Initial states and branching ratios in p\bar{p} annihilations at rest

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p\bar{p} annihilations at rest occur from initial states with \( J^{PC}=0^{-+}, 1^{--}, 1^{+-}, 0^{++}, 1^{++}, 2^{++} \). These initial states are S- and P-wave atomic levels of protonium atoms that are populated by the atomic cascade which follows the atomic capture of the antiproton. Each of the six fractions \( F_{j\varepsilon c} \) of annihilations from initial states of protonium with quantum numbers \( J^{PC} \) depends on the density of the target where antiprotons are brought to rest, and is not known individually at present at any target density. Here we show that the values of the six annihilation fractions \( F_{j\varepsilon c} \) can be determined by measuring at four target densities the values \( f_{\text{channel}} \) of the frequencies of annihilations for six exclusive annihilation channels. The same measurements would provide, for the channels exploited for the \( F_{j\varepsilon c} \) determination, the values \( B^{\text{channel}}_{j\varepsilon c} \) of the branching ratios of annihilation from each \( J^{PC} \) type of initial states from which the channels are allowed.

1. Introduction

p\bar{p} annihilations at rest occur from the singlet \( 0^{-+} \) and the triplet \( 1^{--} \) S-wave atomic states of protonium with principal quantum number \( n \) higher or equal to 1, and from the singlet \( 1^{+-} \) and triplet \( 0^{++}, 1^{++}, 2^{++} \) P-wave hyperfine states with \( n \) higher or equal to 2. Annihilations do not occur from D-wave and higher orbital angular momentum states because of the negligible overlap of the p and \( \bar{p} \) wave functions.

The populations of the levels of protonium and in particular of those from which annihilation takes place depend on the density \( \rho \) of the \( H_2 \) target where antiprotons are brought to rest. The distribution of the initial states of annihilation according to their \( J^{PC} \) value depends then on the target density.

Experimental data from LEAR show indeed that variations of the density of the target are accompanied by variations of the frequencies of some exclusive channels of annihilation, as e.g. \( K^+K^- \), \( K_SK_L \), \( K_SK_S \), \( \pi^+\pi^- \) [1,2] by variations of the ratio between some of these frequencies [1–4], and by variations of the absolute and relative yields of radiative transitions between protonium levels [5–11].

These phenomena are usually discussed within a simplified framework where only two types of annihilation are considered: S-wave and P-wave annihilations. The ASTERIX experiment deduced within this framework the fraction of S-wave and of P-wave annihilations in a \( H_2 \) target at NTP and in a liquid \( H_2 \) target, and the values of the branching ratios of some exclusive final states for what was called pure S-wave annihilation and pure P-wave annihilation [1,2]. The experimental inputs were:

(i) the frequencies \( f_{\text{channel}} \) measured by ASTERIX in a \( H_2 \) gas target at NTP for the two-body final states \( \pi^+\pi^- \), \( K^+K^- \), \( K_SK_S \) and \( K_SK_L \).

(ii) the frequencies \( X_{\text{channel}} \) measured by ASTERIX for the two-body final states \( \pi^+\pi^- \), \( K^+K^- \), \( K_SK_S \) and \( K_SK_L \), produced in coincidence with an X-ray in the 1–4 keV energy window in the same \( H_2 \) gas target at NTP,

(iii) the ratio between the yield of protonium L X-rays and the yield of background X-rays due to internal bremsstrahlung and fluorescence caused by the charged mesons produced by the p\bar{p} annihilation or by decays of neutral mesons,

(iv) the frequencies \( f_{\text{channel}} \) measured in liquid \( H_2 \) targets for the channels \( \pi^+\pi^- \), \( K^+K^- \), \( K_SK_S \) and \( \pi^0\pi^0 \).

The ASTERIX analysis was based on the simplified schematization of annihilation in terms of only
two components and relied on a tacitly assumed hypothesis [12] of which the most critical one was the assumption that the fractions of annihilations from the four P-wave sublevels with principal quantum number \( n = 2 \) were proportional to the four \( F_{7/2} \) fractions of annihilation for P-wave annihilations at the same target density. Within the context of the simplifications mentioned above, the ASTERIX analysis permitted one to determine a significant value for the ratio between S-wave and P-wave annihilations in H\(_2\) gas at NTP, and the value of that ratio in liquid H\(_2\) that was much more uncertain because it depended critically on the magnitude of the fraction \( \frac{\text{annihilation}}{\text{in liquid}} \), whose experimental values varied from \((4.8-1.4) \times 10^{-4}\) to \((2.06-2.5) \times 10^{-4}\) [13-16] and have grown recently to \(6.9 \times 10^{-4}\) [17].

The derivation of the fraction of P-wave annihilation in H\(_2\) gas at 15 atm was obtained by CPLEAR [4] following the track of the ASTERIX analysis. The values derived for the ratio between S- and P-wave annihilations in gas and in liquid are exploited in quantitative analyses of pp annihilations at rest with three and more mesons produced in the final state and with conventional and exotic mesons in the intermediate states [18-23] and in the determination of the electromagnetic form factor of the proton at threshold in the timelike region [24,25].

