Weak Coupling Phase from Decays of Charged $B$ Mesons to $\pi K$ and $\pi\pi$

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The theory of $CP$ violation based on phases in weak couplings in the Cabibbo-Kobayashi-Maskawa (CKM) matrix requires the phase $\gamma \equiv \arg V_{ub}^{*}V_{cd}$ (in a standard convention) to be nonzero. A measurement of $\gamma$ is proposed based on charged $B$ meson decay rates to $\pi^+K^0$, $\pi^0K^+$, $\pi^+\pi^0$, and the charge-conjugate states. The corresponding branching ratios are expected to be of the order of $10^{-5}$.

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At present direct evidence for $CP$ violation comes exclusively from the decays of neutral $K$ mesons. One theory of this phenomenon is based on phases in the Cabibbo-Kobayashi-Maskawa (CKM) matrix $V_{ij}$, which describes the weak charge-changing couplings of left-handed quarks $i = (d,s,b)$ of charge $-1/3$ with left-handed quarks $\alpha = (u,c,t)$ of charge $2/3$. By choosing five relative quark phases, one can take the elements of $V$ along and just above the diagonal to be real (see, e.g., [2]). In this convention, taking account of the observed magnitudes of elements, only $V_{ub}$ and $V_{td}$ can have significant nonzero phases. The observed decays $K_L \rightarrow \pi^+\pi^-$ and $\pi^0\pi^0$ of the long-lived neutral kaon and the charge asymmetry in semileptonic $K_L$ decays can be ascribed to a $CP$-violating mixing of $K^0$ and $\bar{K}^0$ arising from these phases. The CKM model of $CP$ violation also predicts small differences in the ratios $\eta_{++} \equiv A(K_L \rightarrow \pi^+\pi^-)/A(K_S \rightarrow \pi^+\pi^-)$ and $\eta_{00} \equiv A(K_L \rightarrow \pi^0\pi^0)/A(K_S \rightarrow \pi^0\pi^0)$. Two recent experiments [3,4] reach different conclusions about whether $\eta_{++} = \eta_{00}$, and a satisfactory alternative remains a "superweak" theory of direct $K^0-\bar{K}^0$ mixing [5].

A fertile ground for testing the CKM model of $CP$ violation involves the decays of mesons containing the fifth $(b)$ quark [6]. Unequal rates for decays of the mesons $B^0 \equiv \bar{bd}$ and $B^0 \equiv bd$ to $CP$ eigenstates like $J/\psi K_S$ can be interpreted crisply in terms of the weak phase $\arg V_{td}$, without complications from strong final-state interactions. However, the presence of $B^0-\bar{B}^0$ mixing, needed for the rate asymmetry, complicates the identification of neutral $B$ mesons.

The decays of charged $B$ mesons can manifest $CP$ violation in the form of unequal rates for such processes as $B^+ \rightarrow \pi^0K^+$ and $B^- \rightarrow \pi^0K^-$. While the charge of a $B$ meson is easily determined, strong final-state interactions are required for such rate differences. Differences in strong final-state phases among different eigenchannels are expected to be small and uncertain. Thus, except in a few particular cases [7], it has usually been assumed that information on CKM phases cannot be extracted from the study of charged $B$ decays alone. Such decays can play useful auxiliary roles in the separation of final-state interaction effects from weak phases when decays of neutral $B$ mesons to $CP$ eigenstates are also measured [8-10].

In this Letter we describe a way to obtain the weak phase $\gamma \equiv \arg V_{ub}^{*}V_{cd}$, from the rates for the decays of charged $B$ mesons to $\pi^+K^0$, $\pi^0K^+$, $\pi^+\pi^0$, and the charge-conjugate states. We expect equal rates for $B^+ \rightarrow \pi^+\pi^0$ and $B^- \rightarrow \pi^-\pi^0$ on rather general grounds, and equal rates for $B^+ \rightarrow \pi^+K^0$ and $B^- \rightarrow \pi^-K^0$ as a result of a specific assumption to be noted below. The rates for $B^+ \rightarrow \pi^0K^+$ and $B^- \rightarrow \pi^0K^-$ can differ if $CP$ is violated, but it is not necessary to measure a $CP$-violating observable in order to obtain $\gamma$. The corresponding branching ratios are expected to be of the order of $10^{-5}$, which is the level at which decays of $B$ mesons to two light pseudoscalars have already been seen [11].

