BREAKING CHIRAL SYMMETRY*

S. L. Glashow
Physics Department, Harvard University, Cambridge, Massachusetts

Steven Weinberger†
Laboratory for Nuclear Science and Physics Department,
Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received 1 November 1967)

This note will explore what can be learned about chiral-symmetry breaking under two assumptions: (1) The Lagrangian is invariant under SU(3)cgp SU(3) except for a term which transforms under the representation (3, 3*) ⊕ (3*, 3), and (2) certain vertex functions are as smooth as possible. No assumption is made concerning the magnitude of the symmetry-breaking term. We find the natural appearance of a pseudoscalar nonet π, K, η, and η*, and of a scalar kaon κ lying below or just above the Kπ threshold. In addition, we obtain a refinement of the Ademollo-Gatto theorem for the Ee3 form-factor \( f^+ (0) \).

We will begin by proving theorems about a general broken symmetry group \( G \). The Lagrangian is assumed to take the form

\[
\mathcal{L} = \mathcal{L}_0 + \epsilon \sum_i \phi_i^a \phi_i^a
\]

(1)

where \( \mathcal{L}_0 \) is invariant under \( G \), and the \( \phi_i^a \) are local fields forming a basis for some definite real representation \( R \) of \( G \). It follows that the currents \( J_{a}^{\mu} \) constructed according to Noether's theorem satisfy the partial-conservation and commutation laws

\[
\begin{align*}
\epsilon \sum_i j_{a}^{\mu} & = \epsilon \sum_i T_{a i j} \phi_j^a, \\
\int d^4 x \phi_i^a \phi_j^a & = T_{a i j} \phi_j^a (y, t) - T_{a j i} \phi_j^a (x, t),
\end{align*}
\]

(2)

where \( T_{a i j} \) is the real antisymmetric matrix representing the generator \( T^a \) in the representation \( R \). We define 1-, 2-, and 3-point functions

\[
\begin{align*}
\lambda_i & \equiv \langle \phi_i^a \rangle, \\
\Delta_{ij} (p^2) & \equiv \int d^4 x e^{-i p \cdot x} \langle T^a \phi_i^a (x, t), \phi_j^a (0) \rangle \theta_0, \\
(p + p')^{\mu} & f_{ija} (p, p', q) + q^{\mu} f_{ija} (p, p', q) \\
& = \Delta_{ilk} (p^2) \Delta_{jlm} (p^2) \int d^4 x d^4 y e^{-i (p + p') \cdot x - i p' \cdot y} \langle T^a \phi_k^a (x), \phi_l^a (y), \phi_m^a (0) \rangle \theta_0, \\
g_{ijk} (p^2, p'^2, q^2) & = \Delta_{il} (p^2) \Delta_{jm} (p^2) \Delta_{kn} (q^2) \int d^4 x d^4 y e^{-i (p + p') \cdot x - i p' \cdot y} \langle T^a \phi_l^a (x), \phi_m^a (y), \phi_n^a (0) \rangle \theta_0,
\end{align*}
\]

(3)

where \( q \equiv p - p' \). Equations (2) and (3) impose on these the conditions

\[
\begin{align*}
\epsilon \sum_i T_{a i j} \lambda^a_j = & \ 0, \\
\Delta_{ij} (0) \lambda^a_j = & \ \epsilon^a_j, \\
(p^2 - p'^2) f_{ija} (p, p', 0) = & \ \lambda_k^a g_{ijk} (p^2, p'^2, 0) + \Delta_{ilk} (p^2) T_{a i k} - \Delta_{i k j} (p^2),
\end{align*}
\]

(4)

where

\[
\begin{align*}
\Delta_{ij} (p^2) & \equiv \int d^4 x e^{-i p \cdot x} \langle T^a \phi_i^a (x, t), \phi_j^a (0) \rangle \theta_0, \\
\Delta_{il} (p^2) & \equiv \int d^4 x d^4 y e^{-i (p + p') \cdot x - i p' \cdot y} \langle T^a \phi_l^a (x), \phi_m^a (y), \phi_n^a (0) \rangle \theta_0,
\end{align*}
\]

(5)

(6)

(7)

(8)

(9)

(10)
where
\[ \lambda_j^a = T^a_{jk} \lambda_k^j, \quad \epsilon_j^a = T^a_{jk} \epsilon_k^j. \] (11)

Equation (8) follows upon taking the vacuum expectation value of Eq. (2). Equation (9) follows upon multiplying Eq. (5) by \( b_j^a \), using Eq. (2), integrating by parts using Eqs. (3) and (4), and setting \( p_{\mu} = 0 \). Equation (10) follows upon multiplying Eq. (6) by \( q_{\mu} \), setting \( q^2 = 0 \), and using Eqs. (2), (3), and (7).