Here we express the structure of the \( J^{PC} \) distribution of the initial states of pp annihilation at rest in terms of atomic states of protonium. We show that the six annihilation fractions \( F_{J^{PC}} \), which are defined in subsection 2.4, can be determined by measuring the frequencies of some exclusive annihilation channels and the angular distributions of the particles measured in the final state. This task requires measurements at several target densities and will simultaneously yield the branching ratios for annihilation from each allowed \( J^{PC} \) initial state for the channels exploited. The method that we suggest to derive the annihilation fractions and the branching ratios can be completely independent from any hypothesis about the atomic cascade and the effects of strong interactions on the atomic levels of protonium. It can be simplified by using explicitly one dynamical hypothesis about the evolution of the atomic cascade. We show also that the main hypothesis exploited implicitly in the derivation of the S-versus P-wave annihilation ratios by ASTERIX is not justified.

This work elaborates on points anticipated in ref. [26] \(^2\) and discussed [27] at the Nucleon–Antinucleon Conference NAN '91. The additional physics information that can be extracted by performing a similar program in coincidence with detection of L and K X-rays of protonium is discussed elsewhere [28].

The determination of the annihilation fractions is necessary for a precise measurement of the electromagnetic form factor of the proton at the threshold of the timelike region [29] and for quantitative differential measurements in the spectroscopy of light mesons and exotics [30,31]. The determination of the branching ratios of annihilation channels from the different \( J^{PC} \) initial state from which they are allowed is a crucial experimental input for the study of the dynamics of nucleon antinucleon annihilations. Refs. [26,32-34] give experimental and theoretical reviews about this topic. The approach outlined in the present paper does not require one to enter deeply into the details of the atomic cascade of protonium. The interested reader is therefore referred to the papers where the main mechanisms which make the atomic cascade of protonium density dependent were discussed [35-37], to recent reviews of protonium physics [38-40] and cascade calculations [41].

2. Protonium levels, cascade and annihilation

2.1. General points

Each antiproton brought at rest in a H\(_2\) target forms a pp atom in a highly excited state. The pp atom undergoes its atomic cascade, which finishes with annihilation in an S- or P-wave level of protonium. Protonium is analogous to positronium, but has a much larger reduced mass, leading to a Rydberg energy of 12.5 keV, and strong interactions perturbing considerably its S- and P-wave levels. In the ground state, that has a lifetime of about \(10^{-18}\) s, the separation between p and \( \bar{p} \) is about 50 fm and grows as \( n^2 \) with increasing principal quantum number \( n \). Annihilations occur only only from S- and P-states because for states with higher orbital angular momentum the

\(^2\) See in particular pp 227, 233, 238, 248, and p. 252 of ref. [26].
overlap of the p and \( \bar{p} \) wave functions is negligible.

For the same reason annihilations is much stronger in S-waves than in P-waves. The values of the shift \( \Delta E \) and broadening \( \Gamma \) caused by strong interactions to the protonium levels are expected to vary with the principal quantum number \( n \) according to the overlap of the wave functions and to depend on the spin and the angular momentum (see ref. [34] and references therein):

\[
\Gamma_{nJPC} = f(n) \Gamma_{JPC}, \quad \Delta E_{nJPC} = f(n) \Delta E_{JPC}
\]

where for S-states \( f(n) = n^{-3} \) with \( \Gamma_{JPC} = \Gamma_{n=1,JPC} \), and for P-states \( f(n) = \frac{3n}{2} (n^{-3} - n^{-5}) \) with \( \Gamma_{JPC} = \Gamma_{n=2,JPC} \).

None of the 12 independent quantities \( \Delta E_{JPC} \) and \( \Gamma_{JPC} \), that characterize the global effects of strong interactions on the two \( n = 1 \) S-levels and the four \( n = 2 \) P-levels of protonium has been measured directly so far. This implies that there are 12 parameters that require experimental determination, and which are a necessary input for a complete calculation of the cascade of protonium.

The values reported there are those calculated in the model of ref. [42] and are used just to illustrate the orders of magnitude and to stress that the values of \( \Delta E \) and \( \Gamma \) are expected to depend on the \( n_{JPC} \) quantum numbers of the S- and P-levels. From experiments measuring X-rays of protonium [6–11], average values are available for \( \Delta E \) and \( \Gamma \) of the ground state (which correspond to different experimental conditions in which the averages have been made), and the average width \( \Gamma \) for the \( n = 2 \) P states.

