisms are, of course, completely analogous to the mechanisms for charm production. For the dominant mechanism we expect that, well above threshold for $t$-production:

$$\sigma(\nu_\mu + d \rightarrow \mu^- + t)/\sigma(\nu_\mu + d \rightarrow \mu^- + c) \approx \tan^2 \theta_2.$$ \hspace{0.5cm} (4.65)

Using the crude estimate of eq. (4.48) we then find:

$$\sigma(\nu_\mu + d \rightarrow \mu^- + t)/\sigma(\nu_\mu + d \rightarrow \mu^- + c) < m_c/m_t.$$  \hspace{0.5cm} (4.66)

For $m_t \approx 5$ GeV and $E_\nu \approx 200$ GeV we would expect at most 1% of all $\nu N$ events to contain a $t$-quark.

The decay modes of the $t$-quark are very similar to those of the $c$-quark (as long as $m_t < m_b$). The dominant decays involve strange particles:

$$t \rightarrow s + e^+ + \nu_e; \quad t \rightarrow s + \mu^+ + \nu_\mu; \quad t \rightarrow s + u + \bar{d}.$$  \hspace{0.5cm} (4.67)

The lowest lying mesons containing $t$-quarks are

$$T^+ \equiv (t\bar{d}), \quad T^0 \equiv (t\bar{u})$$

and their most likely decay modes

$$T \rightarrow K^+ + e^+ + \nu_e + \text{pions}$$

$$T \rightarrow K^+ + \mu^+ + \nu_\mu + \text{pions}$$

$$T \rightarrow K^+ + \text{pions}.$$  \hspace{0.5cm} (4.68)
An interesting *indirect* source of t-quarks (for $m_t < m_b$) is the production of b followed by the $b \rightarrow t$ decay. A typical process would be:

$$\bar{\nu}_\mu + u \rightarrow \mu^+ + b \rightarrow t + \begin{cases} e^- + \bar{\nu}_e \\ \mu^- + \nu_\mu \\ d + \bar{u} \end{cases}.$$  \hspace{1cm} (4.72)

If $m_t < m_b$, practically all b-decays will include a t-quark (unless the b-t mass difference is small).

The experimental signature of t-quarks produced in neutrino experiments is very similar to the signature of the charmed quarks. For instance, t-production in $\nu N$ scattering would lead to $\mu^- \mu^+$ and $\mu^- e^+$ events similar to those generated by c-production. The only different feature which might be observable would be the average transverse momentum of the $\mu^+$ or $e^+$ relative to the direction of the allegedly exchanged W-boson. The transverse momentum of the $\mu^+$ in t-decay is likely to be larger than that of the c-generated $\mu^+$. Consequently, a high statistics experiment might detect a two-slope structure in the $p_T$ distribution of the $\mu^+$'s. The second slope will gradually appear only at high energies, above the t-threshold.

All the above remarks relate to the case $m_t < m_b$. However, assuming that the T particle is a b$\bar{b}$ state (see section 4.6), it is more likely that $m_b < m_t$. In that case, the $A$ matrix of eq. (4.41) together with eq. (4.50) predicts that all dominant t-decays will include a b-quark:

$$t \rightarrow b + e^+ + \nu_e; \quad t \rightarrow b + \mu^+ + \nu_\mu; \quad t \rightarrow b + u + \bar{d}.$$ \hspace{1cm} (4.73)

Since most b-decays involve a charmed quark, t-decays will often exhibit a remarkable cascade, such as:

$$t \rightarrow b + \mu^+ + \nu_\mu \rightarrow c + \mu^- + \bar{\nu}_\mu \rightarrow s + \mu^+ + \nu_\mu.$$ \hspace{1cm} (4.74)

