Search for charm

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A systematic discussion of the phenomenology of charmed particles is presented with an eye to experimental searches for these states. We begin with an attempt to clarify the theoretical framework for charm. We then discuss the $SU(4)$ spectroscopy of the lowest lying baryon and meson states, their masses, decay modes, lifetimes, and various production mechanisms. We also present a brief discussion of searches for short-lived tracks. Our discussion is largely based on intuition gained from the familiar—but not necessarily understood—phenomenology of known hadrons, and predictions must be interpreted only as guidelines for experimenters.

CONTENTS

I. Prologue 277
II. Spectroscopy 279
A. Quarks 279
B. Baryons 279
C. Mesons 280
III. Masses of Charmed Hadrons 281
A. The nature of SU(4) symmetry breaking 281
B. Pseudoscalar mesons 282
C. Vector mesons 283
D. Spin 1/2 baryons 283
IV. Decays of Charmed Particles 283
A. Mesons—leptonic decays 283
1. Two-body decays 284
2. Three-body decays 284
3. Multipion decays 285
4. Inclusive semileptonic estimate 285
B. Mesons—nonleptonic decays 285
1. Quark model 285
2. A specific two-body final state 286
3. "Statistical" model 286
4. $D^0-ar{D}^0$ Mixing 286
5. Mass spectra and exotic combinations 287
C. Baryon decays 287
D. Vector meson decays 288
V. Production of Charmed Particles 289
A. Diffractive production of $\phi$ 290
B. Neutrino reactions 290
1. Quasielastic scattering 290
2. Diffractive production of vector mesons 290
3. Deep inelastic production 290
C. $e\mu$ annihilation 293
D. Lepton production at high momentum transfer 294
E. Associated production in strong interactions 296
VI. Detection of tracks of charmed particles 296
VII. Summary and conclusions 296
Acknowledgments 297
Note Added in Proof 298

I. PROLOGUE

Both theoretical developments in the study of spontaneously broken gauge theories and the experimental observation (Hasert et al., 1973; Benvenuti et al., 1974; Aubert et al., 1974; Barish et al., 1974a, b; Lee et al., 1974) of neutral currents point in the direction of a unified, renormalizable theory of weak interactions. However, other ingredients are necessary for the successful realization of such a theory; one possibility involves a fourth "charmed" quark, (Amati et al., 1964a; Bjorken and Glashow, 1964; Maki and Ohnuki, 1964; Hara, 1964; Glashow et al., 1970; Weinberg, 1971; Bouchiat et al., 1972) implying the existence of a new spectrum of hadron states.

Let us review the current status of the theoretical background on charmed particles. In order to present conflicting views (which exist even among ourselves), we shall utilize a fictitious dialogue between two researchers—an enthusiast and a devil's advocate.

A: So if one adopts the view that the Weinberg–Salam model (Weinberg, 1967; Salam, 1968) is essentially correct, a viewpoint consonant with the observations of neutral current effects at various laboratories (Hasert et al., 1973; Benvenuti et al., 1974; Aubert et al., 1974; Barish et al., 1974a, b; Lee et al., 1974), then one seems to be driven to the conclusion that some new degrees of freedom—new fields—must be present in the theory, in order to accommodate the absence of strangeness-changing neutral current. I understand that a four-quark scheme will do. Please explain this to me.

B: Forget about the strong interactions for the moment, and consider weak and electromagnetic interactions as manifestations of a single "weak" force. Then all fields are characterized by weak isospin and weak hypercharge. The world consists of the left-handed isodoublets

$$\begin{pmatrix} \nu_e \\ \mu^- \end{pmatrix}_L, \begin{pmatrix} \nu_e \\ \mu^- \end{pmatrix}_R, \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L,$$

and the right-handed fields are isosinglets. Leptons and hadrons are distinguished by their weak hypercharge. When Higgs couplings are turned off, all these fields are massless and couple to massless vector bosons: a triplet which couples to weak isospin and a singlet which couples to weak hyper-
charge. Don't you agree that this picture is more appealing than the old one with only one quark isodoublet \((u, d')\) and one leftover quark \((s', \bar{s}')\) which doesn't couple at all to charged vector bosons?

A: Perhaps. But you're not talking about the real world yet. Once you put in Higgs couplings and strong interactions, you break all that symmetry anyway. Don't you have to arrange things in an artificial way to get strangeness-changing charged currents but not neutral ones?

B: Not really, if you accept the conservation of electric charge as fundamental. Strangeness-changing couplings arise because the states with well-defined masses are not the eigenstates of the weak interactions. Masses arise from Higgs couplings which can also mix fields with the same electric charge: \(e \leftrightarrow u, s \leftrightarrow d\). There are really two Cabibbo angles, but one of them is not observable, since nothing changes if we mix \(c, u\) and \(s, d\) by the same amount. By convention, we speak of \(s, d\) mixing. Neutral currents are diagonal in the fields \(u, d, c, s\). Since \(d'\) and \(s'\) have the same weak quantum numbers, they always appear in neutral currents in the combination \(\hat{s}'s' + \hat{d}'d'\) which is invariant under the Cabibbo rotation.

A: But why should the Higgs mesons pick on quarks? Why don't they mix \(c\) and \(\bar{u}\)?

B: They might, but you'll never know it, as long as the neutrinos remain massless. By definition, \(\nu\) is the neutrino to the physical electron.

A: What do the strong interactions do? Are you really led to an SU(4) symmetry?

B: Renormalization of the weak interactions requires that the strong interactions be invariant under the weak gauge group \((u \leftrightarrow d', c \leftrightarrow s')\). It turns out that they are also invariant under strong isospin \((u \leftrightarrow d)\). Putting all these symmetries together, you are led to SU(4) invariant couplings. SU(4) is, of course, broken by the masses.

A: Nevertheless, your lepton–hadron symmetry is broken by strong interactions. And what about color?

B: Perhaps color should be regarded as an extra degree of freedom for quarks which allows them to couple to color gauge bosons, giving rise to strong interactions. An over-all symmetry may emerge in a larger scheme; Georgi and Glashow (1974) have recently discovered that the weak gauge group, together with color, can be accommodated in an SU(5) gauge group; Pati and Salam (1974) proposed another scheme in which hadrons and leptons are placed in a common multiplet.

A: I see. But why do you advocate the four-quark scheme? After all, aren't there other schemes which dispense with the fourth quark?

B: Yes, for example, the Berkeley M-meson model (Bars et al., 1972, 1973; deWit, 1973) postulates a large number of scalar mesons which carry both hadronic and leptonic characteristics. To me, the M-meson dynamics needed to accomplish suppression of the strangeness-changing neutral current appears extremely arbitrary and unaesthetic. Besides you recall Weinberg's (Weinberg, 1973, 1974a) remark that the suppression of parity- and strangeness-violations to order \(\alpha\) is "unnatural" in theories such as this. Furthermore, models of this type do not seem to lead naturally to an eventual unification of leptons and hadrons. Other models can be constructed, but something is always artificial in them--

A: Well, we seem to be talking about aesthetics, rather than physics substance. By the way, can't you make the fourth quark—charm, as you call it—very massive, so that its existence doesn't matter at energies we are, and are likely to be, accustomed to?

B: No, I am afraid not. Clues on the mass of charmed quarks come from the study of strangeness-changing second-order weak processes, such as \(K_L \rightarrow \mu\bar{\mu}\), \(K^+ \rightarrow \pi^+\pi^0\) and the \(K_LK_S\) mass difference. As you know, in a gauge theory of weak and electromagnetic interactions, the magnitude of a second-order weak amplitude is in general \(G_{\mu\bar{\mu}}\) so in order to explain, for example, the observed magnitude of the \(K_L \rightarrow \mu\bar{\mu}\) amplitude which is of order \(G_{\mu\bar{\mu}}\), we need a suppression mechanism. It is gratifying that in the Weinberg–Salam model the charmed quark, which was invented to remove first-order strangeness-changing neutral current effects, suppresses higher-order effects as well.

A: Does this mean then that if the charmed quark were degenerate in mass with the usual quarks, there would be no strangeness-changing neutral current effects in any order?

B: Precisely. In any case, in the four-quark version of the Weinberg–Salam model the magnitudes of the processes mentioned earlier are all of order \(G_{\mu\bar{\mu}}(m_c^2 - m_u^2)/m_N^2 \ll G_{\mu\bar{\mu}}^2/m_N^2\) assuming \(m_N^2 > m_c^2 > m_u^2\), where \(m_c\) and \(m_u\) are the masses of the \(c\) and \(u\)-quarks.

A: So you should be able to make an estimate of \(m_c\) from the known rates of the aforementioned processes?

B: Yes. The known \(K_LK_S\) mass difference of 0.54 X \(10^{-10}\) \(\text{sec}^{-1}\) implies \(m_c\) of about a few GeV, except that--


B: Well, in the case of \(K_L \rightarrow \mu\bar{\mu}\), something extraordinarily happens. There are two mechanisms for this decay in second order. One is through \(K_L \rightarrow W^+W^- \rightarrow \mu\bar{\mu}\), the other \(K_L \rightarrow \pi \rightarrow \mu\bar{\mu}\). It turns out that these two diagrams cancel exactly to order \(G_{\mu\bar{\mu}}^2/m_N^2\), so it seems that the amplitude for \(K_L \rightarrow \mu\bar{\mu}\) is of order \(G_{\mu\bar{\mu}}^2\) independently of \(m_c\).

A: Hm! That’s very interesting. But isn’t the cancellation you referred to very sensitive to the way you treat strong interactions?

B: Perhaps. What I have said is based on the calculations of Gaillard and Lee (Gaillard and Lee, 1974a; Vainshtein and Khriplovich, 1973; Ma, 1974; Gavriilides, 1974), who deduced the operator for \(\lambda \rightarrow \mu\bar{\mu}\) in a free quark model, and then estimated the matrix element of the operator between the \(K_L\) and vacuum states using PCAC. Recently Joglekar (1974) constructed a renormalizable phenomenological model of SU(4) pseudoscalar mesons coupled to Weinberg–Salam gauge bosons. He computed the \(K_L \rightarrow \mu\bar{\mu}\) decay in this model, and found again that this amplitude vanishes to order \(G_{\mu\bar{\mu}}^2/m_N^2\). Here \(m_c\) is the mean mass of the charmed pseudo-scalar mesons.

A: That’s very intriguing. If I may backtrack, it seems to me an estimate for \(K_L \rightarrow \pi\pi\) based on a free quark model is less reliable than that for \(K_L \rightarrow \mu\bar{\mu}\). The point is that for

Rev. Mod. Phys., Vol. 47, No. 2, April 1975
of charmed quarks. \( K_0 \rightarrow \bar{K}_0 \), one should also worry about \((W^+W^- + \text{hadrons})\) as well as the \(W^+W^-\) intermediate states. So if you discard the quark model calculation for the \(K_LK_S\) mass difference as being unreliable, then there seems to be no need for a small charmed quark mass. Isn’t that right?

**B:** In a way, yes. But it is hard to imagine that the quark model calculation of the \(K_LK_S\) mass difference is misleading even as to the order of magnitude. Secondly, in the absence of a symmetry argument, the cancellation of the \(K_L \rightarrow \mu \bar{\nu}\) amplitude appears purely fortuitous, so when strong interactions are taken into account, some suppression of this amplitude may in any case be necessary.

**A:** By the way, how about \(K^+ \rightarrow \pi^+\eta\)?

**B:** A good question. For this process, the \(W^+W^-\) and \(Z\) exchange diagrams do not cancel, and the order of magnitude of the amplitude is

\[
\sim \frac{1}{\pi} G_{PQ} \left( \frac{m_e}{38 \text{ GeV}} \right)^2 \ln \left( \frac{m_{\pi^0}}{m_e} \right) \sin^2 \theta_w
\]

in Joglekar’s calculation. Here \(m_e\) is the average mass of the charmed pseudoscalar mesons. Unfortunately the present upper bound on this rate, \(\Gamma(K^+ \rightarrow \pi^+\eta) / \Gamma(K^+ \rightarrow \pi^0\eta) \lesssim 10^{-8}\) (Cable et al., 1974), implies only a suppression of order \(\alpha^2\) with respect to the allowed three-body decay:

\[
\Gamma(K^+ \rightarrow \pi^+\eta) / \Gamma(K^+ \rightarrow \pi^0\eta) \lesssim 10^{-8}.
\]

In other words, \((m_e/38 \text{ GeV})^2 < 1\), which is not too useful.

**A:** I see. In that case, an improvement of the bound by an order of magnitude or two, short of setting a rate for this process, seems highly desirable.

**B:** I agree with you completely there.

**A:** I would think it worthwhile to study the spectroscopy, decay modes, and production mechanisms of the charmed particles, assuming their masses are within reach at Fermilab, Super CERN and ISR, or at the next generation of accelerators like PEP, etc., even though I personally am not convinced of their existence.

**B:** Thanks, that’s precisely what I am working on now.2

In the following, we shall interpret “charmed particles” in the narrow sense—these are quarks associated with the fourth quark introduced by Bjorken, Glashow, Iliopoulos and Maiani (Bjorken and Glashow, 1964; Glashow et al., 1970), and incorporated into the Weinberg–Salam model to banish strangeness-changing neutral currents. In Secs. II and III, we discuss energy levels of low-lying mesonic and baryonic charmed states based on SU(4) considerations. Sections IV and V deal with, respectively, decay modes and production mechanisms of these particles.5 Section VI deals with the possible detection of low-mass charmed particles via their tracks, particularly in emulsions. Section VII contains a summary and conclusions.

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2 Charmed particle searches have previously been discussed by Carlson and Freund, 1972; Snow, 1973; and Glashow, 1974.

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**Table 1.** Charmed 1/2+ baryon states.

<table>
<thead>
<tr>
<th>Label</th>
<th>Quark content</th>
<th>Isospin</th>
<th>Strangeness</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C = 1)</td>
<td>(G_{1+}^{++})</td>
<td>(c)</td>
<td>(0)</td>
</tr>
<tr>
<td>(C^+)</td>
<td>(c(d)_{sym})</td>
<td>(T = 1), (T_s = 0)</td>
<td>(\frac{1}{2}^+)</td>
</tr>
<tr>
<td>(S^+)</td>
<td>(s(c)_{sym})</td>
<td>(T = \frac{1}{2}), (T_s = \frac{1}{2})</td>
<td>(-1)</td>
</tr>
<tr>
<td>(A^+)</td>
<td>(c(s)_{sym})</td>
<td>(T = 1), (T_s = \frac{1}{2})</td>
<td>(-\frac{1}{2})</td>
</tr>
<tr>
<td>(C = 2)</td>
<td>(X_{++}^{++})</td>
<td>(c)</td>
<td>(0)</td>
</tr>
<tr>
<td>(X_{+}^{++})</td>
<td>(c)</td>
<td>(T = 0)</td>
<td>(-\frac{1}{2})</td>
</tr>
</tbody>
</table>

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**II. SPECTROSCOPY**

**A. Quarks**

We shall extend notions of the color triplet quark model of hadrons to the case of four quarks; \(u, d, s\), and the charmed \(c\) [Fig. 1(a)].

**B. Baryons**

According to this model, the ground state baryons (\(J^P = 1/2^+\)) are bound states of three quarks, completely antisymmetric in their color indices, and symmetric under the simultaneous interchange of spin and quark labels of any pair. Those states containing only uncharmed quarks \(u, d, s\), and \(s\) form the familiar octet of \((1/2)^+\) baryons. Baryon states containing only one charmed quark \(c\) may be either symmetric or antisymmetric in the remaining two ordinary quarks. There are altogether six and three such states, respectively. These states carry the charm quantum number \(C = +1\).

Baryon states with two charmed quarks contain one of the ordinary quarks. That is, there is a triplet of \(1/2^+\) baryons with \(C = +2\). These states are listed in Table I.

There are altogether 20 states of \(1/2^+\) baryons. They form an irreducible representation of SU(4).4 They form a truncated tetrahedron in the three dimensional plot of \(I_s, I_s,\) and \(C\) (weight diagram) [see Fig. 1(b)]. The truncated tetrahedron has four hexagonal faces, each representing an octet of baryons which transforms irreducibly under an SU(3) subgroup of SU(4), acting on a set of three (out of four) quarks. Thus, the ordinary baryons, \(p, n, \Lambda, \Sigma^{2/3, \frac{3}{2}}\text{??}\) form an octet under the SU(3) acting on \(u, d, s\) and \(s\); the baryons, \(p, n, C_1^{1+}, C_1^{2+}, C_0, X_{++}^+, X_{+}^+, X_{++}^{--}\) for example, form an octet of SU(3) acting on \(u, d, s\) and \(c\). This observation turns out to be useful in deducing the \((G_{A}/G_{V})\) ratios for weak semileptonic transitions from an ordinary nucleon to a charmed baryon (see Sec. V.B and Fig. 4).

4 Construction of SU(4) representations has been discussed in detail by Amati et al., 1964b, and also by Lipkin, 1965a, b.
An inequivalent 20' of SU(4) may be found by symmetrizing the three-quark system in SU(4) indices. The weight diagram of this representation (to which one may expect the 3/2+ baryons to belong) is a tetrahedron. A three-quark system can also belong to a 4 of SU(4) (whose weight diagram is an inverted tetrahedron), but this multiplet is not expected to occur in the ground state baryons and will not be discussed further.

C. Mesons 15

In this picture mesonic states are formed as bound states of a quark and an antiquark, and we are led to consider 15-plets and singlets of mesons of SU(4). A 15-plet of mesons consists of the usual octet and singlet of SU(3) with $C = 0$, 3 and 3 which carry $C = +1$ and $-1$, respectively. 15-plet singlet [of SU(4)] mixing, as well as octet–singlet mixing of SU(3), depend on the nature of SU(4) breaking. This matter will be discussed at some length in the next section under a set of well-defined dynamical assumptions and what we know about spectroscopy of ordinary 0− and 1− mesons. We list these mesons in Tables II and III; the quark content assignments to neutral mesons, $\eta, \eta', \eta''; \omega, \phi, \phi_0$ are approximate, and motivated in Sec. III. Figure 1(c) shows the weight diagram of the singlet +15-plet of SU(4), containing the pseudoscalar mesons.
TABLE II. Charmed $0^-$ mesons.

<table>
<thead>
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<th>Label</th>
<th>Quark content</th>
<th>Isospin</th>
<th>Strangeness</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C = 1$</td>
<td>$D^+$</td>
<td>$c\bar{d}$</td>
<td>$T = \frac{1}{2}, T_z = \frac{1}{2}$; $S = 0$</td>
</tr>
<tr>
<td></td>
<td>$D^0$</td>
<td>$c\bar{u}$</td>
<td>$T = 0; S = +1$</td>
</tr>
<tr>
<td></td>
<td>$F^+$</td>
<td>$c\bar{c}$</td>
<td>$T = 0; S = 0$</td>
</tr>
<tr>
<td>$C = 0$</td>
<td>$\eta'$</td>
<td>$\sim (6)^{-1/2}(u\bar{u} + d\bar{d})$</td>
<td>$T = 0; S = 0$</td>
</tr>
<tr>
<td></td>
<td>$\eta''$</td>
<td>$\sim (12)^{-1/2}(u\bar{u} + d\bar{d} + s\bar{s} + c\bar{c})$</td>
<td>$T = 0; S = 0$</td>
</tr>
<tr>
<td>$C = -1$</td>
<td>$D^-$</td>
<td>$\bar{c}\bar{u}$</td>
<td>$T = \frac{1}{2}, T_z = \frac{1}{2}; S = 0$</td>
</tr>
<tr>
<td></td>
<td>$D^-$</td>
<td>$\bar{c}\bar{d}$</td>
<td>$T = 0; S = -1$</td>
</tr>
</tbody>
</table>

TABLE III. Charmed $1^-$ mesons.

<table>
<thead>
<tr>
<th>Label</th>
<th>Quark content</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C = 1$</td>
<td>$D^{++}$</td>
</tr>
<tr>
<td></td>
<td>$D^0$</td>
</tr>
<tr>
<td></td>
<td>$F^{++}$</td>
</tr>
<tr>
<td>$C = 0$</td>
<td>$\omega = (2)^{-1/2}(u\bar{u} - d\bar{d})$</td>
</tr>
<tr>
<td></td>
<td>$\phi = s\bar{s}$</td>
</tr>
<tr>
<td></td>
<td>$\phi = c\bar{c}$</td>
</tr>
<tr>
<td>$C = -1$</td>
<td>$D^+$</td>
</tr>
<tr>
<td></td>
<td>$D^-$</td>
</tr>
<tr>
<td></td>
<td>$F^+$</td>
</tr>
<tr>
<td></td>
<td>$F^-$</td>
</tr>
</tbody>
</table>

III. MASSES OF CHARMED HADROMS

A. The nature of SU(4) symmetry breaking

In the spontaneously broken gauge theory considered here, strong interactions must be invariant under the group $U(4) \otimes U(4)$, except for quark mass terms:

$$3c_{masses} = \sum_i m_i |q_i$$

where $m_{0,8,16}$ are linear combinations of the $m_i$, and $u_\alpha = \bar{q}_\alpha q_\alpha$.

The $\lambda_\alpha$ are $4 \times 4$ generalizations of the familiar SU(3) matrices; in particular:

$$\lambda_0 = \frac{1}{2}; \lambda_8 = 3^{-1/2}$$

$$\lambda_{16} = 6^{-1/2}$$

The form of SU(4) breaking is thus severely restricted, and we may exploit this property in order to gain some insight into the expected mass spectrum of charmed states. We wish to emphasize however that:

(a) mass relations derived to lowest order in SU(4) breaking are expected to be much more badly violated than those of SU(3). Nevertheless, they may serve as a useful guide in guessing, for example, whether or not a particular state will decay strongly into a lower mass charmed state;

(b) our predictions are semiempirical in that we base our intuition on the successes of SU(3). An example is the assumption of quadratic mass relations for mesons and linear relations for baryons.

The Hamiltonian density (3.1) may be treated perturbatively on three different levels.