The population of a level of protonium from which annihilation may occur is indicated here \( \rho_{nJPC} \). \( \rho_{nJPC} \) represents the probability that the level be reached in the course of the atomic cascade. The population of a level depends on the atomic cascade and on the distribution of initial states of protonium at capture. For any level this population depends on the density \( \rho \) of the target, to which the number of collisions per unit time of protonium atoms with the molecules of the surrounding medium is proportional. The precise calculation of \( \rho_{nJPC} \) is not possible at present because the 12 strong interaction values that characterize the strong interaction effects on the levels of protonium are not known individually. For the purpose of this paper it is however sufficient to indicate explicitly the fact that the values of \( \rho_{nJPC} \) are density dependent.

Any \( n_{JPC} \) level is depopulated by two types of transitions. The first type of transitions are:

(i) radiative transitions to lower lying levels of protonium with decay width \( \Gamma_{nJPC}^{rad} \),

(ii) annihilations with decay width \( \Gamma_{nJPC}^{ann} \).

The widths associated to these transitions are controlled only the internal dynamics of protonium.

The second type of transitions include:

(i) chemical transitions (in the highly excited states of protonium) where the antiproton can exchange the proton partner during a collision.

(ii) Auger transitions, where protonium makes transitions to lower lying levels transferring energy to the electrons of the molecules crossed during the collisions.

(iii) Stark transitions induced by mixing with nearby levels of equal \( n \); the Stark mixing is caused by the non-central electric field experienced by protonium when crossing hydrogen molecules in the target.

The second type of transitions have rates proportional to the density of the target.

The total width of a protonium level is then density dependent and can be written as

\[
\rho \Gamma_{nJPC}^{tot} = \rho \Gamma_{nJPC}^{rad} + \rho \Gamma_{nJPC}^{ann} + \rho \Gamma_{nJPC}^{Stark} + \rho \Gamma_{nJPC}^{Auger} + \rho \Gamma_{nJPC}^{chem}.
\]

\[
\text{2.2. Definition of the branching ratio of an annihila-}
\text{tion channel from a level of given } n_{JPC}.
\]

We indicate with \( \Gamma_{nJPC}^{ch} \) the width for the annihilation into a specific channel \( ch \) of the \( n_{JPC} \) level. \( \Gamma_{nJPC}^{ch} \) obeys the same scaling law (1) as the total annihilation width \( \Gamma_{nJPC}^{ann} \) of the level \( n_{JPC} \):

\[
\Gamma_{nJPC}^{ch} = f(n) \Gamma_{JPC}^{ch}.
\]

For a given exclusive annihilation channel \( ch \) we call \( B_{nJPC}^{ch} \) the ratio between the probability of annihilation of a level \( n_{JPC} \) in a channel \( ch \) \( \Gamma_{nJPC}^{ch} \) and the total probability of annihilation \( \Gamma_{nJPC}^{ann} \) of the level \( n_{JPC} \):

\[
B_{nJPC}^{ch} = \Gamma_{nJPC}^{ch} / \Gamma_{nJPC}^{ann} = \Gamma_{JPC}^{ch} / \Gamma_{JPC}^{ann}.
\]

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$B_{\text{ch}}$ is the branching ratio for annihilations into the channel $\text{ch}$. This quantity is the same for all levels with the same $J^P_C$, since it is independent of $n$ and of the atomic cascade. It tells how much strong interactions favour annihilation into the channel $\text{ch}$ for a protonium level with quantum numbers $J^P_C$.

### 2.3. Annihilation from individual atomic levels

The contribution $\rho f_{nJ^P_C}$ of the $nJ^P_C$ level to annihilation is proportional to its population and to the fraction of its disappearance by annihilation over its total disappearance rate:

$$\rho f_{nJ^P_C} = \rho P_{nJ^P_C} \frac{\Gamma_{\text{ann}}}{\Gamma_{\text{tot}}} . \tag{5}$$

The contribution $\rho f_{nJ^P_C}^\text{ch}$ of the $nJ^P_C$ level to annihilation in a specific channel $\text{ch}$ is proportional to its population and to the fraction of its disappearance by annihilation into that channel $\text{ch}$ over its total disappearance rate:

$$\rho f_{nJ^P_C}^\text{ch} = \rho P_{nJ^P_C} \frac{\Gamma_{\text{ann}}^{\text{ch}}}{\Gamma_{\text{tot}}^{\text{ch}}} . \tag{6}$$

### 2.4. Annihilation fractions

We can reexpress (6) by using (4) in the following way:

$$\rho f_{nJ^P_C}^\text{ch} = B_{\text{ch}}^\rho \rho f_{nJ^P_C} , \tag{7}$$

which says that the contribution of level $nJ^P_C$ in a target of density $\rho$ to annihilation into the channel $\text{ch}$ is proportional to the branching ratio of the channel $\text{ch}$ from the $J^P_C$ level times the probability of populating the level $nJ^P_C$ times the probability that the level $nJ^P_C$ undergoes annihilation.

