Method for Determining the $\theta_1 - \theta_2$ Mass Difference

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The existence of a neutral K meson of long lifetime \(^1\) and mixed strangeness \(^2\), as suggested by Gell-Mann and Pais \(^3\), has been well established. The ideas of Gell-Mann and Pais lead to other effects such as the interference between $\theta_1$ and $\theta_2$ states during the lifetime of the particles \(^4\). Furthermore the masses of the particles designated as $\theta_1$ and $\theta_2$ should be slightly different, but an amount of the order of $h/2\pi \tau = 10^{-6}$ ev, where $\tau$ is the half-life of the (short-lived) $\theta_1$ meson. One method to observe the interference phenomenon and thereby measure the mass difference has been suggested by Treiman and Sachs \(^5\). This method utilizes the decays of the $\theta_1$ and $\theta_2$ as a means of detection, and a charge asymmetry in the decays of the $\theta$ and $\bar{\theta}$ is required to observe the effect. The purpose of this Letter is to call attention to another method of detection which does not utilize the decay properties but, rather, depends upon the direct detection of the $\theta$ and $\bar{\theta}$ modes \(^6\).

Since the strangeness is different for the $\theta$ and $\bar{\theta}$, it can be used as a means of distinguishing them. The strangeness, in turn, can be determined by identifying the products of the strong interactions in matter. These charged particles of known strangeness, i.e., $K$ mesons and hyperons, can be recognized readily. This notion has already been used \(^7\) in the separation of the $\theta$ and $\bar{\theta}$ components of the long-lived $\theta_1$ particle. Our suggestion is that the method could be used to determine the relative number of, say, $\theta$'s as a function of time in order to demonstrate the interference phenomenon.

If, for example, the neutral $K$ beam is known to consist initially of only the $\theta$, the state is given as a function of time by

\begin{equation}
\psi(t) = 2^{-1/2} \left[ \theta_1 \exp(-\lambda_1 t/2) + i\theta_2 \exp(-\lambda_2 t/2) \exp(i\Delta \omega t) \right],
\end{equation}

where $\Delta \omega$ is the difference between the natural frequencies (masses) of the $\theta_1$ and $\theta_2$, and $\lambda_1$, $\lambda_2$ are the decay constants of the $\theta_1$, $\theta_2$, respectively. Writing $\theta_1 = 2^{-1}(\theta + \bar{\theta})$ and $\theta_2 = 2^{-1}(\theta - \bar{\theta})$, we find that the $\bar{\theta}$ amplitude is $\frac{1}{2}[\exp(-\lambda_1 t/2) - \exp(-\lambda_2 t/2) \exp(i\Delta \omega t)]$ and the number of $\bar{\theta}$'s will be proportional to

\begin{equation}
1 + \exp(-\lambda_2 t/2) \cos(\Delta \omega t) \exp(-\lambda_1 t/2),
\end{equation}

as long as $\lambda_2 \ll 1$ (note: $\lambda_2 \ll \lambda_1$). Thus the number

![Figure 1](image)

**Figure 1.** Relative intensity of the $\theta$ mode as a function of time (in units of the $\theta_1$ mean lifetime). The $\theta_1 - \theta_2$ mass difference is $\Delta \omega \pi$.
Electron Scattering by Polarized Nuclei

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SOME time\(^1\) ago it was pointed out by the author that a most sensitive way of measuring a nuclear magnetic moment distribution would be double scattering of electrons on polarized nuclei.\(^2\) The double scattering cross section for electrons on polarized nuclei (as well as the single scattering cross sections for longitudinally polarized electrons) was derived in the first Born approximation under the assumption that both the charge and magnetic moment are fixed, static distributions. Recoil and changes in the magnetic moment direction were neglected. At energies presently available and for nuclei which are not too small, the neglect of recoil is justified.\(^3\) The fixing of the magnetic moment direction as though it were classical, however, restricts the calculated cross sections either to nuclei with very high spin, or to experiments in which the final nuclear spin direction is also measured and found to be the same as initially. Clearly it is desirable to remove this restriction. That is readily done as follows.

First, wherever the nuclear magnetic moment vector \(\mathbf{y}\) appears it is replaced by the operator

\[
\mathbf{y} = \mathbf{S} \mu / s,
\]

where \(\mathbf{S}\) is the nuclear spin operator and \(s\) is the spin quantum number. (This definition of the number \(\mu\) is convenient for our purpose.) Second, the matrix element taken must include that between initial and final nuclear spin directions. Since the final nuclear spin is not measured, we sum over it. As a consequence of the completeness of the spin functions this results in the replacement of the cross section for initial nuclear spin direction \(\mathbf{n}\) by the expectation value in the state of nuclear spin direction \(\mathbf{n}\) of the operator obtained by replacing the \(\mu\) number \(\mathbf{y}\) by the operator (1).

The calculation of these expectation values is a straightforward matter and results in the following replacements:

\[
1 \rightarrow (n|n) = 1,
\]

\[
\mathbf{y} \cdot \mathbf{m} \rightarrow (n|\mathbf{S} \cdot \mathbf{m}|n)\mu / s = \mathbf{y} \cdot \mathbf{m},
\]

\[
\mu^2 \rightarrow (n|\mathbf{S}^2|n)(\mu / s)^2 = \mu^2(s+1)s^{-1},
\]

\[
(\mathbf{y} \cdot \mathbf{m})^2 \rightarrow (n|\mathbf{S} \cdot \mathbf{m})(\mu / s)^2 = \frac{3}{2}s^{-1} + \frac{1}{2}(2s-1)s^{-1}(\mathbf{y} \cdot \mathbf{m})^2,
\]

where on the right, now, \(\mathbf{y} = \mu \mathbf{n}\).

The only change in the single scattering cross sections occurs in the non-spin-flip cross section [see (15), reference 1]

\[
\sigma_{\text{ff}} = \delta \left[ \cos^2 \theta_0 + \Delta \right] \phi(\theta_0) \sin^2 \theta_0 \cos \theta_0 \cos \phi(\mathbf{k} \cdot \mathbf{p}_1 \cdot \mathbf{p}_2)
\]

in the notation of reference 1.\(^4\) The quantity which

Fig. 2. Relative intensity of the \(\theta\) mode as a function of time. For comparison with Fig. 1, the vertical scale must be corrected for the relative efficiency for detection of \(\theta\) and \(\bar{\theta}\).

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\(^3\) Baldo-Ceolin, Dilworth, Fry, Greening, Husaita, Limentani, and Sichtrello, Nuovo cimento 6, 130 (1957); Ammari, Friedman, Leve Setti, and Telegradi, Nuovo cimento 5, 1801 (1957).


\(^7\) A quite different method has been suggested by M. L. Good, University of California Radiation Laboratory Report UCRL-3840; Phys. Rev. (to be published).

\(^8\) The results obtained here are strictly valid only if the weak interactions are invariant under either the operation \(C\) or \(CP\).

\(^9\) Fig. 1 shows the solid line for the \(\Delta\) mode and the dashed line for the \(\Delta\) mode. The vertical scale must be corrected for the relative efficiency for detection of \(\theta\) and \(\bar{\theta}\).