(1) SU(4) symmetry breaking. Simple group theoretical arguments allow us to relate the mass of any charmed hadron to the masses of uncharmed states in the same SU(4) multiplet in terms of a single unknown parameter $(m_{16}/m_8)$ or, equivalently,

$$R = (m_c - m_u)/(m_c - m_u).$$

(2) SU(4) $\otimes$ SU(4) breaking. To lowest order in chiral SU(4) breaking, the matrix elements of $u_\alpha$ [SU(4) singlet] are related to the matrix elements of $u_{16}$ and $u_{15}$ (15-plet). In general, there is an additional contribution to hadron masses arising from the chiral invariant part of the Hamiltonian; for this reason no additional constraints are obtained except for the pseudoscalar mesons. Pseudoscalar masses are assumed to arise only from the mass term (3.1), and their smallness presumably reflects the smallness of quark masses on a hadronic scale. This picture is supported by the success of soft pion theorems; low quark masses are also suggested by an analysis of the decay $K_L \rightarrow \gamma \gamma$ (Gaillard and Lee, 1974a; Vainshtein and Khriplovich, 1973; Ma, 1974; Gavrielides, 1974).

(3) U(4) $\otimes$ U(4) breaking. One may also attempt to treat the full symmetry breaking perturbatively. The mass and mixing angle of the SU(4) singlet pseudoscalar are then related to the masses of the 15-plet pseudoscalars. The solution of the relevant equations leads to the prediction of an $I = Y = 0$ pseudoscalar whose mass satisfies the inequality (Weinberg, 1974b)

$$m \leq \sqrt{3}m_c.$$  

This prediction, which is in flat contradiction with experiment, is independent of the existence of charm. The failure of a perturbative treatment in this case is an outstanding problem of the theory. Here we simply regard it as an empirical result that the breaking of chiral U(4) cannot be treated perturbatively.

To lowest order in chiral SU(4) breaking, pseudoscalar meson masses satisfy the relations (Gell-Mann et al., 1968, Glashow and Weinberg, 1968)

$$m_x^2/(m_d + m_u) = m_p^2/(m_s + m_u) = m_p^2/(m_c + m_u)$$  

$$= m_p^2/(m_c + m_u).$$  

A possible resolution has been suggested by Langacker and Pagels, 1973.
which directly relate the scale of charmed hadron masses to quark masses. In particular, if \( m_u = m_d \), we must have

\[
m_u = m_d \simeq m_u/25 \ll m_e.
\]

The estimate (Gaillard and Lee, 1974a; Vainshtein and Khriplovich, 1973; Ma, 1974; Gavriilides, 1974) \( m_e \lesssim \) 1.5 GeV, together with a lower limit on the u quark mass would provide an upper bound on the charged particle mass scale. For example, if we assume only that the u and d quarks are heavier than the electron we obtain [see Eq. (3.2)]:

\[
R \lesssim 120,
\]

\[
m_D \lesssim 5.5 \text{ GeV}.
\]

One appealing possibility is that since quark and lepton masses arise from the same mechanism—coupling to Higgs scalars—their masses may be related (Dittner et al., 1973; Eliezer, 1974). If

\[
m_d/m_u \simeq m_u/m_e \simeq 1/200,
\]

we obtain

\[
R \simeq 8, \quad m_D \simeq 1.4 \text{ GeV}.
\]

These values are, of course, purely speculative since even if the chiral SU(4) relation, Eq. (3.3), is approximately valid, we have no real information on quark masses. However, a lower bound of about 1.5 GeV for charmed hadrons can probably be inferred from the fact that their tracks have not been observed (see Sec. VI). On the other hand, if the mass scale of charmed hadrons is greater than, say, 10 GeV, it becomes difficult to understand the very strong suppression of induced strangeness changing neutral currents and of \( |\Delta S| = 2 \) transitions. We shall take as a reasonable range:

\[
8 \leq R \leq 100
\]

which corresponds to (approximately!)

\[
1.4 \text{ GeV} \leq m_D \leq 5 \text{ GeV}
\]

for the lowest pseudoscalar state and (see below)

\[
2.4 \text{ GeV} \leq m_{\psi} \lesssim 19 \text{ GeV}
\]

for the lowest baryon state. In the remainder of this section we shall display SU(4) mass formulae for baryons and mesons as a function of the scale parameter \( R \). There is also the possibility, of course, that the calculation of the charmed quark mass (Gaillard and Lee, 1974a; Vainshtein and Khriplovich, 1973; Ma, 1974; Gavriilides, 1974) does not constrain charmed particle masses. Nonetheless, we would regard the absence of any charmed particle below 10 GeV as a serious argument against such a scheme.

### B. Pseudoscalar mesons

To lowest order in the SU(4) symmetry breaking term (3.1), the meson masses may be described by an effective Lagrangian of the form:

\[
\mathcal{L}_{\text{mass}} = \mu_0 (\text{Tr} \pi)^2 + \mu_1 (\text{Tr} \pi^2) + \alpha \text{Tr} (\pi \Delta \pi) + \beta (\text{Tr} \pi^2) (\text{Tr} \Delta \pi),
\]

where \( \pi \) is the \( 4 \times 4 \) matrix representation of the pseudoscalar states (15-plet plus singlet), and \( \Delta \) is a traceless diagonal matrix with two independent elements: \( m_s - m_d, m_s - m_u \). There are five independent parameters in expression (3.4): \( \mu_0, \mu_1 \), the "mean" meson mass; \( \mu_1 \), which separates the SU(4) singlet from the 15-plet; \( \alpha (m_s - m_u) \) and \( \alpha (m_s - m_u) \) which determine the mass shifts of strange and charmed states, respectively; and

\[
\gamma = \beta/\alpha
\]

which determines the singlet–15-plet mixing. In group theoretical language, these five parameters correspond to four independent reduced matrix elements:

\[
\langle 1 \mid 1 \mid 1 \rangle, \quad \langle 15 \mid 1 \mid 15 \rangle, \quad \langle 15 \mid 15 \mid 15 \rangle, \quad \langle 1 \mid 15 \mid 15 \rangle
\]

and the mass difference ratio \( R \) [Eq. (3.2)].

Since four independent masses (\( \pi, \eta, K, X^0 \)) are known from experiment, we may eliminate four parameters and express the remaining masses in terms of the scale parameter \( R \).

For states which do not mix with the SU(4) singlet we obtain:

\[
m_D^2 - m_\psi^2 = m_\pi^2 - m_K^2 = R (m_K^2 - m_\pi^2).
\]

Now consider the mixing of \( I = F = 0 \) states (\( \eta, \eta' \) and \( \eta_0 \) in Table II). There are two values of the mixing parameter \( \gamma \) which are of particular physical interest.

(a) The value \( \gamma = 0 \) separates states according to the masses of their constituents in the limit \( \mu_0 \to 0 \). For finite \( \mu_0 \) but \( R \gg 1 \), this choice effectively separates out the \( (\bar{s}c) \) state which becomes much heavier than the others. However, fixing \( \gamma \) also fixes the SU(3) octet–singlet mixing as a function of pseudoscalar meson masses, and the choice \( \gamma \simeq 0 \) does not allow a fit to the observed masses [\( \eta(549) \) and \( \eta'(958) \)].

(b) The value \( \gamma = -\frac{1}{2} \) separates the SU(4) singlet (\( \eta' \equiv \text{Tr} \pi \)) from the 15-plet. The \( \eta, \eta' \) mixing is then determined to be very small:

\[
\eta \to \eta + \epsilon \eta',
\]

\[
\eta' \to \eta' - \epsilon \eta
\]

\[
\epsilon \simeq 2\sqrt{2}/9R \ll 1
\]

for \( R \gtrsim 8 \). The \( \eta' \) mass is given (to order \( \epsilon^0 \)) by:

\[
m_{\eta'}^2 = m_\pi^2 + \frac{5}{3} (R + \frac{1}{2}) (m_K^2 - m_\pi^2) \simeq \frac{5}{3} m_D^2.
\]

If we now wish to account for the deviation of the \( \eta \) mass
from the Gell-Mann–Okubo relation
\[ m_{q} = \frac{(4m_{K}^{2} - m_{\pi}^{2})}{3} \]
by a small \( \eta, \eta' \) and/or \( \eta, \eta' \), mixing, we may allow
\[ \gamma + \frac{1}{3} \equiv 2\delta \neq 0. \]

Then we obtain to lowest order in the mixing:
\[ \eta \rightarrow \eta + \epsilon_{\eta}[ (m_{K}^{2} - m_{\pi}^{2})/(m_{\pi}^{2} - m_{\pi}^{2}) ] \]
with
\[ \epsilon_{\eta} = (2/3)^{1/3}, \quad \epsilon_{\eta'} = \sqrt{2}/3, \]
\[ \epsilon_{\eta} = -\sqrt{3}(R - \frac{1}{3})/2. \]

The mixing parameter \( \delta \) is determined as a function of the physical masses by:
\[ \delta^{2} = \frac{3}{2} \left( \frac{m_{K}^{2} - m_{\pi}^{2}}{m_{K}^{2} - m_{\pi}^{2}} \right)^{2} \left[ \frac{4m_{K}^{2} - m_{\pi}^{2}}{3} - m_{K}^{2} \right] \]
\[ - \frac{2}{9} \left( \frac{m_{K}^{2} - m_{\pi}^{2}}{m_{K}^{2} - m_{\pi}^{2}} \right)^{2}. \]

Positivity of \( \delta^{2} \) requires
\[ m_{\pi}^{2} \geq 930 \text{ MeV}. \]

A priori one could identify \( \eta', \eta \) with the observed state at 958 MeV, in which case the SU(4) singlet is unmixed \( (\delta \approx 0) \) and need not even exist. This is the solution originally discussed by Bjorken and Glashow (1964); however, the masses in Eq. (3.7) are then determined to be unacceptably low. For \( R \geq 8 \), the \( \eta, \eta' \) mixing becomes negligible, and the \( \eta, \eta' \) mixing reduces to the usual treatment.

C. Vector mesons

The treatment for vector mesons is similar to that for pseudoscalars, with the masses again described by a phenomenological Lagrangian of the form (3.4). However, in this case, the solution chosen by nature is
\[ \gamma = \mu_{0} = 0 \]
so that states separate according to their heavy quark content as indicated in Table III. The SU(4) mass relations are
\[ m_{D^{*}e}^{2} - m_{e}^{2} = m_{p^{*}s}^{2} - m_{s}^{2} = \frac{1}{2}(m_{s}^{2} - m_{s}^{2}) \]
\[ = R(m_{K}^{2} - m_{\pi}^{2}). \]

D. Spin 1/2 baryons

The masses of the ground state baryon multiplet (Table I) depend on three reduced matrix elements: One for the SU(4) singlet operator, and two for the 15 operator, corresponding to symmetric (d) and antisymmetric (f) couplings. The values of these parameters can be inferred from the known baryon masses, and the following relations are obtained
\[ m_{C_{1}} - m_{P} = m_{D_{0}} - m_{Z} = m_{S} - \frac{1}{4}(3m_{A} + m_{Z}) \]
\[ = R(m_{Z} - m_{P}), \]
\[ m_{C_{0}} - m_{P} = m_{A} - \frac{1}{4}(3m_{Z} + m_{A}) = R(m_{A} - m_{P}), \]
\[ m_{X_{s}d} - m_{P} = m_{X_{s}} - m_{Z} = R(m_{Z} - m_{P}). \]

In the above relations we have neglected the mixing of \( S \) and \( A \) which arises from SU(3) breaking. This effect is in fact negligible since SU(3) breaking is very small on the charm mass scale. If \( B_{s} \) and \( B_{d} \) denote states which transform according to irreducible representations of SU(3), the physical states are:
\[ A \simeq B_{s} + \epsilon B_{d}, \]
\[ S \simeq B_{d} - \epsilon B_{s}, \]
\[ \epsilon \simeq (4R - 2)^{-1} \leq 0.03, \]
for \( R \geq 8 \). The effect of the mixing on the masses is second order in \( \epsilon \).

IV. DECAYS OF CHARMED PARTICLES

A. Mesons–leptonic decays

Leptonic decays of charmed mesons can be estimated with some confidence since the structure of the hadronic charged current is given by the Weinberg–Salam theory:
\[ J_{\mu}(1 - \gamma_{5})(d \cos\theta_{c} + s \sin\theta_{c}), \]
\[ = \bar{e} \gamma_{
u}(1 - \gamma_{5})(-d \sin\theta_{c} + s \cos\theta_{c}). \]

In the following discussion we shall neglect form factors, although they may have more rapid variation than in the case of \( K \) decay. This is due to the fact that the vector–pseudoscalar mass splitting is expected to be smaller for charmed particles. Then, for example, in the decay \( D \rightarrow Kl\nu \)
a form factor \( f(q^{2}) = m_{D^{*}}/(m_{D^{*}} - q^{2}) \) with \( 0 \leq q^{2} \leq (m_{D} - m_{K})^{2} \) could lead to an appreciable enhancement with respect to the estimate given below.

An important selection rule emerges from the structure of the charm-changing current in (4.1);
\[ \Delta Q = \Delta C = \Delta S, \quad \Delta I = 0, \quad \text{with} \cos\theta_{c} \text{ in amplitude} \]
or
\[ \Delta Q = \Delta C, \quad \Delta S = 0, \quad \Delta I = \frac{1}{2} \quad \text{with} \sin\theta_{c} \text{ in amplitude}. \]

The dominant decay modes \( \sim \cos\theta_{c} \) are represented schematically in Fig. 2(a).
1. Two-body decays

These are analogous to the $K_B$ decays. We define $f_D$ and $f_F$ by

$$
\langle 0 | J_\mu^e | D^+(q) \rangle = i f_{D\mu} \sin \theta_e,
$$

$$
\langle 0 | J_\mu^e | F^+(q) \rangle = -i f_{F\mu} \cos \theta_e
$$

If the SU(4) symmetry is not spontaneously broken, we expect that

$$
f_D \sim f_F \sim f_K \sim f_e.
$$

Thus we obtain

$$
\Gamma(D^+ \to \mu^+\nu) / \Gamma(K^+ \to \mu^+\nu) \approx (m_D/m_K)
$$

and

$$
\Gamma(F^+ \to \mu^+\nu) / \Gamma(K^+ \to \mu^+\nu) \approx (m_F/m_K) \cot^2 \theta_e.
$$

With $\Gamma(K^+ \to \mu^+\nu) \approx 0.5 \times 10^9$ sec$^{-1}$, we predict

$$
\Gamma(D^+ \to \mu^+\nu) \approx 0.5 \times 10^8$ sec$^{-1}$
$$

$$
\Gamma(F^+ \to \mu^+\nu) \approx 0.9(m_F/m_K) \times 10^9$ sec$^{-1}.
$$

2. Three-body decays

An example is $D^+ \to K^\ast \ell^+\nu$, which is analogous to $K^0 \to \pi^-\ell^+\nu$. To the extent that the lepton mass is negligible in comparison to the $Q$ value, we may neglect the second form factor $f_1$. Assuming the $f_1$ form factors are approximately constant and are equal for the $D_B$ and $K_B$ decays, we have

$$
\Gamma(D^+ \to K^\ast\ell^+\nu) / \Gamma(K^0 \to \pi^-\ell^+\nu)
$$

$$
\approx (m_\ell/m_K)^4(f(x_D)/f(x_K)) \cot^2 \theta_e, \quad (4.6)
$$

where

$$
f(x) = 1 - 8x^2 + 8x^3 - x^4 - 24x^4 \ln x
$$

$$
x_D = (m_K/m_D) \quad \text{and} \quad x_K = (m_\ell/m_K)
$$

For $m_D \approx 1.8$ GeV, we have $x_D \approx x_K$; for $m_D \to \infty$, this ratio $f(x_D)/f(x_K)$ is about 2.

Since $\Gamma(K^0 \to \pi^-\ell^+\nu) = \Gamma(K_L \to \pi\ell\nu) \approx 10^9$ sec$^{-1}$, we expect

$$
\Gamma(D^+ \to K^\ast\ell^+\nu) = \Gamma(D^+ \to K^-\ell^+\nu)
$$

$$
\approx 0.5(m_D/\text{GeV})^4 \times 10^9$ sec$^{-1}. \quad (4.7)
$$
Thus for \( m_D \approx 2 \) GeV and for two kinds of leptons \( \mu, \epsilon \), we expect

\[
\Gamma (D^+ \to K^0 \mu^- \nu) \text{ and } \Gamma (e^+ e^-) \approx 3 \times 10^{11} \text{ sec}^{-1}.
\]

In addition, there are decays into nonstrange final states, such as \( D^+ \to \pi^+ \mu^- \nu \). These decays, however, are suppressed by a factor \( \tan^2 \theta_c \approx 0.05 \) compared to decays into \( K \) mesons.

Three-body semileptonic decays of the \( F^+ \) mesons are more complex. The main decay modes are

\[
F^+ \to \eta + l^+ \nu, \quad \eta' + l^+ \nu, \quad K^0 + l^+ \nu.
\]

Presumably, the first decay mode is the most important one, and we estimate

\[
\Gamma (F^+ \to \eta l^+ \nu) / \Gamma (K_L \to \pi l^+ \nu) \approx \frac{3}{8} \cot^2 \theta_c (m_F / m_K)^4 \quad (4.8)
\]

or

\[
\Gamma (F^+ \to \eta l^+ \nu \text{ and } \eta' l^+ \nu) \approx 7 \times (m_F / \text{GeV})^4 \times 10^9 \text{ sec}^{-1}.
\]

3. Multipion decays

In addition to the decays discussed above, there will be decays in which hadronic final states contain many pions, such as

\[
D^0 \to K + n \pi + l^+ \nu.
\]

If the mass of the charmed meson in question is large enough, one expects the decays into multipion final states to occupy a significant fraction of the total rate. However, there is a reason to think that perhaps multipion decays are less frequent than one would guess at first. This is that in the soft pion limit of any of the final state pions, the amplitude vanishes:

\[
\lim_{k \to \infty} \langle K \cdots \pi^a(k) | \bar{\psi}_\mu (1 - \gamma_5) c | D^0 \rangle = (1 / i f) \langle K \cdots \frac{1}{2} \bar{\psi}_\mu (1 - \gamma_5) c, Q_0^a | D^0 \rangle = 0
\]

where \( Q_0^a \) is the chiral charge with isospin index \( a \).

4. Inclusive semileptonic estimate

We view semileptonic decays of a charmed particle as occurring due to the elementary processes \( c \to s + l^+ \nu, c \to d + l^+ \nu \), followed by de-excitation of the remnant of the hadronic matter which results from replacing the initial charmed quark by either \( s \) or \( d \). If we sum over all final states, the rate of semileptonic decays should be given essentially by the elementary process, provided that the mass of the charmed quark is sufficiently large so that the hadronic state which follows lepton-pair emission has a 100% probability to decay into stable hadrons. If this is true, then the total semileptonic decay rate of a charmed particle is given by the same formula as for \( \mu \) decay:

\[
\Gamma_{\text{total}} (\text{charm} \to l^+ \nu + \text{hadrons}) = G_F m_c^3 192 \pi^3, \quad (4.9)
\]

where \( m_c \) is the mass of the \( c \) quark. Previously, the value \( m_c \approx 1.5 \) GeV was suggested. This implies (for summing over leptons)

\[
\Gamma_{\text{total}} (\text{charm} \to l^+ \nu + \text{hadrons}) \approx 10^{13} \text{ sec}^{-1}. \quad (4.10)
\]

A similar estimate can be made for the total nonleptonic decay rate of a charmed particle (see Sec. IV.B.1, below); we find that in this simple-minded model

\[
\Gamma_{\text{total}} (\text{charm} \to l^+ \nu + \text{hadrons}) \sim 2 \tan^2 \theta_c \sim 8 \%.
\]

B. Mesons--nonleptonic decays

To lowest order, nonleptonic decays of charmed particles are induced by a term in the current \( X \) current interaction

\[
\mathcal{L}_W \approx (G_F / \sqrt{2}) \cos \theta_c [\bar{u} \gamma_\mu (1 - \gamma_5) d \gamma^\mu (1 - \gamma_5) c + \text{h.c.} + \theta (\tan \theta_c)], \quad (4.11)
\]

While we do know the form of the interaction responsible for charmed particle decays, we are not in a position to predict their decays reliably, since doing so would entail complete command of hadron dynamics. So the following discussion should only be considered as an educated guess.

We know that some sort of enhancement is necessary to account for the magnitude of nonleptonic decays of ordinary hadrons, and, in particular, the \( \Delta f = \frac{1}{2} \) (or octet) rule. According to results of Gaillard and Lee (1974b) and of Altarelli and Maiani (1974), such an enhancement can arise in color quark models as a renormalization effect due to color gluon exchange. We may extend this argument to the charm decay interaction (4.11), and find that charm decays should also be enhanced, to the same extent as the enhancement of the \( \Delta f = \frac{1}{2} \) part of strange particle decays. Thus, we find that the effective operators responsible for charm particle decays are bigger by \( \cos \theta_c \) than that for strange particle decays. With this in mind, we shall make several guesses.

1. Quark model

The total width of a charmed meson must be proportional to \( (A G_F \cos \theta_c)^2 \) where \( A \) is the enhancement factor alluded to. From phenomenological analyses of nonleptonic decays of ordinary hadrons, we estimate \( A \cos \theta_c \sin \theta_c \approx 1 \).

Now consider a charmed particle as a collection of quarks confined in a finite space region by any one of the mechanisms recently proposed (Chodos et al., 1974; Gell-Mann and Leutwyler, 1973; Bardeen et al., 1974). The nonleptonic decay of this state will be triggered by the process \( c \to s + u + d \) followed by breakup of the confinement (bag) into many stable hadrons. If the geometrical size of the bag is sufficiently large compared to the wavelengths of the final state quarks, so that the density of available final states is nearly equal to the case of a "free" charmed quark decay, the total rate of nonleptonic decays of a charmed particle

Rev. Mod. Phys., Vol. 47, No. 2, April 1975
is given by the rate of the elementary process $c \to s + u + d$. If the mass of $c$ is much bigger than those of the other quarks, the rate for the latter is given by the same formula as for $\mu$-decay:

$$\Gamma_{\text{total}}(\text{charm} \to \text{hadrons}) \simeq (192\pi^2)^{-1}(G_F A \cos\theta_c)^2 m_c^3$$

$$\simeq 18 (m_c/m_D)^4 \Gamma(\mu \to e\nu\nu),$$

$$\simeq (m_c/m_D)^3 \times 10^9 \text{ sec}^{-1},$$

where $m_c$ is the mass of the charm quark. Assuming $m_c \approx 1.5$ GeV as suggested by Gaillard and Lee (1974a), we guess that

$$\Gamma_{\text{total}}(\text{charm} \to \text{hadrons}) \simeq 10^{11} \text{ sec}^{-1}.$$

Just as $f_l$-values of $\beta$ transitions of nuclei vary widely the total decay rates of charmed particles may vary from the above estimate by as much as a few orders of magnitude either way. (The same remark applies to the estimate of the total leptonic rates of Sec. IV.A.4.)

