FLAVORDYNAMICS OF QUARKS AND LEPTONS

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Abstract:

The present theory of flavordynamics is discussed. After giving a general introduction into the field we describe the gauge theory framework and the spontaneous symmetry breaking. Several examples of spontaneously broken theories are studied. We describe the standard theory of leptons and quarks. A special emphasis is given to a discussion of the weak decays of strange and charmed particles. Furthermore the neutral current interaction is studied. We discuss the sequential flavordynamics of leptons and quarks, concentrating on the six flavor scheme, and speculations about extended schemes of flavordynamics. The report concludes with a description of weak interactions at very high energies (production and decays of W, Z or Higgs bosons etc.).

1. Introduction

The development of particle physics during the last ten years is marked by the following two features:

i. Quarks play a fundamental rôle in our understanding of the strong interactions. Leptons and quarks are the basic building blocks of matter.

ii. Non-Abelian gauge theories are of crucial importance for describing all known interactions. At present we have confidence that the electromagnetic and weak interactions are manifestations of an underlying gauge theory. There are many indications that the strong interactions are a manifestation of Quantum Chromodynamics (QCD). Furthermore, Einstein's theory of gravity represents an example of a non-Abelian gauge theory, with the symmetry group of space and time acting as a gauge group.

In this review we describe a part of the evolution in the field of gauge theories during the last years. We intend to concentrate on quantum flavordynamics (QFD), i.e. the electromagnetic, weak and associated interactions. We shall assume throughout this paper that QCD is the correct field theory of the strong interactions.

Furthermore, we shall concentrate our discussion on models on QFD and on phenomenological aspects. For the more formal fieldtheoretic aspects of gauge theories we refer the interested reader to already existing reviews [1.1]. Another review which in some sense is complementary to this article and which the interested reader should consult is the review by Harari [1.2].

In this review we present our own point of view about the subject, which probably is not shared by everybody working in the field of gauge theories. In order to present some sort of a coherent picture we found it essentially impossible to give a just credit to all those quoted in the references. To do so we would have had to disrupt the style of our presentation almost continuously. Furthermore the writing of this review would have taken much longer, and it could have run the risk of being out of date already at the time of its typing. Thus the selection of the references was made in the sense to give assistance to unexperienced readers and students who want to study a particular question in more detail, and not in the sense to quote everybody at the right place and in the right historical order. We apologize for this, and ask those who feel that their work is not sufficiently recognized for their understanding.

Most of the material contained in this review should be understandable to a graduate student specializing in particle physics, and to physicists working in other fields.

1.1. Historical overview of weak interaction theory

Since the first proposition of the effective weak interaction Hamiltonian by Fermi in 1934 [1.3] it took about 24 years until the space-time structure of the charged weak currents was established. In the sixties the internal symmetry structure of the hadronic weak currents was found, which turned out to be of the
form
\[ j^\text{hadr}_\mu = \cos \theta_c (\bar{u} \gamma_\mu d)_L + \sin \theta_c (\bar{u} \gamma_\mu s)_L \]  \tag{1.1}

[\text{L: left-handed, } (\bar{u} \gamma_\mu d)_L \text{ stands for } \bar{u} \frac{1}{2}(1 + \gamma_5)d; \text{ u, d, s-quark fields}] \text{ where } \theta_c \text{ denotes the Cabibbo angle.}

Experimentally the Cabibbo structure of the charged weak current eq. (1.1) is rather firmly established. The value of \( \cos \theta_c \) is extracted from nucleon or nuclear \( \beta \)-decay, and it is found after taking into account radiative corrections [1.4]:

\[
\cos \theta_c = 0.974 \pm 0.002
\]
\[
\theta_c = (13.1 \pm 0.5)^\circ. \tag{1.2}
\]

The parameter \( \sin \theta_c \) is measured in strange particle decay; one finds [1.4]

\[
\sin \theta_c \approx 0.220 \quad \text{(from K-decay)}
\]
\[
\sin \theta_c \approx 0.236 \quad \text{(from hyperon-decay).} \tag{1.3}
\]

In 1976 the charmed particles were discovered. Thus far there is no disagreement between nature and the GIM-hypothesis [1.5], which implies that the weak charm changing current has the form

\[
\tilde{j}_\mu [[\Delta C = 1] = -\sin \theta_c (\bar{c} \gamma_\mu d)_L + \cos \theta_c (\bar{c} \gamma_\mu s)_L.
\]

Summarizing the present status of weak interaction phenomenology we can write down the effective current–current interaction:

\[
\mathcal{L}^\text{weak} = \frac{4G}{\sqrt{2}} (j^+ j^-) + \text{h.c.} \tag{1.4}
\]

where \( G = 1.1666 \times 10^{-5} \text{ GeV}^{-2} = 1.0270 \times 10^{-5} M_p^{-2} \) (Fermi constant), and the weak current \( j_\mu \) is given by

\[
\begin{align*}
\tilde{j}_\mu &= \bar{\nu}_e \gamma_\mu e^-_L + \bar{\nu}_\mu \gamma_\mu \mu^-_L + \bar{u} \gamma_\mu d'_L + \bar{c} \gamma_\mu s'_L, \\
\tilde{j}_\mu^+ &= (\tilde{j}_\mu^-)^+ \\
d' &= \cos \theta_c \ d + \sin \theta_c \ s, \quad s' = -\sin \theta_c \ d + \cos \theta_c \ s.
\end{align*}
\]

Evidence for a new charged lepton \( \tau \) was found in 1975 [1.6]. For all we know at present, the behaviour of the \( \tau \)-lepton is consistent with it being a sequential heavy lepton, entering physics together with its own massless neutrino \( \nu_\tau \).

If we interpret the charged weak currents as raising or lowering operators of the weak isotopic spin, we can describe them by the following weak doublets:

\[
\begin{pmatrix}
\nu_e \\
e^-
\end{pmatrix}_L, \quad \begin{pmatrix}
\nu_\mu \\
\mu^-
\end{pmatrix}_L, \quad \begin{pmatrix}
\nu_\tau \\
\tau^-
\end{pmatrix}_L, \quad \begin{pmatrix}
u_e \\
u_\mu \\
\nu_\tau \\
\nu_s \\
u_s'
\end{pmatrix}, \quad \begin{pmatrix}
u_\tau \\
u_s \\
\nu_s'
\end{pmatrix}_L. \tag{1.5}
\]
The observation of the $Y$-particle [1.7] in the reaction $p + \text{nucleus} \rightarrow Y + \text{anything}$ and in $e^+e^-$annihilation [1.8, 1.9] has led to its interpretation as a bound state of a new heavy quark and antiquark $b$ with a mass of about 5 GeV.

In 1972 the weak neutral current was discovered [1.10]. Since then neutral current effects have been seen in many different reactions (inclusive neutrino-production, elastic neutrino-proton scattering, single pion production by neutrinos, neutrino-electron scattering etc.). A remarkable feature of the experimental results is that most of them are in agreement with the minimal $SU_2 \times U_1$ gauge theory of the weak and electromagnetic interactions which predicts all neutral current processes as a function of one parameter (weak mixing angle $\theta_w$). The present experimental results are in agreement with $\sin^2 \theta_w = 0.23 \pm 0.02$.

One special feature of the neutrino-production experiments on nuclei is that the total cross sections for muon neutrino and antineutrino scattering are significantly different. One finds

$$\frac{\sigma \bar{\nu}_\mu N \rightarrow \bar{\nu}_\mu X}{\sigma \nu_\mu N \rightarrow \nu_\mu X} \approx 0.5.$$  

This result implies that the neutral current cannot be a pure vector or axialvector current, in which case the ratio eq. (1.6) would be 1. If we make the assumption that the effective neutral current Hamiltonian consists only of the product of one neutral current with itself, analogous to the situation for charged currents, the neutral current for the quarks must be a superposition of a vector current $V$ and an axialvector current $A$. In this case the neutral current Hamiltonian must contain terms of the type $V \cdot A$, i.e. terms which induce parity violation, e.g. in the interaction of electrons with nucleons. Recently it has been found that the interaction of electrons with deuterons violates parity [1.11]. The magnitude and sign of the observed parity violation is in agreement with the prediction of the minimal $SU(2) \times U(1)$ theory for $\sin^2 \theta_w \approx 0.23$.

1.2. Historical overview of gauge theories

The idea of non-Abelian gauge theories was formulated by Yang and Mills in 1954 [1.12]. Some years later various attempts have been made to unite the weak interactions and electromagnetism [1.13–1.16]. All these ideas are based on the assumption that the effective current–current interaction describing the weak decays is a manifestation of the exchange of a heavy charged intermediate vector boson which couples basically with electromagnetic strength to leptons and quarks. The effective Lagrangian (1.4) is replaced by

$$L^\text{weak} = g(W^- j^\mu_\pm + W^+ j^\mu_-)$$  

where $W_\mu$ is the field of the charged massive vector boson. The relation between the $W$-mass and $G$ is given by

$$4G/\sqrt{2} \approx g^2/M_{w}^2.$$  

After the structure of the charged weak currents was found it was especially emphasized by Glashow [1.15] and Salam and Ward [1.16] that a unification of the electromagnetism and weak interactions calls
for the existence of another type of weak interaction involving neutral currents, mediated by a heavy neutral vectorboson $Z$.

The main problem of interpreting the weak interactions as a manifestation of a W-boson theory has been the fact that the propagator of a massive vectorboson has the form

$$\frac{q^\mu q^\nu / M_w^2 - g_\mu^\nu}{q^2 - M_w^2},$$

where $q$ denotes the four-momentum of the virtual vectorboson and $M_w$ its mass. For $|q| \ll M_w$ which is the case for weak decays the term proportional to $q^\mu q^\nu$ can be neglected, and we can approximate the W-propagator by $-g^{\mu\nu}/q^2 - M_w^2$. This leads to eq. (1.8).

However, serious problems arise if $|q|$ becomes large compared to $M_w$. In renormalizable field theories the propagators of the various fields (e.g. the photon field in QED) have the property that they do not depend on dimensional parameters like masses if the momenta or energies involved become large. This is not the case for the massive vectorboson propagator (eq. (1.9)), whose asymptotic form is $(q^\mu q^\nu)/M_w^2 q^2$. For this reason theories involving massive vectorbosons are in general not renormalizable; new divergencies occur in each order of perturbation theory, due to vectorboson loops, which cannot be absorbed by redefining the parameters of the theory.

In 1967 it was suggested [1.17] that the divergencies of W-boson theories may be tamed sufficiently such as to render the theory renormalizable if one starts out with a theory in which all vectorbosons are massless, but acquire a mass finally by a spontaneous breakdown of the gauge symmetry.

This mechanism of spontaneous symmetry breaking was proposed and studied in 1964 by many authors [1.18]. It involves scalar fields and a vacuum state which is not invariant under gauge transformations. Since the equations of motion of the theory are identical to the ones obtained in a theory with massless vectorbosons, it was suggested [1.17] that theories in which the gauge boson masses are generated spontaneously are renormalizable. That this is indeed the case was shown in 1971 [1.19] (for a general presentation of the renormalization program and earlier references see e.g. [1.1]).

Since 1971 many papers have appeared, proposing many different models of the weak interactions. Since one is free to invent new quarks and leptons, provided they are sufficiently heavy, there exists a considerable amount of freedom in the building of models. Nevertheless, the experiments carried out in the recent years have produced results which put strong constraints on the building of models, and today only rather few models, besides the standard SU$_2 \times$ U$_1$ model, can be regarded as realistic.

We mention especially two ideas which have played an important rôle in the recent years. One was the proposition to generate the parity violation of the weak interactions by the spontaneous generation of fermion masses (vectorlike theories). Such theories have the property that there exist as many $V - A$ as $V + A$ weak currents (for a review see ref. [1.20]). An important class of such theories which predicted a pure vectorial neutral current is meanwhile ruled out by experiment. A second proposition is to generate the parity violation of the weak interaction by the spontaneous generation of gauge boson masses (chiral weak interactions). This will be discussed later.

Another important development took place in the recent years in strong interaction physics. It appears today that quantum chromodynamics (QCD) is a good candidate of a theory of the strong interactions. QCD is a non-Abelian gauge theory of colored quarks and gluons; the relevant gauge group is SU$_3$.

At first the idea that non-Abelian gauge fields are the source of the strong interactions was put forward in 1966 [1.21], within the framework of the Han–Nambu quark model [1.22]. Here the gluons
are massive, charged vector bosons. In 1972 an alternative model was proposed [1.23], in which quarks and gluons are supposed to be permanently confined, the gauge group (color group SU₃) remains unbroken, and the gluons stay formally massless. This theory was shown to be asymptotically free in 1973 [1.24], and hence unstable in the infrared region. Thus far theorists have been unable to demonstrate that the infrared instability of QCD leads indeed to the conjectured quark and color confinement; to do so remains one of the great challenges for theoreticians at present.

If it is indeed true that the weak and electromagnetic interactions on the one hand and the strong interactions on the other hand are described by QFD and QCD respectively, it may well be possible to describe all interactions (except, perhaps, gravity) by a Unified Gauge Theory, i.e. by a theory whose gauge group unites the flavor and color gauge group into a simple group. In recent years many examples of such theories were studied by various authors. It is typical for such theories that they involve rather large gauge groups (the smallest possible gauge groups are SU₅ and SO(10) [1.25, 1.26], and that some of the gauge bosons must be very heavy (in some cases as heavy as 10¹⁸ GeV). Although it is probably safe to say today that the correct unified theory of nature has not yet been found it may well be that one of the examples studied in the recent years is quite close to the true answer.

2. Gauge invariance and its spontaneous breaking

2.1. Abelian gauge invariance: Quantum electrodynamics

Gauge theories of the conventional type involve internal symmetries. As a trivial example, consider the U(1) group of phase transformations of a free massive fermion field \( \Psi(x) \):

\[ \Psi(x) \rightarrow e^{-i\alpha} \Psi(x), \quad (2.1) \]

where \( \alpha \) is an arbitrary phase parameter, independent of space and time. The corresponding Lagrangian density

\[ L(x) = \bar{\Psi}(x)(i\not{\partial} - m)\Psi(x) \quad (2.2) \]

is invariant under these transformations. The symmetry implies the existence of a conserved current, namely

\[ j_\mu(x) = \bar{\Psi}(x)\gamma_\mu \psi(x) \quad (2.3) \]

\[ \partial^\mu j_\mu = 0. \]

The conserved charge (generator of the U(1) symmetry group) can be written as an integral over the charge density:

\[ Q = \int d^3x j_0(x). \quad (2.4) \]

The invariance of the Lagrangian (2.2) under phase rotation implies that the phase parameter \( \alpha \) has no physical meaning, i.e. it can be chosen arbitrarily. It is therefore unnatural to fix \( \alpha \) uniquely over all
of space and time, and it would be more satisfactory to have the possibility of choosing it locally, without caring about what happens far away, i.e. to have an invariance of the sort
\[ \psi(x) \rightarrow e^{-i\alpha(x)}\psi(x), \]
where \( \alpha \) depends on space and time in an arbitrary way. However, the Lagrangian (2.2) is no longer invariant under the local phase rotations (2.5), since the derivative \( \partial_\mu \psi(x) \) is replaced after the rotation (2.5) by
\[ \partial_\mu \psi(x) \rightarrow e^{-i\alpha(x)}\partial_\mu \psi(x) + (-ie^{-i\alpha(x)}\partial_\mu \alpha(x)) \psi(x). \]

It is well-known how this problem can be solved. One has to introduce a covariant derivative \( D_\mu \), which has the property that \( D_\mu \psi \) transforms under phase rotations like \( \psi \) itself:
\[ D_\mu \psi(x) \rightarrow e^{-i\alpha(x)}D_\mu \psi(x). \]

Such a covariant derivative can only be introduced if there exists another field, a vector field \( A_\mu \), which interacts with the spinor field \( \psi \). The covariant derivative \( D_\mu \) is chosen as
\[ D_\mu = \partial_\mu + igA_\mu, \]
where \( g \) is an arbitrary coupling constant, and \( A_\mu \) transforms under a local phase transformation (gauge transformation) as follows:
\[ A_\mu(x) \rightarrow A_\mu(x) + \frac{1}{g} \partial_\mu \alpha(x). \]

It is easy to verify that the covariant derivative (2.8) fulfills the requirement (2.7). Hence the invariance of the Lagrangian (2.2) under gauge transformations is restored if we replace \( \partial_\mu \) by \( D_\mu \). However, for consistency reasons we have to add the kinetic term of the \( A_\mu \) field, which must be gauge invariant in itself, i.e. can only involve the gauge-invariant field strength
\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \]

Thus one obtains
\[ L = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \]

If we replace \( g \) by \( e \) (electric charge) and identify \( \psi \) with the electron field, \( A_\mu \) with the photon field, we obtain the Lagrangian of QED. Note that the requirement of local gauge invariance allows only the minimal coupling of the photon field to the electron field of the type \( e \cdot j_\mu A^\mu \). Furthermore a mass term for the photon of the type \( m^2 A_\mu A^\mu \) is not allowed to occur in the Lagrangian.

We emphasize that one is led in a natural way, by imposing local gauge invariance, from the free fermion theory described by the Lagrangian (2.2) to the interacting field theory described by the Lagrangian (2.11). The impressive success of QED in describing the interaction of electrons and
photons, which physicists have witnessed during the first half of this century, leads one to suspect that
the local gauge invariance of QED is not only a formal property of the theory, but is a very essential
ingredient of it. Today we are inclined to believe that this is indeed the case. Furthermore one believes
that the theory of QED is not complete in itself, but a subtheory of a larger system of gauge
interactions, which are all following the principle of local gauge invariance.

2.2. Non-Abelian gauge invariance: Quantum chromodynamics

The principle of local gauge invariance has been generalized by Yang and Mills in 1954 [2.1]. In
QED one is dealing with a very simple gauge symmetry: the phase transformations \( \psi \rightarrow e^{-i\alpha} \psi \) constitute
a U(1) group. What happens if a non-Abelian group is involved instead? Let us imagine a fictitious
world, where isospin is an exact symmetry. The Lagrangian for the system of free protons and neutrons
would be

\[ L = \bar{N}(i\partial - m)N \]  

(2.12)

where \( N \) stands for the isospinor \((\frac{1}{2})\). It is invariant under the SU(2) transformations

\[ N \rightarrow e^{-i\alpha(2)}N \]  

(2.13)

where \( \tau = (\tau_1, \tau_2, \tau_3) \) (\( \tau \): Pauli matrices), and \( \alpha \) is an arbitrary “phase vector”. The isotropic spin
currents \( \bar{N}\gamma_{\mu}^{1/2}\tau_iN \) are conserved; the associated charges \( T_i = \int d^3x \bar{N}\gamma_{\mu}^{1/2}\tau_iN \) generate the algebra of
SU(2):

\[ [T_i, T_j] = i\epsilon_{ijk}T_k. \]  

(2.14)

Note that the gauge transformations involving the Pauli matrices \( \tau_1, \tau_2 \), which are nondiagonal, mix
the p and n states with each other. This constitutes no problem, since in an isospin symmetric world
there would be no physical distinction between neutron and proton. What we call p or n is entirely up to
convention. Thus it would be natural to require that the labels p and n can be redefined locally in an
arbitrary way; in other words: we require invariance under non-Abelian gauge transformations:

\[ (\begin{array}{c} p \\ n \end{array}) \rightarrow e^{-i\alpha(x)}(\begin{array}{c} p \\ n \end{array}) \]  

(2.15)

where \( \alpha(x) \) is an arbitrary, space-time dependent phase vector. Doing so we encounter the same
problem as in the U(1) case discussed above: the derivative \( \partial_\mu N \) which appears in the Lagrangian (2.12)
does not transform under local gauge transformations as \( N \) itself, and we have to find a suitable
definition of a covariant derivative. In order to do so one introduces a triplet of vector gauge fields \( W_\mu^i \),
which transforms under an infinitesimal gauge transformation as follows:

\[ W_\mu^i \rightarrow W_\mu^i + \epsilon_{ijk}\alpha^j W_\mu^k + \frac{1}{g} \partial_\mu \alpha^i. \]  

(2.16)

This transformation law is analogous to eq. (2.9). The second term exhibits the local rotation of the
$W^i$ in the isotropic space. The covariant derivative is defined to be

$$D_\mu = \partial_\mu + igW_\mu$$

(2.17)

where $W_\mu = \frac{1}{2}i\tau W_\mu$. The Lagrangian of the system, including the kinetic term of the vector fields, is

$$L = \bar{N}(i\mathcal{D} - m)N - \frac{1}{4}G_{\mu\nu}^i G^{i\mu\nu},$$

(2.18)

where the $G_{\mu\nu}^i$ are the field strength tensors of the vector fields:

$$G_{\mu\nu}^i = \partial_\nu W_\mu^i - \partial_\mu W_\nu^i - g\epsilon_{ijk}W^{jk}W_{\mu\nu}.$$

(2.19)

The method described above can easily be generalized to the case where an arbitrary gauge group and an arbitrary fermion representation are involved. The only changes are:

(a) Replace the isospin matrices $\tau$ by the corresponding matrices describing the transformation properties of the fermions under the gauge group.

(b) Replace $\epsilon_{ijk}$ by the structure constants $f_{ijk}$ of the gauge group.

We emphasize that the fermions can transform as an arbitrary representation of the gauge group, while the vector gauge fields must transform according to the adjoint representation.

An important difference between the Abelian case and the non-Abelian one is that the vector fields in the non-Abelian case interact with each other directly ("they are charged"). This is not the case in the Abelian theory where the vector field is neutral.

The Lagrangian of the non-Abelian gauge theory (2.18) describes the interaction of massive fermions with massless gauge bosons. Like in the Abelian case the masslessness of the gauge bosons is required by gauge invariance.

While QED provides a beautiful example of a genuine Abelian gauge theory, it was believed for a long time, that no example of a genuine non-Abelian gauge theory exists in nature, simply due to the fact that no massless gauge bosons besides the photon are observed. This point of view, however, has changed considerably during the last few years. We have now many reasons to believe that the strong interactions can be described by a genuine non-Abelian gauge theory, by quantum chromodynamics [2.21. Here the gauge group is the color group $SU_3$, the fermions are tricolored quarks, the gauge bosons (gluons) constitute a color octet. The Lagrangian of QCD is

$$L = \bar{q}(i\mathcal{D} - m)q - \frac{1}{4}G_{\mu\nu}^{ij}G^{ij\mu\nu}.$$ 

(2.20)

Until today it is unclear what the particle content (mass spectrum) of a genuine non-Abelian gauge theory is, due to the unknown long-distance properties of such a theory. There exists indications that only singlets of the gauge group are allowed to exist as free particles (confinement). It is the confinement aspect of genuine non-Abelian theories which give us the right to think that the strong interactions are described by QCD. Only if absolute color confinement is true, we have no problem with the feature of QCD that the interaction among quarks is mediated by massless color octet gluons, and yet gluons need not be observed as free massless quanta.

2.3. Spontaneous symmetry breaking

If we assume that the weak interactions are mediated by massive vector bosons, the question arises:
What is the detailed form of the interaction of these vector fields with the fermions (leptons, quarks) or with themselves? It would be desirable if these interactions are also dictated by the requirement of local gauge invariance. However, we have seen above that it is forbidden to introduce vector boson mass terms by hand, since such terms destroy the gauge invariance of the theory in an intrinsic way. Thus one must look for another possibility to introduce masses for the gauge fields in a more subtle way, such that the local gauge invariance is not destroyed. This can be done by generating the gauge boson masses via a spontaneous breaking of the gauge symmetry.

What is spontaneous symmetry breaking? On many occasions in physics it happens that the equations of motion for a certain physical system are symmetric under some symmetry transformation, however the groundstate of the systems is not. For example the Hamilton operator for an infinitely extended ferromagnet is invariant under rotations in space. However, the groundstate breaks the rotational symmetry since the individual spins are always aligned in some arbitrary direction. In fact, there exist infinitely many different groundstates, corresponding to the infinity of choices to align the spins. Other examples are crystal lattices or superconductors. Analogous situations occur often in field theory, and below we shall discuss some examples of spontaneously broken field theories.

Example 1. The simplest relativistic field theory which exhibits the phenomenon of spontaneous symmetry breaking is the theory of a scalar field interacting with itself via a \( \lambda \phi^4 \) interaction. Consider the Lagrangian

\[
L = \frac{1}{2} (\partial \phi \partial \mu \phi) - \frac{1}{2} \mu^2 \phi^2 - \frac{1}{4} \lambda \phi^4.
\]  

For \( \mu^2 > 0 \) it describes a self-interacting scalar field with mass \( \mu \). The potential \( V(\phi) = \frac{1}{2} \mu^2 \phi^2 + \frac{1}{4} \lambda \phi^4 \) has the form described in fig. 1.

Obviously the Lagrangian (2.21) is invariant under the reflection symmetry \( R \), defined by \( \phi \rightarrow -\phi \). The ground state (vacuum state \( |0\rangle \)) of the theory is the one where \( \phi \) vanishes everywhere. It is invariant under the \( R \)-symmetry: \( R|0\rangle = |0\rangle \).

Suppose now we choose \( \mu^2 < 0 \). In this case the potential \( V(\phi) \) has the form shown in fig. 2.
The minimum of the potential is not at \( \phi = 0 \), but at \( \phi = \pm \sqrt{-\mu^2/\lambda} \). Consequently for the ground state of the theory we have \( \langle 0 | \phi | 0 \rangle = \pm \sqrt{-\mu^2/\lambda} \). If we define \( v = \langle 0 | \phi | 0 \rangle \), we either have \( v = + \sqrt{-\mu^2/\lambda} \) or \( v = - \sqrt{-\mu^2/\lambda} \) i.e. the theory possesses two possible ground states. These vacua are not invariant under the \( R \) symmetry since \( v \neq -v \). The \( R \) symmetry is spontaneously broken. It is useful to define a new field \( \phi' \), for which \( \langle 0 | \phi' | 0 \rangle = 0 \), i.e. \( \phi' = \phi - v \). In terms of \( \phi' \) one finds

\[
L = \frac{1}{2}(\partial_{\mu} \phi' \partial^{\mu} \phi') + \mu^2 |\phi'|^2 - \lambda |\phi'|^4 + \text{const.} \tag{2.22}
\]

One discovers that the Lagrangian (2.21) with \( \mu^2 < 0 \) describes selfinteracting scalar particles with mass \( \sqrt{2|\mu|} \).

Example 2. Let us consider the theory of a complex scalar field \( \phi = \frac{1}{2} \sqrt{2}(\phi_1 + i\phi_2) \), described by the Lagrangian

\[
L = (\partial_{\mu} \phi * \partial^{\mu} \phi) - \mu^2 |\phi|^2 - \lambda |\phi|^4 \tag{2.23}
\]

\[
= \frac{1}{2} \partial_{\mu} \phi_1 \partial^{\mu} \phi_1 + \frac{1}{2} \partial_{\mu} \phi_2 \partial^{\mu} \phi_2 - \lambda (|\phi_1|^2 + |\phi_2|^2)^2.
\]

It is invariant under the phase transformations \( \phi \to e^{-i\theta} \phi \). For \( \mu^2 > 0 \) the theory describes a selfinteracting scalar complex field of mass \( \mu \). Suppose now we choose \( \mu^2 < 0 \). In this case the potential \( V(\phi) \) exhibits its minimum at \( (\phi_1^2 + \phi_2^2) = \frac{1}{2} |\phi|^2 = -\mu^2/\lambda \). Thus the minimum of the potential occurs along a circle of radius \( \sqrt{-\mu^2/\lambda} \) around the origin (see fig. 3).

This time we are dealing with an infinite number of possible vacua, since one can pick any point on the circle as the vacuum state.

Let us take one point on the circle, described by the coordinates \( v = (v_1, v_2) \), as the vacuum. Since the Lagrangian is invariant under phase transformations, we can choose this point to lie on the positive real axis, i.e.

\[
v = (\sqrt{-\mu^2/\lambda}, 0), \text{ which implies } \langle 0 | \phi_1 | 0 \rangle = \sqrt{-\mu^2/\lambda}, \langle 0 | \phi_2 | 0 \rangle = 0.
\]

Now we define \( \phi_1' = \phi_1 - \langle 0 | \phi_1 | 0 \rangle \), in which case the Lagrangian (2.23) takes the form

\[
\begin{align*}
\varphi_2 \\
\varphi_1
\end{align*}
\]

Fig. 3. The minimum of the potential in case of a complex scalar field exhibiting a spontaneous symmetry breaking.
\[ L = \frac{1}{2}(\partial_\mu \phi_1)^2 + \frac{1}{2}(\partial_\mu \phi_2)^2 + \mu^2 \phi_1^2 - \frac{1}{2} \lambda v \phi'(\phi_1^2 + \phi_2^2) - \frac{1}{4} \lambda (\phi_1^2 + \phi_2^2)^2, \] 
(2.24)

where \( v = \sqrt{-\mu^2/\lambda} \).

We observe that the field \( \phi_1 \) describes a particle of mass \( \sqrt{2|\mu|} \), but \( \phi_2 \) remains massless.

The reason for the masslessness of the \( \phi_2 \)-field becomes clear if we consider fig. 3. The field \( \phi_2 \) describes the excitations tangentially to the circle (orbit); in this direction no resistance from the potential is felt. Therefore, the field \( \phi_2 \) describes massless excitations.

The system described above provides an example of the Goldstone theorem [2.3] which states that massless Goldstone bosons appear if a continuous symmetry of a physical system described by some group \( G \) is spontaneously broken. In the example discussed above the symmetry group is the group \( U(1) \) of the phase transformations.

The example described above exhibits an invariance under the global gauge group \( U(1) \) which is isomorphic to \( O(2) \). It can be immediately generalized to involve the gauge group \( O(n) \). We consider the Lagrangian

\[ L = \frac{1}{2}(\partial^\mu \phi_i - \frac{1}{2} \mu^2 \phi_i - \frac{1}{4} \lambda (\phi_i^2)^2, \]
(2.25)

\[ i = 1, 2, \ldots, n; \]

(\( \phi_i \): real scalar field; summation over \( i \) is understood).

This Lagrangian is invariant under the group \( O(n) \). For \( \mu^2 < 0 \) the minimum of the potential is at \( v = \sqrt{-\mu^2/\lambda} \). The minimum of \( V(\phi) \) is achieved whenever \( \phi_i \phi_i = -\mu^2/\lambda \), i.e. it occurs on the \( n \)-dimensional sphere of radius \( \sqrt{-\mu^2/\lambda} \) in the \( n \)-dimensional space defined by the fields \( \phi_i \). Due to the \( O(n) \)-invariance of the Lagrangian we can choose our coordinate system of fields such that the vacuum expectation value of the field vector \( \phi_i \) is as follows:

\[ \langle 0 | \phi | 0 \rangle = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ v \end{bmatrix}. \]
(2.26)

The vacuum expectation value of \( \phi \) breaks the \( O(n) \)-invariance in a very specific way. Since the first \( (n - 1) \) components of \( \langle 0 | \phi | 0 \rangle \) are zero, the vacuum is still invariant under the subgroup \( O(n - 1) \). Suppose we consider the point (2.26) in the \( n \)-dimensional space of the fields \( \phi_i \). There exist \( (n - 1) \) linearly independent directions to leave this point, but to stay on the sphere which minimizes the potential. Consequently there must exist \( (n - 1) \) massless Goldstone bosons. Thus the Lagrangian (2.25) describes a massive field of mass \( \sqrt{-2\mu^2} \) and \( (n - 1) \) massless Goldstone bosons.