Assuming that:

$$\frac{\Gamma(t \rightarrow b + \mu^+ + \nu_\mu)}{\Gamma(t \rightarrow \text{all})} \approx \frac{\Gamma(b \rightarrow c + \mu^- + \bar{\nu}_\mu)}{\Gamma(b \rightarrow \text{all})} \approx \frac{\Gamma(c \rightarrow s + \mu^+ + \bar{\nu}_\mu)}{\Gamma(c \rightarrow \text{all})} \approx 10\%$$ \hspace{1cm} (4.75)

and that, well above the threshold for t-production (see eqs. (4.66), (4.67)):

$$\sigma(\nu + N \rightarrow \mu^- + t + \text{anything})/\sigma(\nu + N \rightarrow \mu^- + \text{anything}) \approx 0.5\%$$ \hspace{1cm} (4.76)

we expect the following rates of multimuon events *due to t-production*:

$$R(\mu^- \mu^+), R(\mu^- \mu^+) \approx 10^{-3}; \quad R(\mu^- \mu^-), R(\mu^- \mu^+ \mu^+) \approx 5 \times 10^{-4}; \quad R(\mu^- \mu^+ \mu^+ \mu^+) \approx 5 \times 10^{-5}; \quad R(\mu^- \mu^- \mu^+) \approx 5 \times 10^{-6}.$$ \hspace{1cm} (4.77)

Here $R(\mu^- x)$ is defined as:

$$R(\mu^- x) = \frac{\sigma(\nu + N \rightarrow \mu^- + t + \text{anything}) \cdot \Gamma(t \rightarrow x + \text{anything})}{\sigma(\nu + N \rightarrow \mu^- + \text{anything}) \cdot \Gamma(t \rightarrow \text{all})}.$$ \hspace{1cm} (4.78)
The rates for all of these multimuon signals are quite small. However, some of the signatures are particularly interesting (especially those of $\mu^-\mu^+\mu^+$ and $\mu^-\mu^+\mu^-$).

4.10. CP-violation in the left-handed six-quark model

The incorporation of CP-violation into the gauge theory was actually the first motivation for the introduction of the six-quark model [69]. CP-violation is naturally built into the left-handed six-quark model (see sections 3.9, 4.7). The phase parameter $\delta$ is responsible for all CP-violating effects in the quark matrix elements. Thus, the CP-violating amplitudes are proportional to $\sin \delta$.

If we restrict our attention to CP-violating phenomena involving the three “light” quarks $u$, $d$ and $s$, we immediately see that all such CP-violating amplitudes are proportional to $\sin \theta_3$. This follows from the structure of the matrix $A$ (eq. (4.41)). The relevant matrix elements of $A$ are $A_{22}$ and $A_{32}$, in which $e^{i\delta}$ is always multiplied by $\sin \theta_3$. We can further show that all CP-violating amplitudes involving only external $u$, $d$, $s$ quarks must vanish in the limit $m_c = m_t$. The proof is simple: if $m_c = m_t$ and if the $c$ and $t$ quarks do not appear in the initial or final state of the considered transition, we may always choose one linear combination of $c$ and $t$ which decouples from both the $d$ and $s$ quarks. Consequently, no interference between amplitudes of different phase is possible. We therefore conclude that all CP-violating transitions involving only $u$, $d$, $s$ quarks must be proportional to $m_c^2 - m_t^2$.

The overall conclusion of this discussion is that all CP-violating amplitudes among states containing only $u$, $d$, $s$ quarks must be proportional to:

$$(m_c^2 - m_t^2) \sin \theta_3 \sin \delta.$$  (4.79)

This holds for all CP-violating K-decays as well as for the electric dipole moment of the neutron.

Why is CP-violation a small effect? The present theory does not answer this question. The parameters $\theta_3$ and $\delta$ may be extremely small, but they do not have to be small. All we know is:

$0 \leq \sin \delta \leq 1, 0 \leq \sin \theta_3 \leq 0.28$. This is, of course, consistent with, but does not explain, the magnitude of CP-violation.

We now proceed to calculate the parameters of the CP-violating amplitudes in $K^0 \rightarrow 2\pi$. This was first done by Pakvasa and Sugawara [94] and, independently, by Maiani [95]. The standard formalism starts from the mass matrix of the neutral K-system with matrix elements $M_{kl} - \frac{1}{2}i\Gamma_{kl}$,

$$M_{kl} = M_0 + (k\mid H_w \mid l) + P \sum_x \frac{(k\mid H_w \mid x) (x\mid H_w \mid l)}{M_0 - E_x}.$$  (4.80)

$$\Gamma_{kl} = 2\pi \sum_x \frac{(k\mid H_w \mid x) (x\mid H_w \mid l)}{E_x - M_0} \delta(E_x - M_0).$$  (4.81)