2. A specific two-body final state

Let us consider, as an illustration, the process $D^0 \to K^- + \pi^+$. We estimate the amplitude of this decay by the following approximation:

$$T(D^0 \to K^- + \pi^+) \sim (G_F/\sqrt{2}) \cot\theta_c \langle \pi^- | \bar{u} \gamma_\mu (1 - \gamma_5) d | 0 \rangle$$

$$\times \langle K^- | \bar{s} \gamma^\mu (1 - \gamma_5) c | D^0 \rangle$$

$$\sim (G_F/\sqrt{2}) \cot\theta_c f_{D^0} m_{D^0}^3.$$

Thus

$$\Gamma(D^0 \to K^- + \pi^+) \sim (1/2m_D^2) |G_F|^2 \cot\theta_c (m_D^2/m_{D^0}^2)^{3/8} \approx 6 \times (m_D/\text{GeV})^3 \times 10^{11} \text{ sec}^{-1},$$

so we expect, for $m_D \approx 2$ GeV

$$\Gamma(D^0 \to K^- + \pi^+) \sim 5 \times 10^{11} \text{ sec}^{-1}.$$

Note that the above formula scales only as $M_D^3$, in contrast to the semileptonic rate for $D^0 \to K^0 \ell^+ \nu$ estimated above which scales as $m_D^5$. For very large $m_D$, as will be shown below, the multipion final states are extremely important in nonleptonic decays, and one thus can expect a comparable $M_D^3$ scaling for nonleptonic decays as well.

An exception to the general enhancement rule may occur when the nonleptonic decay leads to an exotic final state, as in the case of the $\cos^2\theta_c$ decay of the $D^+$

$$D^+ \to (K\pi\pi)^+$$

where $m = 1, 2, \ldots$. We have seen such behavior in the decays $K^0 \to \pi^+\pi^-$, where the $\pi\pi$ system must have $I = 2$. Such decays seem to lack the enhancement factor $A$.

### Table IV. Branching ratios of nonleptonic decays of a charmed $0^-$ meson.

<table>
<thead>
<tr>
<th>Final states</th>
<th>$m_D$</th>
<th>1.4</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^0\pi$</td>
<td></td>
<td>71%</td>
<td>51%</td>
<td>25%</td>
<td>11%</td>
<td>4%</td>
<td>2%</td>
</tr>
<tr>
<td>$K^0\pi$</td>
<td></td>
<td>25%</td>
<td>38%</td>
<td>43%</td>
<td>32%</td>
<td>21%</td>
<td>11%</td>
</tr>
<tr>
<td>$K^0\pi$</td>
<td></td>
<td>5%</td>
<td>9%</td>
<td>24%</td>
<td>34%</td>
<td>33%</td>
<td>26%</td>
</tr>
<tr>
<td>$K^0\pi$</td>
<td></td>
<td>1%</td>
<td>7%</td>
<td>16%</td>
<td>25%</td>
<td>29%</td>
<td>20%</td>
</tr>
<tr>
<td>$K^0\pi$</td>
<td></td>
<td>1%</td>
<td>5%</td>
<td>12%</td>
<td>20%</td>
<td>20%</td>
<td>30%</td>
</tr>
<tr>
<td>$K^0\pi$</td>
<td></td>
<td>1%</td>
<td>3%</td>
<td>20%</td>
<td>30%</td>
<td>30%</td>
<td>40%</td>
</tr>
</tbody>
</table>

Note that $\Delta C = \Delta S$ nonleptonic decays obey the rule $|\Delta I| = 1$. Thus two-body decays of $F^+$ consist of $F^+ \to K^+K^0$ and $\pi^+\pi^-$. We guess that all two-body nonleptonic decays of charmed $0^-$ mesons proceed at the rate of approximately $(M/\text{GeV})^2 \times 10^{11} \text{ sec}^{-1}$.

3. "Statistical" model

To estimate the total hadronic (i.e., nonleptonic) decay rate, we shall take the following simple model. We assume that the amplitudes for $D^0 \to \bar{K} + n\pi$ are independent of external momenta and adopt the following current-algebra inspired guess:

$$\Gamma(D \to \bar{K} + n\pi) = c f^*_{D^n} \int d\mathcal{L}_{\text{PS}}(m_D^2; K, P_1, \ldots, P_n).$$

With the notation

$$\Gamma(D \to \bar{K} + n\pi) = \Gamma_n,$$

we find that

$$\Gamma_{n+3}/\Gamma_n \approx (m_D^2/4\pi f_s)^2 [1/n(n + 1)].$$

In Table IV, we give branching ratios of $\bar{K} + n\pi$ as functions of the mass $m_D$. According to this table, the total nonleptonic decay rate is expected to be

$$\Gamma_{\text{total}}(\text{charm} \to \text{hadrons}) \simeq 10^{11} \text{ sec}^{-1}$$

if the mass of the charmed meson is $\approx 2$ GeV. (Note that as $m_c$ increases, the assumption of a constant matrix element becomes more dubious and is made here only for an order of magnitude estimate.)

4. $D^0 - \bar{D}^0$ mixing

Just as $K^0$ and $\bar{K}^0$ mix to produce $K_1$ and $K_2$ as two states with definite lifetime, so do $D^0$ and $\bar{D}^0$ mix. This comes about from the circumstance that both $D^0$ and $\bar{D}^0$ decay into $S = 0$, $\pm 1$ states. However, the product $T(D^0 \to n) T(\bar{D}^0 \to n)$, where $n$ is a common decay state, is proportional to $\sin\theta_c$ for all $n$. Thus, the real and imaginary off-diagonal elements of the $D^0\bar{D}^0$ mass matrix are expected to
be suppressed by a factor of \( \tan^2 \theta_E \) relative to diagonal ones. While

\[
D_1 = (D^0 - D^+) / \sqrt{2} \\
D_2 = (D^0 + D^+) / \sqrt{2}
\]

are eigenstates of the mass matrix, the lifetimes \( \Gamma_1 \) or \( \Gamma_2 \) would be very close to each other, and the mass difference \( \Delta m = m_1 - m_2 \) would be small compared to \( \Gamma_1 \) or \( \Gamma_2 \). This means that \( D^0 \), when it is produced, would decay mostly into \( D^0 \) channels, before it can turn into \( D^+ \). Under these circumstances, then, the effects of \( D^0 - D^+ \) mixing are not very important in decays of these particles.

5. Mass spectra and exotic combinations

The nonleptonic decays arising from \( e \to u \bar{d} s \) (or \( – \bar{c} u \to \bar{d} s \), \( e \bar{s} \to \bar{u} d \), \( c \bar{d} \to u s \), etc.) proceed with rates proportional to \( \cos \theta_E \). These decays are thus the favored ones. They are illustrated in Fig. 2(b).

Explicitly, one expects the following final states from nonleptonic \( \cos \theta_E \) decays

\[
D^+ \to K^- \pi^+ \bar{K}^0 \pi^0, \ldots \\
F^+ \to K^- \pi^+ \bar{K}^0 \pi^0, \ldots \\
D^+ \to K^- \pi^+ \pi^0, \ldots
\]

Note that the last state is exotic. (This circumstance, as mentioned earlier, may lead to the relative suppression of the nonleptonic decays of \( D^0 \), in analogy with the case of \( K^+ \to \pi^+ \pi^0 \) \( (I_{\pi \pi} = 2) \) versus \( K^0 \to \pi^+ \pi^- \) \( (I_{\pi \pi} = 0) \). But more likely \( D^+ \to K^+ \pi^0 \) is suppressed in the exact SU(3) limit by the transformation properties of the enhanced effective interaction. See Note added in proof.)

Note also that to order \( \cos \theta_E \), the nonleptonic decays of \( q – q^* \) charmed mesons do not lead to states with the quantum numbers of \( K^0 \). The presence of a narrow peak in the \( K^0 \bar{m} \bar{e} \) or \( K^0 \bar{m} \bar{e}^* \) distribution and its absence in \( K^0 \bar{m} \bar{e}^* \) or \( K^0 \bar{m} \bar{e}^* \) distributions would thus be a strong indication in favor of charmed particles.

The transitions \( e \to u \bar{d} \bar{s}, e \to u \bar{s} \bar{d} \), etc., can occur to order \( \cos \theta_E, \sin \theta_E \). The final states to which they lead are shown in Fig. 2c.

C. Baryon decays

Charmed baryons may decay weakly according to the selection rules discussed above (valid to the extent that \( \cos \theta_E \sim 1 \))

\[
\Delta C = \Delta S = \Delta Q, \quad |\Delta I| = 0
\]

for leptonic decays, and

\[
\Delta C = \Delta S = -\Delta I, \quad |\Delta I| = 1
\]

for nonleptonic decays. In this case they would appear as

\[
\begin{align*}
T^0 \to \Omega^- \pi^+, & \quad S^+ \to \Xi^0 \pi^+, & \quad C_{1+/1-} \to \Delta^{1+/2+} K^0, & \quad \Sigma^+ \pi^- & \text{etc.} \\
\text{However if the mass formulae of Sec. III have any very sharp resonances in a variety of channels, some with exotic quantum numbers:}
\end{align*}
\]

![FIG. 3. Schematic representation of strong decays of charmed baryons.](image)

**TABLE V. Strong decays of charmed baryons: \( B^+ \to B \to P^+ \).**

<table>
<thead>
<tr>
<th>Parent</th>
<th>( \Gamma (\text{MeV}) )</th>
<th>Decay mode</th>
<th>Branching ratio</th>
<th>Three-body final states</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_0 )</td>
<td>500</td>
<td>( N D )</td>
<td>100%</td>
<td>75%</td>
</tr>
<tr>
<td>( A )</td>
<td>700</td>
<td>( \Delta D )</td>
<td>4%</td>
<td>5%</td>
</tr>
<tr>
<td>( C_1 )</td>
<td>400</td>
<td>( N D )</td>
<td>70%</td>
<td>40%</td>
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<tr>
<td></td>
<td></td>
<td>( \Sigma F )</td>
<td>10%</td>
<td>40%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( C_{\eta} )</td>
<td>20%</td>
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<tr>
<td>( S )</td>
<td>400</td>
<td>( \Delta D )</td>
<td>50%</td>
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<td></td>
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<td>( \Sigma D )</td>
<td>40%</td>
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<td></td>
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<td>( \Xi K )</td>
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<tr>
<td>( A \pi )</td>
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<td>3%</td>
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<tr>
<td>( A_9 )</td>
<td></td>
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<td></td>
<td>7%</td>
</tr>
<tr>
<td>( T )</td>
<td>700</td>
<td>( \Xi D )</td>
<td>70%</td>
<td>80%</td>
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<tr>
<td></td>
<td></td>
<td>( A \bar{K} )</td>
<td>30%</td>
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<tr>
<td>( X_{n,d} )</td>
<td>15</td>
<td>( C_{aD} )</td>
<td>100%</td>
<td>5%</td>
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<td></td>
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<td>( A F )</td>
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<td>( C_{dD} )</td>
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<td>( S F )</td>
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<tr>
<td>( X_s )</td>
<td>30</td>
<td>( A D )</td>
<td>100%</td>
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<td>( S D )</td>
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<tr>
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<td>( T F )</td>
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</table>

**Rev. Mod. Phys., Vol. 47, No. 2, April 1975**
relevance, it is more likely that charmed baryons will undergo charm conserving strong decays through cascade processes as illustrated in Fig. 3. This expectation is a direct consequence of the assumptions of linear mass formulae for baryons and quadratric formulae for mesons: Charmed baryon masses grow linearly with $R$ and meson masses grow as $R^{2}$. Since our intuition here may be wrong (and also because the mass predictions can be badly violated), one should not exclude the possibility that at least some of the charmed baryons may be stable against strong decay.

Nevertheless we are faced with the prospect of a strong and weak decay chain in which charmed particles will appear as broad resonances in multiparticle channels. If masses are relatively low, the dominant decay modes will probably be of the type

$$B_{c} \rightarrow B + P_{c}.$$  \hspace{1cm} (4.14)

Transition rates can be estimated using SU(4) to relate the couplings to the pion nucleon coupling constant. We assume $f_{d}/d \approx 0.6$ as suggested by PCAC and data on semileptonic baryon decays. The partial width is then given by:

$$\Gamma(B \rightarrow B' + P) = \frac{g_{BPP}^{2}}{16\pi} \left( \frac{(M - m)^{2} - \mu^{2}}{M^{2}} \right)^{3/2} \left( \frac{(M + m)^{2} - \mu^{2}}{M^{2}} \right)^{1/2},$$

where $M$ is the mass of the parent particle, and $m, \mu$ are the masses of the decay baryon and meson, respectively. In Table V we list widths calculated using the masses given by Eq. (3.9) for the case $R = 8$. These values should be regarded as lower limits, both because they increase with the scale of charmed particle masses and also because we have neglected other channels which may be open. We have also listed branching ratios calculated with $R = 8$ and in the limit of very large $R$ where mass differences within an SU(3) multiplet become negligible. However if baryon masses are really very high, the decay widths may become so broad that the resonance structure will completely disappear.

Strong baryon decay followed by the weak decay of a charmed meson will lead to a minimum of three particles in the final state as indicated in Table V. Whatever the primary decay mode, since one weak transition is necessarily involved, the over-all decay chain will satisfy the selection rules (4.12) and (4.13). If the widths of charmed baryons are sufficiently narrow, they will appear as resonances in multiparticle channels ($n \geq 3$) with "exotic" quantum numbers:

$$S = -1, \quad Q = 0, +1, +2 \quad (C_{0}, C_{1}),$$
$$S = -2, \quad Q = 0, +1 \quad (S, A),$$
$$S = -3, \quad Q = 0 \quad (T).$$

for $C = +1$ states. States with $C = +2$ will decay into channels with multiplicity $n \geq 5$ and quantum numbers

$$S = -1, \quad Q = +1, +2 \quad (X_{+}),$$
$$S = -2, \quad Q = +1 \quad (X_{s}).$$

In addition to decays of the type (4.14), there may be decays involving charmed or uncharmed vector mesons as well as spin $3/2$ baryons, for example

$$C^{+} \rightarrow \Delta^{+} + D^{0},$$
$$T^{+} \rightarrow \Omega^{+} + 3^{+}.$$

Of course there is also a predicted spectrum of charmed baryons with spin $3/2$, whose properties we have not discussed. The lowest of such states may be expected to form a $20'$ of SU(4) characterized by a tetrahedral structure.

D. Vector meson decays

In contrast to the case for baryons, the mass formulae of Sec. III indicate that vector mesons may have very narrow decay widths and will perhaps be stable against strong decays. Using the empirical relation

$$m_{K}^{2} - m_{s}^{2} \simeq m_{\phi s}^{2} - m_{s}^{2},\hspace{1cm} (4.15)$$

we obtain from Eqs. (3.1) and (3.8)

$$m_{PS} - m_{P} \lesssim m_{D} \simeq (m_{s}^{2} - m_{s}^{2})/(m_{PS} + m_{D}) \lesssim 200 \text{ MeV}$$
$$m_{PS} - m_{D} \simeq (m_{K}^{2} - m_{s}^{2})/(m_{PS} + m_{D}) \lesssim 250 \text{ MeV}.$$  \hspace{1cm} (4.16)

The decay $F^{*} \rightarrow F + \pi$ is forbidden by isospin, and these results indicate that the decays

$$F^{*} \rightarrow \begin{cases} F + \eta \\ D + K \end{cases}$$

will not be energetically possible. The $F^{*}$ may decay electromagnetically to the $F$, for example

$$F^{*+} \rightarrow F + \gamma,$$

and this mode is expected to be dominant.\footnote{In $F^{*} \rightarrow F_{\gamma}$, only the isoscalar current contributes. Assuming the coupling is comparable to that for $\phi \rightarrow \eta\gamma$, we obtain

$$\Gamma(F^{*} \rightarrow F_{\gamma}) \lesssim (m_{d}/m_{PS})^{3} \left( m_{s}^{2} - m_{s}^{2} \right)^{3} \Gamma(\phi \rightarrow \eta\gamma) \simeq (\text{GeV}/m_{PS})^{3} \times 10^{8} \text{ sec}^{-1},$$

where we have used the mass relations (3.6), (3.8), and (4.15).}

The decay

$$D^{*} \rightarrow D + \pi$$

may be allowed. We can estimate the decay width by comparing the available phase space with that for $K^{*} \rightarrow K_{s}$, since the group structure is identical under the substitution $c \rightarrow s$. Neglecting the squared pion mass we have

$$\Gamma(V(M) \rightarrow P(m) + \pi) \sim \frac{f_{a}}{M^{2}} \sim \frac{(M^{2} - m^{2})^{3}}{M^{4}}.$$  \hspace{1cm} (4.16)

Rev. Mod. Phys., Vol. 47, No. 2, April 1975
Then
\[ \Gamma(D^* \rightarrow D\pi) = (m_{D^*}/m_D) \Gamma(K^* \rightarrow K\pi) \lesssim 3 \text{ MeV}, \]
for \( R \gtrsim 8 \). If \( R \gtrsim 20 \), \( D^* \rightarrow D\pi \) is forbidden.

The state \( \phi_c \) carries no charm and therefore can decay strongly into ordinary pseudoscalars. However, we anticipate that \( \phi_c \) is primarily a \( c \bar{c} \) bound state, and its decay into uncharmed particles may be suppressed in the same way that the decay of \( \phi \sim \bar{s}s \) into nonstrange particles is suppressed. According to Eqs. (3.6) and (3.8), the decays
\[ \phi_c \rightarrow DD, FF \]
will not be energetically possible, and the width of the \( \phi_c \) may be comparable to the \( \phi \) width. Very roughly we expect (\( \Gamma \sim M \))
\[ \Gamma_{\phi_c} \simeq (m_{\phi_c}/m_{\phi}) \Gamma(\phi \rightarrow 3\pi, \eta\gamma) \approx 0.2(m_{\phi_c}/m_{\phi}) \Gamma_{\phi} \simeq 2 \text{ MeV} \]  
(4.16)
for \( R = 8 \) \( (m_{\phi_c} \approx 2) \). Furthermore, the suppression of the "favored" decay mode will enhance the leptonic branching ratio. The vector meson coupling to the photon is proportional to the charge of the bound quarks
\[ g_{\gamma\gamma} \simeq -2g_{\gamma\pi}. \]
Thus we expect:
\[ \Gamma(\phi_c \rightarrow \ell^+\ell^-) \approx 4(m_{\phi_c}/m_{\phi}) \Gamma(\phi \rightarrow \ell^+\ell^-) \approx 4(m_{\phi_c}/m_{\phi}) (5 \times 10^{-4}) \Gamma_{\phi}. \]
If the hadronic decay modes of the \( \phi_c \) are suppressed as much as indicated in Eq. (4.16), we obtain for the lepton branching ratio
\[ \Gamma(\phi_c \rightarrow \ell^+\ell^-)/\Gamma_{\phi_c} \approx 1\%. \]

V. PRODUCTION OF CHARMED PARTICLES

A. Diffractive photoproduction of \( \phi_c \)

We have argued that \( \phi_c \) is mostly \( c \bar{c} \) in the same way that \( \phi \) is mostly \( s \bar{s} \). It has been pointed out by Carlson and Freund (1972) that \( \phi_c \) might be produced diffractively by photons. In this subsection, we give a crude estimate for the diffractive production cross section of \( \phi_c \) based on the vector meson dominance and the simple quark model.

In the simple quark model we have been using, the photon vector meson couplings, \( g_{\gamma} \), defined as
\[ \langle V | J^\mu_{\gamma\gamma} | 0 \rangle = m_V^2/g_{\gamma} \quad (5.1) \]
are in the ratio
\[ (g_\pi^2/4\pi) : (g_\rho^2/4\pi) : (g_{\phi}^2/4\pi) : (g_{\phi_c}^2/4\pi) = \frac{1}{3}: \frac{4}{3}: 1: \frac{1}{3}. \]  
(5.2)

According to the usual lore of vector meson dominance, one has
\[ (d\sigma/dt)(\gamma N \rightarrow \phi_c N) = (e^2/g_{\gamma}^2)(d\sigma/dt)(\phi_c N \rightarrow \phi_c N). \]  
(5.3)

In the forward direction, the elastic differential cross section for \( V N \rightarrow V N \) can probably be estimated fairly accurately from the optical theorem using the quark model estimate of the total cross section for \( V N \). The total cross section for \( \phi_c \sim c \bar{c} \) off a nucleon should be the same as for \( \phi \sim s \bar{s} \) at sufficiently high energies where the mass difference between \( s \) and \( c \) may be neglected. Thus the simple quark model gives, with the additivity assumption,
\[ \sigma_T(\phi_c N) = \sigma_T(\phi N) = \sigma_T(K^+N) + \sigma_T(K^-N) - \sigma_T(\pi^-N). \]  
(5.4)

The last equality was recently studied and parametrized by Lipkin (private communication), who gave
\[ \sigma_T(\phi_c N) = \sigma_T(\phi N) = 13.5 + 1.25 \ln(p/20 \text{ GeV}) \text{mb}, \]  
(5.5)

where \( p \) is the laboratory momentum.

By the optical theorem
\[ d\sigma/dt \big|_{t=0} \approx (1/16\pi) \sigma_T^2. \]  
(5.6)

Equation (5.3) implies
\[ (d\sigma/dt)(\gamma N \rightarrow \phi_c N) \big|_{t=0} \approx \alpha(4\pi/g_\gamma^2)(8/9)(1/16\pi) \times \left[ 13.5 + 1.25 \ln(p/20 \text{ GeV}) \right]^2 \text{mb}(1/0.389 \text{ GeV}^2) \]
where use has been made of Eq. (5.2). Thus, at \( p = 200 \text{ GeV} \), one expects
\[ d\sigma/dt \big|_{t=0}(\gamma N \rightarrow \phi_c N) \approx 40 \text{ \mu b}/\text{GeV}^2 \]

If indeed \( \phi_c \) is produced as copiously as \( \phi \) in a photoproduction experiment, we expect the signal in the muon pair channel to be \( \sim 10^8 \) times stronger for the former than for the latter, because, as discussed in Sec. IV.D., the branching ratio for \( \phi_c \rightarrow \mu \bar{\mu} \) is probably larger by about this factor than that for \( \phi \rightarrow \mu \bar{\mu} \), if \( \phi_c \) is stable against decay into two charmed pseudoscalar mesons. As remarked by Carlson and Freund (1972), existing data on the photoproduction of \( \mu \) pairs suggest already that \( M(\phi_c) > 2 \text{ GeV} \) (see Hayes et al., 1970).