We can then reexpress (7), by using (5) as

$$\rho f_{nJ^P_C}^\text{ch} = B_{\text{ch}}^\rho \rho f_{nJ^P_C} . \tag{8}$$

The fraction $\rho F_{J^P_C}$ of all annihilations from all the initial states of protonium of a given $J^P_C$ is given by the sum over $n$ of the contributions $\rho f_{nJ^P_C}$ to annihilation of all the states $nJ^P_C$:

$$\rho F_{J^P_C} = \sum_n \rho f_{nJ^P_C} = \sum_n \rho P_{nJ^P_C} \frac{\Gamma_{\text{ann}}}{\Gamma_{\text{tot}}} . \tag{9}$$

Each annihilation fraction depends on the density of the target and on the 12 strong interaction parameters $\Delta E_{J^P_C}$ and $\Gamma_{\text{tot}}$, via the dependence of the terms $\rho P_{nJ^P_C}$, $\Gamma_{\text{ann}}^{\text{ch}}$ and $\Gamma_{\text{tot}}^{\text{ch}}$. Even if the dependence on the 12 strong interaction terms via the above first and third terms would be only mild, the dependence on the annihilation width $\Gamma_{\text{tot}}^{\text{ch}}$ is strong, as it can be seen rewriting (9) as

$$\rho F_{J^P_C} = \Gamma_{\text{tot}}^{\text{ch}} \sum_n \rho P_{nJ^P_C} \frac{\Gamma_{\text{ann}}^{\text{ch}}}{\Gamma_{\text{tot}}^{\text{ch}}} . \tag{10}$$

The sum of the contributions of all the $n$ levels of the same $J^P_C$ to annihilations in a given channel $\text{ch}$ gives the contribution $\rho f_{J^P_C}^\text{ch}$ of annihilations for that channel from all protonium levels with that $J^P_C$.

We can then write using (8),

$$\rho f_{J^P_C}^\text{ch} = \sum_n \rho f_{nJ^P_C}^\text{ch} = \sum_n B_{\text{ch}}^\rho \rho f_{nJ^P_C} = B_{\text{ch}}^\rho \rho F_{J^P_C} . \tag{11}$$

### 2.5. Experimental determination of frequencies of annihilation and of partial frequencies of annihilation into exclusive channels

In a target of density $\rho$ the frequency $\rho f_{J^P_C}^\text{ch}$ of annihilations into an exclusive channel $\text{ch}$ is given by the ratio between the number $\rho N_{\text{ch}}$ of annihilations into the channel $\text{ch}$ and the number $N_\rho$ of antiprotons annihilating into the target:

$$\rho f_{J^P_C}^\text{ch} = \frac{\rho N_{\text{ch}}}{N_\rho} . \tag{12}$$

If the channel $\text{ch}$ can be produced from more than one $J^P_C$, we can write $\rho N_{\text{ch}}$ as

$$\rho N_{\text{ch}} = \sum_{J^P_C} \rho N_{\text{ch}}^{J^P_C} . \tag{13}$$

If the analysis of the decay angular distributions permits one to determine the individual contributions $\rho N_{\text{ch}}^{J^P_C}$ to $\rho N_{\text{ch}}$ it is then possible to determine experimentally the partial frequencies that are given by the ratio between the number $\rho N_{\text{ch}}^{J^P_C}$ of annihilations into the channel $\text{ch}$ from all the states with quantum number $J^P_C$ and the number $N_\rho$ of antiprotons annihilating into the target:

$$\rho f_{J^P_C}^{\text{ch}} = \frac{\rho N_{\text{ch}}^{J^P_C}}{N_\rho} . \tag{14}$$
3. Determination of annihilation fractions and of branching ratios

3.1. General points

Eqs. (14) and (12) express in terms of experimental observables the same quantities described in terms of sums of the contributions of the individual atomic levels respectively by eqs. (8) and (11). We have then

\[ \sum_{j,c} B_{j,c}^P \rho F_{j,c} = \rho f_{ch} = \rho N_{ch}^P / N_p , \]  
(15)

\[ B_{j,c}^P \rho F_{j,c} = \rho N_{j,c}^P / N_p . \]  
(16)

Eq. (16) tells us that if the annihilation fraction \( \rho F_{j,c} \) at a given target density \( \rho \) is known for the levels with discrete quantum numbers \( J^P \), then the branching ratio \( B_{j,c}^P \) for annihilation into a channel \( ch \) from the levels with discrete quantum numbers \( J^P \) can be derived directly from the measurement of the partial frequency of annihilation \( \rho f_{ch}^P \) of that channel. The same measurement at a second target density permits one to check the value of the branching ratio if the annihilation fraction at this second density is already known, or conversely to determine the annihilation fraction by using the branching ratio value given by the first measurement.

Table 1 shows conditions (15) and (16) for a number of \( pp \) annihilations into simple channels that can be identified unambiguously and reconstructed with little background. Selection rules limit, sometimes drastically, the \( J^P \) initial states that can contribute to annihilations into the channels listed (see ref. [3] and references therein). Each of the channels listed in the table has already been observed experimentally at some target density in at least one exclusive final state \( #3 \). For each \( J^P \) it is possible to find one or more channels that are not forbidden by selection rules and whose \( \rho f_{ch}^P \) be sizeable so that eq. (16) can be exploited sensibly.