If CP is violated, the $M$-matrix is not symmetric and its eigenvalues are proportional to

$$(1 - \epsilon) K_o + (1 + \epsilon) \bar{K}_o; \quad (1 + \epsilon) K_o - (1 - \epsilon) \bar{K}_o.$$  (4.82)

The $\epsilon$-parameter which characterizes the magnitude of CP-violation in the $K_o$ eigenstates is given by:

$$|\epsilon| = (\text{Im} M_{12})/\sqrt{\Delta M^2 + \frac{1}{4}\Gamma_S^2}$$  (4.83)

where $\Delta M = m(K_S) - m(K_L)$ and $\Gamma_S$ is the width of $K_S^0$. The violation of CP in $K_L \rightarrow 2\pi$ may be due
Fig. 14. Dominant diagram for calculating $\text{Im} M_{12}/\Delta M$. Contributions for $x = u, c, t$ should be included.

either to the mixture of opposite $CP$-values in the $K_L$-state (characterized by $\epsilon$) or to $CP$-violation in the decay amplitude itself. The latter is characterized by the parameter $\epsilon'$, where:

$$
\epsilon' = \frac{i}{\sqrt{2}} \exp \{i(\delta_2 - \delta_0)\} \frac{\text{Im} A_2}{A_0}.
$$

(4.84)

$\delta_2, \delta_0$ are the $I = 2$ and $I = 0 \pi\pi$ S-wave phase shifts at $\sqrt{s} = M_K$. The amplitudes $A_2, A_0$ are defined by:

$$
\langle K^0 | \pi\pi | I = 0 \rangle = A_0 e^{i\delta_0},
$$

(4.85)

$$
\langle K^0 | \pi\pi | I = 2 \rangle = A_2 e^{i\delta_2}.
$$

(4.86)

The calculation of $\epsilon$ requires a calculation of $\text{Im} M_{12}/\Delta M$. The numerator is given by the diagram in fig. 14, where $x = c, t$. The denominator is given by the same diagram, but is dominated by the $x = u$ term. In the approximation $\cos \theta_1, \cos \theta_3 \approx 1$, and defining:

$$
\eta = m_c^2/m_t^2
$$

(4.87)

we obtain [96]:

$$
\left| \frac{\text{Im} M_{12}}{M} \right| \approx s_2 c_2 s_3 \sin \delta \left[ s_2^2 \left( 1 + \frac{\eta \ln \eta}{1 - \eta} + c_2^2 \left( \eta - \frac{\eta \ln \eta}{1 - \eta} \right) \right) / \left[ c_2^2 + s_2^2 - 2 s_2^2 c_2^2 \frac{\eta \ln \eta}{1 - \eta} \right] \right].
$$

(4.88)

For any $m_t$ between 5 GeV and 20 GeV and for any $\theta_2$ obeying the bound of eq. (4.48), we find that the numerical value of the expression in the square brackets is between 3 and 7. Hence:

$$
\left| \frac{\text{Im} M_{12}}{M} \right| \approx 5 s_2 c_2 s_3 \sin \delta.
$$

Using this expression and the approximate relation $2\Delta M \approx \Gamma_S$, we find

$$
|\epsilon| \sim 3 s_2 c_2 s_3 \sin \delta.
$$

(4.89)

The small observed magnitude of $\epsilon$ is unexplained (but not inconsistent with the theory). In fact, using:

$$
|\epsilon| \approx 2 \times 10^{-3}
$$

(4.90)

$$
s_3 < 0.28; \quad s_2 < 0.5
$$

(4.91)

we obtain from eq. (4.89):

$$
\sin \delta > 5 \times 10^{-3}.
$$

(4.92)
At least one of the parameters $\theta_2, \theta_3, \delta$ must be small in order to account for the smallness of $\epsilon$.