Similar considerations apply to lepton production of charmed vector bosons and \( \phi_c \). Thus, \( \phi_c \) may be produced diffractively in electron-, muon-, and neutrino (neutral current)-scattering experiments, while charged current effects in neutrino experiments may include diffractive production of \( F^{**} \):
\[ \nu(p) + N \rightarrow \mu^+ (\mu^-) + F^{**} (F^{**}) + N. \]  
Rev. Mod. Phys., Vol. 47, No. 2, April 1975
B. Neutrino reactions

1. Quasielastic scattering

Near the threshold for charmed particle production one might expect the dominant process to be the production of a single baryon state. However, this process is suppressed by the \( \Delta S = \Delta C \) selection rule and occurs at a level of \( \sin^2 \theta_w \). The allowed "elastic" reactions are

\[
\nu + p \rightarrow C_1^+ + \mu^-, \\
\nu + n \rightarrow \begin{cases} C_1^+ + \mu^- \\ C_0^- + \mu^- \end{cases}
\]

Neglecting weak magnetism and assuming a common form factor for the vector and axial vector parts, the hadronic matrix element is of the form

\[
\langle C \mid J_v \mid N \rangle = \sin \theta_w F(q^2) \bar{u} \gamma_q (g_V - g_A \gamma_5 \gamma_q) u_N,
\]

and the differential cross section is

\[
\frac{d\sigma}{dy} = \frac{2}{\pi} G_{V,N}^2 E_N \sin \theta_w \left[ F(q^2) \left( \frac{g_V + g_A}{2} \right) (1 - y_{th}) + \left( \frac{g_A - g_V}{2} \right) (1 - y)(1 - y + y_{th}) \right],
\]

where \( y \) is the fraction of the incident neutrino energy transmitted to the baryon

\[
y = E_\nu/E_N
\]

The threshold conditions for production of a charmed state are:

\[
E \geq E_{th} \sim m_c/2m_N, \\
y \geq y_{th} = E_{th}/E_N.
\]

In Fig. 4 we show the differential cross section for production of a spin 1/2 state for pure \( V - A \) \((g_V = g_A)\) and pure \( V + A \) \((g_V = -g_A)\) couplings, assuming \( m_c \sim 3 \text{ GeV} \), \( E_N \sim 25 \text{ GeV} \) and \( \frac{d\sigma}{dy} = (1 - q^2/m_{de}^2) \) with \( m_{de} \sim 2 \text{ GeV} \). In either case the effect is less than a percent of the total measured cross section which we approximate by the scaling form \((a = a_\mu + a_\mu)\):

\[
\frac{d\sigma_{tot}}{dy} \sim (G_F^2/\pi)m_N E_N \times 0.5.
\]

The couplings for the reactions (5.7) may be determined in the SU(4) limit from the known hyperon decay parameters. We then obtain a mixture of \( V - A \) and \( V + A \) contributions to the cross section as indicated in Fig. 4. The production cross section for \( C_0^- \) is the more favorable, as it is nearly pure \( V - A \). If the mass of this state is as low as, say 2.5 GeV, it could be produced at CERN and Brookhaven energies. Although the distribution of Fig. 4 shows a sharp fall off with \( y \) for fixed neutrino energy, if one averages the \( y \) distribution over a range of energies near threshold, the net effect will be an increase with \( y \) since lower energies may contribute for higher \( y \).

2. Diffractive production of vector mesons

A more copious source of charmed particle production may be through the diffractive process discussed in Sec. V.A. The \( F^* \) couples to the weak current with amplitude proportional to \( g_\rho \). If its leptonic branching ratio is appreciable, it will provide a source of dileptons. The electroproduction of vector mesons is not well understood (Gottfried, 1971), but we might guess that

\[
\frac{\sigma(\nu + N \rightarrow \mu + F^*)}{\sigma(\nu + N \rightarrow \mu + \text{anything})} \sim \frac{\sigma(e + p \rightarrow e + \phi^, \omega + p)}{\sigma(e + p \rightarrow e + \phi^, \omega + p)}
\]

The right-hand side (Berkelman, 1971) is about 10% for \(|Q^2| \leq 1.2 \text{ (GeV/c)}^2\). Similar estimates should obtain for \( \phi^* \) production by neutral current couplings. Decays of \( F^* \) are discussed in Sec. IV.D.

3. Deep inelastic production

For neutrino energies above charm threshold, it is probably reasonable to apply the quark parton model which
appears to describe well deep inelastic production of ordinary hadrons. Estimates of charmed particle production can then be deduced from cross sections for elementary \(\nu\)-quark scattering (Altarelli et al., 1974; Gaillard, 1974):

\[
\nu + d \rightarrow e + \mu^- \quad \sigma \sim \sin^2 \theta_w, \\
\nu + s \rightarrow e + \mu^- \quad \sigma \sim \cos^2 \theta_w
\]

(5.11) (5.12)

and the charge conjugate processes. The crossed processes (e.g., \(\nu + c \rightarrow \mu^+ + s\)) could also occur if charmed partons are present in the nucleon. The total cross section for charmed particle production depends on the distribution of partons within the nucleon. Comparison of electroproduction and neutrino data suggests that the nucleon consists primarily of valence \(u\) and \(d\) partons. If \(F_i\) is defined as the integrated distribution function for the \(i\)-type parton in the proton, the data yield the following constraints (Altarelli, et al., 1974; Gaillard, 1974)

\[
F_u + F_d = (0.05 \pm 0.02) (F_u + F_d), \\
F_u + F_s \lesssim (0.25) (F_u + F_d),
\]

(5.13)

and if charmed partons are included:

\[
F_c - F_d \lesssim (0.06) (F_u + F_d).
\]

(5.14)

The distribution functions are positive definite; for the purpose of discussion we shall assume:

\[
F_c \simeq F_d \simeq 0, \\
F_u \simeq F_s \lesssim 0.1 (F_u + F_d).
\]

(5.15)

However, it is unlikely that the strange quark content is as high as this upper limit, since the \(u\) and \(d\) content is much smaller.

The contribution of an elementary \(\nu\)-parton scattering process to the total cross section is proportional to the elementary scattering cross section and to the distribution function of the parton. Using the relations:

\[
\sigma(v + q) = \sigma(v + \bar{q}) = \frac{1}{2} \sigma(v + q) = \frac{1}{2} \sigma(v + \bar{q}),
\]

we may estimate the relative cross section for charm production. Distribution functions for the neutron are related to those for the proton by charge symmetry: \(u \leftrightarrow d\) for \(n \leftrightarrow p\). Neglecting contributions of order \(\sin^2 \theta_w\) we obtain for neutrino scattering from a heavy nucleus \((A \simeq 2Z)\):

\[
\frac{\sigma_c}{\sigma_{\text{total}}} \simeq 2F_c/[F_u + F_d + \frac{1}{2}(F_u + F_d) + 2F_s] \lesssim 16\%,
\]

(5.16)

\[
\frac{\sigma_c}{\sigma_{\text{total}}} \simeq 2F_c/[F_u + F_d + \frac{1}{2}(F_u + F_d) + 2F_s] \lesssim 35\%.
\]

(5.17)

However, these bounds can be reached only asymptotically. Even for relatively high neutrino energies, thresholds may be important in limiting the allowed region of phase space. We define the usual scaling variables (see Fig. 5)

\[
x = -\frac{q^2}{2\nu}, \\
y = \nu/m_N E_
\]

(5.18)

where \(q\) is the momentum transfer, and \(\nu = p \cdot q = m_N (E_\nu - E_\mu)\). In general \(x\) and \(y\) satisfy

\[
0 \leq x, y \leq 1.
\]

For production of a state with mass \(m_\nu\), the allowed phase space (deRujula et al., 1974) is limited to

\[
E_\nu/E_\gamma \leq y \leq 1, \\
0 \leq x \leq 1 - E_\nu/yE_\nu,
\]

(5.19)

where \(E_\nu \simeq m_N^2/2m_N\) is the threshold energy. If the lowest charmed state has a mass of, say, 3 GeV, we might expect the parton model to become relevant at a slightly higher hadronic mass, say, 5 GeV. Then for \(E_\gamma = 25\) GeV, the parton model would be applicable in the region:

\[
0 \leq x \leq 0.5, \quad 0.5 \leq y \leq 1.
\]

Under these assumptions the contribution of the scattering process (5.12) to the differential cross section takes the form \((\sigma = \sigma^u + \sigma^s)\)

\[
d\sigma^{\nu\gamma}/dx dy = (4G_F^2 m_N E_\gamma/\pi) f(x) \theta(W^2 - m_\nu^2),
\]

(5.20)

where \(f(x)\) is the parton distribution function:

\[
F_i = \int_0^1 dx f_i(x),
\]

\(m_\nu \simeq 5\) GeV is the "scaling threshold" mass, and \(W\) is the total hadronic invariant mass

\[
W^2 = 2m_N E_\gamma (1 - x).
\]

(5.21)

In Figs. 6 and 7 we show the contribution of the cross section (5.29) to the \(y\) and \(W^2\) distributions (shaded areas); we have assumed the parametrization?

\[
f_s = f_t = 0.2(1 - x)^7
\]

\(\dagger\) This parametrization has been suggested by Farrar, 1974 and is consistent with the data (N. Stanko, private communication).
falls off at high \( y \) where charmed particle production will first appear. In Fig. 6, the dashed line shows the expected \( y \) distribution for \( \bar{v} \) scattering in the absence of charm (assuming a 5% \( \bar{u} + \bar{d} \) content). The solid line shows the full distribution assuming the above parameters. Figure 7 shows the invariant mass distributions where we have further assumed simple parametrizations for the remaining structure functions

\[
\begin{align*}
    f_N &= f_u + f_d = 2(1 - x)^2, \\
    f_{\bar{N}} &= f_u + f_{\bar{d}} = 0.2(1 - x)^7.
\end{align*}
\]

In fact, the \( (1 - x)^7 \) behavior for \( f_{\bar{N}} \) is valid only near \( x = 1 \); we must have (see, e.g., Llewellyn-Smith, 1973):

\[
    f_N \rightarrow f_{\bar{N}} \quad \text{for} \quad x \rightarrow 0.
\]

Therefore the contribution from \( f_N \) falls off more rapidly with increasing \( W^2 \) than indicated in Fig. 7; this will further enhance the effect of charmed particle production. Of course, we have neglected "prescaling" contributions to charm production; their effect will certainly be to smooth out the threshold effects shown in Figs. 6 and 7.

The above discussion rests entirely on the assumption that there is a significant strange parton content in the nucleon. If this is the case, since both the production and decay satisfy \( \Delta S = \Delta C \), there will be no net change of strangeness. The production of charm will appear as associated production of strange particles, possibly accompanied by a dilepton in the final state.

If there is no appreciable strange parton content, charmed particles will be produced at a level of \( \sin^2 \theta_c \simeq 4\% \) via the \( \nu \)-parton scattering process of Eq. (5.11). Since strangeness is conserved in the production, the net change of strangeness will satisfy

\[
    \Delta S = -\Delta Q
\]

for nonleptonic decays of the charmed particles. Using parton model estimates, we obtain for the relative cross sections

\[
\begin{align*}
    \sigma_c/\sigma_{\text{total}}^+ &\simeq \tan^2 \theta_c \simeq 4\% \\
    \sigma_{\bar{c}}/\sigma_{\text{total}}^\pm &\simeq 0.5\%.
\end{align*}
\]

The extra suppression in the antineutrino case is due to the fact that the scattering must occur from \( \bar{d} \). For ordinary \( \Delta S = \Delta Q \) transitions the situation is reversed

\[
\begin{align*}
    \sigma_c/\sigma_{\text{total}}^+ &\simeq 0.5 - 1\%, \\
    \sigma_{\bar{c}}/\sigma_{\text{total}}^\pm &\simeq 4\%,
\end{align*}
\]

thus at energies which are asymptotic with respect to the charm threshold one expects to see a predominance of \( S = -1 \) states at a level of about 4 percent in both neutrino and antineutrino events. In neutrino events they may be accompanied by a dilepton associated with the leptonic decay of a charmed particle.
C. $e\bar{e}$ annihilation

Once the energies of electron–positron colliding beams are high enough, pair production of charmed particles, and resonant production of $\phi$'s are expected to proceed without inhibition.

The process $e + \bar{e} \rightarrow \phi$ should be very similar to $e + \bar{e} \rightarrow \phi$; in particular, $e + \bar{e} \rightarrow \mu + \bar{\mu}$ presents a clean way of measuring the mass and width of the $\phi$ meson.

The processes $e + \bar{e} \rightarrow D + \bar{D} + \text{pions}$, or $F + \bar{F} + \text{pions}$, or $D + \bar{F} + \text{pions and kaons}$, or its charge conjugate reaction should occur copiously above threshold. An interesting reaction is

\[
e + \bar{e} \rightarrow D^0 + \bar{D}^0 \quad \text{asymptotic}
\]

which will tell us about the mass of the $D$ mesons unambiguously. [As pointed out by H. Lipkin (private communication), the process $e^+ + e^- \rightarrow D^0 + \bar{D}^0$ is forbidden in the exact SU(4) limit.] Another signature of charm pair production is the observation of single $\mu$'s in coincidence with strange particles: these events can arise from one of the pair decaying leptonically and the other nonleptonically, i.e.,

\[
e + \bar{e} \rightarrow D^+ + D^- + \cdots \quad \text{leptonic}
\]

\[
e + \bar{e} \rightarrow K^- + \cdots \quad \text{hadronic}
\]

All these final states should occur in principle also in $p\bar{p}$ annihilation.

Finally, a remark is in order on the ratio $R$;

\[
R = \frac{\sigma(e + \bar{e} \rightarrow \text{hadrons})}{\sigma(e + \bar{e} \rightarrow \mu + \bar{\mu})},
\]

which is found to be in the neighborhood of 5 at the current SPEAR energies <5 GeV. In the three-color quark model which is asymptotically free, the asymptotic behavior of $R$ is given by (Appelquist and Georgi, 1973; Zee, 1973)

\[
R = 2[1 + \{C_3/\ln(s/\mu^2)\}], \quad C_3 = 4/9 \quad (5.22)
\]

for the quarks of charges $2/3$, $-1/3$ and $-1/3$;

\[
R = (10/3)[1 + \{C_4/\ln(s/\mu^2)\}], \quad C_4 = 12/25 \quad (5.23)
\]

for four quarks of charges $2/3$, $2/3$, $-1/3$, and $-1/3$. We note that the approach to the asymptotic value ($R = 2$ or $10/3$) is from above. It is tempting to conjecture (which can be disproved soon) that: (1) we are in the regime where the asymptotic forms above are valid, and (2) the currently large value of $R$ is in fact associated with the onset of charm production, and therefore with the transition from one asymptotic form to the other. In Fig. 8, we plot experimental values for $R$ with the curves of Eqs. (5.22, 5.23), assuming, arbitrarily, $\mu = 2$ GeV.

The following remarks were made to us by H. Lipkin, and we shall include them here with Professor Lipkin's kind permission.

1. The large charge of the charmed quark leads to a large predicted cross section for the production of charmed particle pairs once threshold effects are no longer relevant. Standard quark parton arguments would suggest that at sufficiently high energy 40% of all events should contain charmed particle pairs.

2. The dominant decay mode of charmed particles is nonleptonic with a strange particle in the final state. This implies that at sufficiently high energies roughly half of all events should contain strange particles in the final state. This is to be contrasted with prediction of the quark parton model for the case where there are no charmed particles which gives strange particles present in only one-sixth of the events.

3. The nonleptonic decay of a charmed particle into nonstrange particles is suppressed by a factor $\sin^2 \theta$, where $\theta$ is the Cabibbo angle. However, if one of a pair of charmed particles decays in the nonstrange mode while the other decays into strange particles there will be an apparent violation of strangeness conservation in the final state which will contain only a single strange particle. If the probability of producing a charmed pair is 40% as given by the quark parton model and either one of the pair has a probability $\sin^2 \theta$ of decaying into nonstrange particles the probability of observing strangeness violation in a given event is

\[
P(\Delta S = 1) = 0.8 \sin^2 \theta.
\]

This is by no means a small probability. Thus if the colliding beam experiments show a large number of events

\[
\text{FIG. 8. Data on the ratio } R = \sigma(e\bar{e} \rightarrow \text{hadrons})/\sigma(e\bar{e} \rightarrow \mu\bar{\mu}), \text{ and predictions of the asymptotically free quark model without charm (lower curve) and with charm (upper curve). (a) CEA data: Litke et al., 1973. Tarnopol'sky et al., 1973. (b) SLAC-LBL data: Richter, 1974. (c) This is taken from Adler, 1974.}
\]
containing strange particles approaching the order of 50% this can be taken as an indication that charmed particles are indeed produced. It would then be worthwhile to make a special effort to examine the events in which strange particles are produced very carefully to note whether there are any cases in which one and only one strange particle is produced. This would involve finding events in which all particles are detected and a positive identification can be made of charge particles as either pions or kaons. Since this may be difficult with the kind of detectors used at SPEAR it could be left as a second stage in the charged particle search. The first stage would be to establish the probability of strange particle production to determine whether there is any anomalous production suggesting the existence of charmed particles.

D. Lepton production at high momentum transfer

The leptonic decays of charmed particles can provide a source of direct leptons in hadronic collisions. The expected rates, of course, depend on two unknown factors—charm production rates and the branching ratios for leptonic decays—but one can try to make some educated guesses.

Consider the chain

\[ p + p \rightarrow C + \text{anything} \rightarrow \mu + 1 + 2 + \cdots + n, \]

where C is a charmed particle with large \( p_L \), and 1, 2, \( \cdots \), n stand for decay products of C other than the observed \( \mu \). Then the \( \mu \)-distribution is given by

\[
E_{\mu} \frac{d\sigma}{dp_L^2} (p + p \rightarrow \mu + \cdots) \\
\times \left( \prod_{i=1}^{n} \int \frac{dp_{L_1}}{(2\pi)^{3/2} E_{p_{L_1}}} \right) E(2\pi)^{\delta}(p - p_{\mu} - \Sigma p_i) \\
\times \frac{1}{2M} \left| T(p; p_{\mu}, p_1, \cdots, p_r) \right|^2 \times E \frac{d\sigma}{dp_L} (p + p \rightarrow C + \cdots),
\]

where \( M \) and \( \Gamma \) are the mass and decay width of C, and \( T \) is the decay amplitude for \( C \rightarrow \mu + 1 + 2 + \cdots + n \). If we assume a distribution of the form \( \text{(Busser \textit{et al.}, 1973; Alper \textit{et al.}, 1973; Banner \textit{et al.}, 1973)} \) \( (p_L > 2 \text{ GeV}) \)

\[
E(d\sigma/dp_L) (p + p \rightarrow C + \cdots) \\
\sim p_L^{-3.2} \exp(-26.1p_L/s^{1/3}), \quad (5.24)
\]

then \( E_{\mu} \frac{d\sigma}{dp_L^2} \) will have a similar shape. The reduction factor \( r \), defined as

\[
r = \frac{E_{\mu} (d\sigma/dp_L) (p + p \rightarrow \mu + \cdots)}{E(d\sigma/dp_L) (p + p \rightarrow C + \cdots)} \times B |_{p \to 0}, \quad (5.25)
\]

where \( B \) is the branching ratio into the channel \( C \rightarrow \mu + 1 + 2 + \cdots + n \), is about \( 10^{-3} \) for a two-body decay (such as \( C \rightarrow \mu \mu \)) (J.-M. Gaillard, private communication). For a three-body decay (J.-M. Gaillard, private communication) such as \( D \rightarrow K \mu \nu \), \( r \) depends on the \( K \cdot D \) mass ratio and on \( p_L \) (see Fig. 9). In the case when the mass of the parent particle is comparable to the observed \( p_L \), one expects important contributions from charmed particles produced at rest or with small \( p_L \), where the distribution (5.24) is no longer valid. The leptons from this source \((p_L < m/2) \) will have a different distribution. Without a precise model for the production distribution, it is difficult to estimate the lepton yield, which is expected to depend sensitively on the laboratory angle.