The group \( O(n) \) has \( \frac{1}{2}n(n - 1) \) generators. The difference "number of generators of \( O(n) \) minus number of generators of \( O(n - 1) \)" = \( \frac{1}{2}n(n - 1) - \frac{1}{2}(n - 1)(n - 2) \) is equal to \( (n - 1) \), which is the number of generators of \( O(n) \) which do not leave the vacuum invariant. On the other hand we have \( (n - 1) \) massless Goldstone bosons, i.e. the number of Goldstone bosons is equal to the number of generators which are broken spontaneously. Although it looks as if this feature is a special property of the \( O(n) \) model we have investigated, it is in fact a general feature of spontaneously broken theories involving
2.4. Spontaneous generation of mass

Let us again consider the Lagrangian

\[ L = \partial^{\mu} \phi^* \partial_{\mu} \phi - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2, \]

which is invariant under the global gauge transformations \( \phi \rightarrow e^{-i\alpha} \phi \) (U(1) gauge group). However, now we request invariance under the local gauge transformations \( \phi \rightarrow e^{-i\alpha(x)} \phi \). This can be achieved by introducing a gauge field \( A_{\mu} \). Following the procedure outlined in section 2.1 we find the Lagrangian

\[ (D^{\mu} \phi)^* D_{\mu} \phi - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2 - \frac{i}{4} F_{\mu\nu} F^{\mu\nu}, \]

where \( F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} \) and \( D_{\mu} = \partial_{\mu} + igA_{\mu} \). We note that the various fields transform under local gauge transformations as follows:

\[ \phi(x) \rightarrow e^{-i\alpha(x)} \phi(x) \]
\[ A_{\mu}(x) \rightarrow A_{\mu}(x) + \frac{1}{g} \partial_{\mu} \alpha(x). \]

In case \( \mu^2 > 0 \) the Lagrangian (2.28) describes simply the system of a massive scalar field, coupled to a massless gauge field \( A_{\mu} \), i.e. for \( g = e \) one is dealing with scalar electrodynamics. However, for \( \mu^2 < 0 \) the gauge symmetry is spontaneously broken, as described in section 2.3. There we have seen that the Lagrangian (2.27) describes a massive scalar field, accompanied by a massless Goldstone boson. What is happening in case of the Lagrangian (2.28)? Making the substitution

\[ \phi = \phi^0 + (0|\phi^0|0) = \phi^0 + \nu, \quad \nu = \sqrt{-\mu^2/\lambda} \]

in eq. (2.28) we find that especially the following new terms appear:

\[ \frac{1}{2} g^2 v^2 A_{\mu} A^{\mu} \]
\[-g v A_{\mu} \partial^{\mu} \phi_2. \]

The interpretation of the term (2.32) is somewhat obscure, since it mixes the gauge field with the "would be"-Goldstone boson \( \phi_2 \). In order to clarify the situation, let us consider the gauge transformation \( \phi \rightarrow e^{-i\alpha} \phi \) in terms of \( \phi^0 \) and \( \phi_2 \).

For an infinitesimal parameter \( \alpha \) we have

\[ \phi \rightarrow (1 - i\alpha)\phi, \quad \phi_1 \rightarrow \phi_1 - \alpha \phi_2, \quad \phi_2 \rightarrow \phi_2 + \alpha \phi_1, \]

and one finds

\[ \phi^0 \rightarrow \phi^0 - \alpha \phi_2, \quad \phi_2 \rightarrow \phi_2 + \alpha \nu + \alpha \phi^0. \]
Thus $\phi_z$ undergoes an inhomogeneous gauge transformation, like the vector field $A_\mu$. Therefore we can use our freedom of gauge in order to set $\phi_z = 0$, in which case the mixing term (2.32) and the Goldstone boson disappear.

Thus the introduction of the gauge field $A_\mu$ and the requirement of local gauge invariance changes the physical situation entirely: While the Lagrangian (2.27) describes a massive scalar field accompanied by a massless Goldstone boson, we are dealing now, in case of the Lagrangian (2.28), with a system consisting of a massive vector meson $A_\mu$ and a massive scalar boson $\phi$. However, the total number of particle states has remained unchanged. We started with a massless vector boson (two polarization states) and two spin zero particles, i.e. four states altogether. We end up with a massive vector boson (three polarization states) and a spin zero state. Thus the Goldstone boson has been "eaten up" by the gauge field; it has been exchanged for the longitudinal component of the massive vector field.

The spontaneous generation of gauge boson masses described above is important in the following respect. In general theories involving massive gauge bosons are nonrenormalizable, due to the $k_\mu k_\nu / m^2$-term in the gauge boson propagator. However, in the original Lagrangian (2.28) the gauge field is formally massless; no problems with renormalizability are present. It turns out that the theory remains renormalizable even after the spontaneous symmetry breaking, since the short distance structure of the theory is not affected by the symmetry breaking [1.19].

We should note that for the Abelian gauge theory described by the Lagrangian (2.28) the detour using the formalism of the spontaneous symmetry breaking was not really necessary in order to achieve renormalizability. Here a mass term for the gauge field can be introduced by hand, without spoiling the renormalizability of the theory, provided the $A_\mu$-field is coupled to a conserved current. This is not true anymore for a non-Abelian theory; here the spontaneous generation of the gauge boson masses is the only way to guarantee renormalizability.

2.5. Examples of spontaneously broken gauge theories

In this section we should like to discuss some examples of spontaneously broken gauge theories. None of them have an immediate physical meaning, but we should like to describe some of the features of spontaneously broken gauge theories in terms of those examples.

**Example 1.** We consider the gauge group $SU(2)$ and choose a doublet representation of complex scalar fields, coupled in a gauge invariant manner to the gauge fields. The Lagrangian of such a theory is given by

$$ L = -\frac{1}{4} G_\mu^a G_\nu^a + \left( \partial^\mu \phi + i \frac{\xi}{2} \tau^i B_\mu^i \phi \right) \left( \partial_\mu \phi + i \frac{\xi}{2} \tau^i B_\mu^i \phi \right) - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2, $$

(2.35)

where the scalar field $\phi$ stands for the $SU_2$ doublet

$$ \phi = (\phi^a, \phi^b), $$

(2.36)

and the $\tau_i$ denote the Pauli matrices.

In case of $\mu^2 > 0$ the Lagrangian given in eq. (2.33) describes a system of massless gauge fields in interaction with massive scalars of mass $\mu$. Now let us choose $\mu^2 < 0$. In this case the minimum of the potential $V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$ occurs at finite values of $\phi$. The manifold of points in the space of fields $\phi^a$, $\phi^b$, for which $V(\phi)$ is minimized is invariant under $SU(2)$-transformations. Consequently we
can choose a particular SU(2) frame for which we have \( \langle 0 | \phi | 0 \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \cdot 1/\sqrt{2} \), where \( v = \sqrt{-\mu^2/\lambda} \). From eq. (2.35) we read off the gauge boson mass term, which is

\[
(g^2/4)[(\tau^i B^\mu_i) \phi] [((\tau^i B^\mu_i) \phi] = (g^2/4)(\phi^+ \tau^i \phi)(B^\mu_i B^{\mu i})
\]

\[
= (g^2/8) \cdot v^2[(B^\mu_1)^2 + (B^\mu_2)^2 + (B^\mu_3)^2]. \tag{2.37}
\]

We conclude: After spontaneous symmetry breaking we obtain massive gauge fields (mass \( \frac{1}{2}gv \)). The gauge boson mass matrix is SU(2) symmetric, i.e. the three gauge bosons are degenerate in mass. This is a special property of the spontaneous symmetry breaking involving a \textit{SU(2) doublet}.

We note the particle content of the theory: three massive gauge fields, one massive scalar. Three of the scalar fields (originally four real fields) are “eaten up” to provide the longitudinal components of the massive gauge fields.

\textbf{Example 2.} We consider the same gauge group SU(2), but choose a SU(2) triplet representation of real scalar fields. The Lagrangian of the theory is

\[
L = -\frac{1}{4} G_{\mu \nu} G^{\mu \nu} + \frac{1}{2} (\partial \phi_i - g \epsilon_{ijk} B^\mu \phi_k)(\partial \phi_i - g \epsilon_{ilm} B_\mu \phi_m) - \frac{1}{2} \mu^2 (\phi_i \phi_i) - \frac{1}{4} \lambda (\phi_i \phi_i)^2. \tag{2.38}
\]

If we take \( \mu^2 < 0 \), spontaneous symmetry breaking occurs. We can always choose our coordinate system of fields such that we have

\[
\langle 0 | \phi | 0 \rangle = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}. \tag{2.39}
\]

This vector remains invariant under rotations generated by \( T_3 \), i.e. the subgroup U(1) \( \subset \text{SU}(2) \), generated by the third generator \( T_3 \), remains unbroken.

The mass term of the vector bosons is given by

\[
\frac{1}{2} g^2 \epsilon_{ijk} B^\mu_i \langle \phi_k | 0 \rangle \epsilon_{ilm} B^\mu_l \langle 0 | \phi_m | 0 \rangle. \tag{2.40}
\]

Using \( \langle 0 | \phi_k | 0 \rangle = v \delta_{k,3} \), we obtain

\[
\frac{1}{2} g^2 v^2 \epsilon_{ijk} \epsilon_{ilm} B^\mu_i B^\mu_l = \frac{1}{2} g^2 v^2 [(B^\mu_1)^2 + (B^\mu_2)^2]. \tag{2.41}
\]

The vector bosons coupled to the broken generators \( T_1 \) and \( T_2 \) have acquired a mass \( M = gv \). No mass is generated for the third boson \( B^\mu_3 \), which is coupled to the conserved generator \( T_3 \).

\textbf{Example 3: SU(3).} In our third example we consider a gauge theory based on the gauge group SU(3). The gauge bosons form an SU(3) octet, which we denote by \( B^\mu_i \) \( (i = 1, \ldots, 8) \). Let us first study the case where the SU(3) symmetry is broken via the coupling of the gauge bosons to a SU(3) triplet of complex scalar fields \( \phi_i = (\phi_1, \phi_2, \phi_3) \).

The potential of the scalar field is e.g.

\[
V(\phi) = \mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2. \tag{2.42}
\]
For $\mu^2 < 0$ $\phi$ develops a vacuum expectation value. We can always choose a frame in the SU(3) space where $\langle 0 | \phi | 0 \rangle = \phi_0$ takes the simple form

$$\langle 0 | \phi | 0 \rangle = v \cdot (0, 0, 1).$$ (2.43)

The gauge boson mass matrix which results is proportional to

$$(\phi^*_a \lambda_b \phi_0) B_a B^b.$$ (2.44)

Since the first two components of $\langle 0 | \phi | 0 \rangle$ vanish, we are left with the subgroup SU(2) of SU(3) which remains unbroken. Consequently the spectrum of the gauge bosons which results is as follows. The bosons $B_1, B_2, B_3$ remain massless. The bosons $B_4, B_5, B_6, B_7$ acquire a common mass $M$, and the eight boson $B_8$ acquires a mass $(2/\sqrt{3})M$. We started out from a triplet of complex scalar fields, i.e. six real fields. Five of them are absorbed in providing the longitudinal components of the gauge bosons $B_4, \ldots, B_8$. We are left with one massive scalar boson.

Now it is easy to see what happens in the case where two scalar triplets $\phi_a, \phi_b$ are introduced. If we choose our frame in SU(3) space such that $\langle 0 | \phi | 0 \rangle = v(0, 0, 1)$, it will not be possible to require that $\langle 0 | \phi | 0 \rangle$ has the same form. In general we shall have $\langle 0 | \phi | 0 \rangle = (v_1, v_2, v_3)$ where $v_i \neq 0$ $(i = 1, 2, 3)$. Since $\phi_1$ generates a breakdown of the SU(3) symmetry such that an unbroken SU(2) symmetry is left over, we can use the residual SU(2) symmetry to choose a special frame such that $v_1 = 0$. Thus we obtain equal but nonzero masses for $B_1, B_2, B_3$ (see example 1). All gauge bosons have become massive, and the various masses depend on $v_2$ and $v_3$.

One may ask the question: Is it possible to arrange the spontaneous symmetry breaking such that all eight gauge bosons acquire the same mass? In this case the local SU(3) gauge invariance would be broken, however, a global SU(3) symmetry is preserved. Note that an analogous situation is realized in case of SU(2) if the spontaneous symmetry breaking is generated by a SU(2) doublet of scalars (see example 1).

A SU(3) symmetric mass term can be generated if we introduce three complex scalar triplets $\phi^a, \phi^b, \phi^c$ which we write in matrix form as follows:

$$\phi = \begin{pmatrix} \phi^a_1 & \phi^a_2 & \phi^a_3 \\ \phi^b_1 & \phi^b_2 & \phi^b_3 \\ \phi^c_1 & \phi^c_2 & \phi^c_3 \end{pmatrix} [SU(3)]^\phi_{\text{local}}. \quad (2.45)$$

The indices $(a, b, c)$ can be viewed as group indices involving a new SU(3) group, which is not gauged and which we denote by $[SU(3)]^\phi_{\text{global}}$. This group acts only in the $\phi$-space ($\phi$ transforms as a $\bar{3}$-representation) and leaves the gauge bosons invariant. Now we require the self-interaction of the scalars to be not only invariant under the group $[SU(3)]_{\text{local}}$, but under the product $[SU(3)]_{\text{local}} \times [SU(3)]^\phi_{\text{global}}$. Especially we can take

$$V(\phi) = \mu^2 \text{tr}(\phi^* \phi) + \lambda \text{tr}(\phi^* \phi)^2.$$ (2.46)

By applying a SU(3) x SU(3) transformation we can always write $\phi$ in the diagonal form
\[\phi = e^{i\delta} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \]  

(2.47)

where \(\alpha, \beta, \gamma\) are real, and \(\delta\) is an arbitrary phase parameter. For \(\mu^2 < 0\) the minimum of the potential occurs at \(\alpha^2 = \beta^2 = \gamma^2 = -\mu^2/2\lambda\), i.e. we have

\[\phi_0 = \langle 0 | \phi | 0 \rangle = \sqrt{\frac{-\mu^2}{2\lambda}} \ e^{i\delta} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}. \]  

(2.48)

The gauge boson mass matrix is proportional to \(\text{tr}(\phi_0^\dagger \lambda_i \phi_0) = (-\mu^2/2\lambda) \ \text{tr}(\lambda_i \lambda_j) \sim \delta_{ij}\), i.e. the resulting mass spectrum is invariant under global SU(3) transformations.

2.6. The general case

After discussing the examples described above, we are in a position to discuss the general features of spontaneously broken gauge theories. Let us consider a Lagrangian which is invariant under local gauge transformations of a group \(G\), which we assume to be simple. The \(N\) generators \(T_i\) of \(G\) obey the commutation relations

\[[T_i, T_j] = if_{ijk} T_k \quad (i, j, k = 1, \ldots, N) \]  

(2.49)

\((f_{ijk}: \text{structure constants of } G)\). An arbitrary infinitesimal transformation of the group \(G\) can be parametrized by \(1 - ie_i T_i\) (\(e_i: \text{infinitesimal parameters}\)).

The scalar fields \(\phi\) are assumed to transform according to a \(n\)-dimensional, in general reducible representation of \(G\).

It is assumed that the fields \(\phi\) are real. Note that a complex field \(\phi\) can always be decomposed into two real ones by considering the fields \(\frac{1}{2}(\phi + \phi^*)\) and \(-i\sqrt{2})(\phi - \phi^*)\). For an infinitesimal transformation of \(G\) one has:

\[\delta \phi = -ie_i S_i \phi \]  

(2.50)

where the \(S_i\) are \(n \times n\) dimensional Hermitian matrices, satisfying the commutation relations:

\[[S_i, S_j] = if_{ijk} S_k. \]  

(2.51)

The Lagrangian is given by:

\[L = -\frac{1}{4} G_{\mu\nu}^\alpha G^{\mu\nu}_\alpha + \frac{1}{2} [(\partial \mu + ig S_i A^\mu_i) \phi]^\dagger [(\partial \mu + ig S_i A^\mu_i) \phi] - V(\phi) \]  

(2.52)

where \(V\) is a quartic potential in \(\phi\), invariant under \(G\).

We assume that the potential is such that spontaneous symmetry breaking occurs and that the potential is minimized by setting \(\phi = v\), where \(v\) is a \(n\)-dimensional vector.
The vectorboson mass matrix is then given by

\[(M^2)_{\nu} = -g^2(S_{\nu}) \cdot (S_{\nu}). \] (2.53)

In general there will exist a \(M\)-dimensional subgroup \(G' (M < N)\) of \(G\), which remains a symmetry of the vacuum. Let \(T_i(G')\) be the generators of \(G'\).

We have \(T_i(G') \nu = S_i(G') \nu = 0\). There exist \(N - M\) generators of \(G\), for which \(T_i \nu \neq 0\), i.e. one has \(N - M\) “would be” Goldstone bosons. Thus the \(N \times N\) dimensional mass matrix denoted in eq. (2.53) is in fact only a \((N - M) \times (N - M)\) dimensional matrix, if we leave out all terms for which \(S_i \nu = 0\) due to the \(S\)-invariance of the vacuum. The mass matrix eq. (2.53) has to be diagonalized if we want to find the massive vector bosons of definite mass. There exist \(N - M\) massive vector bosons. The \(N - M\) “would be” Goldstone bosons are absorbed into the longitudinal components of the \(N - M\) massive vector bosons. The \(M\) remaining vectorbosons stay massless, due to the \(G'\) invariance of the vacuum.

**Spontaneous generation of fermion masses.** If we include fermions in the gauge theory, we have to add to the Lagrangian (2.52) the terms

\[L_{\text{fermion}} = \bar{\psi}_L (i \gamma - g f_L A_i) \psi_L + \bar{\psi}_R (i \gamma - g f_R A_i) \psi_R - (m \bar{\psi}_R \psi_L + \text{h.c.}) \] (2.54)

and

\[L_{\text{int}} = -G\bar{\psi}_R (R\phi) \psi_L + \text{h.c.} \] (2.55)

where \(\psi_L, \psi_R\) denote the lefthanded and righthanded fermion fields. The fields \(\psi_L, \psi_R\) transform under \(G\) as certain irreducible representations (not necessarily the same for \(\psi_L\) and \(\psi_R\)). The matrices \(f_L, f_R\) denote the transformation properties of the lefthanded and righthanded fermion fields. In eq. (2.55) we have included the term describing the Yukawa interaction of the fermion with the scalar fields. The matrices \(R\) are constructed such that \(\bar{\psi}_R (R\phi) \psi_L\) is invariant under the gauge group. In order to construct such an invariant, it is necessary that the product of representations \((\psi^*)_R \times (\psi)_L \times (\phi)\) contains the (1)-representation of \(G\). Of course, this is not always the case. If the direct product denoted above contains several (1)-representations (e.g. if the field \(\phi\) transforms as a reducible representation of \(G\)) one has to include several Yukawa interaction terms, involving several coupling constants \(G\).

We have also included a bare mass term \((m \bar{\psi}_R \psi_L + \text{h.c.})\). Also this term must be \(G\)-invariant. In order to construct such a term, it is necessary that the direct product \((\psi^*)_R \times (\psi)_L\) contains the (1)-representation.

From the eqs. (2.54, 2.55) we can read off the fermion mass matrix after the spontaneous symmetry breaking. One finds

\[L_{\text{fermion mass}} = -G\bar{\psi}_R (R\nu) \psi_L - m \bar{\psi}_R \psi_L + \text{h.c.} \] (2.56)

If several irreducible representations of fermions are involved, one has to add the various contributions to the Lagrangian (2.55). In this case one has, of course, the freedom to choose the various Yukawa coupling constants independently.
2.7. Model building

A large variety of gauge models has been discussed during the last years by many authors, and it seems that the building of models will continue in the future, until a general consensus is reached about what the correct gauge theory of leptons and quarks may be. For this reason we give below the general recipe for building renormalizable gauge theories. The Lagrangian is constructed as follows:

1. Choose the gauge group, the representations of lefthanded and righthanded fermions and the scalar fields.
2. Couple the gauge fields invariantly to the fermions and scalars.
3. Couple the gauge invariant quartic polynomial of the scalar fields such that the potential attains its minimum for nonvanishing vacuum expectation values $v$.
4. Construct the gauge invariant Yukawa couplings between the fermions and the scalars.

The gauge boson mass matrix which results has the structure:

$$ -\frac{1}{2} g^2 v^2 W_{\mu}^2, $$

the fermion mass matrix is

$$ G \cdot v \phi \bar{\phi}. $$

We should like to add the following comment about the mass spectrum of gauge bosons and fermions. If we are dealing with some gauge group $G$ and generate the spontaneous breakdown of the gauge symmetry by using a certain irreducible representation of scalar fields, the gauge boson and fermion mass spectrum which results will be rather specific. However, one has always the freedom to add more representations of scalar fields, i.e. use a reducible representation of $G$. Doing so, a considerable amount of freedom exists to adjust the various masses of the gauge bosons and fermions. Of course, a fairly large number of scalar fields may appear in this case, which is somewhat unsatisfactory. However, it may be that our present methods to generate the spontaneous symmetry breaking are too simple-minded. For example it has been speculated that the spontaneous breaking of gauge symmetries can be generated dynamically, in which case the scalar fields appear as bound states of the basic fermion fields, and one may tolerate a fairly large number of them. Keeping the possibility of dynamical symmetry breaking in mind, we note for our later discussion of the various gauge models that an economy in the sector of scalar fields should not be required a priori for a realistic gauge theory.

2.8. Triangle anomalies

Gauge theories (both unbroken and spontaneously broken ones) are renormalizable, provided one additional condition is fulfilled: No triangle anomalies must exist.

The coupling of the gauge bosons to the fermions involves in general vector and axialvector fermion currents. For the proof of the renormalizability of gauge theories it is essential that these currents are conserved in the absence of spontaneous symmetry breaking. However, the divergencies of axial vector currents receive in general anomalous contributions from fermion triangle graphs [2.4]. For example in massless QED one has

$$ \partial^\mu (\bar{\psi} \gamma_\mu \gamma_5 \psi) = -\frac{e^2}{16\pi^2} F_{\mu\nu} F_{\rho\sigma} e^{\mu\nu\rho\sigma}, $$

(2.57)
i.e. the divergence of the axialvector current contains a term of scale dimension 4. This term arises due to the presence of fermion triangles. A triangle whose points are linked together by fermion lines is a very singular object in the limit where all three points are close in coordinate space. Leaving out the complications due to the \( \gamma \)-matrices, it behaves in this limit like

\[
\frac{1}{(x - y)^3} \frac{1}{(x - z)^3} \frac{1}{(y - z)^3}
\]

\((x, y, z: \text{positions of the three points in the coordinate space})\). The singularities caused by the fermion propagators generate new terms in the divergence of the axialvector currents [2.4]. For example in massless QED one finds

\[
\partial^\mu (\bar{\psi} \gamma_\mu \gamma_5 \psi) = - \frac{e^2}{16 \pi^2} F_{\mu\nu} F_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma} \tag{2.57}
\]

where \( \psi \) denotes the field of the charged fermion, \( F_{\mu\nu} \) the electromagnetic fieldstrength, and \( \epsilon^{\mu\nu\rho\sigma} \) the total antisymmetric \( \epsilon \)-symbol in four dimensions. In fig. 4 we have shown the Feynman diagram which is responsible for eq. (2.57). We emphasize that the divergence of the axialvector current given in eq. (2.57) is called anomalous since it cannot be obtained by formal manipulations of the field equations. Formally the divergence of the axialvector current in massless QED vanishes. The anomalous divergence of the axialvector current is generated since the axialvector current defined by \( \bar{\psi} \gamma_\mu \gamma_5 \psi \) contains a singularity proportional to \( F_{\mu\nu} \) as \( x \to y \). The divergence of this singular term is given by the r.h.s. of eq. (2.57).

The scale dimension of the divergence of the axialvector current is four, i.e. we are dealing with a "hard" breaking of the \( \gamma_5 \)-invariance. This poses no problem for QED itself, since the axialvector current in QED is not coupled to the photon. Suppose we introduce a gauge boson which is coupled to the axialvector current, denoted by \( Z \). Such a theory will not be renormalizable, since due to the nonzero divergence of the axialvector current the gauge invariance, a necessary requirement for renormalizability, is lost. This manifests itself in an unphysical behaviour of certain scattering amplitudes at high energies. For example, we may consider the scattering of a photon and an electron in a modified theory of QED, where a massive \( Z \) is coupled to the axialvector fermion current. In fourth order of the gauge coupling constant there exists a diagram involving a fermion triangle (see fig. 5).
The scattering amplitude given by this graph exhibits an unphysical behaviour at high energies, i.e. it violates the bounds imposed by unitarity. The only way to achieve this is to make sure that the contributions due to the triangle anomalies cancel. This is impossible in a theory involving only one type of fermion. However, if several fermions contribute such cancellations can occur.

The general condition for the cancellation of the anomalies can be obtained as follows: We consider a general gauge group $G$. The coupling of the fermions $\psi$ to the gauge bosons is given by

$$L^{\text{int}} = g B^\mu_{\alpha} (\bar{\psi}_L \gamma_\mu A^L_{\alpha} \psi_L + \bar{\psi}_R \gamma_\mu A^R_{\alpha} \psi_R)$$

(2.58)

where the $B^a_{\mu}$ denote the gauge boson fields, and $A^L_{\alpha}$ are Hermitean matrices.

One has

$$[A^L_{\alpha}, A^L_{\beta}] = i f_{abc} A^L_c, \quad [A^R_{\alpha}, A^R_{\beta}] = i f_{abc} A^R_c$$

(2.59)

where the $f_{abc}$ are the structure constants of the gauge group. Since the lefthanded and righthanded fermions need not transform the same way under $G$, one has in general $A^L_{\alpha} \neq A^R_{\alpha}$. The fermion triangle graph involving three boson vertices has an anomaly which is proportional to

$$A^L_{\alpha \beta \gamma} - A^R_{\alpha \beta \gamma}$$

(2.60)

where

$$A^L_{\alpha \beta \gamma} = \text{tr}(A^L_{\alpha} A^L_{\beta} A^L_{\gamma})$$

(2.61)

and

$$A^R_{\alpha \beta \gamma} = \text{tr}(A^R_{\alpha} A^R_{\beta} A^R_{\gamma}).$$

Note that $A^{L(R)}_{\alpha \beta \gamma}$ is totally symmetric in $\alpha \beta \gamma$, i.e.

$$\text{tr}[A_{\alpha}, A_{\beta}] A_{\gamma} \sim [\text{tr} A_{\alpha} A_{\beta} A_{\gamma}]_{\text{symmetrized}}.$$

The theory is free of anomalies if $A^L_{\alpha \beta \gamma} - A^R_{\alpha \beta \gamma}$ vanishes for all values of $\alpha$, $\beta$, $\gamma$.

One may consider two cases of anomaly-free theories [2.5].

1. $A_L = A_R \neq 0$. In this case the anomalies caused by the lefthanded fermions are cancelled by the anomalies caused by the righthanded fermions. Evidently this is e.g. the case if $A^L_{\alpha} = A^R_{\alpha}$, i.e. if the theory is a vector gauge theory, or if $A^L_{\alpha} = U^{-1} A^R_{\alpha} U$ where $U$ is a fixed unitary matrix. In the latter case one can write the gauge currents in terms of vector currents alone, if we redefine the fermion fields:

$$\bar{\psi}_L \gamma_\mu A^L_{\alpha} \psi_L + \bar{\psi}_R \gamma_\mu A^R_{\alpha} \psi_R = \bar{\psi}_L \gamma_\mu A^L_{\alpha} \psi_L + \bar{\psi}_R \gamma_\mu U^{-1} A^L_{\alpha} U \psi_R = \bar{\psi}_L \gamma_\mu A_a \psi,$$

(2.62)

where $\psi = \psi_L + U \psi_R$, $A_a = A^L_{\alpha}.$

Note that in general the redefinition of $\psi$ generates $\gamma_5$ terms in the fermion mass matrix. Such a theory is called vectorlike.

An example of a vectorlike theory is QCD. Here the lefthanded quarks generate anomalies in the gauge currents, likewise the righthanded quarks. However the sum of both anomalies vanishes. This is
due to the fact that both the lefthanded and righthanded quarks transform as triplet representations under the color group SU(3). This, of course, is necessary since the theory must be parity conserving. Also QED is an example of a vectorlike theory. The anomaly generated by the contribution of the lefthanded electron is cancelled by the contribution of the righthanded electron.

2. $A^L = A^R = 0$. In this case the fermion representation is such that the anomalies cancel separately for the lefthanded and righthanded fermions. This is, for example, the case if the fermion representations are real. For a real representation, i.e. a representation which is equivalent to its complex conjugate, one has

$$A_a = -U^{-1}A^*_a U$$

where $U$ is a unitary matrix. Consequently we have

$$A_{abc} = \text{tr}(\{A_a, A_b\}A_c) = -\text{tr}(\{A^*_a, A^*_b\}A^*_c)$$

$$= -\text{tr}(\{A_a, A_b\}A_c) = -A_{abc},$$

i.e. $A_{abc}$ vanishes. **Real representations are safe: they do not produce anomalies.** However, a safe representation need not be real.

It is useful to investigate which simple Lie algebras have safe representations [2.5].

(a) SU($n$). The group SU(2) has only real representations, which implies that theories based on SU(2) have no anomalies. Higher unitary groups (SU(3), ...) are not safe, since they have complex representations for which $A_{abc} \neq 0$ (e.g. the 3-representation of SU(3)).

(b) SO($n$). The groups SO($2k+1$) ($k \geq 1$) and SO($4k$) ($k > 2$) have only real representations and are safe. The groups SO($4k+2$) ($k \geq 1$) have complex representations, which, however, are all safe, except for the complex representations of SO(6). Note that SO(6) is isomorphic to SU(4).

(c) Sp($2k$). The symplectic groups Sp($2k$) ($k \geq 3$) are safe since they have only real representations.

(d) The exceptional groups G(2), F(4), E(7) and E(8) have only real representations, hence are safe. The group E(6) has complex representations, but it has been shown [2.5] that all representations of E(6) are free of anomalies.

We conclude:

The only simple Lie groups which are not safe are the unitary groups SU($n$) ($n \geq 3$).

### 2.9. Triangle anomalies in SU(2) × U(1) theories

Since the gauge group SU(2) × U(1) seems to be especially important for physics, let us derive the condition for the cancellation of anomalies in this special case. Only the U(1) part of the gauge group causes the existence of anomalies in the SU(2) × U(1) case. Those are of two different kinds:

(a) Anomalies which arise from fermion triangles involving two SU(2) triplet currents and the SU(2) singlet current.

(b) Anomalies which arise from fermion triangles involving three SU(2) singlet currents.

Suppose we place all lefthanded and righthanded fermions in one, in general large, reducible representation, denoted by the vector $\psi_L$ or $\psi_R$.

The generators of the gauge group are described by matrices $T^L_i$ ($i = 1, 2, 3$) and $Y^L$, as well as $T^R_i$ ($i = 1, 2, 3$) and $Y^R$ ($Y$: weak hypercharge U(1) generator). The general condition for the cancellation
of anomalies is
\[ \text{tr}\{A_a^L, A_b^L\}A_c^L - \text{tr}\{A_a^R, A_b^R\}A_c^R = 0 \]  
(2.65)

where \( A_a \) stands for either \( T_i \) or \( Y \).

In SU(2) \( \times \) U(1) theories the electric charge is represented by

\[ Q = T_3 + \frac{1}{2}Y. \]  
(2.66)

We can use this condition in order to derive a simple formula for the cancellation of anomalies. First we consider the condition

\[ \text{tr}(Y'^0)^3 - \text{tr}(Y'^0)^3 = 0. \]  
(2.67)

Using eq. (2.66) we find

\[ \text{tr}[(T_3 + \frac{1}{2}Y)^3 - (T_3 + \frac{1}{2}Y)^3 - \{L \rightarrow R\} = 0. \]  
(2.68)

Since \( \text{tr}(T_3^L)^3 = \text{tr}(T_3^R)^3 = 0 \) and \( Q^L = Q^R \), this condition reduces to

\[ \text{tr}(Q^L)T_3^L - \text{tr}Q^L(T_3^L)^2 - \{L \rightarrow R\} = 0. \]  
(2.69)

Due to the relation

\[ \text{tr}(Q^L T_3^L - QT_3^L) = \text{tr}\{Q(T_3 + \frac{1}{2}Y)T_3 - QT_3^2\} \]
\[ = \text{tr}\frac{1}{2}QYT_3 = \frac{1}{3}\text{tr}((T_3 + \frac{1}{2}Y)YT_3) = \frac{1}{3}\text{tr}(T_3^2Y) \]
\[ = \text{tr}T_3^2(Q - T_3) = \text{tr}QT_3^2 \]  
(2.70)

we find as a necessary condition for the cancellation of anomalies

\[ \text{tr}(T_3^L)^2Q - \text{tr}(T_3^R)^2Q = 0. \]  
(2.71)

It can be shown that this condition is sufficient for the cancellation of all anomalies in SU(2) \( \times \) U(1) theories [2.6].

The cancellation can occur in two different ways:

i. The lefthanded and righthanded anomalies cancel each other:

\[ \text{tr}(T_3^L)^2Q = \text{tr}(T_3^R)^2Q \neq 0. \]  
(2.72)

This is, for example, the case in vectorlike SU(2) \( \times \) U(1) theories, where \( T_3^L = T_3^R \).

ii. The lefthanded and righthanded anomalies cancel separately. Thus we have

\[ \text{tr}(T_3^L)^2Q = 0, \quad \text{tr}(T_3^R)^2Q = 0. \]  
(2.73)
Often it is the case that \((T_3'j_2)\) is equal for all lefthanded and righthanded fermions. In this case the condition is reduced to

\[
\text{tr } Q = 0;
\]

(2.74)
i.e. the sum of all electric charges of the fermions must vanish.