The $\epsilon'$ parameter is experimentally consistent with zero, and is definitely much smaller than $\epsilon$. This is actually predicted in the left-handed six-quark model [94, 96]. There are two classes of diagrams which contribute to $\epsilon'$ (fig. 15). The diagram of fig. 15a involves the conversion of a $c\bar{c}$ or $t\bar{t}$ system into $d\bar{d}$. This is strongly suppressed by the Zweig–Iizuka rule. The suppression factor cannot be accurately determined but is probably of the order of $1-10\%$. The second diagram (fig. 15b) involves a term of order $m_q^2/M_W^2$. For $m_t \lesssim 5-20$ GeV, this gives us another factor of $1\%-10\%$. The estimates of $\epsilon'$ are given by [94, 96]:

\begin{align}
 \text{fig. 15a:} & \quad |\epsilon'| \approx |A_2/A_o| \cdot O(m_q^2/m_W^2) & (4.93) \\
 \text{fig. 15b:} & \quad |\epsilon'| \approx |A_2/A_o| \sin \delta \ s_2 c_2 s_3 (\xi_c - \xi_t) & (4.94)
\end{align}

where $\xi_c, \xi_t$ are the $c\bar{c}$ and $t\bar{t}$ Zweig–Iizuka suppression factors, respectively. We therefore conclude that:

$$|\epsilon'/\epsilon| \approx |A_2/A_o| \quad (1\%-10\%).$$

This prediction is consistent with the experimental situation and represents a nontrivial success of the model.

Predictions of the model for other $CP$-violating $K$-decays have been extensively discussed by Ellis et al. [96]. In all interesting cases the predictions of the model are experimentally indistinguishable from the predictions of the superweak theory.

The electric dipole moment of the neutron is due, in the six quark model, to the diagram of fig. 16.
It is easy to see that here, again, the CP-violating effect would disappear if either \( \theta_3 \) or \( \delta \) would vanish; it would also vanish if \( m_c = m_t \) or \( m_s = m_b \) (since here the s-quark does not appear in the initial or final state). Maiani [95] and Ellis et al. [96] have discussed this process. The predicted dipole moment is [96]:

\[
\left| \frac{D}{e_{\text{n}}} \right| \approx \frac{\alpha}{\pi^3} \sin \delta \frac{s_1^2 s_2 c_2 s_3}{m_w^4} \frac{(m_l^2 - m_s^2)(m_s^2 - m_c^2)}{m_w^4} m_u. \tag{4.96}
\]

Substituting eq. (4.89) into this expression and assuming \( m_c^2 < m_t^2, m_s^2 < m_b^2 \) we obtain:

\[
\left| \frac{D}{e_{\text{n}}} \right| \approx \frac{\alpha s_1^2}{3\pi^3} \frac{m_t^2 m_b^2}{m_w^4} m_u \approx 10^{-30} \text{ cm.} \tag{4.97}
\]

This prediction is, again, not very different from the predictions of the superweak theory, and is, again, consistent with (but far below) the present experimental limit.

The overall emerging picture is the following: CP-violation is naturally incorporated into a gauge theory based on the left-handed SU(2) \( \times \) U(1) six-quark scheme of section 4.7. There is no inconsistency with experiment, but the smallness of the \( \epsilon \)-parameter is not predicted. The relation \( \epsilon' \ll \epsilon \) is a natural consequence of the model and an extremely small electric dipole moment for the neutron is predicted. If and when the b- and t-quarks are discovered, it would be extremely interesting to determine \( \theta_2 \) and \( \theta_3 \) from the weak production and weak decays of the heavy quarks. This would enable us to express all CP-violating amplitudes in terms of a single parameter \( \delta \). At the present time all we know is that, within the left-handed six quark model:

\[
\theta_1 \approx 13^\circ; \quad \theta_2 < 30^\circ; \quad \theta_3 < 16^\circ; \quad \delta > 0.3^\circ. \tag{4.98}
\]

4.11. Right-handed models involving b- and t-quarks

The last four sections (4.7-4.10) were devoted to a discussion of the purely left-handed six-quark model. We assumed (section 4.7) that all right-handed quarks are in SU(2) \( \times \) U(1) singlets. We noted that the two versions of the SU(2)_L \( \times \) SU(2)_R \( \times \) U(1) model of section 2.6 would lead to similar predictions for charged currents, either because \( M_{W_L^+} \ll M_{W_R^+} \) or because of the orthogonality of \( W_L^+ \) and \( W_R^+ \).