We will not attempt to extract from Eq. (10) all the information it contains. Instead, we differentiate with respect to \( p^2 \), set \( p^2 = p'^2 = 0 \), multiply with \( \lambda_i^c \lambda_j^b \), add the same equations with \( cba \) replaced with \( bca \) and \( acb \), and subtract the same equations with \( cba \) replaced with \( cab \), \( bac \), and \( abc \). Using the symmetry of \( g_{ij}(p^2, 0, 0) \) in \( j \) and \( k \), and the antisymmetry of \( f_{iaria}(0, 0, 0) \) in \( i \) and \( j \), we find that
\[ 2\lambda_i^a \frac{c}{j} f_{i i j} = \Delta_{ij}^{-1}(0) \lambda_j^c \Delta(0) \lambda_i^b + C_{abc} \lambda_i^b \lambda_i^c, \] (12)
where \( C_{abc} \) are the real, totally antisymmetric structure constants
\[ \{T^a, T^b\} = C_{abc} T^c \] (13)
and
\[ \Delta_{ij}^{-1}(0) = [d\Delta^{-1}(p^2)/dp^2]^{-1}(0, 0, 0) \] (14)

In order to make contact with physics, we will further assume that the functions \( f^+ \) and \( g \) appearing in Eq. (10) are as smooth as reasonably possible, over some range \( 0 < p^2 < m^2 \), \( -p^2 < m^2 \), i.e., in this range, \( f_{i i a}(p^2, p'^2, 0) \) is nearly equal to the constant \( f_{i i a}(0, 0, 0) \) and \( g_{i i}(p^2, p'^2, 0) \) is at most linear in \( p^2 \) and \( p'^2 \). It follows that \( \Delta_{ij}^{-1}(p^2) \) is linear in \( p^2 \), so
\[ \Delta(p^2) \equiv Z_T^2(p^2 + \mu^2)^{-1} Z^{1/2}(0 < -p^2 < m^2), \] (15)
where \( \mu^2 \) is the physical mass matrix and \( Z^{1/2} \) is a positive wave-function renormalization matrix.\(^7\)

Now we return to the case of interest, \( G = SU(3) \) \( \otimes SU(3) \) broken by \( R = (3, 3^*) \otimes (3^*, 3) \). It is apparent from Eqs. (9)-(12) that our theorems are almost exclusively concerned with the “would-be Goldstone bosons” which have the same isospin, hypercharge, and parity as the \( \lambda_{ij}^a \), i.e., the same as \( \pi \), \( K \), \( \eta \), and \( \kappa \). (Since isospin and hypercharge are not broken, their generators annihilate \( \lambda \).) The representation \( (3, 3^*) \) \( \otimes (3^*, 3) \) contains two states with the quantum numbers of the \( \eta \), and one each with the quantum numbers of the \( \pi \), \( K \), and \( \kappa \). We defer a discussion of the mixed effects in the \( \eta \) channel to a future work, and will concern ourselves here with only the \( \pi \), \( K \), and \( \kappa \) channels.\(^8\) In these channels, \( Z^{1/2} \) and \( \mu^2 \) are diagonal and Eq. (15) therefore tells us that there is just one particle in each of these channels. Letting “a” denote the particle whose quantum numbers are the same as \( \lambda^a \) or \( T^a \), we find from (2) and (9) that
\[ \epsilon_a = \mu_a \frac{2F}{F_a} Z^{-1/2}(0) \] (16)
and
\[ \lambda_a = \mu_a \frac{2F}{F_a} Z^{-1/2}(0) \] (17)
where \( \lambda_a \) and \( \epsilon_a \) are the single nonvanishing components of \( \lambda_i^a \) and \( \epsilon_i^a \), and \( F_a \) is defined by the one-particle matrix element
\[ \langle 0 | J | \mu_a(0) \rangle = F_a \text{p}^{\mu}(2\pi)^{-3} (2E)^{-1/2}. \] (18)