Since each antiproton that is brought to rest annihilates, the following relation links the values of the six annihilation fractions:

\[ \rho F_{0,-} + \rho F_{1,-} + \rho F_{1,+} + \rho F_{2,+} + \rho F_{2,-} + \rho F_{0,+} = 1 . \]  
(17)

Since in the course of the atomic cascade none of the effects that contribute to the cascade acts directly on the total spin of protonium \([44]\), atoms formed

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### Table 1

Decomposition of the frequencies of annihilation \( f_{ch} \) of a number of channels into their \( J^P \) components \( f_{j,c}^P \) for one fixed target density \( \rho \). The symbol \( \rho \) is not shown as superscript left of the \( F \) and terms for briefness.

<table>
<thead>
<tr>
<th>Channel</th>
<th>( B_{0,-}^P \rho F_{0,-} )</th>
<th>( B_{1,-}^P \rho F_{1,-} )</th>
<th>( B_{1,+}^P \rho F_{1,+} )</th>
<th>( B_{2,+}^P \rho F_{2,+} )</th>
<th>( B_{2,-}^P \rho F_{2,-} )</th>
<th>( B_{0,+}^P \rho F_{0,+} )</th>
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<tbody>
<tr>
<td>( B_{0,-}^P \rho F_{0,-} )</td>
<td>+ ( B_{0,-}^P \rho F_{1,-} ) + ( B_{0,-}^P \rho F_{2,+} ) = ( f_{0,-}^P )</td>
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<td>( B_{1,-}^P \rho F_{1,-} )</td>
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<tr>
<td>( B_{1,+}^P \rho F_{1,+} )</td>
<td>+ ( B_{1,+}^P \rho F_{1,-} ) + ( B_{1,+}^P \rho F_{2,+} ) = ( f_{1,+}^P )</td>
<td>+ ( f_{1,+}^P ) + ( f_{1,+}^P ) = ( f_{1,+}^P )</td>
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<td>( B_{2,+}^P \rho F_{2,+} )</td>
<td>+ ( B_{2,+}^P \rho F_{1,-} ) + ( B_{2,+}^P \rho F_{1,+} ) + ( B_{2,+}^P \rho F_{2,+} ) = ( f_{2,+}^P )</td>
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<td>( B_{2,-}^P \rho F_{2,-} )</td>
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<td>( B_{0,+}^P \rho F_{0,+} )</td>
<td>+ ( B_{0,+}^P \rho F_{0,-} ) + ( B_{0,+}^P \rho F_{1,-} ) + ( B_{0,+}^P \rho F_{1,+} ) + ( B_{0,+}^P \rho F_{2,+} ) = ( f_{0,+}^P )</td>
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in the singlet state will annihilate sooner or later in a singlet state, and analogously for atoms formed in triplet states. For an unpolarized p beam stopped in an unpolarized H2 target, atomic capture will populate statistically the spin levels and eq. (17) breaks down into two separate equations:

\[ pF_{0-} + pF_{1+} = \frac{1}{2}, \quad (18) \]

\[ pF_{1-} + pF_{0++} + pF_{1++} + pF_{2++} = \frac{1}{2}. \quad (19) \]

We have written eqs. (18) and (19) under the explicit assumption about the dynamics of the protonium cascade that no effect acts directly on the total spin of the p0 atom. Should this hypothesis be not justified, one can make only use of eq. (17), which is quite general since the antiproton is a stable particle, and we consider a pure H2 target (hence no atomic transfers to nuclei different from the proton). For the time being we assume the validity of eqs. (18) and (19), and we will discuss later on how to check experimentally a posteriori this assumption. Another implication of conditions (18) and (19) is that, if we consider S-wave annihilations in a target of given density only, the ratio between S-wave spin singlet and S-wave spin triplet annihilations need not to be \( \frac{1}{2} \). If that ratio would be \( \frac{1}{2} \) at that given target density it will very likely not be \( \frac{1}{2} \) at a different density. The same holds if one would consider F-wave annihilations only.

3.2. Determination of \( pF_{0-} \) and of \( pF_{1+} \)

Let us select within table 1 two final states that can be produced from pp 0\(^{--}\) and 1\(^{-+}\) initial states, and whose \( f_{1\pm} \) can be measured separately at two target densities exploiting the decay angular distributions. We will have, for example, at the two densities 1 and 2

\[ B_{0^{++}}^{-1} F_{0-} = 1 f_{0^{--}}, \quad B_{0^{++}}^{-2} F_{0-} = 2 f_{0^{--}}, \]

\[ B_{1^{--}}^{-1} F_{1+} = 1 f_{1^{--}}, \quad B_{1^{--}}^{-2} F_{1+} = 2 f_{1^{--}}, \quad (20) \]

from which we can express the four unknown annihilation fractions \( 1 F_{0-}, 1 F_{1+}, 2 F_{0-}, 2 F_{1+} \) in terms of the four measured partial annihilation frequencies and of the two branching ratios \( B_{0^{++}}^{-1} \) and \( B_{1^{--}}^{-1} \).