We now proceed to discuss the possibility of observable right-handed charged currents involving the six quarks, u, d, c, s, t, b. We continue to use the left-handed assignments of section 4.7 but consider possible right-handed doublets. Our discussion of charged currents in sections 3.1 and 3.2 has led us to exclude the following right-handed doublets:

\[
(u, d)_R; \quad (u, s)_R; \quad (c, d)_R. \tag{4.99}
\]

Consequently, the only possible right-handed doublets are:

\[
(t, d)_R; \quad (c, s)_R; \quad (t, s)_R; \quad (u, b)_R; \quad (c, b)_R; \quad (t, b)_R. \tag{4.100}
\]

Our analysis of the neutrals currents in section 2.4 excluded the possibility \( l_3 (d_R) = -\frac{1}{2} \). This excludes any \((q, d)_R\) doublet, including, of course, \((t, d)_R\). The same analysis indicates that \( u_R \) is probably in a singlet, but that conclusion is not yet fully established. Thus, the \((u, b)_R\) doublet is unlikely, but not ruled out.

If the \( \Upsilon \) is a \( b\bar{b} \) state, the mass of the b-quark (and the \( B^- \), \( B^0 \) mesons) is expected to be around
5 GeV. In this case, a right-handed \((u, b)_R\) doublet would lead to a substantial "γ-anomaly" in \(\bar{\nu}N\) scattering, and to a significant rise with energy of the ratio

\[
\sigma(\bar{\nu} + N \rightarrow \mu^+ + \text{anything})/\sigma(\nu + N \rightarrow \mu^- + \text{anything}). \tag{4.101}
\]

The absence of these effects [13] together with the identification \(T = b\bar{b}\) will provide us, if con-

firmed, with an additional convincing argument against a \((u, b)_R\) doublet.

We have discussed the \((c, s)_R\) doublet in detail in section 3.3. There is no evidence for it, but no direct evidence against it. However, the natural conservation of charm by neutral currents (see sections 3.5, 3.6, 3.7) led us to believe that \(I_3(c_R) = I_3(u_R)\). This, presumably, indicates that \(c_R\) is also in a singlet.

If we accept this conclusion we remain with one and only one possible right-handed SU(2) \(\times\) U(1) doublet:

\[
\begin{pmatrix} t \\ b_R \end{pmatrix}.
\]

As long as the b- and t-quarks are not discovered we obviously cannot exclude such a doublet. How-

ever, we have no reason to expect its existence and the natural conservation of b-number and t-

umber by neutral currents would not permit it.

Our overall tentative conclusion is that, within SU(2) \(\times\) U(1), it is unlikely that any right-handed doublets of quarks exist.

Right-handed doublets within various versions of SU(2)\(_L\) \(\times\) SU(2)\(_R\) \(\times\) U(1) are, of course, allowed

In the version with \(M_{W_L^+} < M_{W_R^+}\), right-handed charged currents are always suppressed, and their detection depends on the value of \(M_{W_L^+}/M_{W_R^+}\). In the version with \(M_{W_L^+} = M_{W_R^+}\), the right-handed doublets can be detected, in principle, through complicated decays such as:

\[
b \rightarrow s + u + c
\]

or:

\[
t \rightarrow c + d + s
\]

both proceeding through a \(W_R^+\). The detection of such decay modes and their identification as being induced by \(W_R^+\) would be extremely difficult.

4.12. Summary: third-generation fermions

The basic questions of the third generation of fermions are very simple:

(i) Does the third generation exist?
(ii) If so, does it follow a similar pattern to its two predecessors?
(iii) If so, why?

We have almost conclusive evidence for the \(\tau^-\)-lepton and for its sequential character.

We have convincing indirect evidence for the neutral lepton \(\nu_{\tau}\), but we are not sure about its mass (less than 300 MeV or more than 2 GeV?).

We have good evidence for a new quark in the existence of \(T\) and \(T'\). We have hints that the new quark is the b-quark, but we cannot be certain until it is observed in \(e^+e^-\) collisions.
There is no experimental evidence for any additional quarks. The assumption that third-generation quarks and leptons follow the same pattern of the first and second generation does not contradict any known data. However, there are very little data to support it, at the present time.

