IS ISOTOPIC SPIN A GOOD QUANTUM NUMBER FOR THE NEW ISOBARS?

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Recent numbers of this Journal report discoveries of several relatively long-lived meson and hyperon isobars. Some of these display remarkable regularities in mass beyond what may be anticipated from known symmetry principles—the three members of the \((T = 1, J = 1)\) 750-Mev two-pion resonance\(^1\) (\(\rho\)) differ in mass from the \((T = 0, J = 1)\) 785-Mev three-pion resonance\(^2\) (\(\omega\)) by only 4\%: stranger yet, the masses of \(Y_{(w)}\) (the \(T = 1\) hyperon isobar at 1385 Mev)\(^3\) and \(Y_{(w)}\) (the \(T = 0\) hyperon at 1400 Mev)\(^4\) differ by about 1\%. In both cases there is a near degeneracy in mass between an isotopic singlet and triplet.

Among particles of better pedigree, \(\Sigma\) and \(\Lambda\) hyperons also show this quadruplet structure, with a \(\Sigma, \Lambda\) mass splitting of 7\%. This suggests the speculation\(^5\) that the symmetry group of the strongest interactions is greater than the isotopic rotation group, and is such that these interactions leave degenerate \(\Sigma\) and \(\Lambda\) hyperons. The even more marked quadruplet structure of both vector bosons and hyperon isobars offers additional support to this view.

Whether nearly degenerate quadruplets are portents of symmetries to come, or whether they are mere acts of chance is not for now to say. In either case, the existence of two particles, identical in essential (i.e., rigorously conserved) quantum numbers and with neighboring masses, can cause a novel enhancement of virtual electromagnetic effects.

Suppose that \(\omega\) and \(\rho_0\) differ by no quantum number respected by electromagnetism. What linear combinations of these fields propagate with definite mass and definite lifetime? They are \(\omega\) and \(\rho_0\) themselves, so long as electromagnetism is ignored, for the strong interactions— we presume— conserve isotopic spin. But electromagnetism fails to conserve isotopic spin: The electric current includes both an isoscalar and isovector part. Thus electromagnetism may induce nonradiative transitions between \(\omega\) and \(\rho_0\).\(^6\) Said another way, the mass (and lifetime) eigenstates, when electromagnetism operates, are no longer simply \(\rho_0\) and \(\omega\) but become superpositions of the two fields.

The amount of admixing does not directly depend upon the relative strength of electromagnetic and strong couplings, but rather, upon the relative size of the transition matrix element between \(\rho_0\) and \(\omega\) and their mass difference. Thus the closeness of the two vector mesons' masses may greatly enhance the extent that their decays violate isotopic invariance.

In order to identify the "particles," we must diagonalize a two-by-two effective mass matrix describing the \(\rho_0, \omega\) system: \(M = M_0 + M_1\). The complex diagonal matrix \(M_0\) assigns to \(\rho_0\) and \(\omega\) their masses and lifetimes without radiative correction. The matrix \(M_1\), in general complex and symmetric, describes the electromagnetic effects—its matrix elements are of order 1/137 those of \(M_0\). The off-diagonal matrix element of \(M_1\), which we call \(\eta\), is just half the transition matrix element connecting \(\rho_0\) and \(\omega\).

In any higher symmetry scheme, \(M_0\) describes the \(\rho_0, \omega\) mass splitting caused by the intermediate strong symmetry-breaking interactions (Issbi) as well as their mean mass determined by the symmetric strong interactions. As an approximation, Issbi may be neglected in the evaluation of \(M_1\). Then, the "electromagnetic eigenstates," or eigenvectors of \(M_1\), depend only on the structure of the higher symmetry group (they would be the "particles" were Issbi absent). In global symmetry,\(^7\) they are \((1/\sqrt{2})(\rho_0 \pm \omega)\). In the eightfold way\(^8\) they are \(\frac{1}{2} \rho_0 \pm \frac{i}{\sqrt{3}} \omega\) and \(\frac{1}{2} \omega - \frac{i}{\sqrt{3}} \rho_0\). The determination of the electromagnetic mass splitting between these states (and thereby, of \(\eta\)) is a dynamical problem far beyond the scope of this note.