3. The minimal theory of flavor dynamics

3.1. \(SU(2) \times U(1)\) theory of the electron

The minimal theory of flavor dynamics involves the gauge group \(SU(2) \times U(1)\), where \(SU(2)\) denotes the weak isotopic spin. The simplest version of such a spontaneously broken gauge theory of the weak and electromagnetic interactions, just involving the electron and its neutrino, was proposed in 1967/8 [3.1]. Here the lefthanded electron and the electron neutrino are placed in a doublet of the weak isotopic spin:

\[
L_e = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}. \tag{3.1}
\]

The righthanded electron \(e^ -_R = [(1 - \gamma_5)/2]e^- = R^e\) is assumed to be an \(SU(2)\)-singlet. Note the necessity to have the \(U(1)\)-factor in the gauge group \(SU(2) \times U(1)\) in order to represent the electric charge. The latter cannot be an \(SU(2)\) generator since the photon couples both to the lefthanded and the righthanded electron. It is useful to denote the \(U(1)\) generator as \(Y\) ("weak hypercharge") and to write for the electric charge:

\[
Q = T_3 + \frac{1}{2}Y. \tag{3.2}
\]

According to this decomposition of \(Q\) we have \(Y(e^-_L) = Y(\nu_e) = -1, Y(e^-_R) = -2\). Introducing the gauge bosons \(A_{\mu}^i\), coupled to the \(SU(2)\) generators \(T^i\), and the gauge boson \(B_{\mu}\), coupled to the hypercharge \(Y\), the gauge boson part of the Lagrangian is

\[
L_{g.b.} = -\frac{1}{4}F_{\mu\nu}^i F^{\mu\nu}_i - \frac{1}{2}G_{\mu\nu}G^{\mu\nu} \tag{3.3}
\]

where

\[
F_{\mu\nu}^i = \partial_\mu A_{\nu}^i - \partial_\nu A_{\mu}^i - g\epsilon_{ijk} A_{\nu}^j A_{\mu}^k \tag{3.4}
\]

\[
G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu.
\]

We introduce an \(SU(2)\) doublet of complex scalar fields

\[
\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}. \tag{3.5}
\]
According to eq. (3.2) the hypercharge $Y$ of $\phi$ is +1. Now we are in a position to write down the fermion and scalar parts of the Lagrangian:

$$L_{\text{fermion}} = \bar{L}i(\not{\mathcal{J}} + i \frac{g}{2} \tau A_i - \frac{g'}{2} \mathcal{B})L + \bar{R}i(\not{\mathcal{J}} - ig'\mathcal{B})R$$

$$L_{\text{scalar}} = \left( \partial^\mu \phi^\dagger - \frac{i}{2} g' B_\mu \phi^\dagger - \frac{i}{2} g A_\mu \tau_3 \phi^\dagger \right) \left( \partial_\mu \phi + \frac{ig'}{2} B_\mu \phi + \frac{ig}{2} \tau_3 A_\mu \phi \right) - V(\phi^\dagger \phi). \quad (3.6)$$

Here the SU(2) coupling constant is denoted by $g$, the U(1) coupling constant by $g'$. Both coupling constants are independent of each other, since the gauge group consists of two factors.

Furthermore we add the Yukawa interaction term. This term must be SU(2) × U(1) invariant. The only term which has this property is

$$L^{\text{Y}} = -G_e[\bar{R}(\phi^\dagger L) + (\bar{\phi} R)]. \quad (3.7)$$

Note that this term has in particular $Y = 0$. An interaction term of the form $\bar{R}(\phi \cdot L)$ would be SU(2) invariant, but would have $Y = -2$, i.e. it cannot appear in $L^{\text{Y}}$.

The most general form of the potential $V$ is

$$V = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2. \quad (3.8)$$

Taking $\mu^2 < 0$, we obtain a spontaneous breaking of the SU(2) × U(1) symmetry. We can always choose a SU(2) frame such that the vacuum expectation values of $\phi$ take the form:

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}. \quad (3.9)$$

By this choice we single out the electric charge uniquely. The component of $\phi$ which does develop a vacuum expectation value must be the neutral one.

Since the scalar fields have $Y = +1$, the spontaneous symmetry breaking breaks not only the SU(2) symmetry, but also the U(1) symmetry. The electric charge, which is a linear combination of $T_3$ and $Y$ (see eq. (3.2)) remains unbroken. The vacuum expectation value $v$ can be chosen to be real; it is given in terms of $\mu$ and $\lambda$:

$$v = \sqrt{-\mu^2/\lambda}. $$

We replace in eq. (3.6) $\phi$ by its vacuum expectation value. The quadratic term in the vector meson fields is given by

$$\frac{1}{8} v^2 ((g' B_\mu - g A_\mu^3)(g' B_\mu - g A_\mu^3) + g^2 ((A_\mu^1)^2 + (A_\mu^2)^2)). \quad (3.10)$$

We define the fields

$$W_\mu^\pm = (A_\mu^1 \mp i A_\mu^2)/\sqrt{2}. \quad (3.11)$$
It is easy to read off from eq. (3.10) the mass of the $W^\pm$:

$$M_{W^\pm} = \frac{1}{2} g v. \quad (3.12)$$

The other two mass eigenstates are given by

$$Z_\mu = -g A_\mu^3 + g' B_\mu, \quad A_\mu = \frac{g B_\mu + g' A_\mu^3}{\sqrt{g^2 + g'^2}}. \quad (3.13)$$

The mass eigenvalues are

$$M_Z = \frac{1}{2} v \sqrt{g^2 + g'^2}, \quad M_A = 0. \quad (3.14)$$

It is useful to define the angle $\theta_w$ by $g'/g = \tan \theta_w$ ($\sin \theta_w = g'/\sqrt{g^2 + g'^2}$) in which case we can rewrite eq. (3.12):

$$A = \cos \theta_w B + \sin \theta_w A^3, \quad B = \cos \theta_w A - \sin \theta_w Z$$

$$Z = -\sin \theta_w B + \cos \theta_w A^3, \quad A^3 = \sin \theta_w A + \cos \theta_w Z. \quad (3.15)$$

Note the mass relation:

$$M_{\mu}/M_Z = g'/\sqrt{g^2 + g'^2} = \cos \theta_w. \quad (3.16)$$

Let us rewrite the fermion-gauge boson interaction term in eq. (3.6) in terms of the fields $W^\pm, Z, A$. One finds for the terms involving $W^\pm$:

$$\frac{g}{2} \bar{L}_\gamma^\mu (\tau^1 A_\mu^1 + \tau^2 A_\mu^2) L = \frac{g}{\sqrt{2}} (\bar{\nu} \gamma^\mu e L W^\nu + \bar{\epsilon} \gamma^\mu \nu L W^-)$$

$$= \frac{g}{\sqrt{2}} (j^{-}_\mu - W^\mu_+) + j^+_\mu W^-) \quad (j^\pm_\mu = \bar{\nu} \gamma^\mu e^\mp L). \quad (3.17)$$

At energies small compared to the $W$-masses this interaction leads to an effective current–current interaction

$$\frac{g^2}{2} \cdot \frac{1}{M_{\mu}^2} \cdot (\bar{\nu} \gamma^\mu e L \cdot \bar{\epsilon} \gamma^\mu \nu L + h.c.). \quad (3.18)$$

On the other hand the electron part of the Fermi current–current interaction is

$$G \sqrt{2} \{ (\bar{\nu} \gamma^\mu (1 + \gamma_5) e) \cdot (\bar{\epsilon} \gamma^\mu (1 + \gamma_5) \nu) + h.c. \}. \quad (3.19)$$

Thus we find:
\[ G/\sqrt{2} = g^2/8M_w^2 = 1/2v^2 \]
\[ v = 246 \text{ GeV}. \]

Now let us consider the interaction of the neutral gauge bosons. According to eq. (3.6) we have:

\[ g_{ \mu}^3 A_\mu^3 + \frac{g'}{2} j_\mu^\gamma \cdot B^\mu \]

where \( j_\mu^3 \) is the third component of the isotopic weak current (\( j_\mu^3 = \frac{1}{2} (\bar{\nu}_e \gamma_\mu \nu_e - \bar{\nu}_e \gamma_\mu e) \)), and \( j_\mu^\gamma \) is the hypercharge current (\( j_\mu^\gamma = -\bar{\nu}_e \gamma_\mu \nu_e - \bar{e} \gamma_\mu e_L - 2\bar{e} \gamma_\mu e_R \)). Eq. (3.21) can be rewritten as follows, using the relation \( j_\mu = j_\mu^3 + \frac{1}{2} j_\mu^\gamma \) (\( j_\mu^\gamma \): electromagnetic current):

\[ g_{ \mu}^3 A_\mu^3 + \frac{g'}{2} j_\mu^\gamma B^\mu = g_{ \mu}^3 A_\mu^3 + g' (j_\mu^e - j_\mu^3) B^\mu \\
= g_{ \mu}^3 (\sin \theta_w A^\mu + \cos \theta_w Z^\mu) + g' (j_\mu^e - j_\mu^3)(\cos \theta_w A^\mu - \sin \theta_w Z^\mu) \\
= A^\mu [g \sin \theta_w j_\mu^3 + g' \cos \theta_w (j_\mu^e - j_\mu^3)] + Z^\mu [g \cos \theta_w j_\mu^3 - g' \sin \theta_w (j_\mu^e - j_\mu^3)]. \]

Using our definition of the angle \( \theta_w \), it is easy to see that the first term in eq. (3.22) is simply \( (gg'/\sqrt{g^2 + g'^2}) A^\mu j_\mu^e \), i.e. we find the electromagnetic coupling constant in terms of \( g \) and \( g' \):

\[ e = gg'/\sqrt{g^2 + g'^2} \]
\[ = g \cdot \sin \theta_w = g' \cdot \cos \theta_w. \]

The interaction term involving the Z-boson can be rewritten as follows:

\[ Z^\mu [g \cos \theta_w j_\mu^3 - g' \sin \theta_w (j_\mu^e - j_\mu^3)] \\
= Z^\mu [j_\mu^3 (g \cos \theta_w + g' \sin \theta_w) - g' \sin \theta_w j_\mu^e] \\
= gZ^\mu \left[ j_\mu^3 \left( \frac{\sin^2 \theta_w}{\cos \theta_w} \right) - \frac{\sin^2 \theta_w}{\cos \theta_w} j_\mu^e \right] \\
= \frac{g}{\cos \theta_w} Z^\mu (j_\mu^3 - \sin^2 \theta_w j_\mu^e) = \frac{g}{\cos \theta_w} Z^\mu j_\mu^n \]

where we have defined the neutral current by

\[ j_\mu^n = j_\mu^3 - \sin^2 \theta_w j_\mu^e. \]

Thus the interaction of the massive gauge bosons with the fermions can be written as

\[ (g/\sqrt{2})(j_\mu^n W_\mu^+ + j_\mu^+ W_\mu^-) + (g/\cos \theta_w)(j_\mu^n Z_\mu). \]

The effective current-current Hamiltonian which results from this interaction is
Using eq. (3.20), we can express $M_w$ and $M_Z$ in terms of $\theta_w$ and $\alpha = e^2/4\pi$:

$$M_w = \left(\frac{\pi \alpha}{\sqrt{2} G}\right)^{1/2} \frac{1}{\sin \theta_w} = \frac{37.3}{\sin \theta_w} \text{ GeV},$$

$$M_Z = \frac{M_w}{\cos \theta_w} = \frac{37.3}{\sin 2 \theta_w} \frac{\sin \theta_w}{\cos \theta_w} \text{ GeV} = \frac{74.6}{\sin \theta_w} \text{ GeV}.$$

(3.28)

Note the inequalities

$$M_w \geq 37.3 \text{ GeV}, \quad M_Z \geq 74.6 \text{ GeV}.$$  

(3.29)

The Yukawa interaction term (3.7) generates the mass of the electron; the neutrino stays massless. One finds:

$$m_e = G_e \cdot (v/\sqrt{2}).$$

(3.30)

Since we have $v = [\sqrt{2} G]^{-1/2}$ (see eq. (3.20)), we obtain

$$G_e = 8^{1/4} \cdot m_e \cdot \sqrt{G} = 2.9 \times 10^{-6},$$

(3.31)

i.e. the Yukawa coupling constant is extremely small.

Finally let us mention some further properties of the simplest SU(2) × U(1) model:

(a) The effective weak currents (charged and neutral ones) are determined by the one parameter $\theta_w$. The freedom of choice for this parameter is due to the fact that there exist two independent coupling constants $g$ and $g'$ in this theory.

(b) The masses of the W and Z bosons conspire such that both the strengths of the charged current and the neutral current interaction are described by the same Fermi constant $G$ (see eq. (3.27)). This is due to the fact that the spontaneous symmetry breaking is generated by a doublet of scalars.

Let us consider again the mass term (3.10). The mass matrix of the gauge bosons is proportional to

$$
\begin{pmatrix}
g^2 & 0 & 0 & 0 \\
0 & g^2 & 0 & 0 \\
0 & 0 & g^2 & -g'g \\
0 & 0 & -g'g & g'^2
\end{pmatrix}
\begin{pmatrix}
A_1 \\
A_2 \\
A_3 \\
B
\end{pmatrix}.
$$

(3.32)

In order to have the same Fermi constant for the charged and neutral current interactions it is necessary that the diagonal mass terms for $A_1, A_2, A_3$ are identical; the mass matrix for the SU(2) bosons must be isospin symmetric. As we have seen in the previous section, this is achieved only if the
spontaneous symmetry breaking is generated by an isodoublet of scalars. The fact that in the 
SU(2) × U(1) scheme discussed above we have \( M_Z \neq M_w \) is due to the mixing between the \( A^3 \) and 
\( B \)-field. This leads automatically to a massless \( A \)-field (photon) and a \( Z \)-field with \( M_Z > M_w \). In the 
limit \( \theta_w \to 0 \) the mixing between \( A^3 \) and \( B \) disappears, and we have \( M_Z = M_w \).

What happens, if we allow other SU(2) representations of scalars as well? First of all we note that the 
representation of scalars must include an SU(2) doublet. Otherwise the electron would stay massless, 
since the electron mass term \( m_e \bar{\psi}_e \psi_e \) belongs to an SU(2) doublet (note that \( \bar{\psi}_e \) is an SU(2) 
singlet).

What would be the consequence of other representations of scalar fields for the spectrum of \( W \) and \( Z \)
bosons? Let us allow several representations of scalars, whose neutral members develop vacuum 
extpectation values. Due to the relation \( Q = T_3 + \frac{1}{2} Y \) those neutral scalars which acquire nonzero 
vacuum expectation values have \( T_3 = -\frac{1}{2} Y \). Using the corresponding generalization of eq. (3.10) one finds

\[
\frac{M_Z^2 \cos^2 \theta_w}{M_w} = \rho = \frac{\sum v_i^2 Y_i^2 (1/2)}{\sum v_i^2 (t_i^2 + t_i - Y_i^2/4)}
\]  

(3.33)

where the summation has to be carried out over the various representations of scalar fields, \( v_i \) is the 
vacuum expectation value of the neutral member of the representation \( i \), and \( Y_i, t_i \) are the hypercharge 
and weak isospin respectively of the \( i \)th representations.

The parameter \( \rho \), defined by eq. (3.33), is, of course, equal to unity, if the scalars form a doublet of 
hypercharge +1. Note however, that \( \rho \) remains unity, if several doublet representations of 
hypercharge +1 are present.

The parameter \( \rho \) is zero for an SU(2) triplet representation with \( Y = 0 \). For a triplet representation 
with \( Y = 2 \) (charges \( (2, 1, 0) \)) one finds \( \rho = 2 \). According to eq. (3.33) one can generate essentially any 
value of \( \rho \) if we allow scalar representations with large enough \( t \) and \( Y \). The case \( \rho = 1 \) realized in the 
special SU(2) × U(1) theory of Salam and Weinberg is a special property which holds only if the 
symmetry is broken by a \( Y = 1 \) scalar doublet (or several \( Y = 1 \) doublets). In view of the analysis of 
the experimental data discussed later it is useful to introduce the parameter \( \rho \) as another free parameter of 
the SU(2) × U(1) theory.

(c) The SU(2) × U(1) theory described above is not free of anomalies, since \( \text{tr} \, Q \neq 0 \). Thus, in order 
to have a renormalizable theory, further fermions and/or further weak currents have to be introduced.

(d) We started out with a doublet of complex scalar fields (altogether four degrees of freedom). 
Three of them are absorbed to generate the longitudinal components of the massive gauge bosons. Thus 
we are left with one neutral scalar field ("Higgs boson"). Its properties will be discussed later.

3.2. The theory of four leptons and four quarks

The minimal theory of flavor dynamics described in section 3.1 for the \((\nu_e, e^-)\)-system can easily be 
generalized to involve the observed leptons \( \nu_e, \nu_\mu \,; \, e^-, \mu^- \) and quark flavors \( u, d, s \) and \( c \). We introduce 
the leptonic doublets

\[
\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L
\]
and the quark doublets

\[
\begin{pmatrix}
  u \\
  d'
\end{pmatrix}_L \quad \begin{pmatrix}
  c \\
  s'
\end{pmatrix}_L
\]

where

\[
d' = d \cos \theta_c + s \sin \theta_c
\]

\[
s' = -d \sin \theta_c + s \cos \theta_c
\]  

(\(\theta_c\): Cabibbo angle).

In order to fulfill the relation \(Q = T_3 + \frac{1}{2}Y\), one has to assign the weak hypercharges of the lefthanded and righthanded quarks as follows:

\[
\begin{array}{cccccccc}
\text{Quark} & u_L & d_L & c_L & s_L & u_R & d_R & c_R & s_R \\
Q_e & 2/3 & -1/3 & 2/3 & -1/3 & 2/3 & -1/3 & 2/3 & -1/3 \\
T_3 & 1/2 & -1/2 & 1/2 & -1/2 & 0 & 0 & 0 & 0 \\
Y & 1/3 & 1/3 & 1/3 & 1/3 & 4/3 & -2/3 & 4/3 & -2/3 \\
\end{array}
\]

It is useful to combine the lepton and quark doublets into two units [3.2]:

(a) the “light” fermions

\[
\begin{pmatrix}
  \nu_e \\
  u \\
  e' \\
  d'
\end{pmatrix}_L
\]

(b) the “heavy” fermions

\[
\begin{pmatrix}
  \nu_\mu \\
  \mu \\
  c \\
  s'
\end{pmatrix}_L
\]

We note that according to eq. (2.74) the anomalies cancel within each unit (since \(\Sigma Q = 0\)) if each quark doublet comes in three different colors.

The neutral current is given by eq. (3.25):

\[
j_\mu^n = j_\mu - \sin^2 \theta_w j_{\mu}^e = \frac{1}{2}(\bar{\nu}_e \gamma_\mu \nu_e - \bar{\nu}_e \gamma_\mu \nu_e) + \frac{1}{2}(\bar{\nu}_\mu \gamma_\mu \nu_\mu - \bar{\mu}_\gamma_\mu \mu_\mu) + \frac{1}{2}(\bar{\mu}_\gamma_\mu \nu_\mu - \bar{\mu}_\gamma_\mu \nu_\mu) + \frac{1}{2}(\bar{\nu}_\gamma_\mu c_\mu - \bar{\nu}_\gamma_\mu s_\mu) - \sin^2 \theta_w j_{\mu}^e.
\]  

Especially we emphasize that \(j_\mu^n\) is diagonal in all fermions. The mixing of d and s given by the Cabibbo angle does not affect the third component of the weak isotopic spin since both d' and its orthogonal component s' are (−1/2) components. If s' would be an SU(2) singlet, there would exist a flavor changing term proportional to \(\sin \theta_c \cdot \cos \theta_c \gamma_\mu s_\mu\) + h.c. in the hadronic neutral current. Such a term is highly suppressed, as can be seen e.g. in the reduced decay rate for \(K_{0L} \to \mu^+ \mu^-\). Experimentally
it is found:

\[
\Gamma(K_{OL} \rightarrow \mu^+\mu^-)/\Gamma(K_{OL} \rightarrow \mu\nu) \approx 3.4 \times 10^{-9}. \tag{3.36}
\]

Of course, it was this fact (among others) which originally led to the introduction of the charmed quark [3.2].

We mention the following further aspects of the four-fermion scheme.

(a) **Generation of fermion masses.** The quark masses are generated by the coupling of the quark fields to the scalars. We introduce one scalar doublet \( \phi = \begin{pmatrix} \phi^+ \\ \phi \end{pmatrix} \) with \( Y = 1 \) and note that the doublet \( \tilde{\phi} = i\sigma_2\phi^* = \begin{pmatrix} \phi^0 \\ -\phi \end{pmatrix} \) transforms also as a doublet (with \( Y = -1 \)). The general SU(2) \( \times \) U(1) invariant interaction between the scalars and the quarks is given by

\[
+ G_{1d} \bar{d}_R \phi^* \begin{pmatrix} u_0 \\ d_0 \end{pmatrix}_L + G_{2d} \bar{d}_R \phi^* \begin{pmatrix} u_0 \\ s_0 \end{pmatrix}_L + G_{3d} \bar{d}_R \phi^* \begin{pmatrix} c_0 \\ s_0 \end{pmatrix}_L + G_{4d} \bar{d}_R \phi^* \begin{pmatrix} c_0 \\ s_0 \end{pmatrix}_L \\
+ G_{5d} \bar{d}_R \phi^* \begin{pmatrix} u_0 \\ d_0 \end{pmatrix}_L + G_{6d} \bar{d}_R \phi^* \begin{pmatrix} u_0 \\ s_0 \end{pmatrix}_L + G_{7d} \bar{d}_R \phi^* \begin{pmatrix} c_0 \\ s_0 \end{pmatrix}_L + G_{8d} \bar{d}_R \phi^* \begin{pmatrix} c_0 \\ s_0 \end{pmatrix}_L + \text{h.c.}, \tag{3.37}
\]

where \( u_0, d_0, \ldots \) denote the quark fields which we take as the eigenstates of SU(2) (they are not the eigenstates of quark mass matrix). Note that the interaction of the quarks with the scalars depends on eight Yukawa coupling constants. The neutral components of \( \phi \) and \( \tilde{\phi} \) acquire nonzero expectation values. Thus we obtain the quark mass matrices

\[
\begin{pmatrix} \bar{u}_0 \bar{c}_0 \end{pmatrix}_R \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} \begin{pmatrix} u_0 \\ c_0 \end{pmatrix}_L + \text{h.c.} = (\bar{u}_0 \bar{c}_0)_R \mathcal{M}_{uc} \begin{pmatrix} u_0 \\ c_0 \end{pmatrix}_L + \text{h.c.}, \tag{3.38}
\]

\[
\begin{pmatrix} \bar{d}_0 \bar{s}_0 \end{pmatrix}_R \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} \begin{pmatrix} d_0 \\ s_0 \end{pmatrix}_L + \text{h.c.} = (\bar{d}_0 \bar{s}_0)_R \mathcal{M}_{ds} \begin{pmatrix} d_0 \\ s_0 \end{pmatrix}_L + \text{h.c.},
\]

depending on eight complex parameters \( \alpha_{11}, \ldots, \beta_{22} \). (Note that the \( 2 \times 2 \) matrices in (3.36) are in general not Hermitian.)

Any \( 2 \times 2 \) matrix with complex elements can be transformed into a diagonal matrix with real nonnegative matrix elements by multiplying it from left and right with suitable unitary matrices:

\[
U_{uc}^{-1} \mathcal{M}_{uc} V_{uc} = \begin{pmatrix} m_u & 0 \\ 0 & m_c \end{pmatrix},
\]

\[
U_{ds}^{-1} \mathcal{M}_{ds} V_{ds} = \begin{pmatrix} m_d & 0 \\ 0 & m_s \end{pmatrix}. \tag{3.39}
\]

where \( m_u, m_d, \ldots \) are the quark masses.

Since the matrices \( V_{uc} \) and \( V_{ds} \) transform the various quark fields of the same charge into each other, the mass eigenstates \( u, c, d, s \) do not coincide with the states \( (u_0, c_0) \) and \( (d_0, s_0) \) we started from.
weak charge density with $\Delta Q = +1$ has the form

$$(u_0, c_0)^L \begin{pmatrix} d_0 \\ s_0 \end{pmatrix}_L = (u, c)^L V_{uc} V_{ds} \begin{pmatrix} d \\ s \end{pmatrix}_L. \quad (3.40)$$

The $2 \times 2$ unitary matrix $V_{uc} V_{ds}$ can always be written in terms of one angle $\theta$ and four phase parameters:

$$V_{uc} V_{ds} = \begin{pmatrix} e^{i\alpha} \cos \theta & e^{i\beta} \sin \theta \\ -e^{i\gamma} \sin \theta & e^{i\delta} \cos \theta \end{pmatrix} \quad (3.41)$$

where $\alpha - \gamma = \beta - \delta$. Thus far the phases of the quark fields are undetermined, and we can absorb the phase factors $e^{i\alpha}$ etc. by redefining the phases of the quark fields. Consequently we obtain:

$$V_{uc} V_{dc} \rightarrow \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}. \quad (3.42)$$

The angle $\theta$ is identified with the Cabibbo angle: $\theta = \theta_c$, and we find the conventional form of the charged weak current.

It is useful to see what the special form of the Yukawa couplings are if we choose $u_0 = u$, $c_0 = c$, $d_{0L} = d_L$, $s_{0L} = -d_L \sin \theta_c$, $s_{0L} = s_L \cos \theta_c$, $d_{0R} = d_R$, $s_{0R} = s_R$. In this case one finds

$$\frac{\sqrt{2}}{\nu} \left[ m_u \tilde{\nu}_R \tilde{\phi}^+ (u_0)_{L} + m_c \tilde{\nu}_R \tilde{\phi}^+ (c_0)_{L} + m_d \cos \theta \tilde{d}_R \phi^+ (d_0)_{L} - m_d \sin \theta \tilde{d}_R \phi^+ (c_0)_{L} + m_s \cos \theta \tilde{s}_R \phi^+ (c_0)_{L} \right]. \quad (3.43)$$

Comparing this expression with eq. (3.35) one can write the various Yukawa coupling constants in terms of the quark masses, the Cabibbo angle, and the vacuum expectation value $\nu$.

(b) CP-violation. In the four-quark and four-lepton scheme, involving one doublet of scalar fields only, all complex parameters which might enter (Yukawa coupling constants, vacuum expectation values, etc.) can be transformed into real ones (see our discussion above). Therefore, in this scheme CP is conserved.

(c) The limit $\theta_c = 0$. In the theory described above the connection between the “light” quarks $(u, d)$ and the “heavy” quarks $(c, s)$ is provided by the Cabibbo angle, i.e. by the quark mass matrix. In the limit where $G_1 = G_2 = \cdots = G_6 = 0$ there exists no connection between the $(u, d)$ and the $(c, s)$ system, and in this case there exist two separate conservation laws (of course, in this limit one has $\theta_c = 0$). Both the sum of $u$ and $d$ quarks and the sum of $c$ and $s$ quarks are exactly conserved, i.e. there exist two different types of baryon numbers. Of course, in this limit all quarks are massless.

(d) Conservation of electron- and muon number. What has been discussed above for the quark sector as a hypothetical situation, happens indeed in the lepton sector. Since in the minimal theory the neutrinos are lefthanded Weyl fields, we have $m_{\nu_e} = m_{\nu_\mu} = 0$. This implies that a leptonic version of the Cabibbo angle cannot be defined; such an angle, if introduced by hand, would have no physical meaning; it can always be rotated away. Electron number and muon number are exactly conserved. As
a consequence the decays $\mu \to e\gamma$ and $\mu \to eee$ are forbidden. We mention the present limits on these decays:

\[
B(\mu \to e\gamma) > 1.9 \times 10^{-10} \quad [3.3]
\]

\[
B(\mu \to eee) < 1.9 \times 10^{-9} \quad [3.4]
\] (3.44)

Recently a very stringent limit for the conversion of muons into electrons in atoms (sulphur) has been obtained [3.5]

\[
R(\mu \to e) = \frac{\sigma(\mu^- + S \to S + e^-)}{\sigma(\mu^- + S \to \text{capture})} < 7 \times 10^{-11}.
\] (3.45)

Furthermore there exists a limit for the conversion of muons into positrons, according to the reaction $\mu^- + \text{nucleus} (Z, A) \to e^+ + \text{nucleus} (Z - 2, A) [3.6],$

\[
R(\mu^- \to e^+) = \frac{\sigma(\mu^- + S \to e^+ + ^{32}\text{Si}^*)}{\sigma(\mu^- + S \to \text{capture})} < 9 \times 10^{-10}.
\] (3.46)

We emphasize that our conclusion about the absence of $e \leftrightarrow \mu$ transitions in the minimal model depends on the assumption that only one scalar doublet is present. In this case the neutral scalar boson coupling is directly proportional to the mass matrix and cannot produce off-diagonal transitions like $e \to \mu$. If more than one scalar doublet is contributing, one expects that the muon number is not exactly conserved [3.7]. Here the dominant effect comes from two-loop diagrams, in which a scalar boson is emitted from a lepton and absorbed by a virtual W or Z boson. The branching ratio for the decay $\mu \to e\gamma$ depends on the coupling parameters of the scalars, and it may be as large as $(\alpha/\pi)^3 \sim 10^{-9}$, which is already in some conflict with the experimental data (3.44).

If new leptons, new gauge interactions, neutrino masses etc. are introduced, it is easy to violate the conservation of electron number, muon number (or lepton number in general). However the predictions for processes like $\mu \to e\gamma$, $\mu e$ conversion etc. are very model dependent. For this reason we shall not go into details [see e.g. [3.8] ...].

(e) Quark masses. Although it is assumed that quarks and gluons are confined inside hadrons, the masses of quarks can be defined (see e.g. [3.9]). Crudely speaking, the quark masses are the mass parameters, which describe the quark propagator at small distances (small compared to the confinement length of $\approx 10^{-13}$ cm). In QCD they depend on the renormalization point $\mu$, i.e. it makes no sense to assign a specific number in MeV to a particular quark mass, unless the renormalization point $\mu$ is specified.

The change of a quark mass $m$ under a change of $\mu$ is given by

\[
\frac{dm}{m} = \frac{d\mu}{\mu} \cdot \gamma_m \left( g, \frac{m}{\mu} \right)
\] (3.47)

where $\gamma_m$ is the associated anomalous dimension. In the case $\mu \gg \Lambda$ [$\Lambda$ ~ scale parameter of QCD] and $(m/\mu) \ll 1$ one can apply the result obtained by the one loop expansion of the renormalization group equation [3.9]

\[
\gamma_m = -\frac{g^2(\mu)}{2\pi^2} = -\frac{2\alpha_s(\mu)}{\pi} .
\] (3.48)
In this case one finds for two different scales $\mu_1, \mu_2$ and small $\ln(\mu_1/\mu_2)$:

$$\frac{m(\mu_1^2)}{m(\mu_2^2)} = \frac{1}{1 + (\alpha_s/\pi) \ln(\mu_1^2/\mu_2^2)}.$$  \hspace{1cm} (3.49)

The quark masses shrink to zero as $\mu \to \infty$. This effect is caused by the emission and reabsorption of virtual gluons; as $\mu^{-1} \to 0$ (decreasing distance) less self-energy corrections are seen, and $m$ decreases.

In the region where we can apply the relation (3.49), the ratio of two quark masses of different flavors is independent of $\mu^2$:

$$m_i(\mu)/m_i(\mu) = m_i^0/m_i^0.$$  \hspace{1cm} (3.50)

The ratio $m_i^0/m_i^0$ is defined to be the "bare mass" ratio. It is a pure number, which can be interpreted as the QCD analog of the lepton mass ratios, e.g. $m_\nu/m_{\mu^+}$.

The bare quark mass ratios can be estimated using PCAC. The $(mass)^2$ of a pseudoscalar meson is given by [3.10]

$$M^2(ps) = -\frac{F^2}{F} \int d^4x d^4y \langle 0 | [\mathcal{F}_0^a(x), \bar{q}q] | 0 \rangle$$  \hspace{1cm} (3.51)

where $\mathcal{F}_\mu^5$ is the axial vector current corresponding to the quantum number of the meson, and $F$ is the associated meson decay constant. Using eq. (3.51), one can obtain the mass formula

$$M^2(\pi^+) = M^2(\pi^0) = -4F^2 (m_u(\bar{u}u) + m_d(\bar{d}d))$$  \hspace{1cm} (3.52)

where $\langle \bar{q}q \rangle$ is the vacuum expectation value of the scalar quark density $\bar{q}q$. The scheme of spontaneous symmetry breaking, involving the pseudoscalar mesons as Goldstone particles, implies that the v.e.v. of $\bar{q}q$ does not vanish. We mention that this feature, like the confinement aspect, has not been demonstrated to be true in QCD. We assume that $\langle \bar{q}q \rangle \neq 0$ [3.11].