5. Summary and open problems

5.1. Do we have a successful gauge theory of quarks and leptons?

A surprisingly successful and simple picture of the world of quarks and leptons is obtained within the original example [2] of a gauge theory for the weak and electromagnetic interactions: $SU(2) \times U(1)$. This picture includes the following ingredients:

(i) The gauge group is $SU(2) \times U(1)$.
(ii) We have six quarks and six leptons.
(iii) All left-handed fermions are in $SU(2) \times U(1)$ doublets:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \end{pmatrix}_L; \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L$$

where small Cabibbo-like mixing definitely exists for quarks and may exist for leptons.

(iv) All right-handed fermions are in $SU(2) \times U(1)$ singlets.

(v) CP-violation is incorporated into the gauge theory.

(vi) The pattern of Higgs particles is minimal: there are two $SU(2) \times U(1)$ doublets before symmetry breaking and one remaining physical Higgs particle after symmetry breaking.

(vii) The masses of $W^\pm$ and $Z$ and the Weinberg angle are

$$\sin^2 \theta_W \approx 0.25 - 0.3$$
$$M_{W^\pm} \approx 68 \text{ GeV}$$
$$M_Z \approx 81 \text{ GeV}.$$  

This model is, essentially, the simplest possible gauge theory for the weak and electromagnetic interactions. That such a simple (almost naive!) model works so well, is amazing. It is consistent with almost all established experimental facts including all charged current processes and all neutral current processes involving neutrinos. It is consistent with theoretical prejudices such as the absence of anomalies and "natural" flavor conservation by neutral currents. There are only two problems:

(a) The smallness (or absence) of parity violation in the atomic bismuth experiments [11] implies that the right-handed electron is not a singlet. It could be in a doublet, but then we run into anomalies and into other theoretical unpleasantness.

(b) There is no clear mechanism for parity violation in the theory. The left- and right-handed fermions transform in an asymmetric way, and the theory does not conserve parity at any stage. This is particularly embarrassing in view of the fact that CP-violation is incorporated into the theory in a natural way and is generated by the same spontaneous symmetry breaking mechanism which generates the masses and the Cabibbo angles.
In a well-defined, tightly built model such as the SU(2) × U(1) gauge theory, it would normally be very hard to rectify these two difficulties without losing some of the successes or creating other difficulties. However, the extended SU(2)ₐ × SU(2)ᵢ × U(1) model (section 2.6) can do precisely what we need:

(i) All right-handed fermions continue to be in singlets of SU(2)ₐ but are in doublets of SU(2)ᵢ. Hence, all left-handed fermions are in (½, 0) multiplets and all right-handed fermions are in (0, ½).

(ii) Parity is conserved by the original Lagrangian of the model, and is spontaneously broken by the usual Higgs mechanism which induces left-right asymmetry.

(iii) All neutral currents conserve parity, but all neutrino induced neutral-current processes obey the predictions of the standard SU(2) × U(1) theory, in agreement with experiment. There is no parity violation in atomic physics (to lowest order in the weak interaction).

(iv) We pay a price in the form of a more complicated spectrum of gauge bosons and Higgs particles. This was discussed in detail in section 2.6 and we will not repeat it here.

We believe that the SU(2)ₐ × SU(2)ᵢ × U(1) picture provides an attractive framework for the continued study of gauge theories of the properties of quarks and leptons. New experimental problems may arise. We discuss several such problems in section 5.2. Many theoretical problems remain open. We list some of them in section 5.3.

5.2. Experimental loose ends and open problems

At the time of this writing there are several experimental “loose ends”, concerning the properties of quarks and leptons. At the rate of progress of this field, some of these issues may be resolved and other problems may emerge before this review is published. We feel, however, that a short discussion of these points is both necessary and useful.

(A) Trimuon events. There are several potential sources of events of the type:

\[ \nu_\mu + N \rightarrow \mu^- + \mu^- + \mu^+ + \text{anything.} \]  

The sources include: (i) Trivial background due to K and π decays, accompanied by “ordinary” μ-pairs. (ii) A normal charged-current event, accompanied by a low mass μ⁺μ⁻-pair produced by a virtual photon or by a vector meson. (iii) Associated charm production followed by c → μ⁺ and c → μ⁻ decays. (iv) Production of a b or t quark followed by a cascade of semileptonic quark decays (see eqs. (4.59), (4.74), (4.77)).