The method relies upon an $SU(3)$ relation between the amplitude for $B^+ \rightarrow \pi^+\pi^0$, which has isospin $I = 2$, and the isospin-3/2 amplitude in $B \rightarrow \pi K$.SU(3) breaking is also introduced, assuming that the two-body hadronic decay amplitudes are factorizable. Other applications of $SU(3)$ to decays of $B$ mesons to pairs of light pseudoscalars have been considered in Refs. [12-15]. A more general recent discussion is contained in Ref. [16], where several new tests of the SU(3) assumption are suggested. Other measurements of time-independent $B$ decay rates to pairs of light pseudoscalars also can determine weak and strong phases [17].

The weak phase of the isospin-3/2 $\pi K$ amplitude is expected to be $\pm\gamma$ for $B^\pm$ decays, while the strong phase does not change sign under charge conjugation. The weak phases of the amplitude for $B^+ \rightarrow \pi^+K^0$ and $B^- \rightarrow \pi^-K^0$ are both expected to be $\pi$ under the assumption that weak annihilation graphs do not contribute to the
decay. (We shall suggest a test of this assumption.) Two triangle relations satisfied by amplitudes, which include information from the rates for $B^\pm \to \pi^0 K^\pm$, then allow one to separate out the desired weak phase $\gamma$ modulo a discrete ambiguity.

We consider charmless decays of $B$ mesons to two light pseudoscalar mesons within SU(3) [12,13]. The operators associated with the four-quark transition $b \to q\bar{u}\bar{u}$ and the direct ("penguin") transition $b \to q\bar{q}$ ($q = d$ or $s$), when combined with the triplet of $B$ meson states, lead to a decomposition of all strangeness-preserving and strangeness-changing decay processes in terms of five SU(3) reduced amplitudes. As shown in Ref. [12], this algebraic decomposition is equivalent to a simpler graphical expansion. The six graphs which contribute are illustrated in Fig. 1 [14]. They consist of a "tree" amplitude $T$ ($T'$), a "color-suppressed" amplitude $C$ ($C'$), a "penguin" amplitude $P$ ($P'$), an "exchange" amplitude $E$ ($E'$), an "annihilation" amplitude $A$ ($A'$), and a "penguin annihilation" amplitude $PA$ ($PA'$). The unprimed amplitudes stand for strangeness-preserving decays, while the primed ones represent strangeness-changing processes. These amplitudes are related by simple CKM factors. In particular,

$$ T'/T = C'/C = E'/E = A'/A = r_u , $$

where $r_u \equiv V_{us}/V_{ud} \approx 0.23$. The set of six graphs is overcomplete. They appear in all processes of the type $B \to PP$ in the form of five linear combinations, corresponding to the five SU(3) reduced matrix elements.

To apply SU(3) to the three decay processes, $B^+ \to \pi^+\pi^0$, $\pi^0K^0$, $\pi^0K^+$, we write the corresponding amplitudes in terms of their graphical contributions:

$$ A(B^+ \to \pi^+\pi^0) = -\frac{1}{\sqrt{2}} (T + C) . $$

$$ A(B^+ \to \pi^0K^0) = P' + A' . $$

$$ A(B^+ \to \pi^0K^+) = -\frac{1}{\sqrt{2}} (T' + C' + P' + A') . $$

Here, for instance, the combinations $C' + T'$ and $P' + A'$ form two of the five linearly independent combinations of graphical contributions. We immediately find

$$ \sqrt{2} A(B^+ \to \pi^0K^+) + A(B^+ \to \pi^+K^0) $$

$$ = r_u \sqrt{2} A(B^+ \to \pi^+\pi^0) . $$

This relation is described by a triangle in the complex plane, as shown in Fig. 2. In the above equation, $r_u$ includes the relation between the primed and unprimed amplitudes [Eq. (1)], as well as SU(3)-breaking effects. The left-hand side of (5) corresponds to the $I = 3/2 B \to \pi K$ amplitude ($T' + C'$), which is related to the $I = 2 B \to \pi \pi$ amplitude ($T + C$) by the Weyl reflection which interchanges $s$ and $d$ quarks [15]. Thus, the only place SU(3) breaking can matter is in relating the $T + C$

![Fig. 1. Diagrams describing decays of $B$ mesons to pairs of light pseudoscalar mesons. Here $\bar{q} = \bar{d}$ for unprimed amplitudes and $\bar{q}$ for primed amplitudes. (a) "Tree" (color-favored) amplitude $T$ or $T'$; (b) "color-suppressed" amplitude $C$ or $C'$; (c) "penguin" amplitude $P$ or $P'$ (we do not show intermediate quarks and gluons); (d) "exchange" amplitude $E$ or $E'$; (e) "annihilation" amplitude $A$ or $A'$; (f) "penguin annihilation" amplitude $PA$ or $PA'$.](image)