The vacuum expectation values $\langle \bar{q}q \rangle_0$ are all equal in the limit $M = 0$ ($m_u = m_d = m_s = m_c = \cdots = 0$), and must be a multiple of $\Lambda^3$ where $\Lambda$ is the QCD scale parameter: $\langle \bar{q}q \rangle_0 = \text{const.} \times \Lambda^3$. Since the generation of the nonzero v.e.v. $\langle \bar{q}q \rangle_0$ ("quark condensation") is supposed to be a pure gluon effect, we expect that the introduction of a heavy quark mass, say $m_c$, will not affect significantly the other quarks whose masses are kept zero, i.e.

$$\langle \bar{u}u \rangle_0 = \langle \bar{d}d \rangle_0 = \langle \bar{s}s \rangle_0 = \langle \bar{q}q \rangle_0 \quad [M = 0].$$

However the v.e.v. $\langle \bar{q}q \rangle_0$ of the massive quark $q$ will change. In the limit $m_q \gg \Lambda$ one expects that the quark condensation effects disappear:

$$\langle \bar{q}q \rangle_0 \underset{m_q \to \infty}{\longrightarrow} 0.$$  

Of course, without having a well-defined theory of the condensation effects we do not know how fast $\langle \bar{q}q \rangle_0$ converges to zero. Probably it disappears like a power of $m_c^{-1}$, e.g. $\langle \bar{q}q \rangle_0 \to c m_c^{-1} \Lambda^4$ (c: dimensionless constant). As far as the "light" quarks $u$, $d$ and $s$ are concerned, we expect that the quark masses do
not affect the v.e.v. $\langle \bar{q}q \rangle_0$ in a significant way, i.e. we have

$$\langle \bar{q}q \rangle_{|m_q=0} \approx \langle \bar{q}q \rangle_{|m_u=0}$$

$(q = u, d, s)$, and the SU(3) symmetry $\langle \bar{u}u \rangle_0 = \langle \bar{d}d \rangle_0 = \langle \bar{s}s \rangle$ is preserved to a good approximation.

If we assume further that the v.e.v. of $\bar{q}q$ and the decay constants $F$ obey an SU(3) symmetry (this would be true exactly in the limit $m_u = m_d = m_s$), we find

$$M^2_\pi = (m_u + m_d)M_0 + \gamma(\pi)$$
$$M^2_{K^+} = (m_u + m_d)M_0 + \gamma(K^+)$$
$$M^2_{K^0} = (m_d + m_s)M_0 + \gamma(K^0)$$

(3.53)

$$(M_0 = -4F^{-2}\langle \bar{q}q \rangle).$$

In the relations above we have included the electromagnetic self-energies of the corresponding particles, denoted by $\gamma$. The latter are expected to vanish for neutral mesons in the SU(3) $\times$ SU(3) limit [3.12]: $\gamma(\pi^0) = \gamma(\pi^+) = 0$. Furthermore one expects due to U-spin symmetry:

$$\gamma(\pi^+) = \gamma(K^+)$$

Using these relations, one finds [3.10]:

$$M^2(\pi^0) \approx (m_u + m_d)M_0$$
$$M^2(K^0) \approx (m_d + m_s)M_0$$
$$M^2(K^+) - M^2(K^0) - (M^2(\pi^+) - M^2(\pi^0)) \approx (m_u - m_d)M_0$$

(3.54)

$$M^2(K^0) - M^2(K^+) + M^2(\pi^+)$$
$$2M^2(\pi^0) + M^2(K^+) - M^2(K^0) - M^2(\pi^+)$$

$$\approx m_d \approx 1.8$$

$$m_s \approx 20, \quad m_s \approx 36, \quad \frac{m_s}{m_u} \approx 26, \quad \frac{m_d - m_u}{m_u + m_d} \approx 0.57.$$
One question which has been asked recently is: Could the u-quark be massless like the neutrino [3.14]?

We find this possibility unlikely; it would mean that the last relation in eq. (3.54) gives 2 instead of 0.57. Further evidence against the case \( m_u = 0 \) can be obtained by considering the mass splitting in the baryon octet [3.15].

Taking into account higher order corrections to the chiral symmetry breaking, the best values for the quark mass ratios are:

\[
m_u : m_d : m_s \approx 3 : 5 : 100. \tag{3.55}
\]

It is useful to consider absolute values of quark masses, taking \( m_s \approx 150 \text{ MeV} \) [3.16]. In this case one finds from eq. (3.54)

\[
m_d \approx 7.5 \text{ MeV} \quad \text{(} m_s \approx 150 \text{ MeV}) . \tag{3.56}
\]

\[
m_u \approx 4.2 \text{ MeV} .
\]

Those mass values can be regarded as the quark masses defined at distances of the order of \( M_0 ^{-1} \approx 10^{-14} \text{ cm} \). On the other hand the mass of the charmed quark can be estimated considering the spectrum of charmed particles [3.17]:

\[
M(\psi(J)) - M(\rho) \approx 2m_c \tag{3.57}
\]

\[
m_c \approx 1.2 \text{ GeV} .
\]

Analogously one can determine the mass of the b quark considering the spectrum of the \( \gamma \) states. We shall adopt the value \( m_b = 4.7 \text{ GeV} \).

Finally we should like to address the question: What are the energies at which the values of the quark masses considered above are relevant? For example, the statement \( m_s \approx 150 \text{ MeV} \approx \text{mass of strange particle} - \text{mass of nonstrange particle} \) makes sense only if we set the baryon matrix element of the scalar operator \( \bar{q}q \) to be equal to \( q^\dagger q \), i.e. we set \( \gamma^0 = 1 \). Since the operator \( \bar{q}q \) is not invariant under renormalization, the statement \( \gamma^0 = 1 \) can only be valid at some particular energy. What is this energy? Since the only mass parameter in the system is the baryon mass (proton mass, \( \Lambda \text{ mass}, \ldots \) itself), we expect that the relation \( \gamma_0 \approx 1 \) is valid at the energy of the order of 1 GeV. This implies \( m_s (q^2 \approx 1 \text{ GeV}^2) \approx 0.15 \text{ GeV} \).

For the heavy quarks (c, b, \ldots) one should take the inverse size of the associated quarkonium state as the corresponding reference energy. For example, the mass of the b-quark is set equal to about 4.4 GeV at \( q^2 = 5 \text{ GeV}^2 \).

(f) Neutrino masses. In the minimal theory of flavor dynamics the neutrino \( \nu_e, \nu_\mu \) are lefthanded massless fermions. However, nothing can prevent us to introduce small masses for the neutrinos, analogously to the masses of the quarks, provided there exist righthanded partners of \( \nu_e \) and \( \nu_\mu \). The present experimental limit muon neutrino mass is \( m(\nu_\mu) < 510 \text{ keV} \) [3.18]. Recently evidence for a non-zero electron neutrino mass has been reported [3.19].

Stringent limits on the neutrino masses can be derived from astrophysics, provided the neutrinos are stable or have a lifetime longer than \( \sim 10^{20} \text{ s} \). There limits come from upper bounds on the present...
cosmological mass density [3.20]
\[ \sum \nu m_\nu < 55 \text{ eV}. \] (3.58)

It has also been argued that the missing mass problem in cosmology could be resolved if the neutrinos have a small rest mass \((m_\nu \sim \text{few eV})\) [3.20]. This point of view has recently been challenged by the authors of ref. [3.21], who also derive a more stringent cosmological bound: \(\sum \nu m_\nu < 1.2 \text{ eV}\). However, we should like to stress that all upper bounds on the total energy density in the universe are based on the assumption of the absence of the cosmological term and therefore must be regarded with caution.

The cosmological bounds on the neutrino masses do not exclude the existence of very heavy stable neutral leptons [3.22], which may contribute to the cosmological mass density in a significant way.

Further information about the neutrino masses can be obtained by considering neutrino oscillations. We postpone the discussion of those to later.

(g) Generation of mass by radiative corrections. In the minimal theory of flavordynamics the scale of the theory is set by the vacuum expectation value \(v = \sqrt{-\mu^2/\lambda}\). On the other hand, the mass of the neutral scalar 

\[ M H = \sqrt{2 \mu^2} = \sqrt{2v^2} = \sqrt{2\lambda} \cdot 246 \text{ GeV}. \] (3.58)

Thus e.g. for \(M_H = 11 \text{ GeV}\) one has \(\lambda \approx 10^{-3}\). It is impossible to make \(\lambda\) arbitrarily small, which means to lower \(M_H\) to arbitrarily small values. Since the coupling of the scalar fields to the gauge bosons is required by gauge invariance to be of the order of \(e\), the radiative corrections involving loops of virtual gauge bosons lead to induced self-couplings of the scalars which are of the order of \(e^4\). Thus \(\lambda\) cannot be smaller than \(\sim \alpha^2\), and one finds a lower limit for \(M_H\) which is of the order of \(\alpha \cdot v\). An exact calculation gives as a lower limit for \(M_H\) about 5 GeV (for \(\sin^2 \theta_{\nu} \approx 0.2\)) [3.23].

The radiative corrections modify the effective potential of the scalar fields. Thus one may ask (as done in ref. [3.24]) what happens if we set the scale \(\mu^2\) of QFD to zero.

The most simple situation one can study in this respect is the electrodynamics of a massless scalar field. The Lagrangian of massless scalar QED is given by

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + (\nabla_\mu \phi^*) (\nabla_\mu \phi) - \frac{1}{2} \lambda (\phi^* \phi)^2 \] (3.59)

where \(\nabla_\mu = \partial_\mu - ie A_\mu\).

Taking into account the renormalization effects both due to the electromagnetic interaction and the quartic self-interaction one obtains:

\[ V(\phi) = \frac{\lambda}{4!} \phi^4 + \left( \frac{5\lambda^2}{1152\pi^2} + \frac{3e^4}{64\pi^2} \right) \phi^4 \left( \ln \left( \frac{\phi^2}{M^2} \right) - \frac{25}{6} \right) \] (3.60)

where \(M\) is an arbitrary mass parameter which is used to define \(\lambda\) (renormalization point)

\[ \lambda(M) \frac{d^2 V}{d\phi^2} \bigg|_{\phi = M}. \] (3.61)
A change of the renormalization point $M$ can simply be absorbed by a change of $\lambda$ such that the potential takes the same form as before replacing $\lambda(M)$ by $\lambda'(M')$.

Since $M$ is arbitrary we can choose $M$ to be the position of the minimum of $\langle \varphi \rangle$, which is then the parameter of the theory fixing all mass scales. In this case we obtain

$$V = \frac{\lambda}{4!} \varphi^4 + \frac{3e^4}{64\pi^2} \varphi^4 \left( \ln \left( \frac{\varphi^2}{(\varphi^2)} \right) - \frac{25}{6} \right),$$  

and the $\lambda$ coupling constant is given by $e$:

$$\lambda = \frac{33}{8\pi^2} e^4,$$

$$V(\varphi) = \frac{3e^4}{64\pi^2} \varphi^4 \left[ \ln \left( \frac{\varphi^2}{(\varphi^2)} \right) - \frac{1}{2} \right].$$

One obtains a spontaneous symmetry breaking, i.e. both the scalar boson and the vector boson acquire masses. It follows

$$m^2(\text{scalar}) = V''(\varphi)|_{\varphi=\langle \varphi \rangle} = \frac{3e^4}{8\pi^2} (\varphi)^2,$$

$$m^2(\text{vector}) = e^2(\varphi)^2,$$

$$\frac{m^2(\text{scalar})}{m^2(\text{vector})} = \frac{3}{2\pi} \frac{e^2}{4\pi^2}.$$  \hspace{1cm} (3.64)

The result is surprising, since the scalar boson mass is some sort of radiative correction to the vector boson mass. Furthermore both the vector boson and the scalar boson masses are generated spontaneously by radiative effects.

The same phenomenon occurs in nonabelian gauge theories. The radiative corrections due to the gauge bosons loops and the fermion loops cause a spontaneous breakdown of the theory even in the absence of the negative scalar mass $\mu^2$. For example in the minimal SU(2) $\times$ U(1) theory one finds neglecting the fermion loops:

$$m^2(H) = \frac{3}{32\pi^2} [2g^2m^2(W) + (g^2 + g'^2)m^2(Z)]$$

(H: Higgs meson).

Of course, in deriving this relation we have set $\mu^2 = 0$. Within the SU(2) $\times$ U(1) theory there is no reason to suppose that this is the case, apart from the economical point of view that one parameter less is better than not. However the situation is different in unified theories of the electroweak and strong interactions (for a discussion see ref. [3.25]). Using the relations
\[
g = \frac{e}{\sin \theta_w}, \quad g' = \frac{e}{\cos \theta_w}
\]
\[
m_w = \sin^{-1} \theta_w \cdot M_0
\]
\[
m_z = \sin^{-1} \theta_w \cdot \cos^{-1} \theta_w \cdot M_0
\]
\[
(M_0 = (\pi \alpha/\sqrt{2} G)^{1/2} = 37.29 \text{ GeV})
\]

one finds
\[
m(H) = \left(\frac{3\alpha}{8\pi}\right)^{1/2} \frac{\sqrt{2} + (\cos \theta)^4}{\sin^2 \theta_w} M_0.
\]

This gives \(m(H) = 9.08 \text{ GeV}\) for \(\sin^2 \theta_w = 0.23\). The mass of the Higgs boson is rather sensitive to \(\sin^2 \theta_w\) (see table below):

<table>
<thead>
<tr>
<th>(\sin^2 \theta_w)</th>
<th>0.19</th>
<th>0.20</th>
<th>0.21</th>
<th>0.22</th>
<th>0.23</th>
<th>0.24</th>
<th>0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m(H))[GeV]</td>
<td>10.8</td>
<td>10.3</td>
<td>9.85</td>
<td>9.45</td>
<td>9.08</td>
<td>8.74</td>
<td>8.43</td>
</tr>
</tbody>
</table>

If the Higgs boson mass is indeed of the order of 9–10 GeV, as predicted by the minimal SU(2)×U(1) theory if the mass generation is due to radiative corrections, there exists the possibility to observe it in the \(Y\)-decay. One has
\[
B = \frac{\Gamma(V(\bar{b}b) \rightarrow H + \gamma)}{\Gamma(V(\bar{b}b) \rightarrow \mu + \mu)} = \frac{G m_{\nu}^2}{4 \sqrt{2} \pi \alpha} \left(1 - \frac{m_H^2}{m_{\nu}^2}\right)^{1/2}
\]

(3.68)

where \(V(\bar{b}b)\) is a \(\bar{b}b\) vector state \((Y, Y', Y'')\). The emission of the Higgs boson proceeds via the process described by the diagram fig. 6 [3.26].

The Yukawa coupling of the H meson to the b quark is given by \(m_b\). One expects \(B \sim 10^{-3} - 10^{-4}\) in case of the \(Y\) decays. The branching ratio (3.68) is independent of the quark charge, i.e. formula (3.68) is valid also for a possibly existing heavy t-quark. The branching ratio is quite sizeable for \(m(T) \approx 30\) GeV, namely \(\sim 7\% \) (T: \((t\bar{t})\)-vector meson).

The scalar boson H is expected to decay predominantly into pairs of heavy quarks \((b, c, \ldots)\) or heavy leptons \((\tau, \ldots)\), which in turn lead to rather complicated final states. For this reason it seems impossible to observe the H boson directly by studying the invariant mass of the decay products. The most suitable way to discover the H boson is to study the invariant mass of the system recoiling against the photon in the decay \(V \rightarrow \gamma + \text{anything}\), as discussed above. Of course, this can only be done in an \(e^+e^-\)-annihilation experiment. Furthermore the H boson mass must be rather light, i.e. not heavier than \(\sim 20\) GeV, in order to be seen by the experiments now under way.

Fig. 6. The decay of a \((b\bar{b})\)-meson into a Higgs boson and a photon.
4. Weak interactions of the four light leptons and quarks

4.1. Charged currents as seen by neutrinos

Neutrino–hadron scattering is a tool to study the semileptonic weak interaction. Especially the reaction $\nu_\mu + N \rightarrow \mu^- + X$ (N: nucleon) is used to investigate the charged hadronic weak currents. This reaction is described by three structure functions ($F_1, F_2, F_3$) [4.1]. If we interpret the $\nu_\mu$-hadron inelastic scattering as due to the elastic scattering of the neutrino off structureless spin 1/2 quarks, using the conventional lefthanded weak currents, one can express the structure functions in terms of the quark distribution functions $q(x)$:

\[
\frac{d^2\sigma}{dx\,dy} = 2 \frac{G^2 M E}{\pi} x [(d(x) + s(x)) + (\bar{u}(x) + \bar{c}(x))(1 - y)^2]
\]

where $M$ is the nucleon mass, $E$ the incident energy. The scaling variable $x$ is given by $x = Q^2/2M\nu$ ($Q^2 = -q^2$ = four-momentum transfer squared, $\nu = E - E'$ ($E'$ final lepton energy), and the parameter $y = \nu/E$ denotes the relative proportion of the total energy transferred to the target.

During the years 1976–1979 a relatively clear picture of the inelastic neutrino–hadron scattering has emerged. One has found that the observed cross sections are rather well described by using the conventional lefthanded weak currents. The agreement between theory and experiments works so well that the neutrino–hadron scattering can be regarded not as a tool to study the weak interactions, but as a tool to probe the various details of the strong interactions (e.g. the QCD finestructure of the quarks). We mention the following aspects:

(a) The experimental results are in agreement with the assumption that the hadronic charged weak current has the structure $\bar{u}\gamma_\mu L(d \cos \theta_c + s \sin \theta_c)$, where $\gamma_\mu L = \gamma_\mu (1 + \gamma_5)/2$. There is no indication that righthanded weak currents of the type $(\bar{u}d)_R$ or $(\bar{u}s)_R$ exist. Those would lead to terms in $d\sigma$ proportional to $(1 - y)^2$, which are not seen in the experimental data (for a detailed analysis of the neutrino scattering results see ref. [4.1] and references therein).

Since the production of charmed particles in neutrino scattering can be “seen” indirectly by looking for dimuon events, a careful study of the dimuon events can be used in order to say something about the structure of the charm-changing charged currents. The experiments carried out thus far are consistent with the form $\bar{c}(s \cos \theta_c - d \sin \theta_c)$, but do not rule out that the charm-changing current deviates significantly from the form given above. It is important to check how good the Cabibbo universality for the charm-changing charged current is, since such a test provides stringent constraints on the admixture of further quarks (e.g. t, b) in the weak current. Detailed tests of the Cabibbo universality are now under way, based on an analysis of the dimuon events. To summerize our conclusions we can say: The charged weak currents as tested in neutrino scattering are consistent with the lefthanded charged currents of the GIM scheme. However the constraints on the charm-changing charged current are not very strong, and significant deviations from the GIM current cannot be excluded. Especially we mention that small deviations from the predictions of the conventional GIM scheme are expected in view of the existence of further quarks, besides $u, d, s, c$.

4.2. Weak decays of strange particles

The leptonic and semileptonic decays of the strange particles are very well studied, and we refer the
reader to one of the recent textbooks on weak interactions [4.2]. However the mysteries of the nonleptonic decays of the strange particles are still with us, and we should like to discuss them within our present view about the weak interactions.

We do not wish to review all the details of the nonleptonic decays of the strange particles, but mention only those aspects, which are of particular interest in view of the recent developments. The basic features of the nonleptonic decays of strange particles are:

1. The \( \Delta I = 1/2 \) weak transitions dominate, both for mesons and baryons. The decay \( K^+ \to \pi^+ \pi^0 \) can proceed only via the \( \Delta I = 3/2 \) part of the weak Hamiltonian. Thus a measure of the \( \Delta I = 1/2 \) enhancement is given by the ratio

\[
\frac{\Gamma(K_s \to \pi\pi)}{\Gamma(K^+ \to \pi^+ \pi^0)} = (25.5)^2.
\]

In general one has

\[
\left| \frac{A(|\Delta I| = 1/2)}{A(|\Delta I| = 3/2)} \right| = 25-45
\]

(\( A \): decay amplitude).

2. The \( |\Delta I| = 1/2 \) nonleptonic amplitudes are larger for both baryons and kaons than the associated semileptonic amplitudes by factors varying between \( \sim 15 \) and \( \sim 30 \), e.g.:

\[
\frac{\Gamma(K^+ \to \mu^+ \nu) \sqrt{2}}{\Gamma(K_s \to 2\pi)} \approx \frac{1}{(15)^2}, \quad \frac{\Gamma(\Lambda \to \pi \nu) \sqrt{2}}{\Gamma(\Lambda \to N \pi)} \approx \frac{1}{(32)^2}.
\]

3. The rate for the \( |\Delta I| = 3/2 \) decays is roughly equal to the semileptonic decay rates, e.g.

\[
\frac{\Gamma(K^+ \to 2\pi, 3\pi)}{\Gamma(K^+ \to \pi \mu \nu, \pi e \nu)} \approx 3.5.
\]

4. The Lee–Sugawara relation [4.2]

\[
2A(\Xi^- \to \Lambda \pi^-) + A(\Lambda \to p \pi^-) = \sqrt{3}A(\Sigma^+ \to \pi^0 p)
\]

is approximately valid both for s-waves and p-waves. It hinges on SU(3) and chiral symmetry. For the p-waves a delicate balance between the \( f/d \) ratios of the dominating parity conserving and parity violating octets involved in the \( |\Delta I| = 1/2 \) transitions and the pseudoscalar–baryon coupling constants is necessary to match the accuracy of the Lee–Sugawara relations.

5. The chiral structure of the \( |\Delta I| = 1/2 \) effective Hamiltonian relative to the \( |\Delta I| = 3/2 \) part is given by the Bouchiat–Meyer sum rule which relates the \( K \to 3\pi \) to \( K \to 2\pi \) amplitudes for both \( |\Delta I| = 1/2 \) and \( |\Delta I| = 3/2 \) transitions [4.3]. One concludes that the chiral structure of the effective Hamiltonian in both \( |\Delta I| = 1/2 \) and \( |\Delta I| = 3/2 \) transitions is to a good approximation pure and equal, i.e. lefthanded, as far as the non-strange quarks (u, d) are concerned. This question is now topical in view of new experimental results and it is thus deemed appropriate to discuss it at some length in Appendix 1.

6. In the limit of the SU(3) symmetry the conventional current–current Hamiltonian (or a Hamil-

tonian, which is identical in its $C$ and $P$ properties to the conventional one) does not contribute to the decay $K_s \to \pi\pi$ [4.4].

In the absence of the strong interactions the effective weak Hamiltonian density is given by

$$H^{\text{eff}}(x) = 2\sqrt{2}G : j^+_\mu(x) j^-_\mu(x):$$

(4.7)

where

$$j^+_\mu = \cos \theta_e \bar{u} \gamma_\mu L d + \sin \theta_e \bar{u} \gamma_\mu L S_L + (-\sin \theta_e) \bar{c} \gamma_\mu L d + \cos \theta_e \bar{c} \gamma_\mu L S_L$$

$$j^-_\mu = (j^+_\mu)^*, \quad \gamma_{\mu L} = \gamma_\mu (1 + \gamma_5)/2,$$

(4.8)

and the $:\ $ symbol denotes normal ordering. The currents are supposed to be color singlets, e.g. the current $\bar{u} \gamma_\mu d$ stands for $\Sigma_{\alpha=1}^3 \bar{u}_\alpha \gamma_\mu d_\alpha$ ($\alpha$: color index).

It is customary to group these current products into linear combinations which are eigenstates of the QCD renormalization group [4.5].

In particular the strangeness changing and charm conserving part of the effective weak Hamiltonian is given by

$$H^{\text{eff}}(1|\Delta S| = 1) = 2\sqrt{2}G \cos \theta_e \sin \theta_e \left[ (\bar{u} \gamma_\mu u) (\bar{c} \gamma_\mu c) - (\bar{d} \gamma_\mu S) (\bar{c} \gamma_\mu d) \right].$$

(4.9)

Now we proceed to take into account the strong interactions. In QCD perturbation theory one encounters two different effects.

(a) The weak interaction vertex (four-fermion vertex) is renormalized by radiative corrections (see fig. 7).

(b) In higher orders of $g_s$ (QCD coupling constant) new types of four-fermion operators are brought in which are not present in the bare Hamiltonian (so-called penguin diagrams, see fig. 8 [4.6]). The reader unfamiliar with this question should first consult Appendix 2 for an introduction.

First let us discuss the radiative corrections of the weak vertex. We rewrite eq. (4.9) as follows:

$$H^{\text{eff}}(1|\Delta S| = 1) = \sqrt{2}G \cos \theta_e \sin \theta_e \cdot (O^{(s)} + O^{(c)})$$

(4.10)

![Fig. 7. One of the diagrams describing a radiative QCD correction to the bare effective weak fourfermion interaction. The gluon is denoted by a curly line.](image1)

![Fig. 8. A QCD process which leads to the appearance of new terms in the weak Hamiltonian ("Penguin diagram").](image2)
where we have

$$O^{(\pm)} = [(\bar{s}u)(\bar{u}d) \pm (\bar{s}d)(\bar{u}u)] - [(\bar{s}c)(\bar{c}d) \pm (\bar{s}d)(\bar{c}c)]$$  \(4.11\)

$$((\bar{s}u) \text{ stands for } \bar{s}\gamma_\mu u).$$

It is useful to decompose $O^{(\pm)}$ into $O_1^{(\pm)}$ and $O_2^{(\pm)}$, where one has:

$$O_1^{(\pm)} = (\bar{s}u)(\bar{u}d) \pm (\bar{s}d)(\bar{u}u)$$

$$O_2^{(\pm)} = (\bar{s}c)(\bar{c}d) \pm (\bar{s}d)(\bar{c}c).$$

We have introduced the new operators $(\bar{s}d)(\bar{u}u)$ and $(\bar{s}d)(\bar{c}c)$, which are added in $O^{(+)}$ and subtracted in $O^{(-)}$. Of course, in doing so we have not changed the bare weak Hamiltonian, since the effect cancels out in the sum $O^{(+)} + O^{(-)}$.

It is useful to work with the operators $O^{(+)}$ and $O^{(-)}$, since in terms of $O^{(+)}$ the renormalization of the weak vertex is particularly simple; they are eigenstates of the QCD renormalization group equations. In general we can write the effective weak Hamiltonian as follows, including the renormalization effects at the one-loop level:

$$H_{\text{eff}}(\Delta S|S = 1) = \sqrt{2} G \cos \theta_c \sin \theta_c (c^+(\mu, g) O^{(+)} + c^-(\mu, g) O^{(-)}),$$  \(4.12\)

where the coefficients $c$ multiplying $O^{(+)}$ and $O^{(-)}$ depend on the QCD subtraction point $\mu$ and the coupling constant $g$, like the operators $O$. This dependence must be such that $H_{\text{eff}}$ is independent of the renormalization parameter $\mu$, which can be chosen arbitrarily. Since the mass of the intermediate weak boson $M_w$ is the only distinguished mass parameter in the scheme, it is useful to rewrite the coefficients $c$ as functions of the dimensionless ratio $M_w/\mu$: $c(\mu, g) \rightarrow c(M_w/\mu, g)$. At $\mu = M_w$ we should obtain, apart from small corrections of order $\tilde{g}(\mu = M_w)$ ($\tilde{g}(\mu)$: running coupling constant of QCD), $c^{(+)}(M_w/\mu, g) = c^{(+)}(1, \tilde{g}(M_w)) \approx 1$. This gives [4.5]

$$c^{(\pm)}(M_w/\mu, g) = (\alpha_s(M_w^2)/\alpha_s(\mu^2))^{a^{(\pm)}}$$  \(4.13\)

where $a^{(\pm)}$ are the anomalous dimensions of the operators $O^{(\pm)}$:

$$a^{(+)} = 2/b$$

$$b = 11 - \frac{2}{3} n_t$$

$$a^{(-)} = -4/b$$  \(4.14\)

($n_t$: number of quark flavors). Note that according to eq. (4.14) $c^{(+)}$ and $c^{(-)}$ are related:

$$c^{(-)} = (1/c^{(+)})^2.$$  \(4.15\)

In the free field limit ($g = 0$) we have, of course, $c^{(+)} = c^{(-)} = 1$. The weak decay amplitudes which are measurable quantities cannot depend on the scale $\mu$, which can be chosen arbitrarily. The matrix elements of the operators $O^{(+)}$, $O^{(-)}$ depend on $\mu$, like the coefficients $c$, such that the decay amplitudes themselves are independent of $\mu$. If we choose $\mu = M_w$, the coefficients $c^{(+)}$, $c^{(-)}$ are equal to one, but the matrix elements of the operators $O^{(+)}$, $O^{(-)}$ are defined at $\mu = M_w$, i.e. they contain a lot of hard
gluon effects, which could be taken care of by rescaling. Since we are free to choose the renormalization point $\mu$, we might as well make the most useful choice, normally to choose $\mu$ as small as possible such that the hard gluon effects are taken care of as much as possible by the coefficients $c^{(a)}$. Thus we choose $\mu = 1$ GeV, in the hope that the lowest loop expansion of the QCD renormalization group equations makes sense in the region $1$ GeV $\lesssim \mu \lesssim \mathcal{M}_{\pi}$. In that case we evaluate the matrix elements of $O^{(a)}$ between normal hadrons, say $\langle \pi | O^{(a)} | \pi \rangle$, at $\mu = 1$ GeV, i.e. at an energy where typical hadronic structures are defined and where we make use of our knowledge about the bond state structures of hadrons.

In the leading log approximation we have

$$
\alpha_s(\mu^2) = \frac{4\pi}{b} \cdot \frac{1}{\ln(\mu^2/\Lambda^2)} \quad (b = 11 - \frac{2}{3} n_f)
$$

and

$$
c^{(a)}(\mu, g) = \left( \frac{\ln(\mu^2/\Lambda^2)}{\ln(\mathcal{M}_{\pi}^2/\Lambda^2)} \right)^{2/b}
$$

(4.16)

where $\Lambda$ is the QCD scale. Using $b = 25/3$, $\mathcal{M}_{\pi} = 90$ GeV and $\Lambda^2 = 0.1$ GeV$^2$, we find

$$
c^+(\mu = 1 \text{ GeV}) = 0.68, \quad c^-(\mu = 1 \text{ GeV}) = 2.15. \quad (4.17)
$$

The weak Hamiltonian takes the form

$$
H^\text{eff}(|\Delta s| = 1) \approx \sqrt{2}G \cos \theta_c \sin \theta_c (0.68 O^{(a)} + 2.15 O^{(c)} - 0.68 O^{(a)}) + \text{h.c.} \quad (4.18)
$$

We observe that the contribution of $O^{(-)}$ to $H^\text{eff}$ is enhanced, while the contribution of $O^{(+)}$ is suppressed, as compared to the situation at $\mu = \mathcal{M}_{\pi}$.

The operators $O^{(+)}_1$ and $O^{(-)}_1$ consist of products of currents $(\bar{s}q)$ and $\bar{q}q$ [q = u, d], which are isospin doublets, triplets and singlets. This implies that $O^{(+)}_1$ and $O^{(-)}_1$ consist of terms which transform under SU(2) as $(1/2)$ and $(3/2)$ representations. The operator $O^{(-)}_1$ has $I_3 = -1/2$ and belongs to a $I = 1/2$ representation. This can be seen easily by applying the isospin lowering operator $I^- = f \, d^3 x \, d^* u$. We have $I^- O^{(-)}_1 = 0$.

