Selection Rules for $N\bar{N}$ Annihilation

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Selection rules governing the annihilation of a nucleon-antineucleon pair into pions, photons, and heavy mesons are derived from the conservation of angular momentum, parity, charge parity, and isotopic spin. Special attention is given to the probability of annihilation purely into neutral pions.

We consider annihilation of nucleonium, i.e., a nucleon-antineucleon pair, from a state of definite angular momentum, parity, isotopic spin, and charge parity (if neutral) into systems of pions, photons, and heavy mesons.

STATES OF NUCLEONIUM

A state of a weakly interacting nucleon-antineucleon pair can be described by the quantum numbers of total angular momentum $j$, spin $s$, orbital angular momentum $l$, and by a specification of the isotopic spin states, i.e., $\rho\pi$, $n\bar{n}$, $\rho\bar{
u}$, or $n\bar{n}$. From these we can express the parity $\theta = (-)^j$, the isotopic spin $i=1$ or 0, the charge parity (in the case of neutral nucleonium, i.e., $i_0=0$) $C = (-)^{l+i}$, and the "CT" parity

$$CT = (-)^{l+i}$$. 

If the pair is interacting strongly, then $s$, $l$, and the "isotopic spin specification" are no longer good quantum numbers, but $\theta$, $i$, and $CT$ (and $C$ if $i=0$) are good quantum numbers if we neglect electromagnetic interaction. For instance, if we call a nucleonium state $^jP_s$ we really mean the state having $j=2$, $i=1$, $\theta=+1$, $CT= -1$ (and $C=+1$ if $i=0$).

DECAY INTO PIONS

The pion has $CT$ parity $-1$, so that an $N$-pion state has $CT = (-)^N$. The pion has an intrinsic spatial parity $-1$, so that an $N$-pion state has $\theta = (-)^N \theta_{\text{orbital}}$. For two pions, $j$ equals $l$ because the pion is spinless, therefore $\theta = (-)^l$; for three pions having $j=0$, $\theta_{\text{orbital}}$ equals +1 and therefore $\theta = -1$.

Thus a nucleonium state having $CT = \{+, -\}$ can decay into an \{even, odd\} number of pions; with the qualifications that two pions are impossible if $(-)^\theta = -1$ (which is always the case for spin-singlet nucleonium), and three pions are impossible if the state is $(0+)$. Selection rules based on $CT$ are not absolute in the presence of electromagnetic interaction, but decays violating these selection rules will be slower than the corresponding radiative decays.

Table I lists the least number of pions into which each nucleonium state can decay; a state that decays into some (even or odd) number of pions can decay into any larger (even or odd) number of pions also. For states of higher odd or even $l$, the table would be precisely the same as given for $P$ or $D$ state respectively, except that the forbidding of three mesons from $^jP_0$ is not repeated, since the corresponding higher $l$ states do not have $j=0$. If for a given $l$ one weighs the four spin states equally, and either weights the four isotopic spin states equally ("mixed" nucleonium, e.g., $\bar{n}$ incident on $D$) or gives equal weight to the two $i=0$ states ("neutral" nucleonium, e.g., $\bar{n}$ incident on $H$) one finds the ratios of the number of states with least pion numbers of 2, 3, 4, or 5 given in Table II.

If the annihilation takes place in a small volume, then low orbital angular momenta in the final state will be favored. The lowest-$l$ configurations are listed in Table I for the least pion number states; the lowest-$l$ configuration for a state of 2p additional pions simply has an additional factor of $z^2$.

It is of some interest to estimate the proportion of decays leading to pure $\pi^0$ states, since these will yield the entire nucleonium rest mass as visible energy in a Cherenkov counter. Since $C = +1$ for a system of neutral pions, and (for a neutral system) we can write $CT = (-)^N \theta = (-)^N$, a system of $N$ neutral pions has $i = \{0, 1\}$ if $N$ is \{even, odd\}. If we neglect the space state of the $N$-pion final state, and thus assume that all isotopic spin states are equally populated, we can deduce the probability $P_N^n$ that an $N$-pion state is a pure $\pi^0$ state, given that $i = 0$ and $i = \{0, 1\}$ if $N$ is \{even, odd\}.

$$P_N^n = \sum_{\{0, 1\}} \frac{1}{4} P_N^n \left(\alpha_N^{(0)} + \alpha_N^{(4)}\right)$$

where $N$ is the number of pions, and $\alpha_N^{(0)}$ and $\alpha_N^{(4)}$ are the eigenvalues of charge conjugation and $T$ is the eigenvalue of a rotation of $180^\circ$ in isotopic spin space around an axis transverse to the $3$ or "charge" axis. Only systems with $i = 0$ have a $T$ parity, and then $T = (-)$. $CT$ is the product of $C$ and $T$, and is useful in that, in the usual representation of the $\tau$ matrices [where, under $C$, $\tau_3 \rightarrow \tau_3, \tau_2 \rightarrow -\tau_2$, if the $T$-rotation is taken to be