If we relabel \( B_{0^{++}}^{-1} = 1/x \) and \( B_{1^{--}}^{-1} = 1/y \), the annihilation fractions can be expressed, using eq. (20) in the following way:

\[ 1 F_{0-} = x 1 f_{0^{--}}, \quad 2 F_{0-} = x 2 f_{0^{--}}, \]

\[ 1 F_{1+} = y 1 f_{1^{--}}, \quad 2 F_{1+} = y 2 f_{1^{--}}. \quad (21) \]

We can now write down eq. (18) at the two densities 1 and 2, and we get

\[ x 1 f_{0^{--}} + y 1 f_{1^{--}} = \frac{1}{2}, \]

\[ x 2 f_{0^{--}} + y 2 f_{1^{--}} = \frac{1}{2}. \quad (22) \]

By solving this system of two linear equations with the two unknowns \( x \) and \( y \), one gets the two branching ratios

\[ \frac{1}{x} = B_{0^{++}}^{-1} = 4 \frac{2 f_{0^{--}} - f_{1^{--}}}{2 f_{0^{--}} - 2 f_{1^{--}}}, \]

\[ \frac{1}{y} = B_{1^{--}}^{-1} = 4 \frac{2 f_{0^{--}} - f_{1^{--}}}{2 f_{0^{--}} - 2 f_{1^{--}}}. \quad (23) \]

By using the values of the two branching ratios in expression (23) above, the two spin singlet annihilation fractions can be derived for each of the densities 1 and 2.

A cross-check of these values can be obtained by using other annihilation channels at the same densities 1 and 2. The procedure not only allows the cross-check, but gives \( B_{2^{++}}^{-1} \) for the new channels used.

3.3. Determination of \( pF_{1-}, pF_{0++}, pF_{1++} \) and \( pF_{2++} \)

Let us select within table 1 final states that can be produced from pp 1\(^{-+}\), 0\(^{++}\), 1\(^{++}\) and 2\(^{++}\) initial states, and whose \( f_{1\pm} \) can be measured conveniently at four target densities. We will have, for example, the following set of 16 measured frequencies at the densities 1, 2, 3 and 4:

\[ B_{1\pm}^{-1} F_{1-} = 1 f_{1-}, \quad B_{1\pm}^{-2} F_{1-} = 2 f_{1-}, \]

\[ B_{1\pm}^{-3} F_{1--} = 3 f_{1-}, \quad B_{1\pm}^{-4} F_{1--} = 4 f_{1-}, \]

\[ B_{0^{++}}^{-1} F_{0-} = 1 f_{0^{--}}, \quad B_{0^{++}}^{-2} F_{0-} = 2 f_{0^{--}}, \]

\[ B_{0^{++}}^{-3} F_{0-} = 3 f_{0^{--}}, \quad B_{0^{++}}^{-4} F_{0-} = 4 f_{0^{--}}, \]

For each of the densities 1 and 2.

\[ B_{1^{--}}^{-1} F_{0-} = 1 f_{1^{--}}, \quad B_{1^{--}}^{-2} F_{0-} = 2 f_{1^{--}}, \]

\[ B_{1^{--}}^{-3} F_{1+} = 3 f_{1^{--}}, \quad B_{1^{--}}^{-4} F_{1+} = 4 f_{1^{--}}, \]

\[ B_{1^{--}}^{-5} F_{1+} = 5 f_{1^{--}}, \quad B_{1^{--}}^{-6} F_{1+} = 6 f_{1^{--}}, \]

\[ B_{1^{--}}^{-7} F_{1+} = 7 f_{1^{--}}, \quad B_{1^{--}}^{-8} F_{1+} = 8 f_{1^{--}}, \]

\[ B_{1^{--}}^{-9} F_{1+} = 9 f_{1^{--}}, \quad B_{1^{--}}^{-10} F_{1+} = 10 f_{1^{--}}, \]

\[ B_{1^{--}}^{-11} F_{1+} = 11 f_{1^{--}}, \quad B_{1^{--}}^{-12} F_{1+} = 12 f_{1^{--}}, \]

\[ B_{1^{--}}^{-13} F_{1+} = 13 f_{1^{--}}, \quad B_{1^{--}}^{-14} F_{1+} = 14 f_{1^{--}}, \]

\[ B_{1^{--}}^{-15} F_{1+} = 15 f_{1^{--}}, \quad B_{1^{--}}^{-16} F_{1+} = 16 f_{1^{--}}, \]

\[ (24) \]
B_{2}^{06} + 3F_{2}^{++} = 3F_{2}^{06} , B_{2}^{06} + 4F_{2}^{++} = 4F_{2}^{06} ,

(24 cont’d)

from which we can express the 16 unknown annihilation fractions in terms of the 16 measured partial annihilation frequencies and of the four branching ratios $B_{1}^{06}, B_{2}^{06}, B_{1}^{+} +$ and $B_{2}^{+} +$.