The operator $(\bar{s}d)(\bar{u}d)$ has $I_3 = -3/2$, i.e. it belongs to a $(3/2)$ representation. Applying the isospin raising operator $I^+ = f \, d^3 x \, u^* d$, we find the corresponding term with $I_3 = -1/2$:

$$
I^+(\bar{s}d)(\bar{u}d) = (\bar{s}u)(\bar{u}d) + (\bar{s}d)(\bar{u}u) - (\bar{s}d)(\bar{d}d) = O^+_1 - (\bar{s}d)(\bar{d}d). \quad (4.19)
$$

Thus we obtain:

$$
O^{(+)}_1 = O(3/2) + (\bar{s}d)(\bar{d}d); \quad (4.20)
$$

the operator $O^{(+)}_1$ contains an $I = 3/2$ term. The operators $O^{(a)}_2$ have $I = 1/2$. Therefore the ratio $c^-/c^+$ is a measure of the $|\Delta I| = 1/2$ enhancement. Using the coefficient in eq. (4.17), one finds 2.8 for this ratio. Thus the QCD radiative corrections to the weak decay amplitudes enhance the $|\Delta I| = 1/2$ channel, and suppress the $|\Delta I| = 3/2$ channel, in agreement with observation. However numerically the effect is much too small in order to account for the observed $|\Delta I| = 1/2$ enhancement. While the QCD radiative
corrections give an enhancement of the order of 3 in amplitude, one needs a factor of about 15–20 in order to understand the observed enhancement. We emphasize that we have set the cut-off parameter $\mu$ to 1 GeV, since one can hardly imagine that perturbative calculations make sense for momenta less than 1 GeV. Nevertheless there are uncalculable strong interaction effects which contribute to the renormalization of the operators $O^{(+)}$ and $O^{(-)}$ and which are due to the emission and reabsorption of soft gluons. If we blindly use the lowest order perturbation theory even in the region $\mu < 1$ GeV, the coefficients $c^{(+)}$ and $c^{(-)}$ depart even stronger from their free-field values $c^{(+)} = c^{(-)} = 1$. For example, for $\Lambda^2 = 0.1$ GeV$^2$ and $\mu = 0.5$ GeV one finds $c^{(-)}/c^{(+)} \approx 6$.

We conclude: It seems not impossible to understand the $|\Delta I| = 1/2$ enhancement of the strange particle decay amplitudes, if one takes into account the calculable QCD radiative corrections plus yet uncalculable soft gluon contributions, i.e. $|\Delta I| = 1/2$ enhancement effects due to gluons with a wavelength larger than $M_P^{-1}$.

Let us return to the new induced terms, described by fig. 8 [Penguin diagrams]. In lowest order of QCD perturbation theory one obtains:

$$-H^{\text{induced}} = \frac{G}{\sqrt{2}} \sin \theta_c \cos \theta_c \ln \left( \frac{m_e^2}{\mu^2} \right) \frac{g_s^2}{2\pi^2} \left[ \left( \bar{d} \gamma_{\mu L} \frac{X^a}{2} u \right) \sum_q \left( \bar{q} \gamma^\nu \frac{X^a}{2} q \right) + \text{h.c.} \right],$$

where $g_s$ is the QCD coupling constant, and $\mu$ denotes a hadronic mass scale typical for the process, which is expected to be of the order of a few hundred MeV. The color SU(3) matrices are denoted by $X^a$.

The operator appearing in eq. (4.2) is not an eigenstate of the renormalization group. Mixing occurs with other quark quadrilinears (for a detailed discussion we refer the reader to ref. [4.6]). The net effect is that we have to add two new operators $P_1, P_2$ to the weak Hamiltonian (see Appendix 2)

$$P_1 = \left( \bar{d} \gamma_{\mu L} \frac{X^a}{2} s \right) \left( \bar{u} \gamma^\nu \frac{X^a}{2} u \rightarrow d \rightarrow u \rightarrow s \right)$$

$$P_2 = \left( \bar{d} \gamma_{\mu L} s \right) \left( \bar{u} \gamma^\nu u \rightarrow d \rightarrow u \rightarrow s \right).$$

$$H = 2\sqrt{2} \sin \theta_c \cos \theta_c (\ldots + c_1 P_1 + c_2 P_2).$$

(4.22)

The coefficients multiplying these operators are relatively small, e.g. in ref. [4.6] one quotes $c_1 = -0.28, c_2 = -0.10$. Note that the operators $P_1$ and $P_2$ contain righthanded quark fields.

They come in, since in the penguin diagrams the coupling of the gluons to the quarks enters. The gluons couple both to lefthanded and righthanded quarks.

To calculate the matrix elements of the operators $P_1, P_2$ we perform a Fierz transformation using the identity

$$\bar{\psi}_1 \gamma_{\mu L} \psi_2 \bar{\psi}_3 \gamma_{\mu R} \psi_4 = -2 \bar{\psi}_1 \gamma_{\mu} \psi_4 \bar{\psi}_3 \psi_2 L$$

(4.23)

and the equations of motion

$$i(m_d + m_u)\bar{d} \gamma_\mu d = \partial^\mu (\bar{d} \gamma_\mu d)$$

$$i(m_u + m_s)\bar{u} \gamma_\mu s = \partial^\mu (\bar{u} \gamma_\mu s).$$

(4.24)
Thus we see that matrix elements of the pseudoscalar densities enter. This leads to an enhancement of the matrix elements, especially for the decay $K_s \rightarrow \pi\pi$. The estimate given in ref. [4.6] is

$$\sqrt{2} G \sin \theta_c \cos \theta_c \langle \pi^+ \pi^- | c_1 P_1 + c_2 P_2 | K^0_s \rangle = iGM K M_{\pi} 0.85,$$

(4.25)

while the experimental number for this decay amplitude gives 1.05 instead of 0.85. Thus the penguin term may account for a sizeable part of the observed decay amplitude, despite the smallness of the coefficients $c_1$ and $c_2$. In a similar way the $|\Delta I| = 1/2$ decay amplitudes for the nonleptonic hyperon decays are dominated by the penguin terms.

We add to our discussion of the nonleptonic strange particle decays the following remarks. The $|\Delta I| = 1/2$ part of the conventional nonleptonic current–current Hamiltonian is enhanced, while the $|\Delta I| = 3/2$ term is suppressed by QCD radiative corrections. The relative enhancement factor for the $|\Delta I| = 1/2$ term is about 2–3, i.e. much too small to account for the observed enhancement. Soft gluon effects may be large enough to give an enhancement of the order of ten in amplitude, as required by experiment. The induced terms (Penguin diagrams) are small compared to the conventional current–current terms, but have relatively large matrix elements. It is not excluded that the observed $|\Delta I| = 1/2$ enhancement is largely due to the penguin diagram contributions. However we must emphasize that the estimates of the matrix elements given in ref. [4.6] are rather uncertain, due to the uncertainties about $\alpha_s$, the light quark masses $m_u$, $m_d$, $m_s$, and the assumption that the matrix elements as well as the coefficient functions can be calculated by using a naive approach (factorization etc.). It may well be that the penguin terms constitute only small fractions of the nonleptonic decay amplitudes. An important check of this will be the study of the nonleptonic decays of charmed particles. Since the penguin diagrams cannot contribute to the main charm particle decay modes (the amplitudes are proportional to $\sin \theta_c$), one should not observe a nonleptonic enhancement of the charm particle decay amplitudes, apart from the QCD radiative corrections. In particular the life-times of $D^0$ and $D^+$ should be equal. The new experimental results disagree with this expectation (see our discussion below), and the question whether the penguin diagrams constitute a major fraction of the nonleptonic decay amplitudes for strange particles must be left open at the moment. It may well be that the mysteries of the decays of strange particles will only be understood finally after the detailed studies of the decays of charmed particles.

**Nonleptonic strange particle decays and constituent quark scattering.** We have emphasized above that the essential part of the $|\Delta I| = 1/2$ enhancements of the various nonleptonic decay amplitudes must come from the thus far ill understood effects due to gluons of low momenta, in particular from effects due to the bound-state nature of the hadrons. Thus far we have neglected the fact that we are dealing with the weak decay of an s-quark which is placed inside a hadron (either baryon or meson), together with other constituent quarks or antiquarks. Could it be that the presence of the other constituent quarks influences the matrix elements of the weak Hamiltonian such that the $|\Delta I| = 1/2$ enhancement results?

First we note that the striking difference between the strength of the $K^0$ decay and the $K^+$ decay could be understood if it would be possible to attribute the fast decay of the $K^0$ as due to the presence of the d quark in the $K^0$ (besides the s). At first sight it looks as if this is possible. The point is that the $(\bar{s}d)$ system in the $K^0$ can annihilate to produce a $(\bar{u}u)$ pair (via W exchange). The latter will proceed e.g. to a final $\pi\pi$ state. On the other hand the $(\bar{s}u)$ system in the $K^+$ can annihilate to produce a $(\bar{d}u)$ pair, which has $I = 1$. The latter cannot lead to a $\pi^+\pi^0$-state, since two pions in a s-wave configuration cannot have $I = 1$. Thus the $(\bar{d}u)$ pair can produce only a $3\pi$ state. If it would be possible to provide an argument why the annihilation of the constituent quarks in a K meson should dominate the nonleptonic
weak decay, one would have understood the difference between the $K^0$ and $K^+$ decay. However it seems that on the basis of a simple-minded approach such an argument cannot be given. Suppose we describe the decay of an $s$ quark in a free quark theory. Comparing the annihilation process with the decay $s \rightarrow u \bar{u} d$, one finds

$$\frac{\Gamma[(s\bar{d}) \rightarrow (u\bar{u})]}{\Gamma[s \rightarrow u\bar{u}d]} \approx \frac{8\pi^2}{3} \frac{F_K^2 m^2}{m_s^4}, \quad m = \frac{m_u + m_d}{2}$$

(4.26)

where $F_K$ is the $K$ meson decay constant, and $m_u$, $m_d$, and $m_s$ are the masses of the $u$, $d$, and $s$ quarks.

This ratio is $\sim 0.07$, if we use $(m_s/m) \sim 20$ (PCAC value), and it seems difficult to argue that this ratio should be as large as $\sim 10^2$, in order to account for the observed enhancement. In the following section we shall come back to this point.

The situation looks more promising, if we look at the hyperon decay. The idea is to interpret the nonleptonic hyperon decay due to the reaction $(su) \rightarrow (ud)$ (W-exchange) inside a hyperon [4.7]. We emphasize that in a simple approach one could not understand the $\Sigma^-$ decay, since two $u$ quarks are contained in the $\Sigma^-$ wave function. For this reason we have to use a more sophisticated approach, namely to use PCAC and to relate the non-leptonic hyperon decay amplitudes $B' \rightarrow B\pi$ to the baryon matrix elements $\langle B'| \hat{H}|B \rangle$, where $\hat{H}$ denotes the new weak Hamiltonian obtained after the PCAC transformation. The decay amplitude for the $s$ wave decays is of the order of

$$A(B' \rightarrow B\pi) \sim F_\pi^{-1} G \sin \theta, |\psi(r_{su} = 0)|^2,$$

(4.27)

where $F_\pi$ denotes the pion decay constant, and $|\psi(r_{su} = 0)|^2$ is the probability to find a $(su)$ pair in the corresponding hadron at zero separation. As emphasized in refs. [4.8], the correct magnitudes of the hyperon $s$ wave decay amplitudes are reproduced, if $|\psi(r_{su} = 0)|^2 \sim 10^{-2} \text{GeV}^3$ (for a discussion of the $p$ waves see ref. [4.9]).

On the other hand it is possible to obtain an information of the baryon wave function at the origin comparing the electromagnetic mass differences of the baryons. One finds [4.8]

$$M(\Sigma^0) - M(\Sigma^+) - (M(n) - M(p)) \equiv \alpha \cdot \frac{2\pi}{3m_q^2} |\psi(r_{qq} = 0)|^2$$

(4.28)

where $m_q$ is the constituent quark mass in the SU(3) limit (given e.g. by the magnetic moments):

$m_q \approx 300 \text{ MeV}$. One finds: $|\psi|^2 \approx 1.1 \times 10^{-2} \text{ GeV}^2$. Thus the absolute magnitudes of the hyperon decay amplitudes are reproduced in a satisfactory way using the W exchange mechanism. However several unsolved problems remain, e.g. an understanding of the $D/F$ ratio of the $s$ wave decay amplitudes, a satisfactory treatment of the $p$-waves, an understanding of the asymmetry parameters. Certainly more work is needed in order to arrive at final understanding of the nonleptonic hyperon decays.

$\Omega^-$ decays. Recently the nonleptonic decays of the $\Omega^-$ have been studied in more detail [4.10]. One finds

$$B(\Omega^- \rightarrow \Lambda K^-) = 67.0 \pm 2.2\%$$
$$B(\Omega^- \rightarrow \Xi^0 \pi^-) = 24.6 \pm 1.9\%$$
$$B(\Omega^- \rightarrow \Xi^- \pi^0) = 8.4 \pm 1.1\%$$
$$\tau(\Omega^-) = (0.82 \pm 0.06) \times 10^{-10} \text{ s}.$$
Since the $\Omega^-$ has $J = 3/2$, the final states $\Lambda K$ or $\Xi\pi$ must be either in a P or D wave. The asymmetry parameter for the $\Lambda K^-$ decay was measured: $\alpha = 0.06 \pm 0.14$ \cite{4.10}. This may be taken as an indication that the amplitude for $\Omega^- \to \Lambda K^-$ is parity conserving (P wave).

If the decay Hamiltonian responsible for the $\Omega^-$ decay would be entirely $|\Delta I| = 1/2$, the isospins of the final states $\Xi^0\pi^-$ and $\Xi^-\pi^0$ must be $I = 1/2$, since $I(\Omega^-) = 0$. The $I = 1/2$ configuration of charge $-1$ is given by $(\sqrt{2}\Xi^0\pi^- - \Xi^-\pi^0)/\sqrt{3}$. Thus we expect:

$$B(\Xi^0\pi^-)/B(\Xi^-\pi^0) = 2.$$  \hspace{1cm} (4.29)

The experimental value is $2.93 \pm 0.50$, indicating that the violation of the $|\Delta I| = 1/2$ rule in the $\Omega^-$ decay is larger than in the normal hyperon decays, where it is at most of the order of a few %.

The $\Omega^-$ decays have been analysed in ref. \cite{4.11} taking into account the QCD radiative corrections and contributions from penguin diagrams. One concludes that the $|\Delta I| = 3/2$ terms are more important in the $\Omega^-$ decays than in the normal hyperon decays, in agreement with the experimental results. The lifetime of the $\Omega^-$ is in good agreement with the theoretical expectation.

We should like to add a few comments about the W exchange mechanism discussed above. The latter cannot be applied directly to the $\Omega^-$ decays since the $\Omega^-$ consists of three s quarks (note that the W exchange mechanism leads to a transition $(\bar{u}d) \to (\bar{u}d)$). In order to apply a similar decay mechanism, one would have to make use of the sea of $\bar{q}q$ pairs in the $\Omega^-$. In this case we expect that the decay rates for the $\Omega^-$ decays are reduced by some factor compared to the corresponding decay rates for the hyperon decays. As a result the $|\Delta I| = 3/2$ transitions are expected to become more prominent due to the reduction of the $|\Delta I| = 1/2$ rate. For example, the decay $\Omega^- \to \Xi^-\pi^0$ can proceed via the weak transition $\bar{s}d \to u\bar{u}$ (W exchange), where the $\bar{d}$ is generated out of the $\bar{q}q$ sea; analogously the decay $\Omega^- \to \Xi^0\pi^-$ can proceed via the transition $\bar{s}u \to \bar{d}d$ (annihilation via W). Furthermore those decays can proceed via the direct decay of the s quark $(s \to u\bar{d})$.

The combined decay amplitudes will have both $|\Delta I| = 1/2$ and $|\Delta I| = 3/2$ terms, due to interference effects of the W-exchange or annihilation terms and the direct decay term.

The decay $\Omega^- \to \Lambda K^-$ may also be understood along the lines discussed above. For example, one of the s quarks in the $\Omega^-$ may react with a $\bar{d}$ via W exchange to produce a $(\bar{u}u)$ system. That way a system consisting of $(\bar{u}u\bar{d}d)$ is formed, which can turn into a $\Lambda K^-$ state. It seems that more theoretical work is needed in order to arrive at a clear picture of how the $\Omega^-$ decay proceeds.

Conclusions. We conclude that our present theoretical picture of the strange particle decays is still rather unsatisfactory. It seems likely that the bulk of the observed $|\Delta I| = 1/2$ enhancement comes from effects related to the bound state structure of the hadrons. A possible way to treat those may be to investigate the constituent quark interaction idea discussed above in more detail. Furthermore it would be desirable to have a better understanding of the Lee–Sugawara relation and of the observed asymmetry parameters in the hyperon decays. For example the asymmetry parameter for the decay $\Sigma^+ \to p\pi^0$ is very close to $-1$, the one for the decay $\Lambda \to p\pi^-$ close to $2/3$. It is difficult to believe that these are accidents. Yet in the current theoretical picture they appear as accidents. As mentioned above, the decay $K_s \to \pi\pi$ vanishes in the SU(3) limit. In the present theoretical framework this decay must be attributed to a SU(3) breaking effect. This may well be the case. Nevertheless it would be useful to investigate whether the SU(3) breaking effects could be strong enough to allow a decay of the $K_s$ as fast as observed.

Very often in the past one has attributed the unexplained features of the nonleptonic weak decays to possibly existing new weak interactions, for example new righthanded currents for the quarks etc. In view of the developments during the last five years it seems, however, that no new interactions of the
type needed are present. This gives additional support to those who try to understand the nonleptonic weak decays of strange particles within the conventional weak interaction theory. It remains to be seen whether a satisfactory picture along these lines emerges in the future.

4.3. Weak decays of charmed particles

The transition operator describing the decay of charmed particles in the conventional 4-quark scheme is given by

$$T(|ΔC| = 1) = 2\sqrt{2}G \int d^4x j_μ^+(x, |ΔC| = 1) j_μ^-(x, |ΔC| = 0) + h.c.$$  \hspace{1cm} (4.30)

where

$$j_μ^+(|ΔC| = 1) = \cos θ_c \bar{c}γ_μ LS - \sin θ_c \bar{c}γ_μ d$$
$$j_μ^+(|ΔC| = 0) = \cos θ_c \bar{u}γ_μ d + \sin θ_c \bar{u}γ_μ LS + \bar{e}_cγ_μ L e^- + \bar{ν}_μγ_μ μ^- + \bar{τ}_γ γ_μ τ^-$$
$$j_μ^- = j_μ^{+*}.$$  \hspace{1cm} (4.31)

(a) Semileptonic decays. It follows immediately from isospin considerations in the limit $θ_c = 0$:

$$Γ(D^+ → e^+ + ν_e + X) = Γ(D^0 → e^+ + ν_e + X)$$
$$≈ Γ(D^+ → μ^+ + ν_μ + X) = Γ(D^0 → μ^+ + ν_μ + X).$$

Furthermore in the same limit ($θ_c = 0$) all semileptonic D decays should contain a K meson. Thus we expect that in reality this is true to a good approximation. The present experimental data are consistent with this expectation [4.12].

The final hadronic state can be either a K, K*, another excited meson of quark composition $(q\bar{q})$ $(q = u, d)$, or a multiparticle state $(Kπ, Kππ, \ldots)$. It is not yet known how often one produces simply a K meson in the D decay, as compared to K*, Kππ etc. The energy distribution of the emitted electron or muon in the D decay will, of course, depend on the structure of the final hadronic state. It has been argued in ref. [4.13] that the energy spectrum of the leptons emitted in the D-decay can be calculated within QCD perturbation theory by taking into account the gluon bremsstrahlung processes [4.12].

Recently one has observed the $Λ_c^+$ state of mass ~2.28 GeV [4.14]. This state can decay semileptonically into $e^+ ν_e [μ^+ ν_μ] + Λ_c, Λ_c π, \ldots$ with probability $cos^2 θ_c (≈96\%)$, and into $e^+ ν_e [μ^+ ν_μ] + N, Nπ, \ldots$ with probability $sin^2 θ_c (≈4\%)$. Thus far the semileptonic decays of the $Λ_c^+$ have not been observed (for theoretical discussions see e.g. [4.15]).

Let us consider the Cabibbo suppressed semileptonic D-decays which proceed via the Hamiltonian containing the product $(dc) (\bar{ν} e^- + \bar{ν}_μ^-)$. Two different types of processes can occur:

(a) The decay of the c quark in the D meson:

$$c → d + (e^+ ν_e), \hspace{1cm} c → d + (μ^+ ν_μ).$$

(b) The annihilation of the c and the $\bar{d}$ in the $D^+$ state (see also our previous remarks in case of the
K^0-decay):

(c\bar{d}) \rightarrow "W^+" \rightarrow (e^+\nu_e) + \text{hadrons}, \quad \mu^+\nu_\mu + \text{hadrons}.

The process of type b is normally disregarded, since in a naive approach the annihilation should be such that the final lepton pair is in a \( J = 0 \) state, and the amplitude is suppressed (it is proportional to \( m_e, m_\mu \) respectively, as it is for the purely leptonic decays \( D^+ \rightarrow e^+\nu_e, \mu^+\nu_\mu \), which are strongly suppressed). However it may well be that this argument is false, and that the Cabibbo suppressed semileptonic decay modes of the \( D^+ \) are dominated by the annihilation of the \( (\bar{d}c) \) pair. This would be the case if there is a sizeable probability for the \( D \) meson to consist of a configuration in which the \( (\bar{d}c) \) pair is in a \( J = 1 \) configuration, and the \( D \) state is formed out of the \( (\bar{d}c) \) pair and a \( J = 1 \) gluonic configuration [4.16]. In that case the \( (\bar{d}c) \) pair annihilates into leptons without any helicity suppression, leaving behind the gluonic part which materializes into hadrons (see fig. 9 and our discussion of the nonleptonic decays below).

As argued in ref. [4.17], it may well be that the semileptonic annihilation process is enhanced, compared to the naive estimates, and that 20–40% of all semileptonic \( D^+ \) decays are due to the annihilation process. In that case 20–40% of all semileptonic \( D^+ \) decays should not contain a \( K \) meson, but should be of the type \( D^+ \rightarrow \nu_e e^+ + \eta, \eta', \pi^+\pi^-, K\bar{K}, \text{etc.} \). It remains to be seen if this is true.

Analogously one can treat the semileptonic \( F^+ \) decays. In the case of the \( F^+ \) the \( (\bar{s}c) \) pair can annihilate into a lepton pair without any Cabibbo suppression. As argued in refs. [4.17, 4.18] the semileptonic decay \( F^+ \rightarrow \nu_\mu e^+(\nu_\mu, \mu^+) + \text{hadrons} \) may be enhanced via the annihilation process, compared to the estimates, based on the quark decay: \( c \rightarrow s + (\nu_\mu, e^+) \). In this case the final hadronic state should have \( I = 0 \) and should be produced via the materialization of the gluon component in the \( F^+ \) state. One may expect that it consists primarily of \( \eta, \eta', S*(980), E(1420), \pi^+\pi^- \), etc.

We consider it important to investigate experimentally the details of the semileptonic decays of the \( D \) and \( F \) mesons as well as of the charmed baryons. Interesting properties of the weak and strong interactions could be learned, if detailed experimental data would become available in the future.

(b) Nonleptonic decays. In the conventional approach the effective Hamiltonian for charm changing nonleptonic processes including QCD radiative corrections is given by [4.19–4.24]:

\[
H^{\text{eff}} = 2\sqrt{2} G\left[\frac{f^+ + f^-}{2} (\bar{u}d')(\bar{s}c) + \frac{f^+ - f^-}{2} (\bar{s}'d')(\bar{t}c)\right] + \text{h.c.}
\] (4.32)

where \((\bar{u}d)\) is given by \( \bar{u}_\alpha \gamma_\mu (1 + \gamma_5)/2d_\alpha \) (\( \alpha \): color index, summed over \( \alpha = 1, 2, 3 \)), and \( d' = d \cos \theta_c + s \sin \theta_c, s' = -d \sin \theta_c + s \cos \theta_c \). The coefficients \( f^+, f^- \) involve the QCD coupling constant \( \alpha_s \).

In the free field limit (\( \alpha_s = 0 \)) only the first term in eq. (4.32) contributes; we have \( f_+ = f_- = 1 \). In reality

![Fig. 9. The Cabibbo suppressed decay of the D^+ meson via the annihilation of the \( \bar{d}c \)-pair. The incoming D^+ splits up into a \( (\bar{d}c) \)-pair and a gluonic part (denoted here by two gluon lines). The \( (\bar{d}c) \) pair annihilates into leptons, while the gluonic part evolves into an I = 0 hadronic system (\( \pi\pi, \eta, \eta', \ldots \)).](image-url)
these coefficients deviate from the free field values, due to gluon exchanges among the quark lines in the four quark vertex. One finds using QCD perturbation theory (see our analogous discussion for strange particle decays and eqs. (4.13), (4.14)):

$$f_- \approx \left[ 1 + \frac{1}{4\pi} \beta_0 (m_c^2) \ln \left( \frac{M_W^2}{m_c^2} \right) \right]^{4/\beta}$$

$$b = 11 - \frac{2}{3} n_l, \quad f_+ = (f_-)^{-1/2},$$  \hspace{1cm} (4.33)

where $n_l$ is the number of quark flavors, which contribute in loop integrals involving energies and momenta up to the W mass. The values for $f_+$ and $f_-$ are defined such that a cut-off in the virtual gluon momenta at $q^2 = m_c^2$ is introduced (further corrections due to the quark masses are neglected). The effects involving gluons on a longer wavelength cannot be included in the effective Hamiltonian, but must be regarded as wave function effects.

Using e.g. $\alpha_s(m_c^2) \approx 0.2$, $m_c \approx 1.2$ GeV, one finds $f_- \approx 2.19$, $f_+ \approx 0.68$. The deviation of the coefficients $f_+$, $f_-$ from their free field values affects the strength of the nonleptonic weak Hamiltonian.

Let us calculate the nonleptonic decay rate for a c quark in QCD perturbation theory taking into account the factors $f_+$ and $f_-$. The decay rate in the absence of strong interaction effects is denoted by $\Gamma_{n1}^0$; if we neglect the masses of the u, d and s quarks, we have $\Gamma_{n1}^0$:

$$\Gamma_{n1}^0 = G^2 m_c^5/2^5 \pi^3.$$  \hspace{1cm} (4.34)

Taking into account the factors $f_+$ and $f_-$, we first perform a Fierz transformation on the second term in eq. (4.32):

$$(\bar{s}'_\alpha d'_\alpha)(\bar{u}_\beta c_{\beta}) = \frac{1}{3} (\bar{u}_\alpha d'_\alpha)(\bar{s}_\beta c_{\beta}) + 2 \left( \bar{u} \frac{\lambda}{2} d \right) \left( \bar{s} \frac{\lambda}{2} c \right)$$  \hspace{1cm} (4.35)

($\alpha$, $\beta$: color indices running from 1–3; $\lambda$: conventional color matrices normalized such that $\text{tr} \, \lambda \lambda_i = 2 \delta_{ii}$). We can rewrite the effective Hamiltonian as follows:

$$H_{\text{eff}} = 2\sqrt{2} G \left[ \left( \frac{f_+ + f_-}{2} + \frac{f_+ - f_-}{3} \right) (\bar{u} d')(\bar{s}' c) + \frac{f_+ - f_-}{2} \cdot \sum_{i=1}^{8} \left( \bar{u} \frac{\lambda_i}{2} d' \right) \left( \bar{s}' \frac{\lambda_i}{2} c \right) \right] + \text{h.c.}$$  \hspace{1cm} (4.36)

$$= 2\sqrt{2} G \left[ \left( \frac{f_+}{2} \frac{f_-}{2} \right) (\bar{u} d')(\bar{s}' c) + (f_+ - f_-) \sum_{i=1}^{8} \left( \bar{u} \frac{\lambda_i}{2} d' \right) \left( \bar{s}' \frac{\lambda_i}{2} c \right) \right] + \text{h.c.}$$

The total decay rate is the incoherent sum of the two contributions given above, provided the final states are described by free quarks (the contribution due to the color singlet current product and the one due to the color octet product add incoherently, since quarks of different colors do not interfere). Thus one finds

$$\frac{\Gamma_{n1}}{\Gamma_{n1}^0} = \left( \frac{f_+}{2} + \frac{f_-}{2} \right)^2 + (f_+ - f_-)^2 \cdot \frac{1}{9} \sum_{i,j=1}^{8} \left( \text{tr} \frac{\lambda_i}{2} \frac{\lambda_j}{2} \right)^2$$

$$= \left( \frac{f_+}{2} + \frac{f_-}{2} \right)^2 + (f_+ - f_-)^2 \cdot \frac{1}{9} = \frac{1}{3} (2f_+^2 + f_-^2).$$  \hspace{1cm} (4.37)
Using the values \( f_+ = 0.68, f_- = 2.19 \), one finds e.g. \( \Gamma_{u_0}/\Gamma_{u_1}^0 = 2.3 \). Note however that especially \( f_- \) is very sensitive to \( \alpha_s(m_c^2) \), and \( \Gamma_{u_0}/\Gamma_{u_0}^0 \) can vary between \( \sim 1.5 \) and 3.

If we would describe the decays of the charmed particles simply by letting the c-quark decay into all available quark and lepton channels, we can work out the semileptonic branching ratio \( B(D \to e + X) \approx B(D \to \mu + X) \). One finds:

\[
B = \frac{1}{2 + f_-^2 + 2f_+^2}.
\] (4.38)

The corrections due to the radiation of gluons alter \( B \) in eq. (4.38). In a calculation including short distance corrections to first order in \( \alpha_s(m_c) \) only the result is [4.25]:

\[
B = 0.2(1 - \alpha_s/m_c),
\]

i.e. the correction to the free field result is rather small (\( \sim 10\% \)).

For \( \alpha_s = 0 \) (\( f_+ = f_- = 1 \)) this result agrees with the one obtained by counting all channels in the free quark theory, namely 0.2. The QCD radiative corrections bring this branching ratio down by a factor of the order of \( \sim 1.5 - 2 \). Taking into account the possible uncertainties in the values of \( m_c \) and \( \alpha_s \), one may conclude that the semileptonic branching ratio for the charmed particles should be \( \sim 10-15\% \) [4.26]. This result was in agreement with the previous data, especially with the average of the branching ratios for \( D^- \) and \( D^0 \), for which values between 8 and 10\% were quoted [4.12]. However recently one has measured the branching ratios both for \( D^- \) and \( D^0 \). Surprisingly it has turned out that the branching ratios for \( D^- \) and \( D^0 \) are significantly different. One has found [4.27]

\[
B(D^-) = (23 \pm 6)\%, \quad B(D^0) < 4\%.
\] (4.39)

This clearly states that something must be wrong with the prescription to describe the weak decays of charmed particles by taking into account solely the decay of the c quark inside the D meson and disregarding the other constituent quark which is treated merely as a spectator. We conclude that in some way the other constituent quark (\( \bar{d} \) in \( D^+ \), \( \bar{u} \) in \( D^0 \)) must play an important rôle in the dynamics of the weak decay. Furthermore we note that the result about the branching ratios mentioned above could be taken as an indication that the \( D^0 \) decays faster than the \( D^- \), i.e. its nonleptonic rate is enhanced. Indeed there are indications from various emulsion experiments that the \( D^- \) has a longer lifetime than the \( D^0 \) [4.28]:

\[
\tau(D^+)/\tau(D^0) \approx 4-10.
\] (4.40)

One is reminded of a similar situation in the case of K decays, where one has:

\[
\tau(K^+)/\tau(K^0) \approx 140.
\] (4.41)

For this reason we find it desirable to have a unified picture by which one can understand both the enhancement of the nonleptonic \( K^0 \) decay and the \( D^0 \) decay. One possibility is to explain the enhancement of the amplitude for the nonleptonic \( K^0 \) decay by taking into account the presence of the penguin diagrams as discussed in the previous section. There exists also a contribution of the penguin
diagrams to the charm decay amplitudes. However it is easy to see that those are strongly suppressed; in particular they are proportional to \( \sin \theta_c \), as they are in the case of strange particle decays. For the latter this does not cause any harm, since all weak decay amplitudes for the strange particles are proportional to \( \sin \theta_c \), however it implies a strong suppression for the charm particle decays, which are dominated by amplitudes proportional to \( \cos \theta_c \). If the presence of the Penguin diagrams would be the major reason why the \( K^0 \) decays as fast as observed, we would expect that the same effect does not repeat itself for the \( D^0 \). The experiments seem to be in disagreement with this expectation, which at least causes the suspicion that the fast decay of the \( K^0 \) could not be due to penguin diagrams.

A picture which is consistent with both the situation in the strange particle decay and the charm particle decay is the annihilation process discussed for the \( K^0 \) at the end of the last section, and for the semileptonic decay just above.

Before we enter the discussion about the annihilation process in the nonleptonic charm particle decay, we mention that the difference between the \( D^0 \) and \( D^+ \) lifetimes could also be due to a suppression of the \( D^+ \) lifetime compared to the \( D^0 \) lifetime. Such a situation arises if the term multiplying \( f^- \) in the weak Hamiltonian \(-\langle ud \rangle (\bar{c}e) - \langle sd \rangle (\bar{u}c)\) dominates (note that this would require \( f^- \gg f^+ \), which cannot be understood within the approach based on QCD perturbation theory). This term leads to a negative interference in the \( D^+ \) decay amplitude, but not in the \( D^0 \) decay, if the final hadronic state consists of two color singlet clusters (in the \( D^+ \) case one has \((\bar{d}u)\) and \((\bar{d}s)\), while in the \( D^0 \) case the clusters \((\bar{d}u)\) and \((\bar{d}s)\) or \((\bar{u}u)\) or \((\bar{d}s)\) and \((\bar{u}u)\) occur). For a detailed discussion of this mechanism see ref. [4.29, 4.30].