At energies which are sufficiently high that mass differences become unimportant, we expect charmed particle

\footnote{This is a modification by J.-M. Gaillard of the parametrization suggested by Carey \textit{et al.}, 1974.}

\footnote{This is a modification by Appel \textit{et al.}, 1974a of the CCR fit, given in Eq. (5.24). (See Busser \textit{et al.}, 1973; Alper \textit{et al.}, 1973; Banner \textit{et al.}, 1973).}

\[ \text{FIG. 9. Monte Carlo calculation of } r \text{ [see Eq. (5.25)] as a function of } p_L \text{ for the decay } D \rightarrow K \mu \nu. \text{ The curves are calculated for leptons observed at } 90^\circ \text{ in the production center-of-mass with } d\sigma/dp = 24 \text{ GeV and for two assumed production distributions, chosen to reproduce the measured slope at } p_L = 3 \text{ GeV:} \\
E(d\sigma/dp_L) \simeq (1 + p_L^{3.2}) (1 - p/p_{\max})^4, \text{ dashed curves,}^8 \\
E(d\sigma/dp_L) \simeq (p_L^2 + 1)^{-4.4} \exp(-26p_L/s^{1/3}), \text{ solid curves.}^9 \\
\text{The ratio } r \text{, calculated for both parent and decay lepton at } 90^\circ \text{ in the production center of mass, is not invariant under transformation to the target rest frame. However, the observed lepton–pion yield ratio:} \\
(1/\pi^0) = r \times B_H \times (D/\pi^0) \]

\[ \text{is invariant, since both leptons and pions are effectively massless and undergo the same Lorentz transformation. Here } B_H \text{ is the branching ratio for the } K \mu \nu \text{ mode of } D \text{ decay (Courtesy J.-M. Gaillard).} \]

\[ \text{Rev. Mod. Phys., Vol. 47, No. 2, April 1975} \]
to be comparable to strange particle production. At Fermilab
energies and for transverse momentum

\[ 2 \text{ GeV} < p_T < 6 \text{ GeV}, \]

the observed \( K \) to \( \pi \) production ratios are \( \text{(Cronin et al., 1973)} \)

\[ \frac{K^+}{\pi^+} \simeq 0.5, \]
\[ \frac{K^-}{\pi^-} \simeq 0.2, \] (5.26)

although there is some variation with energy and transverse momentum. Since the initial state carries neither charm nor strangeness, we expect the final state to be invariant under the substitution \( s \to c \). Then we guess that in some asymptotic limit

\[ \bar{D}^0(u\bar{c}) \simeq K^+(u\bar{c}), \]
\[ D^0(c\bar{u}) \simeq K^-(c\bar{u}). \] (5.27)

Another reasonable guess is

\[ \frac{F^+(c\bar{s})/K^+(u\bar{c})}{D^+(c\bar{d})/\pi^+(u\bar{d})}, \]
\[ \frac{F^-(s\bar{d})/K^-(s\bar{u})}{D^-(d\bar{s})/\pi^-(d\bar{u})}. \] (5.28)

In the energy and \( p_L \) regions considered here there is a slight preference for positive pions, but to a good approximation \( \text{(Appel et al., 1974a; Cronin et al., 1973)} \)

\[ \pi^+ \simeq \pi^0 \simeq \pi^- . \] (5.29)

By analogy we assume

\[ D^+ \simeq D^0, \]
\[ D^- \simeq \bar{D}^0. \] (5.30)

Then we obtain for the predicted asymptotic ratios at, e.g., \( p_L \simeq 3 \text{ GeV}/c \),

\[ \frac{\bar{D}}{\pi} \equiv \left( \frac{D^- + \bar{D}^0}{\pi^0} \right) \simeq 1, \]
\[ \frac{D}{\pi^0} \equiv \left( \frac{D^+ + D^0}{\pi^0} \right) \simeq 0.4, \]
\[ F^-/\pi^0 \simeq F^+/\pi^0 \simeq 0.1. \] (5.31)

The decay mode which will give the largest yield of high \( p_L \) leptons is the one with the lowest multiplicity. For the \( D \) meson, the two-body leptonic decay is suppressed by a factor of \( \sin^2 \theta \), as well as by helicity conservation (see Sec. IV).

\[ \Gamma(D_u)/\Gamma(D_u) < 2 \times 10^{-3} \]

for \( m_D > 1.4 \). However, the two-body decay of the \( F \) meson can be important if the mass is low:

\[ \Gamma(F_u)/\Gamma(F_u) \simeq \begin{cases} 0.05 & m_F = 1.5 \text{ GeV} \\ 0.015 & m_F = 2 \text{ GeV} \end{cases} \]

Using the mass values \( m_D = 1.4 \), \( m_F = 1.5 \), and the production rates of Eq. (5.31), we obtain for the predicted yields of high \( p_F \) leptons:

\[ \mu^-/\pi^0 \simeq (1.7 \times 10^{-3}) B_1, \]
\[ \mu^+/\pi^0 \simeq (1.3 \times 10^{-3}) B_1, \]
\[ e^-/\pi^0 \simeq (0.6 \times 10^{-3}) B_1, \]
\[ e^+/\pi^0 \simeq (0.3 \times 10^{-3}) B_1, \]

where \( B_1 \) is the total leptonic branching ratio, assumed to be the same for all charmed pseudoscalars. The \( \mu-e \) asymmetry is due to the importance of the \( F \to \mu \) mode. For high masses, two-body decays are negligible. For three-body decays, the suppression factor is reduced at high mass, but we also expect the branching ratio to decrease since channels such as

\[ D \to (3K)\ell\nu, \]
\[ F \to (2q)\ell\nu, \]

will have reasonable phase space. The charge asymmetry may also be less pronounced than indicated by the above predictions. In the case of strange particles the process

\[ p + N \to N + Y + K^+ \]

is energetically more favorable than \( K^- \) pair production

\[ p + N \to N + N + K + \bar{K} \]

and, presumably this accounts for the observed \( K/\bar{K} \) asymmetry. If charmed baryon masses are such that (see Sec. III)

\[ m_c > m_N + m_D \]

\( D \)-pair production may be relatively less suppressed. Furthermore, if charmed baryons are very heavy, their production at rest and subsequent decay could yield high \( p_L \) \( D^0 \) mesons (but these would have a different \( p_L \) distribution). In any case, the yields quoted above should be regarded as upper limits, particularly for high masses.

Another potentially important source of leptons is the \( \phi_c \). As it can be produced singly, it may be more abundant than charmed mesons. For leptons with \( p_L > m_{\phi_c} \), we expect a yield:

\[ e^\pm/\pi^0 = \mu^\pm/\pi^0 = (\phi_c/\pi^0) \times 5 \times 10^{-4} \]

if the total leptonic branching ratio of \( \phi_c \) is \( \sim 10^{-2} \). For massive \( \phi_c \) we expect a peak in the lepton distribution at \( p_L = m_{\phi_c}/2 \).

Recent experiments at Fermilab and at the CERN ISR show unexpectedly large yields of leptons at high momentum transfer.\(^{10}\)

\[ e^\pm/\pi^0 \simeq \mu^\pm/\pi^0 \sim 10^{-4}. \]

\(^{10}\)Boymond et al., 1974; Appel et al., 1974b. A lower yield \((\leq 0.25 \times 10^{-6})\) was reported by the Serpukhov group at the XVII International Conference on High Energy Physics, London 1974.
There are no significant asymmetries and no observed threshold effects or structure such as might be expected in the case of very massive sources. If the observed signal has anything to do with charm, the most likely candidate is $\phi_c$ with $m_{\phi_c} \lesssim 3$ or 4 GeV.

E. Associated production in strong interactions

One should stress again the similarity of charm with strangeness: ordinary hadronic reactions can produce charmed particles in pairs. Examples of such reactions, as mentioned before, are

$$\pi^- p \rightarrow M_c B_c$$

and

$$pp \rightarrow N^* p \rightarrow M_c B_c.$$  

The first reaction involves charm exchange. If charmed particles are fairly massive and their Regge trajectories are of the usual slope $\alpha' \simeq 1$ GeV$^{-2}$, the intercepts of these trajectories may be fairly low. One would thus expect the associated-production reaction to be most useful not too far above threshold. The “diffractive excitation” reaction $pp \rightarrow N^* p \rightarrow \cdots$ suffers from no such problem, and can be useful at any energy.

VI. DETECTION OF TRACKS OF CHARMED PARTICLES

Sufficiently light charmed particles may be detected via their tracks in emulsions, and (under extremely favorable circumstances) perhaps also in bubble chambers.

We have argued that the semileptonic decay of a charmed particle should lead to

$$\Gamma_{SL} \approx (10^{14}/\text{sec}) \times [M(\text{GeV})]^3$$

and the total decay rate could be anywhere from twice to a hundred times this value. Let us assume for the present that

$$\Gamma_{\text{total}} = 10^{13} \times [M(\text{GeV})]^3 \text{sec}^{-1}$$  

realizing that this estimate could err by a factor of 10 in either direction. Then the mean path length transversed by such a particle of laboratory momentum $p$ is

$$I = \gamma \sigma r = 300 \mu \times (p/M),$$

where $p$ and $M$ are expressed in GeV. Lines of equal path length are shown in Fig. 10.

The shortest track that can be detected in a bubble chamber is a few millimeters. Even at the highest Fermilab energies, one is unlikely to identify a charmed particle of mass greater than about 2 GeV via its track in a bubble chamber. On the other hand, emulsions are sensitive to tracks as short as several tens of microns: one-hundredth

\[ \sigma \sim M^{-2} \tag{6.4} \]

seems the most optimistic. For a 4 GeV charmed particle, this could suppress charmed particle production by nearly $10^{-4}$ relative to pion production. Consequently, only emulsion experiments with at least several thousand events (at Fermilab energies) begin to place useful bounds on charmed particle production.

VII. SUMMARY AND CONCLUSIONS

We have suggested some phenomena that might be indicative of charmed particles. These include:

(a) “direct” lepton production,
(b) large numbers of strange particles,
(c) narrow peaks in mass spectra of hadrons,
(d) apparent strangeness violations,
(e) short tracks, indicative of particles with lifetime of order $10^{-14}$ sec,
(f) di-lepton production in neutrino reactions,
(g) narrow peaks in $e^+ e^-\rightarrow \mu^+ \mu^-$ mass spectra,
(h) transient threshold phenomena in deep inelastic leptonproduction,
(i) approach of the $\frac{(e^+ e^- \rightarrow \text{hadrons})/(e^+ e^- \rightarrow \mu^+ \mu^-)}{3\lambda}$ ratio to 3$\frac{1}{2}$, perhaps from above, and
(j) any other phenomena that may indicate a mass scale of 2–10 GeV.
Unfortunately, we have not answered the most important question of all: “What would constitute a definitive experiment that would lead us to give up the idea of charm (or some new hadronic degree of freedom) altogether?” We have tried to indicate some of the reasons why this question still can’t be answered properly.

(1) The calculations of the charmed quark masses do not give charmed particle masses. To obtain the latter directly one must apparently resort to questionable pole models or spectral-function approaches. The scale in all these approaches, however, is roughly the same: of the order of a few GeV. One must probably keep an open mind for charmed particles as massive as 10 GeV. These are still entirely within the range of present day accelerators.

(2) The strong interactions are not well enough understood to estimate the associated production of a pair of very massive particles. One might expect the (charmed particle)/(pion) ratio to increase at high transverse momenta, in analogy with the case for kaons. (The large number of pions at small transverse momenta may well come from decays of resonances, which greatly prefer to decay to pions rather than to kaons.) One should probably be prepared for charmed particle production to be at least as suppressed as baryon-antibaryon pair production, in any given kinematic region.

(3) The estimates of total charmed particle lifetime are hampered by our ignorance of factors which may enhance nonleptonic decays. By analogy with known cases one can expect such factors to range from 1 to 100, with ~20 a reasonable guess based on the hyperons. It is much easier to estimate leptonic decays, since the basic weak interaction of the charmed quark is specified. Even here, however, the qualitative conclusions differ widely depending on whether the charmed particle mass is 2 or 10 GeV. In the former case a few channels are important, while in the latter some kind of “inclusive” estimate is needed.

We have estimated, for the semileptonic decays

$$\Gamma_{SL} \approx 10^{14} \text{sec}^{-1} [M(\text{GeV})]^3$$

and, for the total decay rate,

$$\Gamma_{\text{total}} \approx 10^{20} \text{sec}^{-1} [M(\text{GeV})]^3.$$  

Certain charmed particles may not have enhanced nonleptonic decays if their final states are “exotic.” The semileptonic decays of such states might compete more favorably with the nonleptonic ones, and such particles (the $D^\pm$) are the ones for which tracks in emulsions are most likely to be found.

Many of the tests suggested above would not even be conclusive evidence for charmed particles if their results were positive. No test is conclusive which does not lead to a measurement of the charmed particle mass. For example, since all semileptonic decays of charmed particles to uncharged ones involve $|\Delta Q| = 1$, direct lepton production cannot be invoked by itself as evidence for charm, since (because of the missing neutrino) a mass measurement will not be possible.

The most convincing evidence for charmed particles would come from observation of short tracks. These have been looked for in bubble chambers and the results are negative (Ferbel, private communication) so far. However, further emulsion searches are desirable.

It is quite likely that charmed particles might live too short a time to make any visible track. In that case, the most conclusive evidence for their existence would be the detection of narrow peaks in multi-particle mass spectra. Missing-mass spectra are generally not adequate since the charmed particles are produced in pairs or via neutrinos (which do not lend themselves to missing-mass studies). Consequently, high resolution, high-statistics effective-mass multiparticle spectrometers hold the best promise for detection of charmed particles if they are too short-lived to make visible tracks.

One can only hope that we shall be rescued from the problem of charm either by experimentalists—who find it—or by ingenious theorists—who show us how to do without it while still accounting for the remarkable existence of neutral $\Delta S = 0$ currents.

Finally, we ask, “Could any charmed particles have been seen?” There are a few candidates, and we shall discuss them briefly.

(1) In the experiment of Christenson et al. (1970) at AGS, the reaction

$$\rho + U \rightarrow \mu^+ + \mu^- + \text{anything}$$

was studied. The differential cross section in the effective mass of the muon pair was found to have a “shoulder” in the mass region near 3.5 GeV/$c^2$. The authors commented that the observed spectrum could be reproduced as a composite of a narrow vector boson resonance and a steep continuum when the single-particle mass resolution and efficiency were properly introduced into their analysis. Could this phenomenon be due to the production of $\phi$, which decays copiously into muon pairs? If it is, the production cross section of $\phi$ is about $10^{-22}$ cm$^2$ at $\rho \approx 30$ GeV/$c$, assuming the branching ratio of a few percent into the muon pair channel.

(2) Niu et al. (1971) reported on a cosmic ray event in which a heavy particle decayed into a charged particle and a neutral, which subsequently decayed into $2\gamma$. They associate the $2\gamma$ with $\pi^0$ decay. The $2\gamma$ carried the energy of $3.2 \pm 0.4$ TeV, and the charged particle 0.59 TeV. The transverse momentum carried by the decay particles with respect to the flight direction of the parent was $(627 \pm 90)$ MeV/$c$. It is tempting to speculate that this event was a two-body decay of a charmed particle, for example, $D^+ \rightarrow \pi^+ + \pi^0$ or $F^+ \rightarrow \pi^+ \gamma \rightarrow \pi^+ \gamma$. If this is correct, then the mass of the parent is about 2 GeV/$c^2$, and its lifetime about $10^{-14}$ sec.

(3) Dimuon events reported by the Harvard–Pennsylvania–Wisconsin collaboration (Rubbia, 1974) at Fermilab could be due to the $\rho$ production of charmed particles which decay leptonically.

ACKNOWLEDGMENTS

We have enjoyed many beneficial discussions with our colleagues. Particularly we wish to thank S. L. Adler, R. Dashen, T. Ferbel, J.-M. Gaillard, L. Lederman, H. Lipkin,

NOTE ADDED IN PROOF*

1. Discovery of narrow resonances at 3.1 and 3.7 GeV

Since this article was written there have been several dramatic observations of narrow resonances in $e^+e^-$ and other channels.

* This survey was completed January 5, 1975.
In the reaction $p + Be \rightarrow e^+ + e^- +$ anything, Aubert et al., (1974a) have observed a sharp enhancement at $M(e^+e^-) = 3.1$ GeV. The experiment was performed at the Brookhaven 30 GeV alternating-gradient synchrotron.

In independent experiments at the same time, a SLAC-LBL group has observed sharp resonant peaks around 3.1 GeV in the colliding beam processes: $e^+ + e^- \rightarrow \mu^+ + \mu^-$ and $e^+ + e^- \rightarrow$ hadrons, at SPEAR (Augustin et al., 1974). The mass and width as measured at SPEAR are$^{11}$

$$m = 3.105 \pm 0.003 \text{ GeV},$$  
$$\Gamma \leq 1.9 \text{ MeV.}$$  

(A1)  

(A2)

Later observations at SPEAR of the interference between the one-photon and resonant contributions to $e^+ + e^- \rightarrow \mu^+ + \mu^-$ suggest that the 3100 MeV resonance has $J^{PC} = 1^{--}$ (B. Richter, private communication).

The observed width (A2) is a convolution of the actual width with the beam energy resolution and the radiative correction due to soft photon emission (Bonneau and Martin, 1971; Jackson, 1974a; Yennie, 1974). By integrating over the resonant part of $\sigma(e^+ + e^- \rightarrow$ hadrons), one obtains (for example, Jackson, 1974a)

$$\frac{\Gamma(3.1 \text{ GeV} \rightarrow e^+ + e^-) \Gamma(3.1 \text{ GeV} \rightarrow \text{hadrons})}{\Gamma_{\text{total}}(3.1 \text{ GeV})} \simeq 5 \text{ keV.}$$

(A3)

The ratio of the integrated resonant cross sections for $e^+ + e^- \rightarrow \mu^+ + \mu^-$ and $\rightarrow$ hadrons give

$$\frac{\Gamma(3.1 \text{ GeV} \rightarrow \mu^+ + \mu^-)}{\Gamma(3.1 \text{ GeV} \rightarrow \text{hadrons})} \simeq 7\%.$$  

(A4)

Thus, if we assume that there are no purely neutral decays, and $\Gamma(e^+e^-) = \Gamma(\mu^+\mu^-)$ for which there seems to be some support, i.e., $\Gamma_{\text{total}} = \Gamma(\text{hadrons}) + 2\Gamma(\mu^+\mu^-)$, then we have

$$\Gamma(3.1 \text{ GeV} \rightarrow e^+ + e^-) \simeq 6 \text{ keV,}$$

$$\Gamma_{\text{total}}(3.1 \text{ GeV}) \simeq 92 \text{ keV.}$$

(A5)

The resonance at 3.1 GeV has also been observed in colliding experiments at ADONE (Frascati) by Bacci et al. (1974) and at DORIS (Hamburg) by Braunschweig et al. (1974).

Within 10 days of its first discovery, the SLAC-LBL group at SPEAR found another resonance in the process

$$e^+ + e^- \rightarrow \text{hadrons} \quad (\text{Abrams et al., 1974}).$$

The second resonance is quoted to have

$$m = 3.695 \pm 0.004 \text{ GeV,}$$

$$\Gamma \leq 2.7 \text{ MeV.}$$

(A6)  

(A7)

By the integration method used to deduce (A3) and (A4), one obtains

$$\frac{\Gamma(3.7 \text{ GeV} \rightarrow \mu^+ + \mu^-)}{\Gamma(3.7 \text{ GeV} \rightarrow \text{hadrons})} < 2\%,$$

$$\Gamma(3.7 \text{ GeV} \rightarrow \mu^+ + \mu^-) \simeq 2.4 \text{ keV,}$$

and

$$\Gamma_{\text{total}}(3.7 \text{ GeV}) > 125 \text{ keV.}$$

This resonance was not seen in the reaction $p + Be \rightarrow \mu^+ + \mu^- +$ anything at Brookhaven (Aubert, 1974b).

The decays that have been identified so far include:

$$3100 \rightarrow e^+e^-, \mu^+\mu^-,$$

$$\rightarrow 2\pi^+2\pi^- (\rho^0),$$

$$\rightarrow \omega\pi^+\pi^-,$$

$$\rightarrow \rho\rho \bigl\{\text{tentative,}\bigr\}$$

$$3700 \rightarrow 3100 + \pi^+ + \pi^-.$$  

(A9)  

(A10)  

(A11)  

(A12)  

(A14)

(B. Richter, private communication). The decays (A11) and (A14) indicate that the G parity of both resonances is odd (unless the decay (A11) is occurring electromagnetically). A very little 3100 $\rightarrow 2\pi^+2\pi^-$ is seen, further strengthening the odd G-parity assignment. In the process (A14), the $\pi^+\pi^-$ system seems to be in the $I = 0, J = 0$ state; it is very similar to $\rho' (1600) \rightarrow \rho + \pi^+ + \pi^-$, and is suppressed compared to $\rho' \rightarrow \rho + \pi^+ + \pi^-$ to the same extent as the width of the 3.1 GeV resonance is compared to the width of $\rho$. It is likely that the decay (A14) is about 30% of the visible decays of the 3.7 GeV resonance (Jackson, 1974b). In fact, observations are in agreement with the decay scheme $1^- \rightarrow 1^- + 0^+(\pi\pi)$.

2. The new states as (c\bar{c}) bound states

Many interpretations have been advanced for the new resonances. It is not our purpose here to discuss them all. We shall concentrate on the possibility that the new states correspond to:

3100 MeV: $\phi \equiv a_2 S_1 c\bar{c}$ state, partner of $\rho, \omega, \phi$ (see Table III and Secs. 3.3, 4.4),

3700 MeV: $\phi' \equiv$ a radially excited $a_2 S_1 c\bar{c}$ state, possible partner of $\rho'(1600)$.
If this is the case, the new particles should be accompanied by a host of others with nonzero charm (the present ones have charm = 0 since they are made of $c\bar{c}$). (See also Harari, 1974; CERN Meson Workshop, 1974). The charmed particles should have properties even more dramatic than those of the above states. The lowest ones should decay only weakly and have lifetimes of order $10^{-34}$ sec. Their masses should lie between 2 and 2.5 GeV. This is a considerable narrowing of the range of our earlier estimates. It is made possible by the fact that the new particles have set the mass scale in a charm model. If these new particles are not associated with charm, the scale reverts to that we have mentioned previously.

The particle at 3100 MeV has been called $J$ by the MIT-Brookhaven group (Aubert et al., 1974a), and $\psi$ by the SLAC-LBL group (Augustin et al., 1974). We shall call it $\phi_c$ for the remainder of this discussion, while realizing that another name may well have been chosen by the time this article is in print.

The narrow widths of the $\phi_c$ and the $\phi'_c$ can be understood qualitatively if they lie below threshold for production of a pair of charmed hadrons. The hadronic vertices of "normal" strength always appear to involve connected quark graphs. Processes which do not involve such graphs, such as $\phi \to (\text{nonstrange hadrons})$, and presumably $\phi_c \to (\text{charmless hadrons})$, are subject to considerable suppression.\(^{13}\) In the case of $\phi \to \mu^+\mu^-$, this suppression is a factor of at least $10^6$. Nonetheless, one must assume that the suppression for $\phi_c$ is about 20 times stronger than for the $\phi$, as a comparison of (A4) with our prediction (4.16) shows. This is either a serious problem or an important result, depending on one's point of view. In any case, understanding the so-called Zweig's rule is a serious challenge to theorists. For example, the tremendously small width of $\phi_c$ may be evidence that the strong interactions are getting weaker at high energies (see Appelquist and Politzer, 1975; De Rújula and Glashow, 1975). In "asymptotically free" theories the mixing of $\phi$ with nonstrange states and of $\phi_c$ with uncharmed states is governed by the annihilation of the quark-antiquark pair ($s\bar{s}$ or $c\bar{c}$) into three gluons (the minimum number consistent with color and charge-parity conservation). The rate for this process is proportional to the twelfth power of the running coupling constant, which gets weaker for high masses and short distances, if the system is "Coulombic."\(^{14}\) By suitable rescaling of this coupling constant using the renormalization group, one can reduce the prediction (4.16) to agree with (A4). Since there is some question as to how many powers of the running coupling constant should actually be scaled in such an exercise, we shall not quote details, which may be found in the papers just mentioned.