![Fig. 2. SU(3) triangles involving decays of charged $B$'s which may be used to measure the angle $\gamma$. Here $A^{0\pm} \equiv A(B^+ \to \pi^0K^\pm)$, $A^{+0} \equiv A(B^+ \to \pi^+K^0)$, $A^{-0} \equiv A(B^- \to \pi^0K^-)$, $A^{-+} \equiv A(B^- \to \pi^-K^0)$, $A^{00} \equiv A(B^+ \to \pi^-\pi^0)$, $A^{\pi\pi} \equiv A(B^- \to \pi^-\pi^0)$. The lower figure shows one of the triangles flipped about the horizontal axis. This solution must be chosen when $|A^{0+}| = |A^{0-}|$ if $\gamma \neq 0$.](image)
contribution in $B \to \pi \pi$ to the $T' + C'$ contribution in $B \to \pi K$. This can be taken into account by noting that the factorized amplitude of $B^+ \to \pi^+ \pi^0$ involves the pion decay constant $f_\pi$, whereas the $I = 3/2$ amplitude in $B \to \pi K$ involves a factor $f_K \approx 1.2 f_\pi$. We therefore set $\tilde{r}_\pi = r_\pi / f_K / f_\pi$. Additional SU(3) breaking in form factors for recombination of the spectator quark with one of the $b$ quark decay products is likely to be small and has been neglected. Also, any resonances in the $I = 3/2 \pi K$ and $I = 2 \pi \pi$ channels would be exotic (not formed of a quark and an antiquark). Since no exotic resonances have been seen, such effects are unlikely to disturb the SU(3) relation much.

The charge-conjugate processes also form a triangle relation. As we will see below, the two triangles are related in a simple way, under an additional assumption.

The diagrams denoted by $E$, $A$, $PA$ involve contributions to amplitudes which should behave as $f_B / m_B$ in comparison with those from the diagrams $T$, $C$, and $P$ (and similarly for their primed counterparts). This suppression is due to the smallness of the $B$ meson wave function at the origin, and it should remain valid unless rescattering effects are important. Such rescatterings indeed could be responsible for certain decays of charmed particles (such as $D^0 \to K^0 \phi$), but should be less important for the higher-energy $B$ decays. In addition, the diagrams $E$ and $A$ are also helicity suppressed by a factor $m_{u,d,s} / m_B$ since the $B$ mesons are pseudoscalars.

A simple test of the suppression of the amplitudes $E$, $A$, $PA$ would be the following. If rescattering effects are small and the diagrams $E$, $A$, and $PA$ can be neglected, the rate for $B^0 \to K^+ K^-$ will be suppressed relative to $B^0 \to \pi^+ \pi^-$, since the amplitudes for these processes are given by

$$A(B^0 \to \pi^+ \pi^-) = -(T + P + E + PA),$$

$$A(B^0 \to K^+ K^-) = -(E + PA).$$

Assuming that the amplitude $A'$ can be neglected in (3) and (4), the phases in the decay amplitudes and those for the charge-conjugate processes have simple relations to one another. The phase of the $P'$ amplitude, which is expected to be dominated by the top quark loop [18], should be approximately $\text{Arg} V_{tb}^* V_{ts} = \pi$. Then we may denote

$$A(B^+ \to \pi^+ K^0) = A(B^- \to \pi^- K^0) = P' = -a_P e^{i \delta P},$$

where $a_P$ is real. Note that the rates for the process and its charge conjugate are equal, which would not necessarily be so if $A' \neq 0$ in Eq. (3). The equality of these rates thus helps to test our assumptions. Using this assumption [and SU(3)], the triangles corresponding to the processes in (5) and their charge conjugates have a side in common ($P'$), as shown in Fig. 2.

In addition, taking account of the factor which relates $T + C$ to $T' + C'$ [including SU(3) breaking] and using $\text{Arg} V_{tb}^* V_{ts} = \gamma$, we find

$$\tilde{r}_\pi \sqrt{2} A(B^+ \to \pi^+ \pi^0) = -(T' + C') = a_T e^{i \delta_T} e^{i \gamma},$$

while

$$\tilde{r}_\pi \sqrt{2} A(B^- \to \pi^- \pi^0) = a_T e^{i \delta_T} e^{-i \gamma},$$

with $a_T$ real. The rates for these two processes are equal because they involve a single weak phase and a single strong phase. The difference in phase between these two amplitudes is just $2 \gamma$.