All the above sources are consistent with the overall view of section 5.1. It is not entirely clear whether the present data [97] on trimuon events in νN scattering (reaction 5.5) are consistent with the combined contributions of the above mechanisms. It is possible that of the fifteen or so presently observed trimuon events, few are due to each of these mechanisms. At the same time, it is possible that at least some of the observed trimuons are of a different source. A particularly interesting hypothesis which has attracted attention is that some trimuon events are generated by a mechanism in which all three muons are associated with the leptonic vertex and are due to a sequence of decays of a new lepton [98]. Such a new lepton M⁻ must have a significant charged current coupling to ν_μ. It cannot be accommodated within the simple theories of section 5.1. Several ways out have been proposed:

(i) An SU(3) × U(1) gauge group in which SU(3) triplets of quarks and leptons appear [57]. In particular, a left-handed triplet including (ν_μ, μ⁻, M⁻) is assumed. The ν_μ → M⁻ transition is induced by a charged gauge boson V⁺ which is different from the usual W⁺. With the aid of an extra discrete
symmetry, the SU(3) \times U(1) model can be arranged to agree with all charged and neutral current data as well as with the absence of anomalies and the "natural" conservation of flavors in neutral currents [57]. The SU(3) \times U(1) model appears to be the most attractive model which can accommodate the M^- lepton. However, if such a lepton is not needed by the trimuon events, there is no compelling motivation for pursuing an SU(3) \times U(1) scheme.

(ii) Within SU(2) \times U(1), the M^- lepton can be accommodated only at the expense of artificial concoctions involving universal mixing of all fermions [99]. The idea is simple: a significant \( \nu_\mu \rightarrow M^- \) transition requires that M^- has a large component in the same SU(2) \times U(1) doublet as \( \nu_\mu \). This doublet can have the form:

\[
\begin{pmatrix}
\nu_\mu \\
\mu^- \cos \alpha + M^- \sin \alpha
\end{pmatrix}_L.
\]  

However, the \( \nu_\mu \rightarrow \mu^- \) transition will now have a strength \( G \cos \alpha \), inconsistent with \( \mu-e \) universality and with the Cabibbo universality relating \( \mu \)-decay to \( \beta \)-decay. This can be rectified if all "standard" \( I_2 = -\frac{1}{2} \) fermions (e^-, d, s) are mixed with new heavy fermions, with the same universal mixing angle \( \alpha \). We consider this to be an ugly scheme.

(iii) Other models for the M^- lepton involve various versions of SU(2)_L \times SU(2)_R \times U(1) [100], SU(3) \times SU(3) [32] or larger groups. We will not discuss these models in detail.

(B) Parity violating neutral currents. At present, there is no experimental proof of parity violation by the neutral current. All neutrino-induced reactions have a left-right asymmetry built into the initial state. Hence, the final state is not parity invariant, but the current may still conserve parity. Within the framework of the left-handed SU(2) \times U(1) model, the data tell us that neutral currents must violate parity. On the other hand, in the SU(2)_L \times SU(2)_R \times U(1) model, all neutral currents conserve parity. The best experimental way to probe this issue is to look for parity violation in neutral current processes which do not involve neutrinos. These include atomic and nuclear physics experiments and asymmetry measurements in \( e^+ + e^- \rightarrow \mu^+ + \mu^- \) and \( e + p \rightarrow e + \) anything. All of these experiments may be performed in the near future. Table 2 (section 2.6) serves as a guide on this subject. In particular, it is important to improve both the theoretical analysis and the experimental accuracy of the atomic bismuth experiments in order to find whether their result is, indeed, an order of magnitude below the predictions of the standard SU(2) \times U(1) model.

(C) Right-handed charged currents. There is no positive experimental evidence for right-handed charged currents. There are, however, confusing hints. One of them is the "\( \nu \)-anomaly" which was claimed by one experiment and is not seen by others [13]. It would be important to settle this dispute and to rule out a right-handed \( u \leftrightarrow b \) transition in \( \bar{\nu}_\mu N \) scattering.

The possible existence of a \( (c, s)_R \) doublet is still an open question (section 3.3). Studies of D, \( \Lambda_c \) and \( F^+ \) decays as well as the \( \gamma \)-distribution of \( \bar{\nu} + N \rightarrow \mu^- + \mu^+ + \) anything should help to clarify this question.