\(^{13}\) This rule has come to be known as "Zweig's rule" on the basis of partly oral tradition. It was first stated explicitly by (Okubo, 1963). It follows naturally in many dual models, for example in any theory which describes the decay of a resonance by the fissioning of a string into two strings.

\(^{14}\) In this picture, the Coulombic interaction is mediated by "weakened" color gluon exchange. In the nonrelativistic approximation, the $c\bar{c}$ annihilation takes place at the origin, so the probability of annihilation is proportional to the square of the wave function at the origin which is proportional to the sixth power of the coupling constant. This depletion of the wavefunction at the origin is a direct consequence of the rather large spatial size of a Coulombic system. For phenomenological purposes, it suffices in most cases to assume that the $c\bar{c}$ wavefunction is small at the origin, for some reason.

The leptonic decay widths of $\phi_c$ were estimated in Sec. 4.4, by assuming that the photon-$\phi_c$ coupling behaved as $e\phi_c^2/\gamma_c$ and applying the quark model to the ratio

$$\gamma_{\phi_c^{-1}}:\gamma_{\phi}^{-1} = 2:1.$$  \hfill (A15)

In this manner, for $m_{\phi_c} = 3.1$ GeV, we estimated

$$\Gamma(\phi_c \to \mu^+\mu^-) = \Gamma(\phi_c \to e^+e^-),$$

$$= \frac{4(m_{\phi_c}/m_\phi)\Gamma(\phi \to e^+e^-)}{12(3.2 \times 10^{-2}) (4.2 \text{ MeV})},$$

$$\simeq 16.4 \text{ keV}. $$  \hfill (A16)

This is to be compared with the experimental value in (A5)

$$\Gamma(\phi_c \to \mu^+\mu^-) = \Gamma(\phi_c \to e^+e^-)$$

$$\simeq 6 \text{ keV}. $$  \hfill (A17)

It has been argued that one should not apply the quark model to the quantities $\gamma^{-1}$ [as in Eq. (A15)] but rather to the combinations $m_\phi/\gamma_c$. These arguments date from 1967 (Das et al., 1967b) and rely on the use of the first spectral function sum rule of Weinberg (1967b) and Das et al. (1967a), for asymptotic $SU(3)$. A trivial extension of this sum rule to asymptotic $U(4)$, continuing to use single-particle saturation, would predict\(^{16}\)

$$m_{\phi_c}/\gamma_{\phi_c},m_\phi/\gamma_\phi = 2:1,$$  \hfill (A18)

and hence [comparing with Eq. (A16)]

$$\Gamma(\phi_c \to \mu^+\mu^-) = \Gamma(\phi_c \to e^+e^-),$$

$$= (m_{\phi_c}/m_\phi)(16.4 \text{ keV}),$$

$$\simeq 1.8 \text{ keV}. $$  \hfill (A19)

The two predictions bracket the experimental result. Symmetry-breaking effects of this sort will not be calculable without precise dynamical models for the $c\bar{c}$ system (see Yennie, 1974).

One can make a similar range of predictions for the leptonic decays of $\phi'_c$ by assuming that it is the charmed analogue of the $\rho'$ (1600) and that both states are $3S_1$ radial excitations. Instead, we shall use the ratio of (A17) to (A8) to extract the coupling ratio: in the "naive" model

$$\gamma_{\phi'_c}^{-1}:\gamma_{\phi_c}^{-1} = \left[\Gamma(\phi_c \to \text{ leptons})/\Gamma(\phi'_c \to \text{ leptons}) \right] ^{16}$$

$$\propto (m_{\phi'_c}/m_{\phi_c}) \simeq 2. $$  \hfill (A20)

This is to be compared to the ratio obtained from $\rho'$ photo-production

$$4 < \gamma_{\phi'_c}^{-1}:\gamma_{\phi_c}^{-1} < 8$$  \hfill (A21)

\(^{16}\) As the present article is meant primarily as a guide to experimentalists, we must reluctantly omit a large number of references to theoretical papers which perform this and similar calculations based on the direct extension of well-known principles.

\(^{16}\) One such model, considered by (Eichten et al., 1975) uses (A17) as an input which determined the square of the $c\bar{c}$ wave function at the origin.

Rev. Mod. Phys., Vol. 47, No. 2, April 1975
As long as the $\phi'$ is below charm–anticharm threshold, it will remain narrow. (This is an important difference between the $\phi'$ and the $\rho'$, which has a number of open channels into which to decay and is consequently very broad.) The existence of a narrow $\phi'$ at 3700 MeV means that the lowest charmed particle must lie above 1850 MeV in mass. In a deep nonrelativistic square-well potential, for example, the second radial excitation would lie $4\hbar^2$ times as high above $\phi'$ as $\phi'$ lies above $\phi$, i.e., at 4700 MeV. In almost any other potential, one can imagine, a second radial excitation would lie no higher above $\phi'$ than $\phi'$ lies above $\phi$. The total cross section has a peak about 10 nb high and 200 MeV wide at 4.1 GeV (Augustin et al., 1975). This might be a second radial excitation of $\phi'$ above threshold. In any case, the charm–anticharm threshold probably lies somewhere between 3.7 and 4.7 GeV. The lowest-mass charmed hadron probably lies between 1850 and 2350 MeV, unless the potential between $c$ and $\bar{c}$ is of a very unusual sort, or unless the higher discrete states are considerably broader than those at 3.1 and 3.7 GeV.

3. Other $c\bar{c}$ States and their Decays

The Coulombic $c\bar{c}$ system has been referred to as “charmonium” by Appelquist and Politzer (1975) in analogy with positronium. In this picture, the $\phi$ and $\phi'$ are states of “orthocharmonium” ($\phi_S$); they are expected to have hyperfine “paracharmonium” ($\phi_S$) partners, which we shall call $\eta_c$ and $\eta_c'$. These states have $J^{PC} = 0^{-+}$.

Quite independently of the Coulombic nature of the force acting between $c$ and $\bar{c}$, it is expected that the $c\bar{c}$ system has a structure of levels below charm–anticharm threshold associated with radial and orbital excitations. All such levels are expected to be narrow. Because of this, they will provide the first test of detailed quark–antiquark potentials (see for example, Appelquist et al., 1975; Eichten et al., 1975; Callan et al., 1975; Schnitzer, 1974).

These states other than the ones with $J^{PC} = 1^{-+}$ cannot be produced directly in $e^+e^-$ interactions via one–photon intermediate state. However, some of them could be more easily produced in hadronic reactions such as that utilized by Aubert et al., (1974). This is because some of them can communicate with the charmed hadron world via two–gluon exchange, whereas ($c\bar{c}$) states of odd charge conjugation parity communicate via three–gluon exchange (see De Rújula and Glashow, 1975). If the gluon–quark coupling constant is weak, as one might infer from the narrowness of $\phi$, this can give a considerable advantage to $\eta_c$ production.\footnote{In positronium, $\Gamma(\Sigma^0)/\Gamma(\Sigma^+)$ = 9$\sqrt{2}/9$ = 1.15, 1115, a substantial enhancement over the expected scale of $\alpha^{-1} = 137$. A related enhancement is expected for charmonium (see Appelquist and Politzer, 1975): $\Gamma(\Sigma_c)/\Gamma(\Sigma^+)$ = 27$\sqrt{2}/8$ = 1.65 for $\alpha_s = 0.3$.} Note that this observation is independent of the Coulombic nature of ($c\bar{c}$) states.

Further indications of the relative strengths of two–gluon and three–gluon processes may be obtained by comparing the mixing of the “ordinary” vector and pseudoscalar mesons. The vector mesons are nearly “ideally” mixed $[\omega \sim (u\bar{u} + d\bar{d})/(2)^{1/2}; \phi \sim s\bar{s}]$ while the pseudoscalars are nearly pure members of $SU(3)$ multiplets $[\eta \sim (u\bar{u} + d\bar{d} - 2s\bar{s})/(2)^{1/2}; \eta' \sim (u\bar{u} + d\bar{d} + s\bar{s})/(2)^{1/2}]$. This can be understood if, in addition to a quark mass term, the mass operator contains an additional term associated with the transition $q\bar{q} \rightarrow (two \ or \ three \ gluons) \rightarrow q'\bar{q}'$ in the s channel. The pseudoscalar mesons are mixed via two–gluon exchange (strongly) and the vector mesons via three–gluon exchange (weakly).

If this picture is extended to the charm model, one again expects the $0^-$ states to be more strongly mixed than the $1^-$ states. However, in the text (Sec. 3.2) we have proposed a solution in which the $\eta'$ (958) is an $SU(4)$ singlet, and thus spends one-fourth of the time as a $c\bar{c}$ pair. This solution now seems unlikely, as it would entail a width for $\phi_c \rightarrow \eta'$ exceeding the total width of the $\phi$. This process is related to $\omega \rightarrow \pi\gamma$ by a Clebsch–Gordon coefficient: neglecting kinematic factors,

$$\Gamma(\phi_c \rightarrow c\bar{c}(0^-) + \gamma)/\Gamma(\omega \rightarrow \pi\gamma) = 16/9,$$

where $\Gamma$ is the width with kinematic factors divided out. If one adopts the prescription of Gilman and Karliner (1974), $\Gamma \sim \Gamma(p,3)$, one finds that the $\eta'$ must be spending less than $10^{-8}$ of the time as $c\bar{c}$. A similar conclusion follows from vector dominance with the coupling prescription (A15). If $\Gamma \sim \Gamma_{\pi\gamma}$, one still finds that the $\eta'$ cannot be more than one or two percent $c\bar{c}$. Similar arguments apply to the $\eta$. Consequently, there must exist another $0^-$ state which is dominantly $c\bar{c}$, though probably considerably less pure than the $\phi_c$. It is this state to which we shall refer as $\eta_c$.

The choice $\eta_c \sim c\bar{c}$ was dismissed in our article as giving a poor fit to pseudoscalar masses. Recently this fit was re-examined by Lee and Quigg (1974). Two solutions were found, based on a value of $R = (m_c - m_u)/(m_c - m_s)$ consistent with the $\phi_c$ mass.

$$\eta \sim 0.8[1 + (u\bar{u} + d\bar{d})/(2)^{1/2}] - 0.6s\bar{s}, \quad m_c = 508 MeV,$$

$$\eta' \sim 0.6[1 + (u\bar{u} + d\bar{d})/(2)^{1/2}] + 0.8s\bar{s}, \quad m_{\eta'} = 969 MeV,$$

$$\eta_c \sim 1.00c\bar{c}, \quad m_{\eta_c} = 3122 MeV,$$

(A23)

with $c\bar{c}$ admixtures in $\eta$ and $\eta'$ of less than a percent, and

$$\eta \sim 0.66[1 + (u\bar{u} + d\bar{d})/2] - 0.75s\bar{s}, \quad m_c = 551 MeV,$$

$$E \sim 0.75[1 + (u\bar{u} + d\bar{d})/2] + 0.60s\bar{s}, \quad m_E = 1398 MeV,$$

$$\eta_c \sim 1.00c\bar{c}, \quad m_{\eta_c} = 3066 MeV,$$

(A24)

with $c\bar{c}$ admixtures for $\eta$ and $\eta'$ less than a percent. The fit (A23) is not particularly close to the $\eta$ mass, but with such large symmetry breaking perhaps one should not expect better. The fit (A24) requires that one identify the $E(1420)$ with the ninth member of the $0^-$ nonet. In either case, however, the $\eta_c$ is relatively pure $c\bar{c}$ and is fairly close to the $\phi_c$.\footnote{In post

Rev. Mod. Phys., Vol. 47, No. 2, April 1975
The Coulombic estimate (Appelquist et al., 1975) for the hyperfine splitting between \( \phi_e \) and \( \eta_e \) gives

\[
m_{\phi_e} - m_{\eta_e} = \left[ \frac{3}{2} \right] \left[ \frac{1}{\epsilon_e} \right] \left[ \frac{\Gamma(\phi_e \rightarrow e^+e^-)/m_{\phi_e}}{1/2m_{\phi_e}} \right] \approx 80 \sim 90 \text{ MeV, (A25)}
\]

with a smaller splitting expected between \( \phi_e' \) and \( \eta_e' \). A similar estimate is obtained more phenomenologically by assuming \( m_{\phi_e'}^2 - m_{\eta_e'}^2 \approx m_2^2 - m_2^2 \). Several older experiments have seen bumps between 3.0 and 3.1 GeV (French, 1968; Alexander et al., 1970; Braun et al., 1971) and the MIT–Brookhaven group may also have observed the effect. In all of these experiments the \( p\bar{p} \) channel plays a crucial role.

The experiments mentioned by French (1968) involve \( p\bar{p} \rightarrow \pi^0 \) at \( 5.7 \text{ GeV/c} \). Among the numerous peaks mentioned, there is one in the \( (4\pi)^0 \) system at \( 3.08 \text{ GeV} \). In a similar experiment at \( 7.0 \text{ GeV/c} \), Alexander et al. (1970) see peaks in the \( (6\pi)^0 \) channel at \( 3.035 \) and \( 3.4 \text{ GeV} \). It is amusing that \( I = 0, G = + \) are precisely the quantum numbers one expects for the \( c\bar{c} \) states that can be produced via two-gluon exchange, and hence the most likely quantum numbers for \( c\bar{c} \) states coupled to charmless hadrons. In order for the above states to have anything to do with charm, they must be much narrower than quoted in the literature.

Braun et al. (1971) see a peak in the \( p\bar{p} \) distribution at \( 3.05 \text{ GeV} \) in the reaction \( \phi \rightarrow p\bar{p} \pi^0 \) at \( 5.5 \text{ GeV/c} \). Its statistical significance is marginal.

The \( p\bar{p} \) channel is actually an ideal one for the study of \( 0-\)\( c\bar{c} \) states, independently of the above experiments. It is the most readily accessible two-body channel; another is \( \gamma \gamma \), which we shall discuss shortly, and still another is \( \Delta \Delta \). Since one expects the \( c\bar{c} \) state to be a unitary singlet, the \( \Delta \Delta \) decay rate should equal the \( p\bar{p} \) rate modulo slowly varying kinematical factors: \( \Gamma_{\Delta \Delta}/\Gamma_{p\bar{p}} = (m_{c\bar{c}}^2 - 4m_p^2)^{1/2}/(m_{c\bar{c}}^2 - 4m_p^2)^{1/2} \approx 0.87 \).

Several of our colleagues (for example, R. Cahn) have suggested that the bump in total \( p\bar{p} \) cross sections around \( E_{CM} = 1.93 \text{ GeV} \) (Carroll et al., 1974) might be related to the \( \eta_e \). If this is the case, it cannot be the hyperfine partner of the 3100 MeV resonance. Aside from the estimate based on (A25), one can place a lower bound on the mass of an \( \eta_e \sim c\bar{c} \) by using (A22). (We have argued that \( \eta \) and \( \eta' \) have very little \( c\bar{c} \), so that there must exist such a state).

The results are shown in Table A.1.

The value 100 keV, underlined in the Table, corresponds to the total width of the \( \phi_e \). The two barrier factors correspond to the prescription of Gilman and Karliner (1974) and to the assumption of vector dominance with couplings obeying (A18), respectively.\(^9\) The width for \( \phi_e \rightarrow \eta \gamma \) predicted by Appelquist et al. (1974) is even smaller than any of the above values: about 0.03 keV.

On the basis of Table A.1, the \( \eta_e \) must lie above 2.7 GeV, or the \( \phi_e \) would be too wide. Let us assume the \( \eta_e \) has a mass of 3.5 GeV and estimate its \( \gamma \gamma \) width. Scaling the \( \pi\pi \) width up as \( m^2 \) [this follows in a treatment of the axial-vector anomaly, or in a vector dominance model with "naive" couplings as in (A15)]; the \( m^2 \) ansatz works well for \( \pi^0, \eta \rightarrow 2 \gamma \) decays] and using \( \Gamma(\eta_e \rightarrow \gamma \gamma) = (32/9) \Gamma(\pi^0 \rightarrow \gamma \gamma) \), one obtains \( \Gamma(\eta_e \rightarrow \gamma \gamma) \sim 300 \text{ keV} \). The experimental photon–photon coupling, as deduced from a comparison of (A16) and (A17), is probably about a factor of 1.7 to 2 smaller than the "naive" value, leading to the modified estimate based on vector dominance of

\[ \Gamma(\eta_e \rightarrow \gamma \gamma) \sim 20 \sim 40 \text{ keV. (A26)} \]

Appelquist et al. (1974) estimate the electromagnetic-to-hadronic branching ratio directly by comparing two-photon emission with two-gluon emission

\[ \Gamma(\eta_e \rightarrow \gamma \gamma)/\Gamma(\eta_e \rightarrow \text{hadrons}) = \Theta(\alpha_e^2/\alpha_c^2) \approx 10^{-3}. \]

(A27)

With their estimate\(^9\)

\[ \Gamma(\eta_e \rightarrow \text{hadrons}) = 6.5 \text{ MeV, (A28)} \]

this implies

\[ \Gamma(\eta_e \rightarrow \gamma \gamma) = \text{ few keV. (A29)} \]

Vector dominance together with the coupling estimate (A18) implies \( \Gamma \sim \Gamma/m \), which would seriously contradict the large \( (\gamma \rightarrow \gamma)/(\pi \rightarrow \gamma \gamma) \) ratio. (See Browman et al., 1974).

The estimate (A26) may be large enough to permit the observation of the \( \eta_e \) using the Primakoff effect. Using the value \( \Gamma(\eta_e \rightarrow \gamma \gamma) = 100 \text{ keV} \), Lee and Quigg (1974) have predicted that on Pb, \( \sigma(\gamma \rightarrow \eta_e) \sim 170 \text{ nbarns} \) at \( E_{\gamma} = 100 \text{ GeV} \). Hence, very roughly,

\[ \sigma_{\gamma \gamma}(\gamma \rightarrow \eta_e)/\text{nbarns} \sim 1.7 \Gamma(\eta_e \rightarrow \gamma \gamma)/\text{keV, (A30)} \]

since the cross section scales linearly with the \( \gamma \gamma \) width. The coherent production cross sections on various targets are shown in Figs. A1 as functions of the incident \( \gamma \) energy. The best channels for observing the \( \eta_e \), as we have mentioned, would be \( p\bar{p} \) and \( \Delta \Delta \). In analogy with the decay of the \( E(1420) \), we might also expect to see \( \eta_e \) in the \( K^+\pi^-K^0 \) channel. The mode \( \pi^+\Delta^2 \) may also be important (French, 1968).

The excited states of the \( c\bar{c} \) system have been discussed by many authors, including Appelquist et al. (1975); Callan et al. (1975); Eichten et al. (1975), and Schnitzer (1974). A rough guess as to their masses and decays is shown in Table A.2. (See Table A.4 for a wider range of possibilities.)

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\(^9\) The second choice of barrier factor also corresponds to the non-relativistic quark model, as in the calculation of Callan et al., (1975).

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**TABLE A.1.** Predicted widths for \( \phi_e \rightarrow \eta \gamma \), keV.

<table>
<thead>
<tr>
<th>( m_{\phi_e} ) (GeV)</th>
<th>( \Gamma \sim \Gamma(p\bar{p})^3 )</th>
<th>( \Gamma \sim \Gamma(p\bar{p}^3/m_p^2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.7</td>
<td>1600</td>
<td>100</td>
</tr>
<tr>
<td>2.95</td>
<td>100</td>
<td>6</td>
</tr>
<tr>
<td>3.05</td>
<td>5</td>
<td>0.3</td>
</tr>
</tbody>
</table>

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**Rev. Mod. Phys., Vol. 47, No. 2, April 1975**
The electromagnetic transitions among the states in Table A.2 should result in numerous monochromatic photons with varied energies in the range of several tens to several hundreds of kiloelectronvolts. The detection of such photons will be crucial both to verify the charm scheme and to determine the laws governing hadron structure. If the charm scheme is correct, the $c\bar{c}$ system is a unique gift of nature. Its study is likely to provide us with the long-sought (probably non-Coulombic) "Bohr theory" of the hadrons.

We have already discussed the decay $\phi \rightarrow \eta \gamma$. It is a quark–spin-flip ($M1$) transition. The decay $\phi'(3695) \rightarrow \eta \gamma$ also is an $M1$ transition. Estimates of its rate depend on the overlap between the $n = 1$ and $b = 2$ wavefunctions, where $n$ is the principal quantum number:

$$\Gamma(\phi' \rightarrow \eta \gamma) \approx 40 \text{ keV} \quad \text{(Appelquist et al., 1975)},$$

$$= 25 \text{ keV} \quad \text{(Callan et al., 1975)},$$

$$= 1 \text{ keV} \quad \text{(Eichten et al., 1975)}.$$  \hspace{1cm} (A31)

We recall that from experiment $\Gamma_{\text{em}}(\phi') \lesssim 2.7 \text{ MeV}$. If the first two estimates in (A31) are closer to the truth, the best place to produce $\eta_0$ will be in colliding $e^+e^-$ beams with $E_{\text{cm}} = 3.7 $ GeV. If the smaller estimate is correct, one should turn to the Primakoff effect or some hadronic process (e.g., $\bar{p}p \rightarrow \eta_0, \pi^-\pi^+ \rightarrow \eta_0 n, \ldots$) to produce $\eta_0$.

Certain hadronic decays of the states in Table A.2 may proceed faster than others. One class of decays which may have already been observed in $(c\bar{c}) \rightarrow (c\bar{c})^* + (2 \text{ gluons})$ color singlet. This could be the mechanism for the reaction (A14), which accounts for a sizeable fraction of all the decays of the $\phi'(3695)$. According to Appelquist et al. (1975), a similar chain might be important for $\eta'_0 \rightarrow \eta_0 + 2\pi$.

Another class of hadronic decays which might not be negligible might be $(c\bar{c})\rightarrow\rightarrow (2 \text{ gluons}) \rightarrow \text{hadrons}$. Appelquist et al. (1974) estimate this to be an important process only for the $L = 0$ states $\eta_0$ and $\eta'_0$. Perhaps the two peaks seen by (Alexander et al., 1970) correspond to these states. However, if one admits appreciable effects from the gradient of the wavefunction at the origin, the $L = 1$ states of positive charge parity can also decay in this manner. The position of the second peak seen by Alexander et al. (1970), at $3.4 \text{ GeV}$, is indeed consistent with that of the $(0^+, 1^+, 2^+)$ group in Table A.2.

