Another odd thing about unparticle physics

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Abstract

The peculiar propagator of scale invariant unparticles has phases that produce unusual patterns of interference with Standard Model processes. We illustrate some of these effects in $e^+e^- \rightarrow \mu^+\mu^-$. © 2007 Elsevier B.V. All rights reserved.

1. Introduction

In a previous paper [1], I argued that a scale invariant sector that decouples at a large scale is associated with “unparticles” whose production might be detectable in missing energy and momentum distributions. In this note, we consider some of the leading virtual effects of unparticles. In particular, we write down the unparticle propagator and consider the interference between Standard Model amplitudes and amplitudes involving virtual unparticles. As emphasized long ago by Eichten, Lane and Peskin [2], this kind of interference can be a sensitive probe of high-energy processes. In particular, the interference terms are effects of leading nontrivial order in the small couplings of unparticles to Standard Model particles. We make crucial use of the simplifications that result by working to lowest nontrivial order in the small couplings of unparticles fields to Standard Model fields in the effective field theory below $\Lambda_U$. This allows us reliably to calculate some important quantities without having to understand in detail what unparticles look like. We will return to some of these questions at the end of the Letter.

To illustrate the interesting properties of the unparticle propagator, we consider the example of the low energy effect of the following interaction terms.

\[ \begin{align*}
&C_{VU}A_{UL}^{k+1-d_U}M_{UL}^{-k}e\gamma_\mu eO^\mu_U + C_{AU}A_{UL}^{k+1-d_U}M_{UL}^{-k}e\gamma_\mu\gamma_5 eO^\mu_U,
\end{align*} \]

where the unparticle operator is hermitian and transverse, \( \partial_\mu O^\mu_U = 0 \).

I stress that this is just an example. Unparticle operators with different tensor structures can be dealt with in a similar way. In the notation of [1], the transverse 4-vector unparticle propagator is given by

\[ \int e^{i Px} \langle 0| T\left(O^\mu_U(x)O^\nu_U(0)\right)|0\rangle d^4x \]
\[
\left( M^2 \right)_{d\ell t} - 2g^{\mu\nu} + \frac{P^\mu P^\nu}{P^2 - M^2 + i\epsilon} = \frac{i}{2} \left( \frac{d\ell_t}{d\ell_t(\pi)} \right) \sin^2 \left( \frac{\sqrt{s} (GeV)}{d\ell_t} \right) + 5|c_{\text{att}}|^2 \]

\[
-5|c_{\text{att}}|^2 \]

Fig. 1. The fractional change in total cross-section for \( e^+e^- \rightarrow \mu^+\mu^- \) versus \( \sqrt{s} \) for \( d\ell_t = 1, 1.3, 1.5, 1.7 \) and 1.9 for non-zero \( c_{\text{att}} \) and \( c_{\text{U}} = 0 \). The dash-length increases with \( d\ell_t \).

\[
\Delta_{\gamma y}(q^2) \equiv \sum_{j=1}^{5} d^\gamma_{s_j} d^\mu_{s_j} \Delta_j(q^2), \quad \text{where} \ x, y = V \text{ or } A, \quad (8)
\]

with the \( d's \) given in the following table:

<table>
<thead>
<tr>
<th>( d_{s_j} )</th>
<th>( \gamma )</th>
<th>( Z )</th>
<th>( U )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V )</td>
<td>( e )</td>
<td>( e )</td>
<td>( \frac{\epsilon}{\sin^2 \theta} )</td>
</tr>
<tr>
<td>( A )</td>
<td>( 0 )</td>
<td>( \frac{\epsilon}{\sin^2 \theta} )</td>
<td>( \frac{c_{\text{AU}}}{M^2_{Z}} )</td>
</tr>
</tbody>
</table>

and the \( \Delta_j's \) being the \( \gamma, Z \) and \( U \) propagators,

\[
\Delta_j(q^2) = \frac{1}{q^2} \left( \frac{1}{M^2_{Z} + i M^2 \gamma Z} \right) \frac{A_{\Delta t}}{2 \sin(d\ell_t(\pi))} (q^2)^{d\ell_t - 2} e^{-i(d\ell_t - 2)\pi} \quad (10)
\]

We have tacitly assumed in (9) that the unparticle interactions are lepton-flavor-blind, so that we do not have to keep track of the \( e \) and \( \mu \) superscripts on the \( c's \), and we will continue to assume this in the graphs below. But (7) is entirely general and does not depend on this assumption.

As a first example of the interesting structure of (7), consider the total cross section in the LEP region. We are used to thinking that the \( Z \) pole is not a good place to look for interference with the effects of small non-renormalizable interactions because the amplitude is dominantly imaginary on the pole. This prejudice is not warranted for unparticle interactions. The unparticle amplitude can interfere with both the real and imaginary parts of the Standard Model and can therefore contribute both on and off the pole.