If we relabel $B_{1}^{06} = 1/a, B_{2}^{06} = 1/b, B_{1}^{+} + = 1/c, B_{2}^{+} + = 1/d$ and we write down eq. (19) at the four densities used, we get the following system of four linear equations in the four unknowns $a, b, c$ and $d$:

\[
\begin{align*}
  a f_{1}^{+} + b f_{2}^{0} + c f_{1}^{0} + d f_{2}^{0} &= 3, \\
  a^{2} f_{1}^{+} + b^{2} f_{2}^{0} + c^{2} f_{1}^{0} + d^{2} f_{2}^{0} &= 3, \\
  a^{3} f_{1}^{+} + b^{3} f_{2}^{0} + c^{3} f_{1}^{0} + d^{3} f_{2}^{0} &= 3, \\
  a^{4} f_{1}^{+} + b^{4} f_{2}^{0} + c^{4} f_{1}^{0} + d^{4} f_{2}^{0} &= 3 .
\end{align*}
\]

(25)

By solving this system of four linear equations with the four unknowns $a, b, c$ and $d$ one gets the values of the four branching ratios $B_{1}^{06},$ $B_{2}^{06}, B_{1}^{+} +$ and $B_{2}^{+} +$ in terms of the 16 frequencies measured.

By inserting the values of the branching ratios in eq. (24) above, the four spin triplet annihilation fractions can be derived for each of the densities 1, 2, 3 and 4.

Again, cross-checks of these values can be obtained by using other annihilation channels at the same four densities. The procedure not only allows the cross-check, but gives the BR for the new channels used.

If the cross-checks were not to be satisfactory, then one should question the dynamical assumption that has permitted one to break eq. (17) down into eqs. (18) and (19). Determining branching ratios and annihilation fractions would then require one to express the six annihilation fractions $\rho F_{j}^{ec}$ in terms of six different branching ratios $B_{j}^{06}$, to measure at six different densities identified by a label $d = 1-6$, the six annihilation frequencies $f_{j}^{06}$ associated to the six chosen branching ratios, to write eq. (17) at the six different densities $d$, to get a system of six linear equations in the six unknowns corresponding to the inverse of the six branching ratios used, to solve that system in order to get the six values of the branching ratios selected, and finally to use these values of the branching ratios in order to get the six annihilation fractions for each of the six target densities employed.

4. Comments

4.1. S-wave and P-wave annihilation

Given a target at a density $\rho$, the fractions $\rho F_{S}$ of S-wave and $\rho F_{P}$ of S-wave annihilations and the fractions $\rho F_{S}^{ch}$ and $\rho F_{P}^{ch}$ of S-wave and P-wave annihilations into a given channel $ch$ are given respectively by

\[
\begin{align*}
  \rho F_{S} &= \rho F_{00} + \rho F_{11} , \\
  \rho F_{S}^{ch} &= \rho F_{00}^{ch} + \rho F_{11}^{ch} , \\
  \rho F_{P} &= \rho F_{11} + \rho F_{00} + \rho F_{11} + \rho F_{22} , \\
  \rho F_{P}^{ch} &= \rho F_{00}^{ch} + \rho F_{11}^{ch} + \rho F_{22}^{ch} + \rho F_{11}^{ch} + \rho F_{22}^{ch} + \rho F_{22}^{ch} .
\end{align*}
\]

Comparison of expressions (26) and (27) and of expressions (28) and (29) shows immediately that in general $\rho F_{S}$ is not proportional to $\rho F_{S}^{ch}$ and $\rho F_{P}$ is not proportional to $\rho F_{P}^{ch}$. This could only occur for a channel allowed for all S or P-wave quantum numbers and with the same branching ratio for all quantum numbers.

4.2. Annihilations from the n=2 levels of protonium

The n=2 levels of protonium are populated dominantly by radiative transitions from levels with higher $n$. These radiative transitions can be detected in coincidence with the products of annihilation from the n=2 levels (or of annihilation from n=1 levels populated by a radiative transition from n=2 to n=1).

The fractions of annihilation from the n=2 levels of protonium can be expressed by specialising eq. (5):

\[
\begin{align*}
  \rho F_{2}^{ec} &= \rho P_{2}^{ec} \Gamma_{2}^{ec} / \rho \Gamma \Gamma_{2}^{ec} , \\
  \rho F_{2}^{ec} &= \Gamma_{2}^{ec} f(2) \rho P_{2}^{ec} / \rho \Gamma \Gamma_{2}^{ec} , \\
  \rho F_{2}^{ec} &= \Gamma_{2}^{ec} \sum_{n} f(n) \rho P_{n}^{ec} / \rho \Gamma \Gamma_{2}^{ec} .
\end{align*}
\]

Eqs. (10) and (30) and their comparison permit one to extract a number of conclusions which are of general nature and are not dependent on any assumptions about the evolution of the atomic cascade.