There are only two nonvanishing \( \epsilon_a \) and two nonvanishing \( \lambda_a \), corresponding to the two vectors of \( (3, 3^*) \otimes (3^*, 3) \) with even parity and zero isospin and hypercharge. Thus there is one relation among the three \( \epsilon_a \) and among the three \( \lambda_a \), which turns out to say that \( \epsilon_{\pi} = \epsilon_{K} + \epsilon_{\kappa} \) and \( \lambda_{\pi} = \lambda_{K} + \lambda_{\kappa} \). We then conclude from (16) and (17) that
\[ \mu_{\pi} \frac{2F}{F_\pi} Z^{-1/2}(0) = \mu_{K} \frac{2F}{F_K} Z^{-1/2}(0) + \mu_{\kappa} \frac{2F}{F_\kappa} Z^{-1/2}(0) \] (19)
and
\[ F_\pi Z^{1/2}(0) = F_K Z_{K}^{1/2}(0) + F_\kappa Z_{\kappa}^{1/2}(0). \] (20)

It is not possible to eliminate all of the unknown \( Z_a \), but we can deduce that the \( \mu_a |F_a| \) are not subject to a triangle inequality, i.e., either
\[ \mu_{\pi} \geq \mu_{\pi} |F_\pi + \mu_{K} |F_K|/|F_K| \] (21a)
or
\[ \mu_{\kappa} \geq \mu_{\kappa} |F_\kappa + \mu_{K} |F_K|/|F_K| \] (21b)
where (21a) applies only when \( F_\pi \) and \( F_K \) have the same sign, and (21b) applies when they are of opposite sign. Also, by letting \( a, b, c \) in Eq. (12) correspond to \( K \), \( \pi \), and \( \pi \), and dividing by \( 2\Delta_{\pi \kappa} Z_{\pi}^{-1/2} Z_{\kappa}^{-1/2} \), we find that the renormalized \( K_{\pi} \) form factor is
\begin{align*}
&f^+(0) = (F_\pi^2 + F_K^2 - F_\kappa^2)/2F_K F_\pi. \quad (22)
\end{align*}
Note that in the limit of exact SU(3), \( F_F = F_K \), and \( F_K = 0 \); so \( f^+ (0) = 1 \). Further, \( |f^+ (0) - 1| \) is of second order in the SU(3)-breaking parameters \( F_K - F_F \) and \( F_K \), in agreement with the Ademollo-Gatto theorem. The Cabibbo\(^{15} \) theory relates \( f^+ (0) \) to the measured amplitudes for \( K \rightarrow \pi \), \( \pi \rightarrow \pi \), and \( \pi \rightarrow \mu \). We favor the first inequality because it is the one that holds when \( F_F \) is of the same sign as \( F_K \), as would be expected for weak SU(3) breaking.\(^{15} \)

The \( K \pi \) threshold is at 630 MeV, and if the \( \kappa \) lies below this mass (as we suspect\(^{16} \), then it would only decay electromagnetically by \( \kappa \rightarrow \pi + 2 \gamma \). The existence of such a particle is not precluded by current experimental evidence.

\(^{15} \)Research supported in part by the Atomic Energy Commission Contract No. AT(30-1)2098 and by the Office of Naval Research Grant No. NONR 1866(65).

\(^{16} \)On leave from the University of California, Berkeley, California.

\(^{17} \)M. Gell-Mann [Physics (N.Y.) 1, 63 (1963)] has considered chiral SU(3) \( \otimes \) SU(3) as a weakly broken symmetry group.

\(^{18} \)The association of a scalar kaon with the divergence of the strange vector current originates with Y. Nambu, Phys. Rev. Letters 4, 380 (1966).

\(^{19} \)Only the continuous connected component of \( G \) is relevant to our analysis.

\(^{20} \)Equation (9) reduces to a proof of the Goldstone theorem for the case \( e = 0 \) (cf. Eq. (18) of J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. Letters 12, 965 (1962)).

\(^{21} \)We originally deduced Eq. (9) in Lagrangian field theory without making use of current algebra.

\(^{22} \)The same smoothness assumption is used by H. Schnitzer and S. Weinberg, Phys. Rev. 164, 1828 (1967).