<table>
<thead>
<tr>
<th>Initial nucleon state</th>
<th>C</th>
<th>CT</th>
<th>Pions*</th>
<th>KK*</th>
<th>like K mesons</th>
<th>unlike K mesons</th>
<th>Scheme A: the θ is (0+)</th>
<th>Scheme B: the θ is (1⁻)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1S1</td>
<td>-</td>
<td>+</td>
<td>2p</td>
<td>like</td>
<td>p₁p</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1S1</td>
<td>-</td>
<td>-</td>
<td>3p</td>
<td>like</td>
<td>p₁p</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1S0</td>
<td>+</td>
<td>-</td>
<td>3p</td>
<td>unlike</td>
<td>s₁s</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2P₁</td>
<td>+</td>
<td>+</td>
<td>4p</td>
<td>unlike</td>
<td>s₁s</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2P₁, 2s, 2p, 2d</td>
<td>-</td>
<td>+</td>
<td>2p</td>
<td>like; unlike; like</td>
<td>-</td>
<td>s₁p</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2P₁, 2s, 2p, 2d</td>
<td>+</td>
<td>+</td>
<td>2p</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>3D₁, 3s, 3d</td>
<td>-</td>
<td>+</td>
<td>2p</td>
<td>like; unlike; like</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

where the αₙ^⁽⁽m⁾⁾ are the branching ratios for decay into \( N \) pions of a state that decays into at least \( m \) pions \((N-m)\) even). The estimation of the \( \alpha_l \) by the Lepore-Neuman statistical theory⁵ indicates that they are rapidly decreasing functions of \( N \); i.e., \( \alpha_l^{+2}(m) \ll \alpha_l^{(m)} \), so that \( \alpha_l^{(m)} \approx \delta_{3m}. \) Further, since the phase space for a final state of two heavy mesons (see next section) is nearly the same as for two pions, the at-least-four-pion decay states would not decay into pions on this theory. Taking the estimations⁵ \( \alpha_l^{(2)} = 3/7, \alpha_l^{(3)} = 1/7, \) and all others zero, we find 0.033 for the probability that neutral nucleonium decays (directly) into \( \pi^0 \)'s only.

## DECAY INTO HEAVY MESONS

We assume Gell-Mann's scheme, assigning \( i = 1/2 \) to both the \( \theta \rightarrow 2\pi \) and \( \tau \rightarrow 3\pi \), which we call collectively \( K \) mesons. The spin and parity of the two are known to the following extent. The \( \theta \) must have \( \phi = (1/2) \) in order to decay into two pions, and further, \( j \) must be even for \( \theta \) to decay into two neutral pions.²

The \( \tau \) seems to be \( (0-\) on the basis of Dalitz's analysis of the observed correlation in the final state. We shall accept the \( \tau \) as \( (0-) \) and consider the possibilities that the \( \theta \) is either \( (0+) \) or \( (1-) \), calling the first assumption \( \theta \) is \( (0+) \) scheme \( A \), the second \( \theta \) is \( (1-) \) scheme \( B \).

³ J. Osher (unpublished) has evidence from an experiment similar to that of G. B. Collins [Proceedings of the Fifth Annual Rochester Conference on High-Energy Physics (Interscience Press, New York, 1955), p. 139] that the \( \theta \rightarrow \pi^0 + \pi^+ \) decay does occur, with lifetime and abundance of the same order as for \( \theta \rightarrow \pi^+ + \pi^- \). Furthermore, from the theoretical point of view, if the \( \theta \) has odd spin its two-pion final state would necessarily have \( i = 1 \), so that the different lifetimes of \( \theta^+ \) and \( \theta^- \) would be inexplicable under charge independence.

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* The prefixed number is the least possible number of pions; the lowest-\( l \) configuration is given for this least number state.
* 'Like' signifies that the pair \( KK \) must be either \( 0+ \) or \( 1- \); 'unlike' that the pair is \( 1- \) or \( 0+ \).
* The lowest-\( l \) configuration is given, the first letter being the orbital state of the \( KK \) pair, with its isotopic spin as a subscript; the second letter is the orbital state of the pion around the \( K \)-meson pair. Where the isotopic spin of the \( KK \) pair can be either 0 or 1, it is not stated.

⁵ The lowest \( l \) orbital state is given.

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In either case, a state of two $K$ mesons has $i=0$ or 1. In Scheme $A$, a pair $\theta\theta$ or $\tau\tau$ (which we call like pairs) has $\varphi=\theta^2=(-\varphi)^1$, and $CT=\pm 1$ [and $C=(-1)^i$ if $i=0$. The pairs $\theta\tau$ or $\tau\theta$ (unlike pairs) are presumably interconvertible, as there is no selection-rule bar between them, and thus we can form linear combinations having $\varphi=\theta^2=(-\varphi)^1$, and $CT=\pm 1$. In Scheme $B$, a pair $\theta\theta$ has spin states $s_i=0$, 1, or 2, so that $CT=\pm 1$ for a given $i$. The unlike pairs in Scheme $B$ have $\varphi=\theta^2=-\varphi^1$. Thus for a given $j\neq 0$, $\varphi$ can be either $\pm 1$ ($CT$ likewise); but if $j=0$, then $l=\pm 1$, therefore $\varphi=-1$.