Because of the likelihood that at least the $0^+ c\bar{c}$ states (and possibly others) couple to hadrons with widths of the order of MeV, we urge the systematic study of $p\bar{p}$ spectra and $p\bar{p}$ direct channel processes in the interesting range $2.5 \text{ GeV} \leq E_{\text{cm}} \leq 4 \text{ GeV}$. \textit{A priori}, the $p\bar{p}$ system can communicate with any $c\bar{c}$ system, and we have argued that it is most likely to do so for states of positive charge-parity (and hence positive $G$ parity).

The argument that charmless hadrons communicate with the charmonium $(c\bar{c})$ world via $C = +$ states may help to explain why the MIT–Brookhaven group do not see the $\phi'$ in their experiment at anything greater than 1% of the rate of $\phi$ production (Leong, 1974). Suppose that the process they observe goes via the chain

$$p + Be \rightarrow (C = +, c\bar{c} \text{ state}) + X \rightarrow \phi(3105) + (\gamma \text{ or hadrons}) \rightarrow e^+e^-.$$  \hspace{1cm} (A32)

The ratio of $\phi' \rightarrow \phi$ production will then depend on the spectrum of available $C = +$ parent states and their branching ratios into the appropriate vector mesons. If the only available parent state for the $\phi'$ lies above charm–anticharm threshold, for example, the $\phi'$ will not be produced at all. Instead, the parent will decay strongly into a pair of charmed mesons. Apart from this, the small branching ratio of $\phi' \rightarrow e^+\gamma_e^-$ is a major reason for the suppression.

4. Mass estimates of charmed particles

We now turn to a discussion of mass formulas. So far we have been occupied entirely with the $c\bar{c}$ system, and we now turn to the states $(c\bar{s}, c\bar{d}, c\bar{s}, \text{and their charge conjugates})$ and the baryonic states $csu, cuds, \text{and cdd}$. These states are the most likely to be observed in the near future, and the reader may deduce the consequences for others from the main body of our paper.

The mass formulas in our article are equivalent to the following simple quark-model rules for the charged vector mesons $D^{++} = cd, D^{*0} = cs, F^{*+} = c\bar{s}, \text{and their charge conjugates}$

$$m_{D^{*0}} = \frac{1}{2}(m_{s}^2 + m_{d}^2) \Rightarrow m_{D^{*0}} = 2.26 \text{ GeV},$$

$$m_{F^{*+}} = \frac{1}{2}(m_{s}^2 + m_{d}^2) \Rightarrow m_{F^{*+}} = 2.31 \text{ GeV}. $$  \hspace{1cm} (A33)

If one were to use instead a linear interpolation formula the masses of the charmed vector meson would be around 2 GeV.

---

TABLE A.2. Excited states of the $c\bar{c}$ system.

<table>
<thead>
<tr>
<th>$L$</th>
<th>mass $(\text{MeV})$</th>
<th>$J_p$</th>
<th>mass $(\text{MeV})$</th>
<th>$J_p$</th>
<th>mass $(\text{MeV})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{1/2}$</td>
<td>3050</td>
<td>$1^-$</td>
<td>13400</td>
<td>$1^-$</td>
<td>3700</td>
</tr>
<tr>
<td>$S_0$</td>
<td>0$^+$</td>
<td>$1^-$</td>
<td>13400</td>
<td>$1^-$</td>
<td>3700</td>
</tr>
</tbody>
</table>

All of the states in Table A.2 are expected to lie below charm–anticharm threshold. Their electromagnetic decays thus can compete favorably with their hadronic decays: $\Gamma((c\bar{c}) \rightarrow (c\bar{c})' + \gamma)$ should be of order several hundred keV for many of the transitions between the states in Table A.2, while for the negative charge-parity states one expects $\Gamma(c\bar{c} \rightarrow \text{hadrons}) \lesssim \Gamma((c\bar{c}) \rightarrow \text{hadrons})$ for the positive charge-parity states $\Gamma(c\bar{c} \rightarrow \text{hadrons}) \lesssim \Gamma((c\bar{c}) \rightarrow \text{hadrons})$ = (few MeV). In specific dynamical models the suppression of the $L \neq 0$ $c\bar{c}$ wavefunctions will reduce the rates for decays into charmless hadrons even further.

We now turn to a discussion of mass formulas. So far we have been occupied entirely with the $c\bar{c}$ system, and we now turn to the states $(c\bar{s}, c\bar{d}, c\bar{s}, \text{and their charge conjugates})$ and the baryonic states $csu, cuds, \text{and cdd}$. These states are the most likely to be observed in the near future, and the reader may deduce the consequences for others from the main body of our paper.

The mass formulas in our article are equivalent to the following simple quark-model rules for the charged vector mesons $D^{++} = cd, D^{*0} = cs, F^{*+} = c\bar{s}, \text{and their charge conjugates}$

$$m_{D^{*0}} = \frac{1}{2}(m_{s}^2 + m_{d}^2) \Rightarrow m_{D^{*0}} = 2.26 \text{ GeV},$$

$$m_{F^{*+}} = \frac{1}{2}(m_{s}^2 + m_{d}^2) \Rightarrow m_{F^{*+}} = 2.31 \text{ GeV}. $$  \hspace{1cm} (A33)

If one were to use instead a linear interpolation formula the masses of the charmed vector meson would be around 2 GeV.
To estimate the masses of the singly charged pseudoscalar mesons one can use the analogue of (A33), assuming some value for the mass of $\eta_c$. We shall take $m_{\eta_c} = 3.05$ GeV. We then obtain

$$m_{D^2} = \frac{1}{2}(m_{\eta_c}^2 + m_s^2) \Rightarrow m_D = 2.16 \text{ GeV},$$
$$m_{p^2} = m_{D^2}^2 + m_{K^+}^2 - m_s^2 \Rightarrow m_p = 2.21 \text{ GeV}. \quad \quad \text{(A34)}$$

The mass of the $D$ is compatible with the guess made above that charm–anticharm threshold lies below 4.7 GeV. If one uses linear interpolation one estimates $m_D = 1.6$ GeV. This lies below the bound set by the narrowness of the $\phi^*$, $m_D \geq 1.85$ GeV.

A convenient mnemonic for $L = 0$ ground state meson masses roughly equivalent to the above estimates is

$$m_{\pi^0} \approx 0.02 + 0.23 n_s + 4.53 n_c + 0.56 S_q, \quad \quad \text{(A35)}$$

where $n_s$ is the number of strange quarks, $n_c$ is the number of charmed quarks, and $S_q$ is the quark spin (0 or 1). We must stress that all these estimates apply standard first-order symmetry breaking to much greater splittings than those encountered previously. Hence we should not be at all surprised if our charmed particle mass estimates were off by as much as 100–200 MeV. There is no substitute for dynamical calculations, which we do not perform.

From the predicted mass of the $D$ in (2.16), one can obtain the parameter $R$ introduced in (3.2):

$$R = \frac{m_s - m_\pi}{m_s - m_\eta} = \frac{m_D^2 - m_\pi^2}{m_K^2 - m_\pi^2} \approx 20. \quad \quad \text{(A36)}$$

This value permits us to estimate the masses of the charmed baryons, using the linear formulae (3.9) and their analogue for $3/2^+$ states. We can also estimate baryon masses by assuming that (3.9) applies to squares of masses, and finally, we can try linear formula for both mesons and baryons. The results are shown in Table A.3.

The underlined state in Table A.3 is stable with respect to strong and electromagnetic decay. Its favored two-body nonleptonic modes (no sin $\theta_c$ factors) are $\Lambda \pi^+$, $\Sigma \pi^+$, $\Sigma^+ \pi^+$, and $p K^0$. It has two-body nonleptonic modes with a sin $\theta_c$ suppression consisting of $n K^+$, $\Lambda K^+$, $\Sigma K^+$, and $\Sigma^+ K^0$. Given our estimate (A34), any charmed baryon below 3 GeV should be stable with respect to strong and electromagnetic decay. [The states on the last line of Table A.3 will decay strongly if Eqs. (A34) are replaced by linear formulae.]

One oddity of Table A.3 is the inversion of the $C_{+}^+ + (1/2^+)$ and $C_{+}^+ + (3/2^+)$ masses with respect to (say) the $\Sigma(1/2^+)$ and $\Sigma^*(3/2^+)$ masses. If this could be confirmed, it would be dramatic evidence for first-order symmetry breaking, to say the least. More likely, none of the entries in Table A.3 is particularly correct, and one might just as well estimate charmed baryon masses by adding about 1.5 GeV ($m_{\eta_c}/2$) to the corresponding ones for charmless baryons:

$$m_{C_{++}^+} \approx m_Z + 1.5 \text{ GeV} \approx 2.7 \text{ GeV},$$
$$m_{C_{+}^+} \approx m_A + 1.5 \text{ GeV} \approx 2.6 \text{ GeV},$$
$$m_{C_{+}^{3+}} \approx m_{X_{+}} + 1.5 \text{ GeV} \approx 2.9 \text{ GeV}. \quad \quad \text{(A37)}$$

<table>
<thead>
<tr>
<th>Table A.3. Attempts to guess the mass of charmed baryons.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meson mass Baryon mass formula</td>
</tr>
<tr>
<td>--------------------------------</td>
</tr>
<tr>
<td>Quadratic Linear</td>
</tr>
<tr>
<td>Quadratic Quadratic</td>
</tr>
<tr>
<td>Linear Linear</td>
</tr>
</tbody>
</table>

In this case the doubly-charged, singly-charmed baryons also become (meta) stable. Their weak nonleptonic decays include $\Sigma^+ \pi^+$, $\Lambda \pi^+ \pi^+$, and $p K^0 \pi^+$ (not suppressed by $\sin^2 \theta_c$ factors, but possibly suppressed since these are exotic channels$^{18}$), and $p \pi^+$ and $\Sigma^+ K^+$ (suppressed by $\sin^2 \theta_c$).

A comment on nonleptonic decays of charmed particles: According to the enhancement mechanism alluded to in the text (Sec. 4.2), the enhanced piece of the $\Delta c = \Delta s = 1$ interaction has the form $(\xi_2 \xi_2)$, $\xi_2 (\xi_2 \xi_2)$, and therefore has $\Delta V = 0$. (Recall $\xi_2$ spin acts on $u$ and $s$.) The $D^+$ has $V = 0$, so its decay into $K^0 \pi^+$, whose symmetric S-wave combination has $V = 1$, is forbidden in the $\text{SU}(3)$ limit (see also Altarelli et al., 1974; Kingsley et al., 1975 for more general and exhaustive considerations based on $\text{SU}(3)$). However $D^+ \to K \pi$ is in general allowed; we do not think that the total decay rate of $D^+$ is suppressed relative to that of $D^0, D^+$ by more than a factor of 2, say. After all, though, the enhancement of nonleptonic weak interactions might arise from a quite different source (see, for example, Lee and Treiman, 1971); it is possible that nonleptonic decays of charmed particles arise effectively from the metamorphism $c \to u$. (We thank J. D. Bjorken for reminding us of this). In such a case, the D mesons will decay mostly into nonstrange states, and the F mesons into strange ones.

If the charmed baryons are all unstable with respect to decay into ordinary baryons and charmed mesons, their identification may be very difficult. Nonetheless, the discovery of the resonances at 3100 and 3700 MeV, and their identification with $c\bar{c}$ states, has considerably reduced the highest mass at which we expect the lowest charmed baryon to occur: from 19 GeV (see Sec. 3) to around 4 GeV.

The orbital and radial excitations of charmed mesons are of interest primarily as an aid to Regge phenomenology. If the intercepts of the trajectories of the $D^*$ and its tensor partner lie high enough, associated production reactions such as

$$\pi^+ + p \to M_c + B_c \quad \quad \text{(A38)}$$

may not be suppressed at high energy as strongly as indicated in Sec. 5.5.

An optimistic estimate of the $D^*$ intercept has been obtained by Field and Quigg (1975). Let us denote the 2$^+$

---

18 The fact that $\Gamma(K^+ \to \pi^+ \pi^+) / \Gamma(K^0 \to \pi^+ \pi^-) \sim 1.5 \times 10^4$ indicates that this suppression may be important. On the other hand, exoticity of final states may have nothing to do with the suppression of $K^+ \to \pi^+ \pi^+$; it may simply be a reflection of transformation properties (such as $\Delta I = \frac{1}{2}$) of the interaction.
partner of the $D^*$ as $D_{s}^*$. If one is entitled to use Eq. (3.8) for tensor mesons with the same value of $R$, one finds

$$
\frac{m_{D_{s}^*}^2 - m_{A_2}^2}{m_{K*}^2 - m_{A_2}^2} = \frac{1}{2} \left( \frac{m_{D_{s}^*}^2 - m_{K^*}^2}{m_{D_{s}^*}^2 - m_{K^*}^2} \right)
$$

i.e.,

$$m_{D_{s}^*} = 8.29 \text{ GeV}^2 \text{ or } m_{D_{s}^*} = 2.88 \text{ GeV.} \quad (A39)$$

Further assuming the $D^*$ and $D_{s}^*$ to lie on a single exchange-degenerate trajectory, one finds this trajectory to be

$$\alpha_{D^*}(t) = -0.61 + 0.32i. \quad (A40)$$

This exercise clearly depends on taking seriously the small discrepancy between $m_{K}^2 - m_{\pi}^2 \approx 0.21$ and $m_{D_{s}^*}^2 - m_{A_2}^2 \approx 0.30$; its validity is probably no greater than the baryon predictions of Table A.3. Given $m_{D_{s}^*}$, we can then estimate the mass of $\phi_{s}(2^+)$ using

$$m_{\phi_{s}^*} = 2m_{D_{s}^*} - m_{A_2}^2 \quad (A41)$$

and find$^{19}$

$$m_{\phi_{s}^*} = 15.17 \text{ GeV}^2; \quad m_{\phi_{s}^*} = 3.87 \text{ GeV} \quad (A42)$$

corresponding to a trajectory (assumed exchange-degenerate)

$$\alpha_{\phi_{s}^*}(t) = -0.79 + 0.19i. \quad (A43)$$

The $F^*$ trajectories should be fairly close to the $D^*$ trajectories in any model, and we shall not estimate them separately.

The next estimate we can give for Regge trajectories (always assuming a straight line form, which may be questionable) is based on taking the harmonic-oscillator spectrum, for which $\phi_{s}'$ is degenerate with $L = 2$ excitations. In this case the $\phi_{s}$ trajectory has roughly half the usual slope. One interpolates for the $D_{s}^*$ mass using (A41) to find the $D^*$ trajectory. Finally, one can assume the usual slope $a' \approx 0.9 \text{ GeV}^{-2}$. (Dual models for $\pi \pi \rightarrow DD$ require charmed particle trajectories to have the same slope as charmless ones, as pointed out to us by F. Freund.) The results of all three methods are collected in Table A.4.

One might expect the estimates of the $2^+$ masses given here to be valid for all of the $L = 1$ states: $2^F_s, 2^P_s$, and $1^P_t$, as well as $1^P_s$. For example, one might expect to be able to produce an axial-vector $F_{s}^{*+}$ (the analogue of the elusive $A_1$) in the diffractive reaction

$$\nu + p \rightarrow \mu^- + F_{s}^{*+} + p. \quad \text{(target)} \quad \text{(slow)} \quad (A44)$$

A likely mass for this $F_{s}^{*+}$ on the basis of Table A.4 would be $2.6$ to $2.7 \text{ GeV}$. Its most likely decay would be to $F^*(2.21) + \gamma$, and the $F^*$ could be detected by its non-

<table>
<thead>
<tr>
<th>Method</th>
<th>$\alpha_{D^*}$</th>
<th>$m(D^*)$ (GeV)</th>
<th>$\alpha_{\phi_{s}^*}(t)$</th>
<th>$m(\phi_{s}^*)$ (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.6 + 0.32i</td>
<td>2.9</td>
<td>-0.79 + 0.19i</td>
<td>3.9</td>
</tr>
<tr>
<td>2</td>
<td>-2.4 + 0.66i</td>
<td>2.6</td>
<td>-3.8 + 0.50i</td>
<td>3.4</td>
</tr>
<tr>
<td>3</td>
<td>-3.6 + 0.91i</td>
<td>2.5</td>
<td>-7.7 + 0.91i</td>
<td>3.3</td>
</tr>
</tbody>
</table>

* The mass of tensor meson $2^+(2^+)$. There may also be $0^+, 1^+, 1^+ -$ mesons nearly degenerate with $2^+$. leptonic mode as a peak in effective mass (e.g., of three charged pions).

The $L$-excited $D^*$ states in Table A.4 contain nonstrange quarks. They are allowed to decay to $D^*_\pi$ and/or $D\pi$ both by Zweig's rule and by phase space, and presumably do so most of the time. By contrast, the rates for $F^*(L = 1) \rightarrow \{F^*(L = 0) + 2\pi \text{ or } F(L = 0) + 2\pi\}$ are expected to be at least as small as that for $\phi_{s} \rightarrow \phi_{s} + \pi$, and other hadronic channels are probably closed. If one can ever produce them, the lowest excited states of the $F$ should bear some resemblance to charmonia ($\not{\Xi}$ systems).

5. Estimates of production cross sections

The mass scale set by the $\phi_{s}$ and $\phi_{s}'$ allows us to make firmer estimates of production processes. We turn first to the photoproduction of these two vector mesons.

In Sec. 5.5 we estimated a differential cross section for $\phi_{s}$ photoproduction at $t = 0$ of $\sim 40 \mu b/\text{GeV}^2$. With a slope at 200 GeV of about 6 GeV$^{-2}$ (see Moffit, 1973) this would imply a total cross section of $6-7 \mu b$ for $\phi_{s}$ photoproduction. This is many times larger than the present experimental upper limit (M. Perl, private communication).

$$\sigma(\gamma N \rightarrow \phi_{s} N) \lesssim 30 \mu b, \quad (A45)$$

but several factors could work to reduce our estimate: (1) If one simply assumes $\sigma(\phi_{s} N)_{200 \text{ GeV}/c} \approx \sigma(\phi_{s} N)_{0 \text{ GeV}/c}$ and uses data at $\gamma N \rightarrow \phi_{s} N$ and the coupling estimate (A15), one obtains$^{30}$

$$\left. (d\sigma/dt) \right|_{t=0} (\gamma N \rightarrow \phi_{s} N) = (\gamma^2/\gamma_{\phi_{s}^*}) (d\sigma/dt) \left|_{t=0} (\gamma N \rightarrow \phi_{s} N) : = 4(2.85 \pm 0.2 \mu b/\text{GeV}^2) \approx 11 \mu b/\text{GeV}^2. \quad (A46)$$

The larger value in Sec. 5.5 depends on using (5.5) for the total $\phi_{s}$ cross section, which is larger than the value implied by present low-energy $\phi$ photoproduction data. (2) The square of the photon-$\phi_{s}$ coupling, evaluated at $m_{\phi_{s}}^2 = q^2$, seems to be about a factor of 3 less than that estimated from (A15). If this same suppression holds at $q^2 = 0$, one might expect a corresponding suppression factor (about 3) in the cross section for $\phi_{s}$ photoproduction. (3) A

$^{19}$ This mass is not far from that of the $\phi_{s}'$. The two would be degenerate for a Coulomb potential.

$^{30}$ See Table 1 of (Moffit, 1973).
\[ \phi^N \text{ to } \phi N \text{ total cross section to be} \]
\[
\left[ \frac{\sigma_{\phi^N} \left( \phi^N \right)}{\sigma_{\phi N} \left( \phi N \right)} \right]^n = \left[ \frac{\alpha_{\phi^N} \left( 0 \right) - \alpha_{\phi^N} \left( 0 \right)}{\alpha_{\phi N} \left( 0 \right) - \alpha_{\phi N} \left( 0 \right)} \right]^n \approx 1/10 \text{ to } 1/75
\]

(A47)

where the estimates of the \( \phi, \phi^N \) trajectory intercept are taken from Table A.4. These ratios may also have to be applied to our prediction. To summarize, a likely range for the total production cross section is

\[
\sigma(\gamma N \rightarrow \phi N) = \text{(initial estimate)} \times (\text{photon coupling suppression}) \times (\text{pomeron coupling suppression}) \\
= (2 \text{ to } 7 \mu b) \times (1/3) \times (1/10 \text{ to } 1/75) \\
= 10 \text{ to } 200 \text{ nb} \quad \text{(A48)}
\]

if the Pomeron models of Carlitz et al. (1971) ("f-dominated Pomeron") are correct. This range is consistent with the estimate (A45), and probably also with the forthcoming result from Fermilab (W. Lee et al., private communication).

Using Eq. (A20), we would estimate the cross section for \( \phi^N \) photoproduction to be about a factor of 2 less than that for \( \phi^N \).

If one assumes that the photon-\( \phi^N \) coupling does not vary too much between \( q^2 = m_{\phi^N}^2 \) and \( q^2 = 0 \), the photoproduction of \( \phi^N \) becomes an experiment to measure the Pomeranchuk coupling to \( \phi^N \). This is of tremendous importance in estimating the production of charmed particle pairs. In the central region of rapidity space, the asymptotic rate of production of any particle \( A \) is governed by the forward three particle-to-three particle amplitude with two Pomeron pairs (Fig. A1): A particular model for the Pomeranchuk trajectory considered by Farrar and Rosner (1974), generalizing an approach by Cahn and Einhorn (1971), relates the coupling of particles in Fig. A1 to Pomeron–Pomeron–particle couplings in such a way that

\[
\sigma(D)/\sigma(K) \big|_{\text{central}} = \sigma_{\phi N} \left( \phi N \right) / \sigma_{\phi N} \left( \phi N \right) \\
\sim 1/3 \text{ to } 1/9 \quad \text{(A49)}
\]

if we use (A47). These are enormous values for charmed particle production. They conflict strongly with thermodynamic estimates based on the formula of Hagedorn (1971):

\[
\sigma(M_\perp) \sim \exp \left( -M_\perp/T_0 \right) \\
M_\perp = (\rho^2 + M^2)^{1/2} \\
T_0 \sim 160 \text{ MeV} \quad \text{(A50)}
\]

However, the asymptotic limits (A49) are not likely to be approached until well above ISR energies in a multiperipheral model (e.g., Einhorn and Nussinov, 1974). This is because charmed particles are presumably produced in pairs, with a cluster mass of at least \( 2m_q = 4 \) to 5 GeV.