One may suspect that the annihilation process, involving gluons, has something to do with the enhancement of nonleptonic decays, if one notes that the process \( \pi^+ \to e^+ \nu_e \) is strongly suppressed by mass factors. One has:

\[
\Gamma(\pi^+ \to e^+ \nu_e)/\Gamma(\pi^+ \to \mu^+ \nu_\mu) \approx \left( \frac{m_\mu}{m_e} \right)^2 \cdot \left( \frac{m_e^2 - m_\mu^2}{m_e^2 - m_\mu^2} \right)^2,
\]

but the process \( \pi^+ \to e^+ \nu_e \gamma \) is not. Of course, the latter process is suppressed by the additional factor of \((a/\pi)\), since it is a radiative correction to the process \( \pi^+ \to e^+ \nu_e \), and by the low energy theorem for soft photon emission. (In case of \( K^+ \to e^+ \nu \gamma \) and \( K^+ \to \mu^+ \nu \gamma \) the suppression of the electron decay mode is less than for pions.)

In a nonleptonic decay the rôle of the photon can be taken over by a gluon, which has a strong coupling to the quarks. For example, we may calculate the virtual process \( D^0 \to s\bar{d} + \) gluon (i.e. the decay of the \( D^0 \) in a free \( s\bar{d} \)-pair and a gluon) in lowest order of QCD perturbation theory.

One finds

\[
\Gamma(D^0 \to s + \bar{d} + \text{gluon}) \approx \frac{G^2}{192 \pi^3} \cdot \left( \frac{8}{27} g^2 \right) M_{D^0}^4 \cdot I^2
\]

where \( g^2 \) is the QCD coupling constant, and \( I \) is an integral involving the \( D_0 \) wavefunction [4.16, 4.31]. In a nonrelativistic approach \((|M_{D^0} - m_c - m_u| \ll m_u)\) one has

\[
I^2 \approx F_D^2 M_D/4m_u^2
\]

\((F_D: \text{D meson decay constant, analogous to } F_K, \text{ } m_u: \text{constituent } u \text{ quark mass})\). Using \( m_u = 300 \, \text{MeV}, \text{ } m_c = 1.4 \, \text{GeV}, \text{ } m_{D^0} = 1.87 \, \text{GeV}\), one finds

\[
\frac{\Gamma(D^0 \to s + \bar{d} + \text{gluon})}{\Gamma(D^0 \to \mu + \nu_{\mu})} \approx \left( \frac{m_\mu}{m_e} \right)^2 \cdot \left( \frac{m_e^2 - m_\mu^2}{m_e^2 - m_\mu^2} \right)^2 \cdot \left( \frac{8}{27} \right) M_{D^0}^4 \cdot I^2
\]

Thus, this process is strongly suppressed by the mass factors. One may therefore expect that the decays \( D^0 \to s\bar{d} + \) gluon will be even more suppressed than \( D^0 \to \mu + \nu_{\mu} \), and indeed the experiments confirm this expectation. A similar argument can be made for the \( D^+ \) decay.

For the semileptonic decays, it is not possible to use the same argument, since the final states are not color singlet clusters. One may, however, use the same argument as above for the \( K^0 \) decays, and expect a strong suppression for the charm decays.

In conclusion, we have seen that the annihilation process is a possible explanation for the fast decay of the \( K^0 \) and \( D^0 \) mesons. The suppression of the \( D^0 \) decay compared to the \( D^+ \) decay can also be explained by this process. The experiments seem to be in agreement with this expectation, which at least causes the suspicion that the fast decay of the \( K^0 \) could not be due to penguin diagrams.
$F_D = 200-300 \text{ MeV}$, one has $I^2 \approx 200-500 \text{ MeV}$. It is useful to compare $\Gamma(D^0 \rightarrow s + \bar{d} + \text{gluon})$ with the free $c$-quark decay amplitude $\Gamma(c \rightarrow s + u + d) = (G^2 m_c^3/2a) \cdot \pi^3$. One finds:

$$r = \frac{\Gamma(D^0 \rightarrow s + \bar{d} + \text{gluon})}{\Gamma(D^0 \rightarrow s + u + d)} \approx 4\pi \alpha_s \cdot \frac{8I^2}{81} \left( \frac{M_D}{m_c} \right)^5.$$

(4.43)

Using $I^2 = 0.5 \text{ GeV}$ and $\alpha_s = 1$, one finds $r \approx 2$, i.e. the radiative gluonic decay is slightly stronger than the free quark decay. Of course, our estimate is extremely crude, due to the uncertainty in the correct values of $m_c$, $\alpha_s$, and $I^2$ to be taken in the calculation. Furthermore the use of QCD perturbation theory is very questionable since the gluon momenta are small. Nevertheless the calculation above is instructive since it shows that one can overcome the suppression by quark mass factors in the annihilation process by taking into account the emission of gluons.

A slightly different point of view is as follows [4.16]. Effectively the $(\bar{u}c)$ pair in the $D^0$ state annihilates into a $\bar{d}s$ pair and a gluon. The latter is emitted off the $u$-quark in the $D^0$, before the $(\bar{u}c)$ pair annihilates. Thus we may regard the gluon as a spectator in the $D^0$ wave function whose rôle is to absorb a unit in angular momentum in order to allow the $(\bar{u}c)$ pair to annihilate in a $J = 1$ state. In other words: We may assume that the $D^0$ consists part of the time of a $(\bar{u}c)$ pair in a $J = 1$ configuration, and one or several gluons carrying $J = 1$ such that the $J = 0$ meson can be formed. Let us define $\bar{\rho}$ as the probability for the $D^0$ to be in such a configuration. In this case we have [4.16]

$$\Gamma(D^0 \rightarrow \bar{d}s + \text{glue}) \sim \bar{\rho} \cdot \frac{G^2}{6\pi} \cdot \tilde{F}_D^2 m_c^3.$$

(4.44)

where $\tilde{F}_D$ is a decay constant analogous to $F_D$, which describes the probability to find the $(\bar{u}c)$ pair near each other. One obtains

$$r = \frac{\Gamma(D^0 \rightarrow s + \bar{d} + \text{glue})}{\Gamma(c \rightarrow s + \bar{d} + u)} \sim \bar{\rho} \cdot \left( \frac{\tilde{F}_D}{m_c} \right)^2 105.$$

(4.45)

For example, taking $F_D \sim 0.4 \text{ GeV}$, $\bar{\rho} \approx 25\%$ and $m_c \approx 1.2 \text{ GeV}$, one finds $r = 3$; the annihilation process dominates over the decay process. We conclude:

(a) Despite all uncertainties in the estimates given above, we feel that it is justified to assume that the annihilation process plays an important rôle in the charm particle decay. It may be that the $D^0$ decay is dominated by the annihilation of the $(\bar{u}c)$ pair in the $D^0$, leaving behind a newly created $(\bar{d}s)$ pair and gluons. Since the $D^+$ cannot do the same in its Cabibbo favored decay mode, one has

$$\Gamma(D^0 \rightarrow \text{all})/\Gamma(D^+ \rightarrow \text{all}) > 1,$$

(4.46)

in agreement with the observations, which give $4-10$ for this ratio.

(b) Taking into account the color quantum number and assuming that the annihilation of a $q\bar{q}$-pair in a charm meson can only proceed by emitting one gluon (two or several gluons are supposed to be suppressed), the $F^+$ cannot decay via annihilation since the $(\bar{s}c)$-pair must be in a color singlet in order to annihilate. It can annihilate if we include the QCD radiative corrections, described by the factors $f^+$ and $f^-$, in which case the $F^+$ can decay via the second term in eq. (4.32); its decay amplitude is proportional to $f^+ - f^-$. However we find it dangerous to use lowest order perturbation theory in the
gluonic annihilation process, since evidently one is in a region where strong coupling effects play a significant rôle. For this reason we would not be surprised if it turns out that the $F^+$ has about the same life-time as the $D^0$. In general we would expect that one has

$$\tau(D^+) > \tau(F^+) \approx \tau(D^0).$$

(4.47)

(c) An important test of the annihilation hypothesis is to consider the Cabibbo suppressed $D^+$ decays. The point is that the Cabibbo suppressed nonleptonic decay of the $D^+$ can proceed via an annihilation like the Cabibbo favored $F^+$ decay. If the latter is indeed enhanced compared to the Cabibbo favored $D^+$ decay, we expect that the Cabibbo suppressed $D^+$ decay is enhanced in the same way, i.e. it proceeds mainly via the annihilation $D^+ \to (ud + \text{glue})$. The final state has $S = 0$ and $I = 1$, i.e. it cannot contain a single $K$ meson and cannot consist of $\pi^+ \pi^0$. Probably the final state consists mainly of three or more $\pi$-mesons. The annihilation decay mode of the $D^+$ contributes more to the total decay rate than expected on naive grounds, i.e. significantly more than $\sin^2 \theta_c \sim 5\%$. Estimates, given in ref. [4.17], range between 20–40\%. An unexpectedly large portion of the nonleptonic $D^+$ decay should lead to a final state without $K$ mesons.

A further interesting test of the weak decay mechanism discussed above is to consider the $\Lambda_c^+$ decay. Like in the case of the hyperon decays the nonleptonic decays of the $\Lambda_c^+$ involving the weak interaction of two constituent quarks may be of particular importance. Those decays are such that the (cd) diquark system in the $\Lambda_c^+$ turns itself via the weak interaction (W-exchange) into a $(us)$ system, i.e. the $\Lambda_c^+$ decay proceeds via the process $cdu \to suu$. The hadronic final system is then given by the hadronization of the $(suu)$ system giving rise to final states like $\Lambda \pi^+, \Lambda \rho^+, \Sigma^+ \pi^0, \bar{K}^0 p, K^+ n$, etc.

In the nonrelativistic quark model the decay rate for the decay $\Lambda_c^+ \to suu$ (the quarks are treated as free Dirac particles) can be calculated [4.32]. Because of the color antisymmetry of the baryon wave function the operator proportional to $f^-$ does not contribute, and therefore the decay rate is multiplied by $(f^-)^2$:

$$\Gamma(\Lambda_c^+ \to suu) \approx \frac{G^2}{2\pi} f^2 |\psi(0)|^2 \cdot m_c^2$$

(4.48)

(we have set $\theta_c = 0$, and have neglected the effects due to the light quark masses), where $\psi(0)$ denotes the $\Lambda_c^+$ wave function at the origin.

The major source of uncertainty in the relation above is $|\psi(0)|^2$. Using e.g. the value $|\psi(0)|^2 = 7.4 \times 10^{-3}$ GeV$^3$, as derived in ref. [4.32] by taking into account the hyperfine splitting of the baryons interpreted within QCD, one finds

$$\Gamma(\Lambda_c^+) \approx 2 \times 10^{-12} \text{ GeV} \approx 0.3 \times 10^{13} \text{ s}^{-1}. \quad (4.49)$$

Furthermore one expects [4.32] the ratio $\tau(D^+)/\tau(\Lambda_c^+)$ to be of the order of 2–3. In general one can say that the weak interaction of two constituent quarks inside the $\Lambda_c^+$ leads to a decay rate which is larger (factor 2–3, perhaps even more) than the one derived on the assumption that the $\Lambda_c^+$ decays weakly via the c quark decay. It is not excluded that the $\Lambda_c^+$ lifetime is equal or only slightly larger than the $D^0$ lifetime.

It remains to be seen if the various predictions of the annihilation hypothesis are verified in the future or not. If they are, one could say that a significant step towards a better understanding of the
nonleptonic decays of the charmed and strange particles has been made. One faces the prospect that it is necessary to study the weak decays of the charmed particles in order to arrive at an understanding of the puzzles of the nonleptonic strange particle decays.

4.4. The neutral current of quarks as seen by neutrinos

All the experimental information obtained thus far is consistent with the possibility that the neutral current conserves the quark flavor, i.e. it never connects two flavors of the same electric charge, but of different mass. We note that the same is true for the electromagnetic current. The neutral current is in general not conserved; in particular it contains axialvector terms. For this reason there is no need that the neutral current must be flavor conserving. Nevertheless it seems to be, which can be seen, for example, in case of the strangeness changing neutral current. Here one has e.g.

\[ \Gamma(K_L^0 \to \mu^+ \mu^-)/\Gamma(K_L^0 \to \text{all}) \approx 10^{-8}, \]  

(4.50)
i.e. the possibly existing terms in the neutral current proportional to \( \langle \bar{s}d \rangle \) must be very strongly suppressed. Furthermore there is evidence that the neutral current does not change charm. A term in the neutral current proportional to \( \langle \bar{c}u \rangle \) would lead in particular to \( D^0 - \bar{D}^0 \) mixing, which is not observed in the \( e^+ e^- \)-annihilation experiments [4.33]. Furthermore the \( D^0 - \bar{D}^0 \) mixing would lead to \( \mu^+ \mu^- \)-events ("wrong sign dimuon events") in the neutrino scattering experiments, which are not observed in the predicted magnitude [4.34].

During the last five years the usual way to present the experimental results of the neutrino scattering experiments on neutral current phenomena was to compare the experimental findings with the predictions of a specific model, e.g. the minimal SU(2) \( \times \) U(1) theory, and to fit the parameters of the theory, e.g. \( \sin^2 \theta_w \). However recently a large amount of new data has become available, and it is possible to determine the neutral current coupling constants without reference to a specific model. The analysis goes roughly as follows (for a detailed presentation of the arguments see ref. [4.35]). It is well-known that the contributions of strange and charmed quarks to the nucleon wave function are very small. Thus we neglect the \( \bar{s}s \) and \( \bar{c}c \) terms of the neutral current.

In this case we can write down the effective neutrino–quark interaction as follows:

\[ H^{n.e.} = \frac{4G}{\sqrt{2}} (\bar{\nu}_\mu \nu_\mu)_\nu [U_L(\bar{u}u)_L + U_R(\bar{u}u)_R + D_L(\bar{d}d)_L + D_R(\bar{d}d)_R], \]  

(4.51)

where \( U_L, U_R, D_L, D_R \) are the neutral current coupling constants, describing the coupling of the neutrino current to the various lefthanded or righthanded quark fields.

The inclusive neutral current results in case of isoscalar nuclear targets measure the square of the coupling constants averaged over \( u \) and \( d \) quarks, i.e. \( U_L^2 + D_L^2 \) and \( U_R^2 + D_R^2 \). Furthermore the elastic (anti)neutrino–proton results, the semi-inclusive \( \pi^+ \) and \( \pi^- \) production, as well as the single \( \pi \) production (\( A \) production) can be used to determine the four neutral current coupling constants both in magnitude and sign. For details of this analysis we refer the reader to the papers given in [4.35]. Especially we mention that the inclusive \( \pi \) production results of the bubble chamber groups can be used in order to discriminate between the \( u \) and \( d \) quarks. The point is that the \( u \) and \( d \) quark fragmentation functions are relatively well-known, especially from \( e^+ e^- \)-annihilation and electron or muon production. In particular a \( u \)-quark prefers to fragment into a \( \pi^+ \) (quark composition \( \bar{d}u \)) and is reluctant to
fragment into a $\pi^-$ (quark composition $\bar{u}d$). Thus one can distinguish neutral current processes in which a $u$-quark was hit from the processes in which a $d$-quark was hit [4.36]. This information, together with the information on the elastic $\nu-p$ scattering and the inclusive neutral current results is enough to determine the magnitudes and signs of the four coupling parameters, except the sign of $U_R$. Thus one is left with two discrete solutions. At this point the single $\pi$-production data ($\Delta$-production) is used in order to discriminate between these two solutions. The final result is given by the following matrix of coupling constants:

$$
\begin{pmatrix}
U_L & U_R \\
D_L & D_R \\
\end{pmatrix} = \begin{pmatrix}
0.35 \pm 0.07 & -0.19 \pm 0.06 \\
-0.40 \pm 0.07 & 0.0 \pm 0.11 \\
\end{pmatrix}
$$

(4.52)

We note that the overall sign of these coupling constants is free; it is convention to take the coupling constant $U_L$ positive. For comparison we give the result in the minimal $SU(2) \times U(1)$ theory for $\sin^2 \theta_W = 0.23$:

$$
\begin{pmatrix}
U_L & U_R \\
D_L & D_R \\
\end{pmatrix} \bigg|_{\sin^2 \theta_W = 0.23} = \begin{pmatrix}
0.35 & -0.15 \\
-0.42 & 0.08 \\
\end{pmatrix}
$$

(4.53)

The agreement of the minimal model with experiment is impressive. Especially we would like to emphasize that the coupling constants given in eq. (4.53) disagree with the possibilities that the $u$-quark current is either $V-A$, $V$, $A$, or $V+A$, and that the $d$-quark current is $V$, $A$, or $V+A$. Thus the neutral current is not "pure", i.e. of a particular simple structure, but is, if not identical, very close to the one predicted within the minimal $SU(2) \times U(1)$ theory, namely some mixture of the neutral $I_3$ current $(\bar{u}u-\bar{d}d)_L$ and the electromagnetic current.

4.5. The neutral current of electrons as seen by neutrinos

Three different types of experiments on neutrino–electron elastic scattering have been reported, using $\nu_\mu$, $\bar{\nu}_\mu$ or $\nu_e$ beams [4.37–4.40]. Each cross-section measurement determines an area in the $g_V-g_A$ plane, where $g_V$ and $g_A$ are the vector and axialvector coupling constants describing the neutral current interaction of electrons and neutrinos. The effective neutrino–electron interaction describing neutrino–electron elastic scattering at relatively low energies is described by

$$
H^{\text{eff}} = \frac{G_F}{\sqrt{2}} (\bar{\nu}_\mu \gamma^\rho (1 + \gamma_5) \nu_\mu + \bar{\nu}_e \gamma^\rho (1 + \gamma_5) \nu_e) (g_V \bar{e}^\gamma \gamma_\rho e^- + g_A \bar{e}^\gamma \gamma_\rho \gamma_5 e^-) + \frac{G_F}{\sqrt{2}} \bar{\nu}_e \gamma^\rho (1 + \gamma_5) e^- \cdot \bar{e}^\gamma \gamma_\rho (1 + \gamma_5) \nu_e.
$$

(4.54)

Note that the first term in eq. (4.54) is the neutral current interaction while the second term describes $\nu_e-e$ elastic scattering mediated by the charged current. After a Fierz rearrangement of the second term one obtains

$$
H^{\text{eff}} = \frac{G_F}{\sqrt{2}} (\bar{\nu}_\mu \gamma^\rho (1 + \gamma_5) \nu_\mu (g_V \bar{e}^\gamma \gamma_\rho e^- + g_A \bar{e}^\gamma \gamma_\rho \gamma_5 e^-) + \frac{G_F}{\sqrt{2}} \bar{\nu}_e \gamma^\rho (1 + \gamma_5) \nu_e [(g_V + 1) \bar{e}^\gamma \gamma_\rho e^- + (g_A + 1) \bar{e}^\gamma \gamma_5 \gamma_\rho e^-].
$$

(4.55)
In the minimal SU(2) x U(1) theory one has

\[
g_V = -\frac{1}{2} + 2 \sin^2 \theta_w, \quad g_A = -\frac{1}{2}.
\]  

(4.56)

The differential cross section for neutrino–electron scattering can be written as follows:

\[
\frac{d\sigma}{dy} = \frac{G^2 \cdot m_e}{2\pi} \cdot E_e \left[ A + B(1 - y)^2 - C \frac{m_e}{E_e} \cdot y \right]
\]

(4.57)

\[
(\frac{y}{y_{\text{final, lab}}} = \frac{\text{energy of final electron in lab.-system}}{\text{energy of incoming neutrino in lab.-system}})
\]

where the parameters \(A, B\) and \(C\) relevant to \(\nu_\mu e^-\)-scattering can be expressed in terms of \(g_A\) and \(g_V\) as follows:

\[
A = (g_V + g_A)^2, \quad B = (g_V - g_A)^2, \quad C = (g_V^2 - g_A^2) = -\sqrt{AB}.
\]

(4.58)

For the other reactions the parameters \(A, B\) and \(C\) are obtained as follows:

\[
\begin{align*}
\bar{\nu}_\mu e^- &\to \bar{\nu}_\mu e^- \quad g_V \to g_V, \quad g_A \to -g_A \\
\nu_\mu e^- &\to e^- \quad g_V \to g_V + 1, \quad g_A \to g_A + 1 \\
\bar{\nu}_e e^- &\to \bar{\nu}_e e^- \quad g_V \to g_V + 1, \quad g_A \to -(g_A + 1).
\end{align*}
\]

(4.59)

We emphasize that \(\nu_\mu e^-\)–e\(^-\) elastic scattering also takes place via the charged current interaction.

In fig. 10 we describe the results of the various experiments in terms of \(g_V\) and \(g_A\). Each measured cross-section determines an area in the \(g_A-g_V\) plane. The limits of the 90% confidence level domains are

![Fig. 10. The domain of \(g_V\) and \(g_A\) allowed by the \(\nu-e^-\)-scattering experiments. The solid line at \(g_A = -1/2\) denotes the prediction of the minimal SU(2) x U(1) model as a function of \(\sin^2 \theta_w\).](image-url)
shown in fig. 10. The intersection of the three regions (dark shaded area) is the region allowed by all experiments. The minimal SU(2) x U(1) theory is consistent with all experiments for sin²θw = 0.2–0.3.

Soon new data on νμ–e⁻ scattering will become available from CERN and FNAL. Furthermore it is planned to carry out new reactor experiments and to measure νe–e⁻ scattering with higher precision.

4.6. Parity violation in electron–nucleon interactions

Knowledge about the neutral current interaction was considerably improved recently by the experimental results of the SLAC–Yale–Aachen–Hamburg experiment [4.41] on polarized electron–deuteron scattering. Using polarized electrons one can check if the electron–nucleon interaction conserves parity. If parity is conserved, the asymmetry

\[
A = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L}
\]  

(\(d\sigma_R\): differential cross-section for the process e⁻ (righthanded) + nuclear target → X, etc.) must vanish. (Note that one uses an unpolarized target. Therefore a parity reflection changes \(d\sigma_R\) into \(d\sigma_L\) and vice versa.)

If the neutral current interaction violates parity, one expects \(A\) to be nonzero. The leading parity-violating effect (of order \(G_F\)) will arise from the interference of the electromagnetic and the neutral current interaction, and one expects in a crude order-of-magnitude estimate the asymmetry to be of the order of \(|q^2|/M_Z^2\) (\(q\): four-momentum transfer). Furthermore one can say something about the sign of the asymmetry in the minimal SU(2) x U(1) theory. The neutrino scattering experiments indicate that sin²θw = 0.2, i.e. sin²θw is close to zero. In the case sin²θw = 0 the neutral current is purely lefthanded, i.e. \(d\sigma_R\) receives no contribution from the neutral current interaction. Therefore we expect \(d\sigma_L > d\sigma_R\), i.e. \(A < 0\). In the experiment of ref. [4.41] the typical value of \(|q|^2\) is of the order of 1 GeV². Thus we expect \(A \sim -10^{-4}\).

The simplest way to calculate parity violation effects in deep inelastic scattering of electrons or muons off a hadron target is as described below [4.42–4.44]. The electromagnetic interaction of the quarks and leptons at the γ-fermion-vertex is described by \(Q_f \gamma^\mu \gamma^\nu \gamma_{\mu L} + \gamma_{\mu R}\) where \(Q^\mu\) denotes the charge and \(\gamma_{\mu L}(\gamma_{\mu R})\) stand for the lefthanded (righthanded) projections of \(\gamma^\mu\): \(\gamma_{\mu L} = \gamma^\mu (1 + \gamma_5)/2, \gamma_{\mu R} = \gamma^\mu (1 - \gamma_5)/2\). The coupling of the neutral Z boson to the fermion \(f\) is given by

\[
Q^{Zf}_{\mu L} + Q^{Zf}_{\mu R}
\]

(in general one has \(Q_R \neq Q_L\).

It is well-known from the analogous situation in deep inelastic neutrino scattering that a lefthanded neutrino scatters from a lefthanded quark with a constant \(y\)-distribution, while it scatters from a righthanded antiquark with a \(y\)-distribution proportional to \((1 - y)^2\) \(y = E(\text{final hadrons})/E_\nu\). Analogously a lefthanded electron scatters from a lefthanded quark with a constant \(y\)-distribution, and it scatters from a righthanded quark (or antiquark) with a \(y\)-distribution proportional to \((1 - y)^2\). Correspondingly a righthanded electron scatters from a righthanded quark with a constant distribution, and from a lefthanded quark with a distribution proportional to \((1 - y)^2\). As a consequence one finds:

(a) Scattering of a righthanded electron on a righthanded quark \(q\)

\[
d\sigma \sim \left| \frac{Q^e_{\mu L} Q^\mu_{\nu L}}{q^2} + \frac{Q^{Z\nu}_{\mu R} Q^{Z\nu}_{\mu R}}{q^2 - M_Z^2} \right|^2
\]  

(4.61)
(b) Scattering of a righthanded electron on a lefthanded quark $q$

$$d\sigma \sim \left| \frac{Q_e^* Q_e^* + Q_R^* Q_L^*}{q^2 - M_Z^2} \right|^2 (1 - y)^2$$

(4.62)

c) Scattering of a lefthanded electron on a lefthanded quark $q$

$$d\sigma \sim \left| \frac{Q_e^* Q_e^* + Q_L^* Q_L^*}{q^2 - M_Z^2} \right|^2$$

(4.63)

d) Scattering of a lefthanded electron on a righthanded quark $q$

$$d\sigma \sim \left| \frac{Q_e^* Q_e^* + Q_R^* Q_R^*}{q^2 - M_Z^2} \right|^2 (1 - y)^2.$$  

(4.64)

The scattering asymmetry of longitudinally polarized electrons can be computed by summing over the various quark contributions and integrating over the quark distribution functions. The differential cross sections $d\sigma_R(x, y)$ [$x$: scaling variable; $x = -q^2/2M$, ($M$: proton mass, $\nu = p \cdot q$, $q$: four-momentum of current)] involve an incoherent sum over the lefthanded and righthanded quarks in the target wave function (the target is assumed to be unpolarized).

Neglecting all terms of order $(q^2/M_Z^2)^2$ and keeping only terms of order $(q^2/M_Z^2)$, one obtains

$$A(x, y) = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L}$$

$$= \frac{q^2}{M_Z^2} \frac{1}{2} \sum_q f_q(x) Q^*_e Q^*_q \{Q_R^e Q_R^q - Q_L^e Q_L^q\} + (Q_R^e Q_L^q - Q_L^e Q_R^q)(1 - y)^2\} \left( \frac{1 + (1 - y)^2}{1 + (1 - y)^2} g_{\Lambda e} g_{\Lambda q} \right)$$

$$= \frac{q^2}{M_Z^2} \frac{1}{2} \sum_q f_q(x) (\frac{Q^*_e}{e}) \{ \frac{1 + (1 - y)^2}{1 + (1 - y)^2} g_{\Lambda e} g_{\Lambda q} \} / \sum_q f_q(x) (\frac{Q^*_e}{e})^2.$$

(4.65)

This expression can be rewritten in terms of the vector and axial vector coupling constants as follows:

$$A(x, y) = -\frac{2q^2}{M_Z^2} \sum_q f_q(x) \left( \frac{Q^*_e}{e} \right) \{ g_{\Lambda e} g_{\Lambda q} + \frac{1 - (1 - y)^2}{1 + (1 - y)^2} g_{\Lambda e} g_{\Lambda q} \} / \sum_q f_q(x) (\frac{Q^*_e}{e})^2.\) 

(4.66)

In the minimal SU(2) x U(1) theory one has

$$\frac{G}{2\sqrt{2}\pi a} = (4 \sin \theta_w \cos \theta_w M_Z^2)^{-1}$$

$$Q_L^e = \frac{e}{\sin \theta_w \cos \theta_w} (T_{3L} - Q^* \sin^2 \theta_w)$$

(4.67)

$$Q_R^e = \frac{e}{\sin \theta_w \cos \theta_w} \cdot (Q^* \sin^2 \theta_w)$$

($T_{3L}$: third component of the weak isospin; $T_{3L}(e^-) = -\frac{1}{2}$, $T_{3L}(u) = +\frac{1}{2}$, $T_{3L}(d) = -\frac{1}{2}$).
For an isosinglet target where \( f_n(x) = f_d(x) \) the expression (4.67) simplifies considerably if we neglect the antiquark distributions. One finds e.g. for deuteron targets

\[
A_{sd} = - \frac{Gq^2}{2\sqrt{2} \pi \alpha} \cdot \frac{9}{10} \left\{ (1 - \frac{2}{9} \sin^2 \theta_w) + (1 - 4 \sin^2 \theta_w) \frac{1 - (1 - y)^2}{1 + (1 - y)^2} \right\}. \tag{4.68}
\]

The \( x \)-dependence drops out entirely. In the case of \( \sin^2 \theta_w = 1/4 \) the asymmetry is independent of \( y \). This is easily understood by looking at eq. (4.67); in the case of \( \sin^2 \theta_w = 1/4 \), the Z-coupling to the electron is axial, therefore the relevant Z-quark coupling is vector, like the \( \gamma \)-quark coupling, and both \( d\sigma_R - d\sigma_L \) and \( d\sigma_R + d\sigma_L \) are proportional to \( 1 + (1 - y)^2 \). The ratio \( (d\sigma_R - d\sigma_L)/(d\sigma_R + d\sigma_L) \) is therefore \( y \)-independent.

**Comparison with experiment.** In fig. 11 we show the predictions of the minimal theory, compared with the experimental data. The latter are in a good agreement with the minimal SU(2) \( \times \) U(1) theory for \( \sin^2 \theta_w = 0.224 \pm 0.020 \).

One may ask how sensitive these predictions are to our assumption to neglect the antiquark as well as strange and charmed quark contributions to the nucleon wave function. This question has been addressed recently by various authors [4.45, 4.46]. The conclusion is that the corrections are very minor; one estimates that the overall effect of the presence of antiquarks, s-quarks etc. is to reduce the value of \( \sin^2 \theta_w \) by a very small amount (about 0.005). We emphasize that the asymmetry \( A \) is very sensitive to \( \sin^2 \theta_w \). Thus a precise measurement of \( A \), in particular of the \( y \)-dependence, provides a very sensitive test of the minimal SU(2) \( \times \) U(1) theory and a rather precise determination of \( \sin^2 \theta_w \).

Recently the \( y \)-dependence of the asymmetry has been measured in the range between \( y = 0.15 \) and \( y = 0.4 \) [4.47]. The data are consistent with a flat \( y \)-distribution as expected within the minimal SU(2) \( \times \) U(1) theory (\( \sin^2 \theta_w \approx 0.25 \)).

![Fig. 11. The asymmetry \( A \) as a function of the variable \( y \) and of \( \sin^2 \theta_w \) in the minimal SU(2) \( \times \) U(1) theory (in units of \( 10^{-5}/Q^2[\text{GeV}^2] \)). For the parametrization of \( \frac{A}{Q^2} = a_1 + a_2(1 - (1 - y)^2)/(1 + (1 - y)^2) \) the evaluation of the data [4.47] yields \( a_1 = -9.7 \pm 2.6, a_2 = 4.9 \pm 8.1 \).](image)
4.7. Parity violation in atomic and nuclear physics

One has investigated parity violation effects in atomic physics by searching for an optical rotation of polarized laser light in bismuth vapor. The parity violation in atoms is in general due to the parity violating term in the neutral current interaction, which is proportional to:

\[ j^A \cdot j^V + j^e \cdot j^q \]

where \( j \) denotes the neutral current and \( A, V, e, q \) stands for axial vector, vector, electron and quark respectively. However in heavy atoms the contribution of the second term is negligible compared to the one of the first term; the first term is additive in the number of nucleons \( A \), while the second one is proportional to the nuclear spin, which is small compared to \( A \).

The recent experiments carried out in order to measure parity violation in atomic physics are in agreement with the prediction of the minimal \( SU(2) \times U(1) \) theory. For details we refer the reader to the original literature \([4.48-4.50]\).

The weak forces contribute to the nucleon–nucleon interaction and therefore to the nuclear potential. Their contribution is of order \( Gm_e^2/(g^2/4\pi) \sim 10^{-6} \) (\( g \): \( \pi \)-N coupling constant), i.e. it is so small that it would be lost in the nuclear physics complexities, if it were not for the fact that the weak interaction violates parity. At present there exists evidence that the internucleon forces violate parity on a level expected in the theory \([4.51]\). However due to the complexities of the strong interactions it is very difficult to come to definite conclusions with respect to the weak interactions.