It is instructive to begin by assuming \( c_{\text{VU}} = 0 \) (remember, we are taking the same \( c \) for \( e \) and \( \mu \)) and considering the total cross section. Because the vector coupling vanishes, the interference between the unparticle exchange amplitude and the photon decay amplitude does not contribute to the total cross section, so we expect only interference with \( Z \) exchange. In Fig. 1, I show the fractional change in the total cross section for small non-zero \( c_{\text{AU}} \) for various values of \( d\ell_t \) between 1 and 2.
The dominant effect as expected is the interference term proportional to a single power of \(|c_{\text{UL}}|^2\). But the striking thing about this graph is how sensitively the result depends on the value of \(d_{\text{UL}}\). We can understand qualitatively what is going on by thinking about the phase of the unparticle propagator along the physical cut which is

\[ \phi_{d_{\text{UL}}} = -(d_{\text{UL}} - 1)\pi. \]  

(11)

The real part of (11) is positive for \(1 < d_{\text{UL}} < 3/2\) and negative for \(3/2 < d_{\text{UL}} < 2\). The real part of \(1/(q^2 - M_Z^2 + iM_Z\Gamma_Z)\) is negative below the Z pole and positive above. Thus away from the Z pole, where the imaginary part of \(1/(q^2 - M_Z^2 + iM_Z\Gamma_Z)\) is small, we expect destructive (constructive) interference below (above) the pole for \(1 < d_{\text{UL}} < 3/2\), and vice-versa for \(3/2 < d_{\text{UL}} < 1\). Near the Z pole, the situation is complicated, as illustrated in Fig. 2 because both real and imaginary parts contribute to the interference.

The situation simplifies in a very interesting way for \(d_{\text{UL}} = 3/2\). In this case, the phase from (11) is \(\phi_{d_{\text{UL}}} = -\pi/2\), so the unparticle amplitude interferes only with the imaginary part of the Z-exchange amplitude. This is a smaller effect than we see for values of \(d_{\text{UL}}\) very different from 3/2 because it is proportional to the Z width, rather than \(q^2 - M_Z^2\). It gives constructive interference that peaks on the Z pole and goes to zero far from the pole. This is shown on a different scale in Fig. 3. Here I have also included a few values of \(d_{\text{UL}}\) close to 3/2, for comparison.

Having seen how things work for purely axial vector unparticle couplings, let us now consider what the total cross section looks like for a vector coupling. Now we expect interference with the photon-exchange amplitude, and because the vector part of the leptonic coupling of the Z is (by an “accident” of the value of \(\sin^2\theta\)) very small, there is very little interference with the Z-exchange amplitude. Now we expect constructive interference for \(1 < d_{\text{UL}} < 3/2\) and destructive interference for \(3/2 < d_{\text{UL}} < 1\). The result is shown in Fig. 4. The dip at the Z pole arises simply because we are plotting a fractional change and the large contribution from the pole is in the denominator.

The unparticle interference in the matrix element (7) also gives rise to a complicated pattern of changes in the front-back asymmetry

\[
\frac{\sigma_f - \sigma_b}{\sigma_f + \sigma_b} = \frac{3}{8} \left( \frac{\text{Re}(\Delta_{VV}(q^2)\Delta_{AA}(q^2)) + \text{Re}(\Delta_{VA}(q^2)\Delta_{AV}(q^2))}{|\Delta_{VV}(q^2)|^2 + |\Delta_{AA}(q^2)|^2 + |\Delta_{VA}(q^2)|^2 + |\Delta_{AV}(q^2)|^2} \right).
\]

(12)

This is shown in Figs. 5 and 6. As for the total cross section, the effect for \(d_{\text{UL}} = 3/2\) is smaller and concentrated at the Z pole. In Figs. 7 and 8, we focus down on values of \(d_{\text{UL}} \approx 3/2\).
I hope I have convinced the reader that the unparticle propagator in the time-like region has interesting properties that force us to reexamine many of our preconceived notions about interference. Working to lowest non-trivial order in the couplings of the unparticles in the effective low energy theory, we can make detailed predictions of the form of interference between time-like unparticle exchange amplitudes and Standard Model amplitudes even though we still lack an intuitive or even detailed picture of what an unparticle looks like.

Let me close with a couple of more speculative comments. One might argue that the term “propagator” is not particularly felicitous for the unparticle time-ordered product, (3), because the unparticle does not really propagate in the usual way. It is also worth noting the connection between this analysis and the more confusing issue of unparticle decay. There is a sense in which the unparticle exchange amplitude that we have used in our analysis is associated with unparticle production and decay. But in the process we have studied in this note, the decay process is masked by the leading order (and therefore larger) interference term. And as with the term “propagator”, the term “decay” may be a little misleading for an unparticle because it suggests that the particle was propagating over an large distance before it decayed. I hope to return to these deliciously confusing issues in a future publication.

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References