(1) The fractions of annihilation in the P-wave
from the levels \( n=2 \) of protonium depend on the target density.

(2) The ratios between the four fractions of annihilation in the P-wave from the levels \( n=2 \) of protonium depend on the target density.

(3) The ratios \( R_{P}^{j^Pc}/R_{S}^{j^Pc} \) depend on \( J^Pc \).

(4) The ratios \( R_{P}^{j^Pc}/R_{S}^{j^Pc} \) depend on the target density.

(5) Annihilations at NTP in coincidence with X-rays of protonium, which is characterised by the fractions of annihilation \( NT_P^{j^Pc} \), correspond to a specific distribution of \( J^Pc \) P-wave initial states that is in general different from the distribution of P-wave initial states at NTP, which in turn is characterised by the annihilation fractions \( NT_P^{j^Pc} \).

(6) The assumption made in the ASTERIX analysis that annihilation with L X-rays of protonium in coincidence provides a sample of “pure” P-wave annihilation is arbitrary. There is no ideal target where one can have “pure” P-wave or “pure” S-wave annihilation. In the conditions of the ASTERIX experiment requesting the X-rays in coincidence was surely providing a sample of events where annihilations occurred dominantly from the four P-wave sublevels of the \( n=2 \) state of protonium. However the ratios between the fractions of annihilations of \( n=2, 1^+, 0^+, 1^+ \) and \( 2^+ \) levels in a target at NTP are not known and need not be proportional to the annihilation fractions at NTP for \( J^Pc=1^+, 0^+, 1^+ \) and \( 2^+ \).

5. Conclusions

We have shown that the annihilation fractions and the branching ratios for annihilations into exclusive channels can be determined by measurements of frequencies of annihilations at several target densities with a method that is independent of the hypotheses about the effects of strong interactions on the levels of protonium.

Six target pressures are required to fulfil this program without making any assumptions about the dynamics of the atomic cascade of protonium.

Under the hypothesis that the collisional effects that make the cascade density dependent, do not induce transitions between spin triplet and spin singlet states, two target densities are necessary and sufficient to measure the annihilation fractions of the two types of singlet states and branching ratios of annihilations from singlet states. Under the same hypothesis four target densities are necessary and sufficient to measure the four annihilation fractions of the triplet states and branching ratios of annihilations from triplet states. Measurements at four densities of more than six annihilation frequencies will provide checks on the validity of the above-mentioned hypothesis of conservation of the total spin of protonium until the end of the atomic cascade.

We have also shown that assumptions made implicitly in the available derivations of the relative fractions of S- and P-wave annihilation are not justified and may have caused errors which cannot be estimated at present, especially because the total annihilation widths of the four \( n=2 \) P-wave levels are not known.

We have used the term branching ratio to indicate quantities that are not dependent on the target density, and we have used the term fraction for quantities that do depend on the target density.

We have defined branching ratios so that the sum of the branching ratios of all annihilation channels open to a state with quantum numbers \( J^Pc \) adds up to one. Also the sum of all annihilation fractions adds up to one. The frequency of annihilations into a channel determined experimentally using a specific target is called channel branching ratio in some papers. This name is confusing, because the quantity indicated is density dependent. It is important that the target conditions (and the conditions of selection of initial states, if there are any) in association to any measurement of the frequency of an annihilation channel are spelled out explicitly in order to make the experimental data usable to deduce branching ratios and annihilation fractions.

A number of measurements of the frequencies of annihilation for exclusive channels with charged particles in the final state are available from bubble chamber measurements for a liquid \( \text{H}_2 \) target. ASTERIX has measured a set of branching frequencies in \( \text{H}_2 \) gas at NTP for exclusive final states containing charged particles. Currently three experiments are taking data at LEAR \( ^{**} \) on \( pp \) annihilations at rest using \( \text{H}_2 \) targets of three different densities:

\(^{**} \) Experiments at CERN, Geneva (1991)
Crystal Barrel uses a liquid H₂ target and is analyzing several branching frequencies of exclusive channels with neutrals in the final state; CPLEAR uses a gas H₂ target at normal temperature and 15 atm pressure; OBELIX uses a H₂ gas target at NTP, and can use efficiently a target of lower density also. The three experiments are all equipped with detection systems capable of measuring partial frequencies of annihilations of fully reconstructed final states. The detectors have different specialisation, but a number of final states can be measured by each of them at the same target density for a cross-check of systematic effects. The conditions exist therefore in order that the necessary good statistics data be collected so that the annihilation fractions can be determined at the densities for which they are more urgently needed.

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References