For the \(\rho_0, \omega\) system, we may expect that the electromagnetic matrix elements are small compared to the mass difference due to strong interactions. [Electromagnetic effects are typically several Mev, whereas the masses of the two vector particles differ by \(\text{Re}(\delta m) \sim 35\) Mev. Actually, we must also take into account their difference in lifetime, so that \(|\delta m| \sim 50\) Mev.] Thus, one of the two eigenstates is dominantly \(\rho_0\), the other dominantly \(\omega\). Dropping terms of order \(\eta^2(\delta m)^{-2}\), we find for these eigenstates:

\[\tilde{\rho}_0 = \rho_0 + \eta(\delta m)^{-1} \omega,\]

\[\tilde{\omega} = \omega - \eta(\delta m)^{-1} \rho_0.\]

Because \(\delta m\) is relatively small, the amount of admixing is not at all negligible. When the isotopic spin conserving interactions of \(\rho_0\) and \(\omega\) are expressed in terms of the "particles" \(\tilde{\rho}_0\) and \(\tilde{\omega}\), a
significant probability is revealed for the \( \tilde{\rho}_0 \) to
decay into the three-pion channel and for \( \tilde{\omega} \) to
decay into the two-pion channel. The narrower experimental
width of the three-pion channel means that the second effect is far
more important of the two. With \( \eta \approx 5 \text{ Mev} \) as a pure guess,
about 5% of all \( \tilde{\omega} \) should decay into two pions. Experimentally,
this effect might be misinterpreted as evidence for a second \( \rho_0 \) at 790 Mev, except
that it is absent for the charged partners of \( \rho \).

The case for \( Y_{(0)} \) and \( Y_{(0)} \) is similar. The mass
difference is smaller, \( |0m| \approx 20 \text{ Mev} \), so that the departures from charge independence could be
more pronounced. With \( \eta \approx 5 \text{ Mev} \), the eigenstates,

\[
\tilde{Y}_{(0)} = Y_{(0)} + \eta(0m)^{-1}Y_{(0)}
\]

and

\[
\tilde{Y}_{(0)} = Y_{(0)} - \eta(0m)^{-1}Y_{(0)}
\]

involve admixtures of 25% in amplitude. The principal decay mode of \( \tilde{Y}_{(0)} \) is \( \tilde{Y}_{(0)} \to (\Sigma + \pi)^T = 0 \)
and that of \( \tilde{Y}_{(0)} \) is \( \tilde{Y}_{(0)} \to \Lambda + \pi^0 \). Radiative correc-
tions allow the isospin-violating modes \( \tilde{Y}_{(0)} \to \Lambda + \pi^0 \) and \( \tilde{Y}_{(0)} \to (\Sigma + \pi)^T = 0 \). With the \( (\Sigma + \pi)^T = 0 \)
channel assumed three times narrower than the \( \Lambda + \pi \) channel (in accordance with phase space—for experimentally there are as yet only upper bounds for the widths), we find for the branching ratios of the anomalous modes: 2% for \( \tilde{Y}_{(0)} \) and 20% for \( \tilde{Y}_{(0)} \). Since we have guessed both \( \eta \) and the
relative widths of the two states, this result could be very much in error. However, the existence
of such an effect as we discuss depends critically on the assumption that \( Y_{(0)} \) and \( Y_{(0)} \) share the same
values of spin and parity. The observation of a significant branching ratio for \( \tilde{Y}_{(0)} \to \Lambda + \pi^0 \) would be very convincing evidence for this assertion.

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6. Should both \( \rho_0 \) and \( \omega \) contribute to electromagnetic form factors, the virtual sequence \( \rho \to \gamma \to \omega \) is allowed and generates a nonvanishing matrix element connecting \( \omega \) and \( \rho_0 \). An estimate of the transition rate between \( \omega \) and \( \rho_0 \) due to this diagram may be made if their contributions to the nucleon form factors, their coupling constants to nucleons, and their widths are known. Our guess, \( \eta \approx 5 \text{ Mev} \), is consistent with such an estimate, with current experimental data, the value \( \langle g^2/4\pi \rangle_{\pi NN} \approx 0.3 \) [W. K. R. Watson (private communication)], and a similar value for the strength of the \( \omega \)-nucleon coupling. However, this estimate is not reliable. It requires knowledge of form factors and coupling constants near the \( \omega \) or \( \rho_0 \) mass shells, not at low momentum transfer; furthermore, diagrams with other intermediate states like \( \rho_0 \to \gamma + \pi^0 \to \omega \) cannot be neglected.


8. The relative size of the two-pion peaks due to \( \tilde{\rho}_0 \) and \( \tilde{\omega} \) depends upon the experiment considered—the anomalous peak would be more striking if more \( \tilde{\omega} \) were produced than \( \tilde{\rho}_0 \).

9. The decay mode \( Y(1) \to (\Sigma + \pi)^T = 1 \) conserves isotopic spin, but mysteriously seems to be absent. It could be forbidden by assuming \( R \) invariance. See J. J. Sakurai, Phys. Rev. Letters 7, 426 (1961).