(D) \( \tau \)-decays. The \( V, A \) structure of \( \tau \)-decays is not fully understood. This must be settled in the near future. The potential problem [81] of not observing \( \tau^- \rightarrow \tau^- + e^- + \nu \) should be carefully studied. We hope to hear soon that the decay has been observed at the expected level. The properties of \( \nu_\tau \) are not known, but their understanding will probably require several years of study.

(E) Heavy \( q \bar{q} \) systems. There are still some important open problems in the charmonium spectrum and, now, also in the T-family. In charmonium, the X(2.85) and \( \chi(3.45) \) states are extremely puzzling [101] and seem to defy any sensible classification. In the T-family, the relative \( \mu^+ \mu^- \) branching ratios of \( T^- \) and \( T' \) are incompatible with the \( T^-T' \) mass difference (section 4.6). All of these prob-
lems should be clarified soon. We normally assume that the \( \psi \)-family represents one new quark (c-quark) and the \( \Upsilon \)-family represents one additional quark (probably b). It is probable, but not absolutely clear, that only one new quark is involved in each of these families. Only a clarification of the remaining theoretical and experimental “loose ends” in both families will exclude the possibility of having more than one new quark in each family.

(F) Neutrino masses and mixing. Finally, the entire question of leptonic Cabibbo-like mixing as well as the possibility of neutrino oscillations depend on whether \( \nu_e, \nu_\mu, \) and \( \nu_\tau \) are massless. The existence of zero mass particles usually has a profound meaning and it is extremely important to improve on the mass limit of all neutral leptons.

5.3. Theoretical open problems

We conclude with a short list of open theoretical problems:

(A) Which is the correct gauge group of the weak and electromagnetic interactions? SU(2) \( \times \) U(1) was extremely successful but is now in some trouble. SU(2) \( _L \) \( \times \) SU(2) \( _R \) \( \times \) U(1) is an attractive, promising alternative which preserves much of the beauty of SU(2) \( \times \) U(1). Other groups such as SU(3), SU(3) \( \times \) U(1) and SU(3) \( \times \) SU(3) have been proposed. A direct experimental study of the correct gauge group must involve the discovery of the charged and neutral gauge bosons. Their number depends on the group, but their mass pattern depends on the Higgs particles and their vacuum expectation values. The observation of weak phenomena in the next generation of e\( ^+e^- \) machines (PEP and PETRA) should enable us to have some feeling for the masses of the W and Z bosons, although it is very unlikely that these masses will be directly accessible at such installations.

(B) Is there any theoretical limit on the number of fundamental fermions? The asymptotic freedom condition (if it survives at large \( q^2 \)) dictates [102] that the number of quark flavors must be less than 17. Another argument, based on cosmology considerations, limits the number of massless neutrinos to seven [103]. Assuming that every such neutrino is in a doublet and that the numbers of quark and lepton flavors are the same, we end up with an upper limit of seven or eight “generations” of fermions. These limits are, at present, almost in the domain of science fiction. We do not have any other useful limit on the number of flavors, but any profound understanding of the structure of matter must explain why the number of fundamental building-blocks is what it is.

(C) Perhaps the most difficult open problem is the calculation of masses and angles. All quark and lepton masses, as well as all Cabibbo-like angles, \( CP \)-violation parameters and, possibly, parity-violation parameters, are presumably generated by the same mechanism of spontaneous symmetry breaking. We have very little understanding of what determines the values of all of these mass and angle parameters. All attempts to calculate them have essentially failed, so far. The total number of such free parameters is large (of order 20 or more in the case of three generations of fermions), and it is clear that no satisfactory theory can afford to leave them as arbitrary quantities.

(D) The connection between quarks and leptons and the possible grand unification of weak, electromagnetic and strong interactions is another open problem. We discussed it in some detail in sections 2.8, 2.9, 2.10 and 3.11. A major theoretical breakthrough is clearly needed here. We believe that the next chapter in our pursuit of a deep understanding of the structure of matter must establish a clear connection between quarks and leptons. Such a connection will be an important step towards a unified picture of the fundamental building-blocks of matter and their interactions.
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