The third side of each amplitude triangle is provided by the rate for the decay $B^+ \to \pi^0 K^+$ or $B^- \to \pi^0 K^-$, as shown in Fig. 2. Here $A^{0+} \equiv A(B^+ \to \pi^0 K^+)$, $A^{+0} \equiv A(B^+ \to \pi^+ K^0)$, $A^{00} \equiv A(B^- \to \pi^0 K^-)$, $A^{+0} \equiv A(B^- \to \pi^+ K^0)$, $A^{00} \equiv A(B^- \to \pi^0 K^-)$. Modulo a twofold ambiguity which corresponds to flipping one triangle about the horizontal axis, the rates determine the shapes of the triangles and hence the difference $2 \gamma$. The flipping of one triangle corresponds to interchanging $\gamma$ and $\delta_T - \delta_T$. In general, $CP$ violation is expected to show up as a difference in rates between $B^+ \to \pi^0 K^+$ and its charge conjugate, since two CKM amplitudes with different phases interfere in this process. The crucial point in determining $\gamma$ is that the magnitudes of these two amplitudes are separately measured in $B^+ \to \pi^0 K^+$ and $B^+ \to \pi^+ \pi^0$. If $\delta_T - \delta_T = 0$, we will not observe such a difference in rates. In that case, however, we would have to choose the lower part of Fig. 2, since only this configuration would correspond to a nonzero value of $\gamma$.

Figure 2 will permit the measurement of $\gamma$ if each of the decay rates can be measured with sufficient accuracy. Explicitly, defining $a \equiv |A^{0+}| = |A^{-0}|$, $b \equiv (f_K / f_\pi) r_\pi \sqrt{2} |A^{+0}| = (f_K / f_\pi) r_\pi \sqrt{2} |A^{00}|$, $c \equiv \sqrt{2} |A^{+0}|$, $c' \equiv \sqrt{2} |A^{00}|$, one has

$$4ab \sin \gamma = \pm [(a + b)^2 - c^2][c^2 - (a - b)^2]^{1/2}$$

$$= \pm (c \leftrightarrow c').$$

The present data on $B^0$ decays to pairs of pseudoscalars [11] do not allow one to distinguish between $\pi^- K^+$ and $\pi^+ \pi^-$ final states. The combined branching ratio is about $2 \times 10^{-5}$, with equal rates for $\pi^- K^+$ and $\pi^+ \pi^-$ being most likely. If this is true, the amplitudes $T$ and $P'$ have about the same magnitude, so that the short sides of the triangles in Fig. 2 are probably about 1/4 to 1/3 [1/3 $\approx (f_K / f_\pi) r_\pi$] the lengths of the other two sides. Then the "long" sides of the triangle must be measured with fractional accuracies of about $(f_K / f_\pi) r_\pi$ in order to achieve an accuracy of $\delta_T$ in the angle $\gamma$. For example, to measure $\gamma$ to a statistical accuracy of about 10°, one probably needs fractional errors of about 1/20 in amplitudes, or 10% in rates. This would require at least 100
decays in each channel of interest. We end with some comments about other ways of measuring weak phases.

(1) Another measurement of $\gamma$ from charged $B$ decays uses the processes $B^{\pm} \to K^{\pm} D^0$, $K^{\pm} D^0$, where $D_{CP}$ denotes a $CP$ eigenstate [7,19]. The three $B^+$ amplitudes and their charge conjugates obey two triangle relations similar to the above. Here too the angle $\gamma$ can be measured without an observation of $CP$ violation in $B^{\pm} \to K^{\pm} D_{CP}$, even when the final-state phase differences are too small to detect. While $B^+ \to K^+ D^0$ may be strongly color suppressed, all the measured rates are expected to be of comparable magnitudes in the method presented here.

(2) The present measurement uses $B$ decay modes with rates similar to $B^0 \to \pi^+ \pi^-$ decays. The use of $\pi^+ \pi^-$ decays requires tagging the neutral $B$ meson flavor at time of production, and suffers from uncertainties associated with penguin amplitudes [8]. These uncertainties can be eliminated by a complete isospin analysis of all charge states in $B \to \pi \pi$ decays [9], or at least estimated by relating via SU(3) the rates of $B^0 \to \pi^+ \pi^-$ and $B^0 \to \pi^- K^+$ [15]. Information from additional $\pi\pi$, $\pi K$, and $K\bar{K}$ branching ratios of charged and neutral $B$'s can be combined with the rates mentioned here to further eliminate ambiguities and constrain other weak phases [16,17].

To summarize, we have shown that measurements of the rates for charged $B$ decays to $\pi K$ and $\pi\pi$, together with a simple SU(3) relation, suffice to specify the geometry of amplitude triangles from which one can extract the weak phase $\gamma = \text{Arg} V_{ub}^*$, where $V_{ub}$ describes an element of the CKM matrix. No final-state-interaction phases need be specified. A nonzero value of $\gamma$ in accord with other analyses of parameters in the CKM matrix would provide valuable confirmation of a popular model of $CP$ violation.

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