We mention the following cases where evidence for parity violation in nuclear physics was found:

(a) Parity mixing in nuclei. There exists meanwhile a great deal of evidence that there are small parity impurities in nuclear states \([4.52]\).

(b) Circular polarisation in the reaction \( n + p \rightarrow d + \gamma \). If parity is conserved, one expects the circular polarisation of the emitted photon \( P_\gamma = (S \cdot p)/(p) \) (\( S \): photon spin, \( p \): photon momentum) to vanish. The experiment by Lobashov et al. gives \([4.53]\):

\[ P_\gamma = -(1.3 \pm 0.45) \times 10^{-6} \]

This result seems too large, in view of various theoretical estimates \([4.54]\) which give

\[ P_\gamma \sim 10^{-7} - 10^{-8} \]

(c) Asymmetry in \( p-p \) scattering with longitudinally polarized protons. The ratio

\[ A_p = (\sigma_+ - \sigma_-)/(\sigma_+ + \sigma_-) \]

(\( \sigma_+(-) \); cross section for protons of positive (negative) helicity) would be zero, if parity were exactly conserved. One finds \([4.55]\)

\[ A_p = (-3.2 \pm 1.1) \times 10^{-7} \].
The order of magnitude of this result is in agreement with theoretical estimates [4.51].

(d) Photon asymmetry in the capture of polarized neutrons by protons. The photon asymmetry

$$A_\gamma = \langle \sigma(n)p(\gamma) | p(\gamma) \rangle$$

in this reaction is expected to be of the order of $10^{-8}$, which is consistent with the present experimental limit $A_\gamma = (0.6 \pm 2.1) \times 10^{-7}$ [4.56].

(e) Parity violations involving complex nuclei. Examples are:

The reaction $^{16}$O($2^-, 8.88$ MeV) → $^{12}$C + $\alpha$ (measured width $\Gamma = (1.02 \pm 0.26) \times 10^{-10}$ eV) [4.57], the reaction $^{18}$F($0^-, 1.08$ MeV) → $^{18}$F(groundstate) + $\gamma$ (measurement of the circular polarisation; the present experimental limit is $P_e = (-0.7 \pm 2.0) \times 10^{-3}$ [4.58], one order of magnitude above the theoretical expectation), the process $^{19}$F($1/2^-, 110$ keV) → $^{19}$F($1/2^+$, groundstate) + $\gamma$ [4.59], and the reaction $^{21}$Ne($1/2^-, 2.78$ MeV) → $^{21}$Ne(groundstate) + $\gamma$ [4.60].

Two aspects of parity violations in nuclear physics are of special interest for the particle physicists [4.61]:

(a) One can obtain information about the structure of the $\Delta S = 0$ nonleptonic weak Hamiltonian. Especially it is interesting to find out whether there is a $\Delta S = 0$ analog of the $\Delta I = 1/2$ enhancement. Thus far nothing is known about it.

(b) It seems possible to obtain information about the nonleptonic part of the neutral current interaction. Although nobody doubts the existence of such an interaction, it would be useful to obtain a direct evidence for it and a measurement of its strength.

A S-wave $\pi$–N Yukawa interaction is in general constrained by CP conservation to take the $|\Delta I| = 1$ form $g\vec{N}(\tau \times \pi)_3 N$ [$g$: coupling constant, $N$: nucleon field]. On the other hand the $|\Delta I| = 1$ part of the charged current nonleptonic weak parity violating Hamiltonian is suppressed by a factor $\sin^2 \theta_c$; it has the form

$$H^{c.c.} = \frac{G}{2\sqrt{2}} \sin^2 \theta_c (c^+ O_+ + c^- O_-) \quad (4.69)$$

where the operators $O_+$ and $O_-$ are given by [4.62]

$$O_+ = \frac{1}{2}(\bar{u}\gamma^\mu \gamma_5 u - \bar{d}\gamma^\mu \gamma_5 d)(\bar{s}\gamma_\mu s - \bar{c}\gamma_\mu c) + \frac{1}{2}(\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d)(\bar{s}\gamma_\mu s - \bar{c}\gamma_\mu c) \pm \text{Fierz transform}. \quad (4.70)$$

The coefficients $c_+, c_-$ are unity in the absence of the strong interactions; they are analogs of the coefficients $f_+, f_-$ discussed previously. Gluonic radiative corrections modify them; one has $c_+ \approx [\ln(M/\mu)]^\gamma_{\pm}$ were in case of four flavors $\gamma_+ = -0.24$, $\gamma_- = 0.48$ [4.62], and $\mu$ is a scale of the order of 1 GeV. Besides being proportional to $\sin^2 \theta_c$, the matrix elements of the effective $|\Delta I| = 1$ Hamiltonian (4.59) are sensitive to the strange and charmed quark content of the nucleon, which implies an additional suppression.

The neutral current $|\Delta I| = 1$ contribution to the p.v. Hamiltonian is given by

$$H^{n.c.} = -\frac{G}{2\sqrt{2}} (1 - 2 \sin^2 \theta_w)(c^+ c^O_+ + c^- c^O_- + \frac{2}{3} \sin^2 \theta_w c^O_0 O_0) \quad (4.71)$$

where

$$O_0 = (\bar{u}\gamma_\mu \gamma_5 u - \bar{d}\gamma_\mu \gamma_5 d)(\bar{u}\gamma^\mu u + \bar{d}\gamma^\mu d + \bar{c}\gamma^\mu c + \bar{s}\gamma^\mu s),$$
and the coefficients $c^\gamma$ are analogously to the $c^\omega$ defined:

$$c_{+,0}^\gamma = \left[ \ln(M_{\rho}/\gamma) \right]^{\gamma_{+}.0} \quad (\gamma_0 \approx 0.85). \tag{4.72}$$

Since the exponent $\gamma_+$ is negative, the matrix elements of $O_+$ are suppressed. Neglecting those and disregarding the mass difference between $M_{\rho}$ and $M_{\omega}$, one finds that the neutral current contribution enhances the $|\Delta I| = 1$ parity violating interaction. The enhancement factor is given by

$$1 + \frac{1 - 2 \sin^2 \theta_{\rho} + N \cdot \sin^2 \theta_{\omega}}{\sin^2 \theta_c} \tag{4.73}$$

where $N = (c_0/c_-)$ times a factor involving the ratio of the matrix elements of $O_0$ and $O_-$. The second term in (4.73) is of the order of 10, and does not depend upon unknown quantities like the strange and charmed quark content of the nucleon. The third term involving $N$ is difficult to estimate; typical estimates for $N$ range between 0.6 and 3 (see e.g. [4.63]).

In general one expects that the neutral current interaction enhances the $\Delta I = 1$ parity violation in nuclei by a factor of the order of ten. The experiments carried out thus far ($^9F$, $^{41}K$, $^{175}Lu$, $^{181}Ta$) indicate that the observed parity violation is indeed larger than expected on the basis of the $W$-exchange interaction. Further, more refined experiments are expected to clarify the situation.

Thus far we have mentioned only nuclear physics tests of the nonleptonic weak interaction. There exist a number of possibilities to use nuclear transitions in order to investigate the semileptonic weak interactions, by considering neutrino induced reactions (both charged and neutral current processes) and electron induced reactions (polarized beams). We refer the reader to a previous Physics Reports article [4.64].

5. Sequential flavordynamics of leptons

During the last two years a remarkable development has taken place. One has found direct evidence for the existence of a new charged lepton ($\tau$) with a mass of 1.78 GeV [5.1, 5.2, 5.3], and indirect evidence for a new neutral lepton $\nu_\tau$, associated with the $\tau^-$ lepton. Furthermore one has obtained indirect evidence for the existence of a new quark flavor $b$ ($e(b) = \pm 1/3$, $m(b) \approx 4.74$ GeV) interpreting the $Y$-states as $b\bar{b}$ bound states [5.4]. It is one of the important tasks for experimentalists and theorists to find out what kind of weak interaction properties the new degrees of freedom possess.

5.1. The $\tau$-lepton family

The standard model for the $\tau$-family is obtained by placing the lefthanded $\tau^-$ and its neutrino $\nu_\tau$ into a doublet of the weak isotopic spin:

$$\begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix} _L,$$

and assuming that the $\tau$-neutrino is massless. All experimental data obtained in $e^+e^-$ annihilation experiments ($e\mu$-events, inclusive muon production, inclusive electron production) are consistent with
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This model. The mass of the $\tau$ lepton is $M_\tau = \left(1.782^{+0.002}_{-0.007}\right) \text{GeV}$ (for a review see [5.3]). One expects the life time of the $\tau$-lepton to be

$$T(\tau) = BR(\tau \to e\nu\bar{\nu})(M_\mu/M_\tau)^5 \quad T(\mu) = 2.8 \times 10^{-13} \text{s}$$

(5.1)

where $T(\mu)$ and $M_\mu$ are the muon lifetime and mass, and a branching ratio of 16.7% has been used. The experimental value of $BR(\tau \to e\nu\bar{\nu}) = BR(\tau \to \mu\nu\bar{\nu})$ is $(16.7 \pm 1.0)\%$. The experimental limits on the $\tau$ lifetime are [5.2]

$$T_\tau < 3.5 \times 10^{-12} \text{s}.$$  

(5.2)

The form of the leptonic spectrum depends on the chirality of the charged current which leads to $\tau$ decay. The experiments favor the $V-A$ interaction. A $V+A$ form for the $(\tau^-, \nu_\tau)$-current is excluded, while $V$ or $A$ currents are not excluded. The upper limit on the $\nu_\tau$ mass is 250 MeV [5.2].

The $\tau$-decay provides an excellent tool to measure the spectral functions of the vector and axial vector currents. Both the decays $\tau^- \to \nu_\tau \rho^- \text{ and } \tau^- \to \nu_\tau A_1$ have been observed. The corresponding branching ratios are in good agreement with theoretical estimates (for references see the reviews [5.3]).

The decay $\tau \to \nu\pi$ is especially important, since it can be unambiguously predicted in terms of the pion coupling constant $F_\pi$:

$$\frac{\Gamma(\tau \to \pi\nu\bar{\nu})}{\Gamma(\tau \to e\nu\bar{\nu})} = 12\pi^2 F_\pi^2 \cos^2 \theta_c M_\tau^{-2}.$$  

(5.3)

With $BR(\tau \to e\nu\bar{\nu}) = 16.7\%$, $F_\pi = 0.132 \text{ GeV}$, $M_\tau = 1.8 \text{ GeV}$ this gives

$$BR(\tau \to \pi\nu) = 9.5\%.$$  

(5.4)

On the other hand the observed branching ratio is:

$$BR(\tau \to \pi\nu) = 8.3 \pm 3.0\%,$$  

(5.5)

in good agreement with the theoretical prediction.

The semileptonic decay modes $\tau \to \nu_\tau X$ ($X$: any hadronic state) measure the spectral functions of the vector and axial vector currents $\rho_\nu$ and $\rho_A$ respectively. In the chiral limit $m_u = m_d = 0$ of QCD, these functions obey the sum rules [5.5]

$$\int (\rho_\nu - \rho_A) dQ^2/Q^2 = F_\nu^2, \quad \int (\rho_\nu - \rho_A) dQ^2 = 0.$$  

(5.6)

The vectorial spectral function could be measured in $e^+e^- \to \text{ hadrons}$. Both functions $\rho_\nu$ and $\rho_A$ appear in the inclusive $\tau$ decay rate:

$$\frac{d\Gamma(\tau \to \nu_\tau X)}{dQ^2} = \frac{G_F^2 \cos^2 \theta_c (m_\tau^2 - Q^2)^2 (m_\tau^2 + Q^2)}{32\pi m_e^2 Q^2} \left(\rho_\nu(Q^2) + \rho_A(Q^2)\right)$$  

(5.7)
(X: nonstrange final state), where $Q^2$ is the invariant mass of the emitted hadronic system. At present the sum rules (5.6) can only be tested by assuming that the spectral functions are dominated by the lowest lying resonances. In that case one predicts $B(\tau \to A_1 \nu_\tau) \approx 0.09$, in good agreement with the observed value $0.10 \pm 0.03$ [5.3] (for details on spectral function sum rules see e.g. [5.6]).

The possibility that the $\tau$ lepton is not coupled to a new neutrino has been discussed [5.7]. In this case the $\tau$ lepton can decay only via mixing with the "old" leptons $e$ and $\mu$. Due to the mixing one obtains lepton flavor changing neutral currents, which in particular cause the decay of $\tau$ into three charged leptons (e.g. $e^- e^+ e^- e^-$). One predicts

$$BR(\tau \to 3 \text{ charged leptons}) \approx 0.05.$$ 

On the other hand one knows from experiment [5.2]

$$BR(\tau^- \to 3 \text{ charged leptons}) < 0.6\% \ [90\% \ c.L],$$

which excludes the possibility of $\tau^-$ decay via mixing in the absence of $\nu_\tau$.

Furthermore it has been discussed whether the neutral lepton $\nu_\tau$ can be heavier than the $\tau$ [5.8]. In this case the $\tau^-$ can decay only via lepton flavor mixing. However the troublesome flavor changing neutral current contributions mentioned above are absent. The constraints from universality restrict the mixing angles to be fairly small such that one has [5.8]

$$T(\tau) > 10^{-11} \text{ s}. \ (5.8)$$

The present limit on the $\tau$ lifetime is slightly smaller (see eq. (5.2)), and we have to conclude that the possibility of $\tau$ decay via mixing is unlikely.

Summary: We conclude that the $\tau$ lepton comes in association with a new neutrino $\nu_\tau$ which must be lighter than 250 MeV. The $(\tau, \nu_\tau)$ charged current is likely to be of the structure $V - A$. All experimental data are in agreement with the possibility that the weak interaction responsible for the $\tau$ decay is of the conventional type. No new interactions (new currents etc.) are needed. Very likely the $\tau$ lepton and its neutrino forms another doublet of weak isotopic spin, and the weak interaction properties of the leptons are described by three doublets:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L.$$

We emphasize, however, that not much is known about the third neutrino $\nu_\tau$, besides the constraint $m(\nu_\tau) < 250 \text{ MeV}$. Of course, it is quite possible that $\nu_\tau$ like the other neutrinos $\nu_e, \nu_\mu$ is massless. If $\nu_\tau$ is massive, we regard it as unlikely that $\nu_\tau$ is stable. One expects it to decay via mixing: $\nu_\tau \to e^- e^+ \nu_e, \nu_\tau \to \mu^- e^+ \nu_e, \nu_\tau \to e^- \pi^+$. In any case the lifetime of $\nu_\tau$ is expected to be more than $10^{-6} \text{ s}$, depending on the magnitude of the $\nu_\tau$ mass and the relevant weak mixing angle. The $e^+ e^-$ annihilation experiments are not able to detect the possible decay of $\nu_\tau$ due to the relatively long lifetime, and it will be very difficult to observe such a decay in other experiments (e.g. $\tau^+ \tau^-$-production in $\mu$-nucleon scattering).

5.2. Neutrino oscillations

If the neutrinos do have a mass, there is no reason why the neutrino mass matrix should be diagonal
in the weak interaction eigenstates. As a consequence we expect that there exist leptonic analogs of the weak interaction mixing angles (Cabibbo angle etc.). They lead to neutrino oscillations [5,9]. We illustrate those by considering first the case of two neutrinos ($\nu_e, \nu_\mu$) and assuming that they are both massive Fermi–Dirac particles. The leptonic weak currents are given by the following weak doublets:

$$
\begin{pmatrix}
\nu_e' \\
\nu_\mu'
\end{pmatrix} = \begin{pmatrix}
\nu_1 & \nu_2
\end{pmatrix} = \begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
$$

(5.9)

$(\nu_1, \nu_2$: neutrino mass eigenstates). In this case the neutrino state emitted in $\beta$ decay is a mixture of two mass eigenstates, and the amount of mixing is given by the angle $\theta$ (the leptonic analog of the Cabibbo angle). Suppose we look at a $\nu_\mu$ beam which is produced by the weak decay of pions or kaons in a decay tunnel. If we consider this beam right at the beginning, i.e. immediately after the decay, it is a pure $\nu_\mu$-beam. The time development of $\nu_\mu$-beam with definite momentum is given by:

$$
\nu_\mu(t) = -\sin \theta \nu_1 e^{-iE_1 t} + \cos \theta \nu_2 e^{-iE_2 t}
$$

(5.10)

$(t$: time of propagation, $E_i = \sqrt{p^2 + m^2}$, where $p$ is the beam momentum and $m_i$ are the neutrino masses). At the time $t$ (i.e. at the distance $ct$ in case of $p/c \gg m_i$) the probability to find the combination $\nu_\mu'$ is given by

$$
|\langle -\nu_1 \sin \theta + \nu_2 \cos \theta | \nu_\mu(t) \rangle|^2 = \sin^4 \theta + \cos^4 \theta + 2 \sin^2 \theta \cos^2 \theta \cos(E_2 - E_1)t,
$$

(5.11)

i.e. one obtains an oscillating behaviour. In the relativistic limit $E_i \gg m \cdot c^2$ the corresponding oscillation length is given by

$$
l = 4\pi p/\Delta m^2 \quad (\Delta m^2 = |m_1^2 - m_2^2|)
$$

(5.12)

$$
\approx \frac{p[\text{MeV}]}{4\Delta m^2[\text{eV}^2]} \times 10 \text{ m}.
$$

For example, in case of $\Delta m^2 = 1 \text{ eV}^2$ one finds:

<table>
<thead>
<tr>
<th>$p$</th>
<th>$1 \text{ MeV}$</th>
<th>$1 \text{ GeV}$</th>
<th>$100 \text{ GeV}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l$</td>
<td>2.5 m</td>
<td>2.5 km</td>
<td>250 km</td>
</tr>
</tbody>
</table>

Neutrino oscillations of the type considered above can be discovered, for example, by observing the $\nu_\mu$-beam, produced in an accelerator laboratory, turning into a $\nu_e$-beam, and vice versa.

The simple mixing scheme described above is not the only one which one can imagine. For example it may be that the neutrinos are massive Majorana particles instead of Fermi–Dirac particles. In this case oscillations of the type $\nu_{eL} \leftrightarrow \bar{\nu}_{eL}$ may occur ($\bar{\nu}_{eL}$ is the lefthanded counterpart of $\bar{\nu}_{eR}$—it is not coupled to the electron (positron) by the conventional weak current).

Neutrino oscillations have been looked for in various experiments.

(a) Nuclear reactors. A reactor emits $\bar{\nu}_e$. Neutrino oscillations will lead to a reduction of the $\bar{\nu}_e$ flux and to oscillations on top of of the $1/r^2$ attenuation law. The $\bar{\nu}_e$ spectrum will be distorted, since the
oscillations affect neutrinos of different energy in a different way. Thus far no clear evidence for oscillations is seen at distances of ~5–15 m from the reactor core [5.10]. This implies e.g. $\Delta m^2 \approx 0.3 \text{ eV}^2$ for $\theta = 45^\circ$. New experiments, now under way, will soon provide better information.

Recently it has been claimed that neutrino oscillations are observed in the reaction $\bar{\nu}_e + d \rightarrow n + n + e^+$ (deuteron break-up) [5.11].

Experimentally one finds:

$$\left(\frac{\bar{\sigma}_{e.c.}}{\bar{\sigma}_{n.c.}}\right)_{\text{exp.}} = 0.191 \pm 0.073$$

(5.13)

where $\bar{\sigma}_{e.c.}$, $\bar{\sigma}_{n.c.}$ is the cross section for charged current ($\bar{\nu}_e + d \rightarrow n + n + e^+$) or neutral current ($\bar{\nu}_e + d \rightarrow n + p + \nu_e$) deuteron break-up, averaged over the neutrino spectrum. The ratio given above is largely independent of the incident neutrino flux. It is useful to introduce the double ratio

$$r = \left(\frac{\bar{\sigma}_{e.c.}}{\bar{\sigma}_{n.c.}}\right)_{\text{exp.}} / \left(\frac{\bar{\sigma}_{e.c.}}{\bar{\sigma}_{n.c.}}\right)_{\text{th.}}$$

(5.14)

where the denominator denotes the cross section ratio expected theoretically. Within the SU(2) x U(1) theory the cross section $\bar{\sigma}_{n.c.}$ is independent of the neutrino type, i.e. eventually existing neutrino oscillations cancel out due to a leptonic analog of the GIM mechanism. Furthermore in case of the low energy reactor neutrinos ($E_e < 10 \text{ MeV}$) the active part of the neutral current is its axial vector part; only the allowed Gamow–Teller transitions ($|\Delta J| = 1$) are relevant, and therefore the cross section is independent of $\sin^2 \theta_w$.

In the absence of neutrino oscillation we expect within the SU(2) x U(1) theory $r = 1$; any deviation from $r = 1$ may be interpreted as due to neutrino oscillations. Experimentally one finds [5.11]

$$r = 0.43 \pm 0.17.$$  

(5.15)

Interpreting this result in terms of neutrino oscillations, one finds e.g. in case $\theta = 45^\circ$ for $\Delta = m_1^2 - m_2^2$ values above 0.3 eV$^2$. It remains to be seen by new experiments whether the result quoted above is indeed correct.

(b) **Sun.** The sun is expected to emit electron neutrinos. If the oscillation length is small compared to the diameter of the core of the sun the neutrino oscillations will lead to an apparent reduction of the neutrino flux. At present it seems that this is the case [5.12]. However it must be emphasized that the experiments are sensitive only to the high energy tail of the solar neutrino spectrum, which cannot be estimated in a reliable way. New experiments are needed which are sensitive to the neutrinos arising from the main thermonuclear reaction in the sun: $p + p \rightarrow d + e^+ + \nu_e$. Such experiments may be done in the near future [5.13].

(c) **Accelerators.** At CERN one has looked for neutrino oscillations using the Gargamelle bubble chamber [5.14] by searching for an excess of $\nu_e$ events in the $\nu_\mu$ beam at a distance of about 50 m from the target. One has found:

$$\frac{\langle \nu_\mu \rightarrow \nu_e \rangle}{\langle \nu_\mu \rightarrow \nu_\mu \rangle} = (-0.03 \pm 0.1) \times 10^{-2}$$

$$\frac{\langle \bar{\nu}_\mu \rightarrow \bar{\nu}_e \rangle}{\langle \bar{\nu}_\mu \rightarrow \bar{\nu}_\mu \rangle} = (0.02 \pm 0.07) \times 10^{-2}$$

(5.16)
where e.g. \(\langle \nu_\mu \rightarrow \nu_e \rangle\) denotes the rate for a \(\nu_\mu\) to produce a \(\nu_e\)-type event. The results are compatible with the absence of oscillations. In case \(\theta = 45^\circ\) one finds an upper limit of about 1 eV\(^2\) for \(\Delta m^2\).

Furthermore we mention a limit on the \(\nu_\mu \leftrightarrow \nu_e\) oscillations [5.15]:

\[
\langle \nu_\mu \rightarrow \nu_\tau \rangle / \langle \nu_\mu \rightarrow \nu_\mu \rangle < 0.025.
\] (5.17)

Recently several beam dump experiments have been performed at CERN [5.16]. Especially one has studied prompt neutrinos. The conventional interpretation of those is that they originate almost exclusively from the semileptonic decays of charmed particles. In that case the flux of prompt electron and muon neutrinos should be the same, due to the fact that charmed particles have equal probabilities to decay into electrons and muons. There exist indications from three different experiments that the electron neutrino flux is less than the muon neutrino flux. For example, the BEBC bubble chamber experiment gives \(\phi(\nu_e)/\phi(\nu_\mu) \approx 0.6 \pm 0.2\). The results could be interpreted as due to oscillations. Since \(\nu_\mu \leftrightarrow \nu_e\) oscillations seem to be absent (see discussion above), one could think of \(\nu_e \leftrightarrow \nu_\mu\) oscillations. Such oscillations would reduce the electron neutrino flux, without changing the \(\nu_\mu\) flux (for a discussion of such a case see ref. [5.17]). We emphasize that \(\nu_e \leftrightarrow \nu_\mu\) oscillations are not the only possibility. For example, one may have \(\nu_e \leftrightarrow \bar{\nu}_{eL}\) oscillations (\(\bar{\nu}_{eL}\): lefthanded counterpart of the conventional righthanded neutrino); such oscillations can be present if the mass term of the neutrinos contains Majorana terms [5.18] (see our discussion below).

Let us consider neutrino oscillations in the case of more than two neutrinos. For illustration we discuss the possibility to have mixings between the three neutrinos \(\nu_e, \nu_\mu\) and \(\nu_\tau\). The weak doublets are \((\nu_e, e^-)_L, (\nu_\mu, \mu^-)_L\) and \((\nu_\tau, \tau^-)_L\). The weak interaction eigenstates \(\nu_e, \nu_\mu\) and \(\nu_\tau\) are related to the mass eigenstates \(\nu_1, \nu_2, \nu_3\) by a unitary transformation:

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} =
\begin{pmatrix}
c_1 & s_1 c_3 & s_1 s_3 \\
-s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i \delta^l} & c_1 c_2 s_3 + s_2 c_3 e^{i \delta^l} \\
s_1 s_2 & -c_1 s_2 c_3 - c_2 s_3 e^{i \delta^l} & -c_1 s_2 s_3 + c_2 c_3 e^{i \delta^l}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}. \tag{5.18}
\]

Here \(c_1\) stands for \(\cos \theta^l_1\), \(s_1\) for \(\sin \theta^l_1\), etc. The unitary transformation is described by three angles \(\theta^l_1, \theta^l_2, \theta^l_3\) and a phase parameter \(\delta^l\). We have used the same parametrization of the unitary mixing matrix as the one which describes the mixings of the quark flavors to be discussed in the next section. However we emphasize that in general there exists no connection between the leptonic mixing parameters \(\theta^l\), \(\delta^l\) and the corresponding ones for the quarks.

As in the quark case, the presence of the phase angle \(\delta^l\) implies a violation of \(CP\). As far as the neutrino oscillations are concerned, a violation of \(CP\) will lead e.g. to \(\langle \nu_e \rightarrow \nu_\mu \rangle \neq \langle \nu_\mu \rightarrow \nu_e \rangle, \langle \nu_e \rightarrow \nu_\tau \rangle \neq \langle \nu_\tau \rightarrow \nu_e \rangle\) [5.19].

Neutrino oscillations will occur if at least one mass difference \(m_i - m_j\) is different from zero. We have remarked earlier that there are rather stringent limits on the existence of oscillations involving the muon neutrino. Thus one may be inclined to assume that the muon neutrino \(\nu_\mu\) is to a good approximation a mass eigenstate. This would imply that mixing can only occur between \(\nu_e\) and \(\nu_\tau\) and the mixing matrix reduces to a simple rotation matrix:

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}. \tag{5.19}
\]
It seems that the best way to look for neutrino oscillations is to carry out careful measurements of the neutrino flux and the neutrino spectrum at nuclear reactors, and to investigate the neutrinos produced in beam dump experiments. If neutrino oscillations are found, the implications for particle physics, and especially for astrophysics, would be striking.

We would like to mention that neutrino oscillations between $\nu_e$ and $\nu_\tau$ in neutrino beams emitted off nuclear reactors can only occur if the $\nu_\tau$ mass is less than a few MeV, i.e. less than the typical energy of a neutrino emitted. In order to avoid the cosmological bound discussed above, a mass of $\nu_\tau$ of the order of 1 MeV or larger can only be present if the $\nu_\tau$ is unstable [5.20]. In that case the only limit on the $\nu_\tau$ mass comes from the $e^+e^-$-annihilation experiments [5.3]: $M(\nu_\tau) < 250\text{ MeV}.$

Let us discuss the possibility of an unstable $\nu_\tau$ in more detail. In the absence of new interactions the $\nu_\tau$ can decay via its mixing with $\nu_e$ and $\nu_\mu$. For $m(\nu_\tau) < 2m_e$ the decay of the $\nu_\tau$ will proceed mainly via the radiative decay: $\nu_\tau \rightarrow \gamma + \nu_e(\nu_\mu)$, which, however, is very slow. The amplitude for this decay is of the order of $Ga \cdot (m_e/M_W)^2 \sin^2 \varphi$ where $\varphi$ is a small mixing angle. In case of $m(\nu_\tau) > 2m_e$ the decay of the $\nu_\tau$ will mainly proceed via the process $\nu_\tau \rightarrow e^+ e^- \nu_e$. For $m(\nu_\tau) > m_\mu + m_\mu$ the decay $\nu_\tau \rightarrow \mu e\nu$ is possible and for $m(\nu_\tau) > m_\tau + m_\tau$ the decay $\nu_\tau \rightarrow \tau^+ \tau^-$ can occur.

In ref. [5.21] the various experimental constraints on the leptonic mixing angles etc. were taken into account. The result is that $\nu_\tau$ masses between 50 eV and 10 MeV are excluded; the $\nu_\tau$ must therefore be lighter than 50 eV or heavier than 10 MeV. The lifetime is in any case much larger than the lifetimes which can be seen in the present experiments ($\gg 10^{-4}$ s).

Since neutrinos are neutral particles, the neutrino mass term need not be lepton number conserving, i.e. the neutrino mass matrix can contain Majorana terms. (A Dirac mass term turns a lefthanded fermion state into a righthanded one, and vice versa, while a Majorana mass term turns a lefthanded fermion state into a righthanded antifermion state. Evidently in case of a Majorana mass term it is not possible to define a conserved fermion number.) If the neutrino mass matrix contains Majorana terms, new types of oscillations are possible, which one would not have otherwise. As an example we consider a situation, in which we are dealing with the three four-component fields $\nu_e, \nu_\mu$ and $\nu_\tau$. If the neutrino masses are of the Dirac type, the neutrino oscillations will preserve the overall lepton number, i.e. we have the oscillations $\nu_e \leftrightarrow \nu_\mu, \nu_e \leftrightarrow \nu_\tau$ and $\nu_\mu \leftrightarrow \nu_\tau$. If Majorana mass terms are allowed, one can have oscillations like $\nu_e \leftrightarrow \bar{\nu}_e, \nu_\mu \leftrightarrow \bar{\nu}_\mu$, etc. Note that in the conventional scheme of the weak interactions the righthanded neutrinos ($\nu_e)_R, (\nu_\mu)_R, (\nu_\tau)_R$ do not take part in the weak interactions, i.e. the oscillations involving those neutrinos can manifest themselves only by an attenuation of the neutrino flux. (We emphasize that neutrino-oscillations cannot change the helicity of a neutrino; a lefthanded neutrino emitted e.g. in $\pi$-decay, stays lefthanded, even if it changes its flavor or fermion number.)

5.3. Constraints from cosmology

One of the unknowns of flavordynamics is the number of lepton doublets, or the number of neutrino states. A constraint on the number of “light” neutrino states can be derived from astrophysics, assuming the standard evolution of the universe (“big bang cosmology”). As pointed out in ref. [5.22], the primordial abundance of $^4\text{He}$ depends on the number of relativistic particles present in the early universe, and therefore on the number of “light” neutrinos (mass $< 1\text{ MeV, } \tau > 1\text{ sec}$). This is so due to the fact that almost all neutrons present at the start of the nucleosynthesis in the early universe are incorporated in $^4\text{He}$ [5.23]. On the other hand the number of neutrons depends on the expansion rate, which in turn is a function of the energy density. If we add a new neutrino, the energy density is increased and the expansion rate speeded up. As a result the amount of $^4\text{He}$ increases. The observations
suggest that the primordial abundance of \(^4\text{He}\) was less than 25\% \[5.23, 5.24\]. This allows just the observed neutrino states \(\nu_e, \nu_\mu, \text{ and } \nu_\tau\), and at most one more neutrino. For example, it seems impossible to have righthanded counterparts to the observed neutrinos, provided the latter are coupled significantly via the neutral current to the lefthanded neutrinos (via the transitions \(\bar{\nu}_e \to \bar{\nu}_e, \bar{\nu}_\mu \nu_\mu, \bar{\nu}_\tau \nu_\tau\)) \[5.25\]). In the conventional scheme of flavordynamics the righthanded neutrinos do not couple to the neutral current. Therefore they are not in thermal equilibrium with the lefthanded neutrinos in the early universe and do not count in the cosmological analysis.