Thus in Scheme $A$, a state having $\varphi=\theta^2=\theta^2$ can decay into a like, unlike pair. In Scheme $B$, any state can decay into a pair $\theta\theta$ or $\theta\theta$, which is the same in both schemes.

Any state can decay into a pair of $K$ mesons plus one or more pions. The lower-limit configuration, along with the isotopic spin of the $K\bar{K}$ pair, is given in Table I.

The proportion of neutral nucleonium decaying into heavy mesons that go ultimately into neutral pions can be easily estimated. In Scheme $A$, the $\theta^2$ would decay into charged or neutral pions in a ratio between 2:1 and 1:2, depending on the ratio of $i=0$ to $i=2$ in the final state; if charge independence is invoked, the hundred-times-shorter lifetime of $\theta^2$ compared to $\theta^2$ implies that $\theta^2$ decays almost exclusively through $i=0$. Therefore the probability that a neutral $\theta\theta$ pair will decay into four neutral pions is 1/18. We now have to estimate the proportion of like $K\bar{K}$ that are $\theta\theta$ pairs. If at high energies "$\theta^2$ particles and "$\tau^2$ particles [i.e., (0+) and (0-) $K$ mesons] are produced in equal numbers, the observed ratio of $\theta^2$, $\tau^2$, and $K_{\pi^\pm}\pi^\pm$ decays, roughly 10:1:20, implies that a $\theta^2$ has a hundred-times-shorter lifetime of the $\theta^2$ insures that a $\theta^2$...
to each such $N$-pion state is a definite probability that all $N$ pions are $\pi^\pm$. The quantities $P^N(i)$ are the averages of these probabilities, taken over all states.

Thus $P^N(i)$ is a sum of terms, each of which is a product of three factors $a, b, c$ which are identified as follows:

Factor $a$: the fraction of the states with $i_N$ formed from states with $i_{N-1}$, i.e., $n^N(i_{N-1})/n^N(i_N)$, where $n^N(i)$ is the number of states of the $N$-pion system with $i_N=i, i_z=0$.

Factor $b$: the probability that $(i_N)_{z}=(i_1)_{z}=0$ for a state of $N$ pions formed from a system of $N-1$ pions with $i_{N-1}$ and one pion with $i_1$. This quantity is symbolized by $\langle(i_0)(00)|_{i,i_{N-1}} \rangle$. It can be shown that

$$\langle(i_0)(00)|_{i,i_{N-1}} \rangle = \frac{i(1-\delta_{ij})+\frac{1}{2}(1+j-\delta_{ij})}{2i+1+2j},$$

where $j=0, \pm 1$.

Factor $c$: the probability that if $(i_{N-1})_{z}=0$, the $(N-1)$-pion system is all $\pi^\pm$. This is just $P^{N-1}(i_{N-1})$.

Multiplying $a$, $b$, and $c$, one obtains

$$P^N(i)=\frac{n^{N-1}(i+1)}{n^N(i)} \cdot \frac{i+1}{2i+3} \cdot \frac{n^{N-1}(i)}{n^N(i)} \cdot \frac{n^{N-1}}{n^N(i)} \cdot \frac{n^{N-1}(i-1)}{2i-1} \cdot P^{N-1}(i-1)$$

$$= \frac{1}{n^N(i)} \left[ \frac{i+1}{2i+3} \cdot \frac{n^{N-1}}{n^N(i)} \cdot P^{N-1}(i+1) + \frac{i}{2i-1} \cdot \frac{n^{N-1}(i-1)}{n^N(i)} \cdot P^{N-1}(i-1) \right].$$

Or, defining

$$f^N=i n^N(i) P^N(i),$$

one has

$$f^N = \left[ \frac{i+1}{2i+3} f^N_{i+1} + \frac{i}{2i-1} f^N_{i-1} \right].$$

The $f^N_i$ must satisfy the conditions that $f^N_0=1, f^N_i=0$ for $N<1$ or $i<0$. Since $f^N$ is the probability that a state of $N$ $\pi^\pm$s has isotopic spin $i$, it follows that $\sum_i f^N_i = 1$.

The recursion relations for the $f^N_i$s may be solved with the help of a generating function. The solution is

$$f^N_i = \frac{N!}{(2i+1)!} \frac{1}{(N-1)!} \frac{1}{(N+i+1)(N+i-1)\cdots(N+1-i)},$$

$$f^N_0 = 1/(N+1) \quad \text{(for $i=0$)},$$

and

$$n^N_i = n^N_{i+1} + n^N_{i-1} \quad \text{(for $i\neq 0$)},$$

with $n^N_0=1$, and $n^N_i=0$ (if $N<i$).

Values of $f^N_i$ and $P^N(i)$ are given in Tables III and IV.

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This was pointed out by R. J. Riddell, Jr., who carried out the solution.