\(^{23} \)Originally, we took \( g \) to be constant for small momenta. This gave the result \( Z = 1 \), with the consequences \( F_K \approx 1.4 F_F \), \( F_K \approx 0.4 F_F \), and \( \mu_K \approx 890 \) MeV. However, these values of \( F \) are in poor agreement with the SU(3)\( \otimes \)SU(3) spectral-function sum rules [S. Glashow, H. Schnitzer, and S. Weinberg, Phys. Rev. Letters 19, 139 (1967); T. Das, V. Mathur, and S. Okubo, Phys. Rev. Letters 18, 761 (1967)], and furthermore, we cannot have \( Z = 1 \) if there are gauge-invariant couplings of vector and axial-vector mesons.

It is for these reasons that we allow \( g \) to have linear dependence on \( p^2 \) and \( p^2 \). Recent work of J. Sucher and C. Woo [Phys. Rev. Letters 18, 723 (1967)] on the \( \sigma \) model does use the hypothesis \( Z = 1 \).

\(^{24} \)When our method is applied to the mixed channel, the principal new result is the relation

\[
F_K^2/F_F^2 = 1.17, \quad F_K/F_F = 0.34.
\]

The inequality (21) thus becomes

\[\mu_K \leq \text{670 MeV or } \mu_K \geq \text{1150 MeV.}\]
EXPERIMENTAL TESTS OF THE VECTOR-DOMINANCE MODEL


Deutsches Elektronen-Synchrotron, Hamburg, Germany, and Department of Physics and Laboratory of Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received 2 January 1968)

In this Letter we report the results of an experiment designed to perform some independent tests of the validity of the vector-dominance model of electromagnetic interactions of hadrons. The vector-dominance model relates the electromagnetic current \( J_\mu(x) \) of hadrons with the phenomenological fields of vector mesons \( \rho_\mu(x) \), \( \varphi_\mu(x) \), and \( \omega_\mu(x) \) via

\[
J_\mu(x) = -\frac{m^2}{2\gamma_\rho} \rho_\mu(x) - \frac{m^2}{2\gamma_{\omega}} \omega_\mu(x) - \frac{m^2}{2\gamma_{\varphi}} \varphi_\mu(x) \tag{1}
\]

It follows from (1) that the electromagnetic form factors of nucleons and pseudoscalar mesons as well as electromagnetic interactions of mesons can all be expressed in terms of measurable quantities \( \gamma_\rho \), \( \gamma_\omega \), and \( \gamma_{\varphi} \), which couple the vector meson to the photon. In particular, the photoproduction of \( \rho^0 \) mesons on complex nuclei can be thought of as via the diagram (Fig. 1) where the photon materializes itself into \( \rho^0 \) with a coupling strength \( \alpha_\pi/\gamma_{\rho} \), and the \( \rho^0 \) meson subsequently scatters diffractively off the whole nucleus. This diagram for photoproduction of \( \rho^0 \) mesons then carries the following two important implications.

(1) A factor \( -m^{-2} \) enters from the \( \rho^0 \) propagator and the \( \rho^0 \) decay spectrum can be shown to be of the form

\[
R(m) = \left( \frac{m}{\rho} \right)^4 f_{\text{BW}}(m) \cdots, \tag{2}
\]

where \( m^2 = p^+ + p^- \), and where \( f_{\text{BW}}(m) \) is the relativistic Breit-Wigner mass formula for the decay \( \rho^0 \rightarrow \pi^+ \pi^- \):

\[
1 = m \Gamma(m) \tag{3}
\]

with

\[
\Gamma(m) = \frac{m}{\rho} \beta \left( \frac{1}{2} m^2 - \frac{1}{2} m^2 \right)^{3/2} \Gamma_0.
\]

Equation (2) provides a mass shift of \( \approx 20 \text{ MeV}/c^2 \) and has been used as an explanation for the difference between the mass \( m_{\rho} = 755 \text{ MeV}/c^2 \) (\( \rho^0 \) mesons produced from \( \pi^- N \) interactions) and that of \( m_{\rho'} = 740 \text{ MeV}/c^2 \) (\( \rho^0 \) mesons produced in photoproduction experiments).

The first purpose of the present experiment is to study the spectrum of \( \rho^0 \rightarrow \pi^+ \pi^- \) in the region of high \( \pi^+ \pi^- \) invariant mass, \( 930 < m < 1130 \) MeV. We have...