---

FIG. A1. Coherent Primakoff production cross section of \( \eta \) as a function of laboratory photon energy on targets Pb and Cu (a); Be and H (b). \( \Gamma(\eta \rightarrow \gamma \gamma) \) of 100 keV and mass of \( \eta = 3.05 \) are assumed. The cross section \( \sigma \) and \( \Gamma(\eta \rightarrow \gamma \gamma) \) are proportional. A large class of models for the Pomeranchuk trajectory (see, e.g., Carlitz et al., 1971) predicts the suppression of Pomeron couplings to particles containing strange or charmed quarks. For example, such models predict the asymptotic ratio for \( \phi^N \) to \( \phi N \) total cross section to be

\[ \left[ \frac{\sigma_{\phi^N} \left( \phi^N \right)}{\sigma_{\phi N} \left( \phi N \right)} \right]^n = \left[ \frac{\alpha_{\phi^N} \left( 0 \right) - \alpha_{\phi^N} \left( 0 \right)}{\alpha_{\phi N} \left( 0 \right) - \alpha_{\phi N} \left( 0 \right)} \right]^n \approx 1/10 \text{ to } 1/75 \]

(A47)

where the estimates of the \( \phi, \phi^N \) trajectory intercept are taken from Table A.4. These ratios may also have to be applied to our prediction. To summarize, a likely range for the total production cross section is

\[ \sigma(\gamma N \rightarrow \phi N) = \text{(initial estimate)} \times (\text{photon coupling suppression}) \times (\text{pomeron coupling suppression}) \]

\[ = (2 \text{ to } 7 \mu b) \times (1/3) \times (1/10 \text{ to } 1/75) \]

\[ = 10 \text{ to } 200 \text{ nb} \quad \text{(A48)} \]

if the Pomeron models of Carlitz et al. (1971) ("f-dominated Pomeron") are correct. This range is consistent with the estimate (A45), and probably also with the forthcoming result from Fermilab (W. Lee et al., private communication).

Using Eq. (A20), we would estimate the cross section for \( \phi^N \) photoproduction to be about a factor of 2 less than that for \( \phi N \).

If one assumes that the photon-\( \phi^N \) coupling does not vary too much between \( q^2 = m_{\phi^N}^2 \) and \( q^2 = 0 \), the photoproduction of \( \phi^N \) becomes an experiment to measure the Pomeranchuk coupling to \( \phi^N \). This is of tremendous importance in estimating the production of charmed particle pairs. In the central region of rapidity space, the asymptotic rate of production of any particle \( A \) is governed by the forward three particle-to-three particle amplitude with two Pomeron pairs (Fig. A1): A particular model for the Pomeranchuk trajectory considered by Farrar and Rosner (1974), generalizing an approach by Cahn and Einhorn (1971), relates the coupling of particles in Fig. A1 to Pomeron–Pomeron–particle couplings in such a way that

\[ \sigma(D)/\sigma(K) \big|_{\text{central}} = \sigma_{\phi N} \left( \phi N \right) / \sigma_{\phi N} \left( \phi N \right) \]

\[ \sim 1/3 \text{ to } 1/9 \quad \text{(A49)} \]

if we use (A47). These are enormous values for charmed particle production. They conflict strongly with thermodynamic estimates based on the formula of Hagedorn (1971):

\[ \sigma(M_\perp) \sim \exp \left( -M_\perp/T_0 \right) \]

\[ M_\perp = (\rho^2 + M^2)^{1/2} \]

\[ T_0 \sim 160 \text{ MeV} \quad \text{(A50)} \]

However, the asymptotic limits (A49) are not likely to be approached until well above ISR energies in a multiperipheral model (e.g., Einhorn and Nussinov, 1974). This is because charmed particles are presumably produced in pairs, with a cluster mass of at least \( 2m_q = 4 \) to 5 GeV.
and the production of such a massive cluster is highly disfavored in multiperipheral models because of $t_{\text{min}}$ effects. While such effects are hard to estimate, they could easily increase the cross section for charm particle production in the central region at Fermilab and CERN II by 10–100 (or even greater) over that at Brookhaven and the CERN PS.

Estimates for charm particle production via the two-gluon production of a $car{c}$ pair have been made by Einhorn and Ellis (1975). These calculations are very sensitive to the assumed gluon spectrum in a hadron. If one takes the gluons to have the same $x$ distribution in the hadron as the anti-partons, for example (i.e., peaked toward low $x$), the cross section for $car{c}$ production can rise by several orders of magnitude between Brookhaven and ISR energies.

Let us now estimate the cross section for charm particle production at Fermilab energies (150–300 GeV for protons) in the diffractive region, as mentioned at the end of Sec. 5. The minimum mass of a diffractively produced cluster in the reaction

\[
\text{meson} + \text{target} \rightarrow (M^*) + \cdots
\]

(or photon)

\[
\rightarrow m_e M_e
\]

is at least $2m_D \approx 4$ GeV, and for a baryonic cluster in

\[
\text{baryon} + \text{target} \rightarrow (N^*) + \cdots
\]

\[
\rightarrow B M_e
\]

is probably $m_c + 2m_D \approx 5$ GeV. Let us assume that the diffractive process is important only for $x = 1 - M^2/s \gtrsim 0.9$, where $M$ is the mass of the cluster. (See, e.g., the review by Leith, 1973.) Then for the process (A52) the interesting range is $25 \leq M^2 \leq 60$ GeV$^2$, for which we estimate the diffractive cross section at a single vertex to be of the order of several hundred microbarns. Of course, either the target or the projectile can undergo diffraction in these processes. Projectile diffraction is ideal for observing short tracks, while target diffraction allows better resolution in plotting effective masses. Observing the target recoil also provides a handle for measuring the effective mass of the cluster.

Given an $N^*$ cluster with masses squared around 40 GeV$^2$, what is its probability of undergoing the decays mentioned in (A52)? Such a cluster can also be produced in the direct channel by pions of about 20–25 GeV/c on nucleons (and this might be as good a source of charmed particles as high-energy diffraction). As a rule of thumb, noting that $m_c/m_s \simeq m_s/m_u$, we shall guess that

\[
\frac{\sigma(\text{charmed})}{\sigma(\text{strange})} \sim \frac{\sigma(\text{strange})}{\sigma(\text{total})} \sim 10-15\%
\]

in $\pi^+p$ interactions at 20–25 GeV/c. (A53)

Combining this estimate with the estimate of several hundred $\mu$b for diffractive $N^*$ production in the mass range of interest, we arrive at an estimate of several $\mu$b for charmed particle production in the reaction (A52). This would correspond to one charmed particle event in several thousand in the emulsion exposures at Fermilab as mentioned at the end of Sec. 6. If marked by a distinctive signature, such as a forward-going doubly charged track (as one would have if the $C^+_1$ were stable), such events would not be too hard to identify. Note that we have assumed that the $C^+_1 +$ might live longer than the average charm particle: even for $m = 3$ GeV, a lifetime of $10^{-15}$ sec would not seem unreasonable.

If the estimate in (A53) is really correct, of course, pion–nucleon interactions at 20–25 GeV/c become an ideal place to look for charm particles. We would imagine that Eq. (A53) should really be applied, in pion–nucleon interactions, to the non-Pomeron contribution to the total cross section, since associated production will be the main mechanism for production of charm particles at such low energies. This non-Pomeron contribution is of the order of a couple of millibarns, which still allows for charm production cross sections of the order of several tens of microbarns. Typical reactions to look for would be

\[
\pi^- + p \rightarrow D^- + [C^+_1]
\]

\[
\rightarrow K^0\pi^- \rightarrow \Delta^0 +
\]

(A54)

or

\[
\pi^- + p \rightarrow \bar{D} + C^+_1 +
\]

\[
\rightarrow \Delta^0 +
\]

(A55)

We remind the reader that the best channels in which to make effective-mass studies may be nonexotic ones (those with the quantum numbers of $K^0$, such as $K^+\pi^-$ and $K_s\pi^+\pi^-$, or $\Sigma^+$, such as $\Lambda^+$ and $pK^0$). The baryons in Eqs. (A54) and (A55) may also be unstable with respect to the strong interactions, decaying to charm = +1 mesons (which give rise to $S = -1$, preferentially) and ordinary baryons.

If one were to estimate particle production cross sections using the formula (Snow, 1973) $\sigma(x) \sim 1/M^2$, the above estimates for diffractive and associated production would read, respectively,

\[
\sigma_{\text{diff}}(\text{charm}) \sim m_c^2/m_D^2 \quad (\text{few hundred } \mu\text{b})
\]

(A56)

around $p_L = 300$ GeV/c

and

\[
\sigma_{\text{assoc}}(\text{charm}) \sim m_c^2/m_D^2 \quad (\text{couple of } \mu\text{b})
\]

\sim several \mu b

(\pi\pi \text{ interactions around } p_L = 20 \text{ GeV/c}). (A57)

Even with such optimistic estimates, the identification of charmed particles from mass spectra in bubble chamber experiments would be marginal. If the exponential spectrum in (A50) is closer to the truth, one will have to rely exclusively on high-statistics, high-resolution counter experi-
ments for such studies. The estimates of Field and Quigg (1975) on two-body associated charm production cross sections based on Regge pole phenomenology are far smaller than (A57) even with the most optimistic Regge trajectory.

What about $e^+ - e^-$ reactions? If the excess in $R$ above our lower curve in Fig. 8 is really due to charm production, we would expect to see charmed particles with a cross section equal to this excess.

One possibility, which we feel deserves some study, is that between the mass of the $\phi'$ and charm threshold the excess above the three-quark value of $R$ is due to nonresonant $c\bar{c}$ production. In that case one would expect the $c\bar{c}$ system to radiate gluons or photons until it reached a narrow resonant state:

$$e^+ + e^- \rightarrow c\bar{c} \rightarrow (\phi, \phi', ...) + \gamma' s$$

or $$\rightarrow (\phi, \phi', ...) + \text{(gluons)}$$

$$\rightarrow \text{pions.} \quad \text{(A58)}$$

In this case the inclusive production of $\phi$, or $\phi'$, might be very large through a wide range of colliding beam energies. It might even predominate over the pair production of charmed particles until somewhat above charm threshold. At $E_{\text{CM}} = 4.8$ GeV, though, there seems to be no sign of a $\phi$, recoiling against $\pi^+\pi^-$ (J. D. Jackson, private communication).

The more straightforward charm-related explanation for the excess of $R$ above the three-quark value would, of course, be the pair production of charmed particles: just above threshold,

$$e^+ + e^- \rightarrow D^+ + \bar{D}^0,$$  \text{(A59)}

$$\rightarrow D^+ + D^-,$$  \text{(A60)}

$$\rightarrow F^+ + F^-,$$  \text{(A61)}

with single- or double-vector meson production and inclusive channels becoming important by a few hundred millielectronvolts above threshold. In the exact $U(4)$ limit, as we have mentioned, reaction (A59) should be suppressed; the contributions of the respective quark charges cancel. This mechanism would also suppress $D^{*+} - \bar{D}^{*0}$ production, though not $D^+ - \bar{D}^{*0}$ production (or its charge conjugate). The rates for (A60) and (A61) would be equal.

Now, we have argued that since the non-$\sin^2 \theta_g$ two-body decay of the charged $D^+$ may be suppressed, perhaps even to the level of the $\sin^2 \theta_g$ decay (which involves channels with the quantum numbers of the singly charged pion). The favored decay of the $F$ also involves quantum numbers of the singly charged pion. Consequently, in the $U(4)$ limit, the best place to look for charmed particles in colliding beam experiments may be in such mass combinations as $(3\pi)^2$, $K^2K_0$, $\pi\pi^2$, and so on. High resolution will be essential to avoid background problems. If the enhancement mechanism of nonleptonic interactions is not what we envisaged here, and if it results in the effective $e \rightarrow \mu$ conversion, the strange particle yield will not increase above the $DD$ threshold. It will increase, somewhat, only after the $FP$ threshold is reached.

If one notes that $U(4)$ is badly broken since the $\phi$, pole lies much closer to the physical region for the reactions (A59–A61) than do the other vector meson poles, the reaction (A59) is not suppressed as much. It is still expected to be less frequent than (A60) or (A61), however. For production of a pseudoscalar-vector pair, the roles are reversed; the analogue of Eq. (A59) would dominate the analogues of Eq. (A60) and (A61), strongly in the exact $U(4)$ limit (by a factor of 16!) and considerably less so if the $\phi$ pole dominated charmed particle production.

At the very least, we would regard the absence of a charmed particle signal in the $K^+\pi^-$ channel in colliding-beam data around 5 GeV as evidence that our Table IV of branching ratios is unreliable. If a $K^+\pi^-$ signal is not even seen at the level of a few percent of all kaon-containing hadronic events at $E_{\text{CM}} \sim 6$ GeV, we would begin to suspect the validity of the charm hypothesis $\phi = \psi, J$ itself.

The photoreaction

$$\gamma + N \rightarrow (\text{charm}) + \overline{\text{charm}} + N \quad \text{(A62)}$$

bears the same relation to colliding $e^+ - e^-$ beam experiments as the diffractive processes (A51) and (A52) bear to associated production (e.g., A38). One would expect the cross section for reaction (A62) above charm threshold to be of the same order as $c\bar{c}$ vector meson production, as this reaction is likely to be dominated by the nearby $\phi$ and $\phi'$ poles. Hence, using (A58) and the fact that total photon–nucleon cross sections are of the order of 100 mb, one might expect $10^4$ to $2 \times 10^4$ of the photoreactions at high energies to involve the process (A62). While this is not a particularly large number, the reaction (A62) may have some intrinsic advantages, for example in emulsions where the use of a neutral beam avoids large numbers of noninteracting tracks. (As we have stressed, the major use of emulsions is in detecting short tracks.)

We would like to add some remarks concerning our estimates of charmed particle production in neutrino reactions (Sec. 5.2). These remarks are based on the mass scale for charmed particles implied by taking the resonances at 3100 and 3700 MeV to be the $\phi$, and $\phi'$. The process illustrated in Fig. 6 and 7 was expressed in parton language as occurring via the transformation of a strange quark or antiquark ($s$ or $\bar{s}$) in the $q\bar{q}$ “sea” of the target nucleon into a charmed quark or antiquark ($c$ or $\bar{c}$). From Fig. 6 one can see that charm production should account for roughly 10% of deep inelastic antineutrino–nucleon interactions under the conditions shown. Since one expects the total deep inelastic charm production cross sections to be equal for neutrinos and antineutrinos, a few

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Rev. Mod. Phys., Vol. 47, No. 2, April 1975
percent of neutrino deep inelastic interactions should contain charmed particles.

The above estimates may be viewed as reflecting cross sections for “inclusive diffractive production” of the states (c$\bar{c}$)$^+$ (by neutrinos) or (c$\bar{c}$)$^-$ (by antineutrinos) off a nucleon target. Indeed, the dynamical assumptions that go into Fig. 6 (strong peaking toward $x = 0$, peaking toward $y = 1$) are just those that arise from $t_{\text{min}}$ effects, which one would not expect to be important in diffraction processes. If the production of charmed particles by neutrinos or antineutrinos is really diffractive, of course, the target should have a small recoil momentum, less than a GeV/c.

The expression for $t_{\text{min}}$ in a deep inelastic process (see Sec. 5.2 for kinematic definitions) may be written

\[-t_{\text{min}} \sim \left[M^2 - (qx)^2\right]/2x^2 = \left[m_p^2 + M^2/2yE_n\right]^2, \tag{A63}\]

where $M$ is the mass of the diffractively produced state coupling to the current. If the cross section for such a process is peaked in $t$, where $t$ is the momentum transfer between the current and the state of mass $M$, Eq. (A63) can introduce considerable peaking toward $y = 1$.

For low-$M^2$ states, only the first term in (A63) is important. This term is probably already taken into account in the phenomenological parton distributions that describe low-$x$ behavior. For high-$M^2$ states, the cross term and the square of the second term in (A63) become important. A calculation was thus performed in which the shape of the $x$-$y$ distribution was described by

\[d\sigma^{n.p.}/dxdy = \text{const} \times 0.2(1 - x)^3 \exp (b t_{\text{min}}), \tag{A64}\]

\[b = 10 \text{ GeV}^{-2}, \tag{A65}\]

\[-t_{\text{min}} = (m_p^2) \left(M^2/2yE_n\right) + (M^2/2yE_n)^2, \tag{A66}\]

where the only modification with respect to the original estimate in Sec. 5 is the exp ($b t_{\text{min}}$) factor.

The qualitative effects of the $t_{\text{min}}$ factor are roughly equivalent to choosing the rather high threshold of $m_p \approx 5 \text{ GeV}$ as done in Sec. 5.2. Note that the absolute threshold for the diffractive process we are considering is only $m_p + m_p \approx 3.2 \text{ GeV}$. For $E_n \approx 25 \text{ GeV}$, most of the events in (A64) occur with muon energies between 2 and 10 GeV, and muon angles with $\cos \theta_\mu \geq 0.98$. This suggests that the diffractive charm production process can probably be enhanced by cutting the data in $v = xy = E_n(1 - \cos \theta_\mu)/m_p$ and selecting (say) $v \leq 0.1$. For charm production in heavy nuclei, where the diffractive slopes are expected to be greater than (A66), we expect the $t_{\text{min}}$ effects to be correspondingly greater, and events will be peaked more strongly toward low $x$ and high $y$. It is even conceivable that one could sort out such effects by looking at differences between neutrino-induced events in materials of two different atomic numbers.

If it is correct that charm production by neutrinos and antineutrinos can be viewed primarily as a diffractive effect, there are other diffractive effects that should be at least as great, such as three-charged pion production in the mass region 1100–1400 MeV (the $A_1$ region):

\[\nu(p) + N \rightarrow \mu^-(\mu^+) + (3\pi)^+[-(3\pi^-)] + N. \tag{A67}\]

There are several reasons why (A67) should have a larger cross section than the corresponding charm production process

\[\nu(p) + N \rightarrow \mu^-(\mu^+) + (c\bar{c})^+[-(c\bar{c})^-] + N \rightarrow P^+ + \gamma \rightarrow P^{*+} + \gamma \rightarrow D^0 K^+ \rightarrow \ldots \tag{A68}\]

(1) The lowest mass state in (A68) may be as much as twice as heavy as that in (A67). We have argued earlier in this addendum, for example below (A44), that an axial vector $F_2^*$ would have a mass of 2.6 to 2.7 GeV. The $t_{\text{min}}$ effects discussed above will thus be more important for (A68) than for (A67). (2) The effective coupling of the weak current to the $c\bar{c}$ system may be smaller than that to the $ud$ system. (3) The Pomeranchuk trajectory may couple less strongly to the $c\bar{c}$ system than to the $ud$ system. For $E_n = 25 \text{ GeV}$, a rough estimate of these effects gives

\[\sigma(F_2^*)/\sigma(A_1) = (1/2) t_{\text{min}} (1/3) \text{current coupling} (1/4) \text{Pomeron coupling} \approx 4\%. \tag{A69}\]

As one expects “$A_1$” production to be less than the total production of charmless hadrons in deep inelastic processes, the estimate (A69) may be somewhat more pessimistic than those based on the parton picture (Sec. 5.2, Figs. 6 and 7).

An estimate (CERN Boson Workshop, 1974) of $\phi_1$ diffractive production in $\nu$ neutral current gives a very small cross section.

The fact that charmed particles are expected to be produced in neutrino and antineutrino reactions with frequency of order several percent means that neutrino experiments are the best ones for emulsion exposures. As we have pointed out, it is only by the detection of short tracks in emulsions that one will be able to tell that a state is present which must decay weakly. Given our mass estimates and Fig. 10, the tracks of charmed particles at present-day energies will be too short to see in bubble chambers, but should definitely be of the order of tens or hundreds of microns: easily detectable in emulsions.

6. SUMMARY

Let us summarize this addendum. The identification of the states at 3100 and 3700 MeV with $\phi_1$ and its first radial excitation narrows considerably the search for charm. If this identification is correct, one is in a much better position than six months ago to propose experiments which will confirm or rule out the charm idea. Detection of short tracks remain the crucial experiment, and becomes feasible now that the hypothetical charmed particle mass scale
has been set. In addition, one must prove that the charmed quark couples more strongly to the strange quarks than to the quark $d$ by a factor of cot $\theta$. (Gell-Mann, 1964; Bjorken and Glashow, 1964). This will require the observation, for example, both of the decay
\[ \bar{D}^0 \rightarrow K^+\pi^- (\sim \cos \theta), \]  
(A70)
and of
\[ \bar{D}^0 \rightarrow \pi^+\pi^- (\sim \sin \theta \cos \theta), \]  
(A71)
or of the pair
\[ \bar{D}^0 \rightarrow K^+\pi^- (\sim \cos \theta), \]  
(A72)
and
\[ \bar{D}^0 \rightarrow \pi^+\pi^- (\sim \sin \theta). \]  
(A73)

As nonleptonic enhancement effects are still not totally understood, the processes (A72) and (A73) may be more reliable for such a test. The process (A72) can lead to charged kaon-lepton coincidences, themselves a powerful indication in favor of charm. The expected kaon-lepton effective mass spectrum in the decay (A72) is shown in Fig. A.3; it appears even possible to determine the mass of the parent through the detection of this spectrum.

The new resonances may, after all, turn out not to be associated with charm. However, in pursuing the experiments we have suggested, we suspect that more new effects are bound to show up. The emerging pattern of the hadrons is likely to be at least as interesting and varied as that we have described here.

The subject of this addendum has been the source of lively discussions with our theoretical and experimental colleagues. We would particularly like to thank F. di Bianca, D. Cline, A. Dolgov, A. Erwin, G. Feldman, G. Goldhaber, J. Lord and F. Vanucci for their patience in explaining to us current limits on the effects we have mentioned. We would like to thank especially J. D. Jackson for sharing his knowledge and wisdom with us, and having gone through our first draft. Sam Treiman's encouragement has a lot to do with our undertaking this somewhat quixotic attempt at an instant review. We thank the members of the Theoretical Physics Department at Fermilab for discussions and enlightenment.

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**Rev. Mod. Phys., Vol. 47, No. 2, April 1975**