We should like to point out that the cosmological considerations mentioned above do not apply in the absence of neutrino degeneracy (number of neutrinos \(\gg\) number of antineutrinos) \[5.24, 5.26\]. For example, a \(\nu_e\) degeneracy drives the weak interactions towards the direction of a decreased neutron-to-proton ratio (note that the relevant reaction is \(p + e^- \to n + \nu_e\)) and therefore to a decreased helium abundance. However one needs a very large degeneracy (|\(\Delta L_e| \gg N(\gamma)\), \(L_e\): electron number, \(N(\gamma)\): number of photons), which is very difficult to understand (in most theories one expects |\(\Delta L| \approx |\Delta B| \approx 10^{-9}N(\gamma)\) (\(B\): baryon number) \[5.27, 5.28\]). For this reason we find it rather unlikely that the cosmological constraints on the number of neutrino states are invalidated by neutrino degeneracies.

6. Sequential flavordynamics of quarks

6.1. Weak currents and six quarks

The simplest way to accommodate the newly discovered b quark flavor is to introduce a new doublet of weak isospin \(\left(\begin{array}{c}t \\ b\end{array}\right)\) where \(t\) is a new flavor of charge \(2/3\). The effective mass of the b-quark is about 4.4 GeV. A way to estimate the latter is to use the relation

\[
M(Y) - M(\Upsilon) = 9.46 \text{ GeV} - 3.10 \text{ GeV} \approx 2(m_\nu - m_\tau).
\]  

(6.1)

Using \(m_\tau \approx 1.2 \text{ GeV}\), one finds \(m_\nu \approx 4.4 \text{ GeV}\).

The t quark flavor remains to be discovered. The present experimental limit on \(m_t\) is about 18 GeV \[6.1\].

If we simply introduce a new \(\left(\begin{array}{c}t \\ b\end{array}\right)\) doublet, there exists no communication between the new quarks t, b and the "old" quarks, u, d; c, s. As a result the t-quark could decay weakly into b, but the latter would be stable. In particular the B\(^-\) mesons of quark composition (\(\bar{u}b\)) would be absolutely stable, while the B\(^0\) mesons of quark composition (\(\bar{d}b\)) would decay via \(\beta\)-decay into B\(^-\): B\(^0\) \(\to B^- + \nu_\tau + e^+\). One has looked for the production of long-lived mesons with a mass of \(\sim 5 \text{ GeV}\) in proton–nucleus scattering \[6.2\]. No such mesons with a lifetime longer than \(10^{-8}\) s have been found, which are produced with a cross section comparable or larger than the cross section for producing Y mesons. (It is expected that \(\sigma(\bar{B}B) > \sigma(Y)\) at 400 GeV laboratory energy.) We conclude that the B meson must have a lifetime less than \(10^{-8}\) s.

In the sequential six quark scheme the B mesons can decay weakly only by weak interaction mixing. The weak doublets are in general

\[
\left(\begin{array}{ccc}u & c & t \\ d' & s' & b'\end{array}\right)_L
\]
where \(d', s', b'\) are mixtures of \(d, s, b\) described by

\[
\begin{pmatrix}
  d' \\
  s' \\
  b'
\end{pmatrix}_L = U
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}_L ,
\]

(6.2)

and the weak currents are given by

\[
(\bar{u} c \bar{t}) U
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}.
\]

The \(3 \times 3\)-matrix \(U\) must be unitary, due to the constraint of universality of the charged weak interaction. In general a unitary \(3 \times 3\)-matrix is described by 9 independent parameters. Since the phase of the quark fields are not fixed yet, we have the freedom to transform each quark field as \(q \rightarrow e^{i\alpha(q)}q\) where \(\alpha(q)\) is a phase parameter depending on the flavor index \(q\). The weak coupling matrix \(U\) does not change under a common phase transformation \(q \rightarrow e^{i\alpha}q\) (\(\alpha\) flavor independent). However it changes under transformations involving different phases for the various quark flavors. Thus we can transform \(U\) into a form in which it depends only on four parameters. One such choice is as follows [6.3]

\[
U = \begin{pmatrix}
  c_1 & s_1 c_3 & s_1 s_3 \\
  -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\
  s_1 s_2 & -c_1 s_2 c_3 - c_2 s_3 e^{i\delta} & -c_1 s_2 s_3 + c_2 c_3 e^{i\delta}
\end{pmatrix}
\]

(6.3)

where \(c_1\) stands for \(\cos \theta_1\), \(s_1\) for \(\sin \theta_1\), etc. The angles \(\theta_1, \theta_2, \theta_3\) are the tree Euler angles which describe the rotation among the three flavors \(d, s, b\). The fourth parameter \(\delta\) is a phase parameter which is chosen such that the transitions \((\bar{u} d), (\bar{u} s), (\bar{u} b)\) as well as \((\bar{c} d)\) and \((\bar{t} d)\) are described by real coefficients.

It is easy to see that the coupling matrix denoted above can be written as a product of three matrices as follows:

\[
U = \begin{pmatrix}
  1 & 0 & 0 \\
  0 & c_2 & s_2 \\
  0 & -s_2 & c_2
\end{pmatrix} \cdot \begin{pmatrix}
  c_1 & s_1 & 0 \\
  -s_1 & c_1 & 0 \\
  0 & 0 & 1
\end{pmatrix} \cdot \begin{pmatrix}
  1 & 0 & 0 \\
  0 & 1 & 0 \\
  0 & 0 & e^{i\delta}
\end{pmatrix} \cdot \begin{pmatrix}
  1 & 0 & 0 \\
  0 & c_3 & s_3 \\
  0 & -s_3 & c_3
\end{pmatrix}.
\]

(6.4)

The angle \(\theta_1\) is essentially the Cabibbo angle, which describes the weak rotation in the \((d, s)\) space. The \((\bar{u} s)\) current is multiplied by \(s_1 c_3\). The universality of the weak interaction requires \(c_1^2 + (s_1 c_3)^2 \approx 1\), i.e. the third angle \(\theta_3\) cannot be very large. Taking into account the errors in the determination of the Cabibbo angle, one finds [6.4]:

\[
0 < \theta_3 < 29^\circ.
\]

(6.5)

In the special case \(\theta_3 = 0\) the Cabibbo universality is exact, and we can rewrite the weak doublets as follows:

\[
\begin{pmatrix}
  u \\
  d \cos \theta_1 + s \sin \theta_1
\end{pmatrix}
\begin{pmatrix}
  c \cos \theta_2 - t \sin \theta_2 & c \sin \theta_2 + t \cos \theta_2 \\
  -d \sin \theta_1 + s \cos \theta_1 & b
\end{pmatrix}
\]

(6.6)
i.e. one is dealing with a Cabibbo mixing in the (d, s) space described by $\theta_1 = \theta_c$, and another mixing in the (c, t) space described by $\theta_2$. We also note that in the limit $\theta_3 = 0$ the phase factor $e^{i\delta}$ can be absorbed by a redefinition of the b-quark phase.

There is no restrictive limit for $\theta_2$. However $\theta_2$ cannot be very large, say close to $\pi/2$. In case $\theta_2 = \pi/2$ the roles of the c and t quarks are interchanged, and it would be impossible to produce charmed particles in $\nu_{\mu}$-scattering off the valence d-quarks since the coefficient $s_1c_2$ vanishes. Since one observes the production of charmed particles in $\nu_{\mu}$ scattering with a rate as expected in the GIM scheme ($\theta_2 = \theta_3 = 0$) we conclude that $\theta_2$ must be fairly small. However the experimental data are not precise enough to provide a good limit for $\theta_2$.

Indirectly we can obtain a useful limit on $\theta_2$ by considering the $K^0$–$\bar{K}^0$ system as discussed below [6.5].

6.2. Weak interaction mixing and the $K^0$–$\bar{K}^0$ system

It is well-known that the extremely small magnitude of the $K^0$–$\bar{K}^0$ transition amplitude (i.e. the small $K_L$–$K_S$ mass difference: $(M_L - M_S)/M_K = 0.7 \times 10^{-10}$) can be understood as a consequence of the strong suppression of induced $\Delta S \neq 0$ neutral currents. The GIM scheme leads to the cancellation of the $|\Delta S| = 1$ neutral currents at the tree level. Furthermore the $K^0$–$\bar{K}^0$ transition amplitude vanishes not only at the tree level (i.e. there are no $|\Delta S| = 1$ terms of order $G$), but also at the one-loop level. As a consequence the transition amplitude is of the order of $G^2\Delta m^2_q$ ($\Delta m_q$: quark mass difference). The calculation is done by working out the transition amplitude $\bar{s}d \rightarrow \bar{d}s$ for the free quark case, i.e. by calculating an effective Hamiltonian. The corresponding diagram is given in fig. 12.

In the approximation where the external quark momenta are neglected compared to the loop momenta and the quark masses $m_u$, $m_c$, $m_t$, one finds:

$$H^{\text{eff}} = 4C(s_L\gamma_\mu d_t\bar{s}_L\gamma^\mu d_L),$$

(6.7)

where the constant $C$ is given by

$$C = \frac{G}{\sqrt{2}} \frac{\alpha}{16\pi \sin^2 \theta_W} \int_0^\infty dx \left[ \sum_{i=1}^n \lambda_i \epsilon_i \right]^2$$

(6.8)

where $\epsilon_i = m^2 q_i/M^2_Z$ and

$$\lambda_i = V_{tr}^* V_{td}$$

Fig. 12. The diagram which describes the $K^0$–$\bar{K}^0$ transition amplitude together with its crossed version. Since the W bosons couple only to the lefthanded quark fields, only the latter contribute. In the sequential flavordynamics the quarks of charge 2/3 (u, c, t, ...) contribute in the intermediate states.
matrix element of the mixing matrix given in case of three quark doublets in eq. (6.6), \( i \) stands for the various quarks of charge 2/3, e.g. \( i = 1 \) denotes the u-quarks, \( i = 2 \) the c-quark etc.; \( n \) is the number of quark doublets.

In the simplest case (two quark doublets) one finds [6.5]

\[
C \equiv \frac{G^2}{16\pi^2} (m_e^2 - m_u^2) \cdot \sin^2 \theta_c \cos^2 \theta_c. \tag{6.9}
\]

For six quarks one obtains using the mixing matrix given above and neglecting \( m_u \):

\[
\begin{align*}
\text{Re } C &= \frac{G^2}{16\pi^2} \left\{ \text{Re } \lambda_e^2 m_e^2 + \text{Re } \lambda^2 m_e^2 + \frac{2\text{Re } \lambda_e \text{Re } \lambda_t m_e^2}{1 - m_e^2/m_t^2} \ln \left( \frac{m_e^2}{m_c^2} \right) \right\} \\
&\quad - \text{Im } \lambda_e \left[ m_e^2 + m_t^2 - \frac{2m_e^2}{1 - m_e^2/m_t^2} \ln \left( \frac{m_e^2}{m_c^2} \right) \right], \\
\text{Im } C &= \frac{G^2}{8\pi^2} \text{Im } \lambda_c \left[ \text{Re } \lambda_e m_e^2 - \text{Re } \lambda_t m_t^2 + \frac{\text{Re } \lambda_t - \text{Re } \lambda_e}{1 - m_e^2/m_t^2} \ln \left( \frac{m_e^2}{m_c^2} \right) \right]. \tag{6.10}
\end{align*}
\]

The scale of the transition described in fig. 12 is set by the heavy quark masses \((m_e, m_t, \ldots)\). Since those are relatively large one ignores the strong interaction corrections to the transition process. (Within QCD one may argue that \( \alpha_s(m_e^2) \) and \( \alpha_s(m_t^2) \) are small.) In this case only the operator given in eq. (6.7) contributes, and one is left with the task to evaluate the matrix element of this operator between physical kaon states: \(<\bar{K}^0|(\bar{s}d)\bar{I}|K^0>\). This matrix element can be obtained by a SU(3)-rotation from the weak nonleptonic decay amplitude for the decay \( K^+ \rightarrow \pi^+ \pi^0 \) using PCAC [6.6].

Introducing a complete set of intermediate states \(|n>\) we can rewrite this matrix element as

\[
<\bar{K}^0|(\bar{s}d)|0> (0|(\bar{s}d)|K^0>.
\]

Of the various matrix elements only the contribution of the vacuum state can be calculated explicitly without further dynamical calculations. One finds:

\[
<\bar{K}^0|(\bar{s}d)|0> (0|(\bar{s}d)|K^0> = \frac{8}{3} \cdot \frac{F_K^2 M_K^2}{2 M_K^2}. \tag{6.11}
\]

\((F_K: \text{K decay constant}; \text{the factor } 8/3 \text{ arises by taking into account the color quantum number and all possible vacuum insertions}).

Neglecting all other intermediate states one finds in the case of two quark doublets [6.5]:

\[
\frac{M_t - M_s}{M_K} = \frac{G^2}{6\pi^2} F_K^2 m_e^2 \sin^2 \theta_c \cos^2 \theta_c. \tag{6.12}
\]

This relation may be used in order to determine \( m_e \) taking into account \( F_K/F_\pi \equiv 1.2, F_\pi = 0.93 M_\pi \):

\[
\frac{M_t - M_s}{M_K} = 0.50 \times 10^{-14} m_e^2/1 \text{ GeV}^2, \quad \left( \frac{M_t - M_s}{M_K^0} \right)_{\text{expt.}} = 0.71 \times 10^{-14}. \tag{6.13}
\]
One finds \( m_e \equiv 1.2 \text{ GeV} \). The agreement between experiment and theory is very satisfactory.

However this agreement must be considered as somewhat accidental. In order to test the reliability of the vacuum insertion approximation one may estimate the contribution of the one pion intermediate state to the matrix element \([6.7]\). This can be done using the experimental information about \( K_{13} \) decays. The result depends on the corresponding form factors and will not be given here. It is of the same order of magnitude as the result obtained by the vacuum insertion, but opposite in sign. Hence there is no reason to assume that the vacuum insertion method gives an accurate estimate of the \( K^0 - \bar{K}^0 \) transition amplitude. Of course, it may be that the one pion contribution and all other multiparticle contributions conspire such that their sum is negligible compared to the vacuum insertion result.

Another way to obtain a reliable result is to evaluate the transition amplitude in a definite bound state model. In ref. \([6.4]\) this was done using the bag model, where one finds

\[
\frac{A(K^0 \leftrightarrow \bar{K}^0)_{\text{bag}}}{A(K^0 \leftrightarrow \bar{K}^0)_{\text{vacuum}}} = 0.42,
\]

i.e. the actual decay amplitude is reduced by more than a factor of two compared to the vacuum insertion amplitude.

Using this result about the \( K^0 - \bar{K}^0 \) transition amplitude one can obtain information about the mixing angle \( \theta_2 \) in the six quark case \([6.8]\). The result depends on \( \text{sgn}(\cos \delta) \), \( m_1 \) and \( \theta_3 \). For example for \( m_1 = 15 \text{ GeV} \) and \( |s_3| = 0.3 \) one finds

\[
0.1 \leq |s_2| \leq 0.3 \quad (\text{sgn}(\cos \delta) = +1)
\]

\[
0.4 \leq |s_2| \leq 0.6 \quad (\text{sgn}(\cos \delta) = -1).
\]

One notes that the second mixing angle \( \theta_2 \) may be rather large. Furthermore the case \( |\theta_2| > 0 \) is favored, although for \( \text{sgn}(\cos \delta) = +1 \) the lower limit is not far from zero. In any case the experimental data available at the present time suggest that the main weak transitions are the ones given by \( u \leftrightarrow d, \ c \leftrightarrow s, \ t \leftrightarrow b \), while all other transitions are somewhat small (i.e. suppressed by relatively small weak mixing angles). It is interesting to see that this pattern of the weak interaction mixing agrees with the pattern of the quark masses: the dominant weak transitions are those which connect quarks whose masses are comparable, while other transitions, connecting quarks of much different masses (e.g. \( u \leftrightarrow b, \ s \leftrightarrow t, \ldots \)) are suppressed.

### 6.3. The lepton-quark mass spectrum and weak interaction mixing

In the conventional gauge theories of flavor dynamics (e.g. in the minimal SU(2) \( \times \) U(1) theory) the quark and lepton masses are generated spontaneously by the coupling of the fermions to the scalar fields. They are essentially free parameters of the theory, and one of the unsolved problems in flavordynamics is to find a way to establish the various masses. If we look at the observed mass spectrum, it becomes clear that the latter is not completely arbitrary. There exists a remarkable order in the fermion mass spectrum. The fact that thus far nobody has proposed a theoretical framework in which this order can be understood in a satisfactory manner indicates that there is yet something missing in the theory.

What are the various phenomenological facts which point towards a definite order in the fermion
mass spectrum? First we denote the various quark and lepton masses (in MeV):

\[
\begin{pmatrix}
    d [7.5] & s [150] & b [4600]
\end{pmatrix}
\]

\[
\begin{pmatrix}
    \nu_e [0.5] & \nu_\mu & \nu_\tau \\
    e^- [0.5] & \mu^- [106] & \tau^- [1780]
\end{pmatrix}
\]

(No masses or limits on the masses are given for the neutrinos.) Furthermore it is important to note that the main weak transitions (i.e. the weak transitions in the limit where the small weak mixing angles \( \theta_1, \theta_2 \) and \( \theta_3 \) are neglected) are the ones between the weak isospin doublets denoted above.

The order which exists in the fermion spectrum can be described as follows. Since the flavor mixing angles are all reasonably small, it makes sense to assign the quarks to three different generations with only small mixings between them. Furthermore we can associate each quark generation to a lepton generation:

\[
\begin{pmatrix}
    \nu_e & u \\
    e^- & d \\
\end{pmatrix} (I) \quad \begin{pmatrix}
    \nu_\mu & c \\
    \mu^- & s \\
\end{pmatrix} (II) \quad \begin{pmatrix}
    \nu_\tau & t \\
    \tau^- & b \\
\end{pmatrix} (III)
\]

This association is such that the masses increase as the generation increases:

\[
m_u < m_c < m_t \\
m_d < m_s < m_b \\
m_e < m_\mu < m_\tau.
\]

Of course, it is purely a matter of convention to assign the leptons and the quarks of a definite charge, say the \( Q = 2/3 \) quarks, in such a way that the inequalities (6.16) arise. However it is not evident that the quarks of the other charge can be ordered as shown above. For example it could have been the case that the dominant weak transitions are \( c \leftrightarrow b, t \leftrightarrow s \) or \( u \leftrightarrow s, c \leftrightarrow d \). That this is not the case indicates strongly that there must exist some connection between the weak mixing angles and the quark masses.

We should like to add a remark about the numerical values of the quark and lepton masses. The values of the fermion masses given above are such that there is no overlap between the different generations: the fermions of a given generation are heavier than the heaviest fermion of the previous generation and lighter than the ones of the next generation. It seems that the ratios of the masses of two fermions which follow each other in the sequence of the generations are somewhat comparable [6.9]:

\[
\frac{m_d}{m_s} \sim \frac{m_d}{m_c} \\
\frac{m_e}{m_\mu} \sim \frac{m_\mu}{m_\tau}.
\]

If such a trend continues for possible further lepton–quark generations we expect that the lightest charged member of a fourth generation is the negatively charged lepton. Its mass will be a few tenth of GeV (\( \sim 20–50 \) GeV). The associated quarks will be very heavy (masses larger than eighty GeV).
Despite our lack of knowledge to calculate the spectrum of quarks and leptons, there exist speculations about relationships between the quark masses and the weak mixing angles. The weak mixing angles arise since in reality the quark states for which the weak interactions are diagonal are not eigenstates of the quark mass matrix. For this reason it is useful to perform a unitary transformation in the space of quark fields such that the weak interactions are diagonal. We discuss such a procedure first in the case of the four flavors u, d; c, s. In terms of mass eigenstates the weak doublets are

\[
\begin{pmatrix}
  u \\
  d \cos \theta_c + s \sin \theta_c \\
  c \\
  -d \sin \theta_c + s \cos \theta_c
\end{pmatrix},
\]

and the mass matrix is diagonal:

\[
\mathcal{M}(u, c) = \begin{pmatrix} m_u & m_c \\ m_c & m_u \end{pmatrix}, \quad \mathcal{M}(d, s) = \begin{pmatrix} m_d & m_s \\ m_s & m_d \end{pmatrix}.
\]

Since \( m_u \) is very small compared to \( m_c \), we may neglect it, i.e. we set \( m_u \) equal to zero. Let us introduce the eigenstates of the weak interactions \( d_0 = d \cos \theta_c + s \sin \theta_c, \ s_0 = -d \sin \theta_c + s \cos \theta_c \). In this new basis the mass matrix \( \mathcal{M}(d, s) \) takes the form

\[
\mathcal{M}(d_0, s_0) = \begin{pmatrix} \cos^2 \theta_c m_d + \sin^2 \theta_c m_s & -\cos \theta_c \sin \theta_c (m_d - m_s) \\ -\cos \theta_c \sin \theta_c (m_d - m_s) & \sin^2 \theta_c m_d + \cos^2 \theta_c m_s \end{pmatrix}.
\]

It is well-known [6.10] that one has approximately \( \tan^2 \theta_c \approx m_d/m_s \), or \( \cos^2 \theta_c m_d \approx \sin^2 \theta_c m_s \). Thus the mass matrix \( \mathcal{M}(d_0, s_0) \) denoted above takes a very special form, namely

\[
\mathcal{M}(d_0, s_0) = \begin{pmatrix} 0 & a \\ a & b \end{pmatrix},
\]

if we assume \( m_d \) to be negative, and \( m_s \) to be positive (or vice-versa). (Note that the sign of a mass eigenvalue for a Fermi–Dirac field \( \psi \) need not be positive; the sign can easily be changed by a \( \gamma_5 \)-transformation, e.g. \( \psi_R \rightarrow -\psi_R, \psi_L \rightarrow \psi_L \)). As a consequence we find a relation between \( m_d, m_s \), and the Cabibbo angle, if the mass matrix \( \mathcal{M}(d_0, s_0) \) has the form given in eq. (6.21):

\[
\theta_c = \arctan \sqrt{\frac{m_d}{m_s}}.
\]

The mass matrix is such that the d quark starts out to be massless in the absence of weak mixing. The latter is introduced by the off-diagonal term (denoted by \( a \) in eq. (6.21)), which at the same time produces the d quark mass [6.11].

Thus far we have neglected \( m_u \). If we perform the same operation as the one described above in the (u, c)-space, one finds

\[
\sin \theta_c = \left| e^{i\beta} \sqrt{\frac{m_c}{m_c + m_u}} \cdot \sqrt{\frac{m_d}{m_s + m_d}} - \sqrt{\frac{m_u}{m_s + m_u}} \cdot \sqrt{\frac{m_s}{m_s + m_d}} \right|
\]

(6.23)

where \( \beta \) is an (unknown) phase parameter. According to this relation the Cabibbo angle is constrained
by the inequalities
\[ \tilde{\theta} - \theta \leq \theta_c \leq \tilde{\theta} + \theta \] (6.24)

where \( \tilde{\theta} = \arctan \sqrt{m_d/m_s} \) and \( \theta = \arctan \sqrt{m_u/m_c} \). Using \( m_c/m_u \sim 300 \), one has \( 10^\circ < \theta_c < 16^\circ \), in agreement with observation.

The question which arises is: How can one arrive at the special form of the mass matrices shown in eq. (6.21)? It is easy to see that there is no way to do so within the SU(2) \( \times \) U(1) theory. One needs at least a flavor gauge group like SU(2) \( \times \) SU(2) \( \times \) U(1), and discrete symmetries must be imposed in addition to the gauge symmetries. For details we refer the reader to the references [6.11, 6.12].

In the case of six quarks the matrix (6.21) must be replaced by a \( 3 \times 3 \) matrix. A natural generalization of eq. (6.21) would be to assume

\[ \mathcal{M}(d_0, s_0, b_0) = \begin{pmatrix} 0 & a & 0 \\ a & 0 & b \\ 0 & b & c \end{pmatrix} \] (6.25)

(and a similar matrix for \( \mathcal{M}(u_0, c_0, t_0) \)).

In this case the three weak mixing angles are related to the quark masses. Especially the relation for \( \theta_1 = \theta_c \) is essentially the same as in the four-flavor case [6.13]. We mention that the ratio \( s_3/s_2 \) is given by:

\[ s_3/s_2 \approx \sqrt{m_u/m_d/m_c/m_d} \approx \frac{1}{4}. \] (6.26)

Since the angle \( \theta_3 \) determines the strength of the weak transition \( b \to u \), and \( \theta_2 \) the strength of \( t \to d \), one concludes that the quark communicates much more with the \( d \) quark than the \( b \) quark with \( u \):

\[ \frac{\text{rate}(t \to d)}{\text{rate}(b \to u)} \approx 16. \] (6.27)

The mass matrices discussed above are not the only ones which may be considered. In the literature a rather extensive study of possibilities to derive relations between quark masses and mixing angles has been made. For details we refer the reader to the references [6.11–6.14]. The consequences of such schemes for flavor changing processes has been emphasized in ref. [6.15].

6.4. Weak decays of \( b \) - and \( t \) flavored particles in sequential QFD

It is expected that the lowest-lying mesons including \( b \) or \( t \) quarks correspond to the isospin doublets

\[ B^0 \sim (b\bar{d}) \quad B^- \sim (b\bar{u}) \]
\[ T^+ \sim (t\bar{d}) \quad T^0 \sim (t\bar{u}). \]

Since we have \( m_b < m_t \), the \( b \)-quark cannot decay via its main weak coupling, but only via mixing. If we neglect the effects due to the finite \( c \)-quark mass, we find
\[
\frac{|b \to u + \text{anything}|^2}{|b \to c + \text{anything}|^2} \approx \frac{|s_1s_2|}{-c_1s_2c_3 - c_2s_3e^{i\delta}}. \tag{6.28}
\]

This ratio is expected to be rather small, unless the values of \(\theta_2, \theta_3\) and \(\delta\) conspire such that \(|s_1s_2| = |c_1s_2c_3 + c_2s_3e^{i\delta}|^2\). For example in case \(\theta_1 = \theta_2 = \theta_3 = \theta_\epsilon, \delta = 0\) one finds 0.03 for this ratio. Thus we expect a sequential weak decay pattern: \(b \to c \to s\).

We would like to draw the attention of the reader to the possibility that the final state in b-quark decay may involve a \(c\bar{c}\)-pair, such as in the case

\[
b \to c + \bar{c} + s. \tag{6.29}
\]

In this case, the \(c\bar{c}\) pair may lead to the formation of a \(\bar{c}c\)-meson, e.g. the \(J/\psi\), and we would find decay modes like

\[
B^-(b\bar{u}) \to J/\psi + K + \pi, \text{ etc.} \tag{6.30}
\]

Such decay modes are particularly interesting, since the electromagnetic decay of the \(J/\psi\) leads to a \(\mu^+\mu^-\) pair which can easily be identified. It is possible to estimate the branching ratio [6.16]. The latter turns out to be unexpectedly large:

\[
\Gamma(B \to \psi(J) + X)/\Gamma(B \to \text{anything}) \approx \text{few \%}. \tag{6.31}
\]

Furthermore one can give arguments [6.17] that the hadronic system emitted in the B decay besides the \(J/\psi\) state consists of relatively few particles. The most favored state is: \(X = K + \pi\). Thus the B particles should have a few \% chance to disintegrate into \((J/\psi)K + \pi\) [6.17, 6.18].

Another possibility to detect the B mesons is to look for the leptonic decays \(B \to \tau\nu\) [6.19]. One expects \(B(B_u \to \tau\nu) \approx B(B_c \to \tau\nu) = 1\text{--}2\%\). The \(\tau\) lepton can decay into \(e\nu\bar{\nu}\) or \(\mu\nu\bar{\nu}\) (the combined branching ratio is expected to be about 0.36). As a consequence the initial B meson is converted into a charged lepton and three neutrinos. In the class of events where initially a pair of B mesons is produced in \(e^+e^-\)-annihilation, and one B meson decays into \(\tau\nu\), exactly half of the available energy is hadronic, while the other half is leptonic and mostly invisible. The signature of the events is quite clear.

Thus far it is not known whether the \(t\) quark exists or not. The experiments performed at PETRA have set a lower limit on the effective \(t\) quark mass which is of the order of 18 GeV [6.1]. Thus the T mesons (mesons of quark composition \(\bar{u}t, \bar{d}t\)) will have a mass of at least 15 GeV. It is expected that T mesons decay predominantly into B mesons, according to the chain \(t \to b \to c \to s\).

Since we have \(m_t - m_b \geq 10\text{ GeV}\), the energy release in the \(t\) decay is large, and the three quarks emitted in a nonleptonic decay, say \(t \to b + \bar{d} + u\), are rather energetic with respect to the \(t\) quark rest frame. Thus one expects the appearance of jets in the weak decay (see e.g. [6.20]). Since the \(t\) quarks are produced in pairs in \(e^+e^-\)-annihilation, one can find events exhibiting up to six jets. Such events would be one of the signatures which could be used to discover \(t\) flavored hadrons.

**Multileptons in \(b\) and \(t\) decays.** According to the discussion given above the weak decays of \(b\) and \(t\) flavored particles will probably involve sequential weak transitions, e.g. \(t \to b \to c \to s\). At each step a virtual W boson is emitted, which can lead to the formation of a quark--antiquark pair or a lepton pair. The chance to emit a lepton pair is not small (\(\sim 20\text{--}30\%\)). Thus the probability to find one or more charged leptons in the final state is quite large. This provides a possibility to look for new flavors in \(e^+e^-\)-annihilation and other reactions [6.21].
6.5. CP violation and the six quark theory

It is well-known that in case of four quarks (two lefthanded weak doublets) any complex phase parameter in the weak currents can be transformed away by a redefinition of the quark fields, i.e. there is no CP violation in the absence of further interactions. With six quarks this is no longer true, and a phase parameter \( \delta \) appears (see eq. (6.3)). This parameter is responsible for CP violating amplitudes, which for \( \delta < 1 \) must be proportional to \( \delta \) [6.22]. By looking at the weak coupling matrix one can easily see that any CP violation involving the light quarks u, d and s must be proportional to \( s_3 \), since \( e^{i\theta} \) is always multiplied by \( s_3 \) in the relevant matrix elements.

The \( K^0-\bar{K}^0 \) system. The off-diagonal element of the neutral K mass matrix is given by

\[
M_{12} = \langle K^0|H_w|\bar{K}^0\rangle + \sum_n \frac{\langle K^0|H_w|n\rangle\langle n|H_w|\bar{K}^0\rangle}{M_{K^0}^2 - m_n^2} + \ldots
\]  

(6.32)

(\( H_w \): weak Hamiltonian, \(|n\rangle\): intermediate state).

The \( \varepsilon \) parameter which characterizes the magnitude of CP violation in the \( K^0 \) states is given by:

\[
|\varepsilon| \cong \text{Im} \frac{M_{12}}{\sqrt{\Delta M^2 + \frac{1}{4}I_0^2}}
\]  

(6.33)

(\( \Delta M = M(K_L) - M(K_S) \), \( I_0 \): width of \( K_0 \)). The violation of CP in the decay \( K_L \rightarrow 2\pi \) can either be caused by the admixture of the "wrong" CP state in the \( K_L \) (given by \( \varepsilon \)) or by the CP violation in the \( K_L \rightarrow 2\pi \) amplitude. The latter is characterized by

\[
\delta = \varepsilon e^{i\delta_0} = \frac{\text{Im} \frac{A_2}{A_0}}{2}
\]  

(6.34)

(\( \delta_2, \delta_0 \): \( I = 2 \) and \( I = 0 \) \( \pi \pi \)-S wave phase shifts at \( E(2\pi) = M_K \); \( A_2, A_0 \): decay amplitudes given by \( \langle 2\pi(I = 0)|H_w|K^0\rangle = A_0 e^{i\delta_0} \), \( \langle 2\pi(I = 2)|H_w|K^0\rangle A_2 e^{i\delta_2} \).

We have argued previously that the \( K^0-\bar{K}^0 \) mass difference is dominated by the contributions given in fig. 11. Assuming this both for \( \text{Re} M_{12} \) and \( \text{Im} M_{12} \), one finds [6.23]:

\[
\frac{\text{Im} M_{12}}{\text{Re} M_{12}} \cong 2s_2c_2s_3 \sin \delta F(x, \theta_2)
\]  

(6.35)

where

\[
F(x, \theta_2) = \left\{ s_2^2 \left( 1 + \frac{x \ln x}{4 - x} \right) - c_2^2 \left( x + \frac{x \ln x}{4 - x} \right) \right\} \left\{ c_2^4 \ln x + s_2^4 - 2s_2^2 c_2^2 x \ln x \right\}
\]  

(6.36)

\( (x = m_2^2/m_\pi^2) \).

For the allowed range of \( \theta_2 \) and \( m_\pi \sim 10-20 \) GeV the function \( F \) is of the order of 5. Experimentally one has \( I_0 \approx 2\Delta M \). Thus one obtains:

\[
|\varepsilon| \approx \left| \frac{\text{Im} M_{12}}{\sqrt{2}\Delta M} \right| \approx \frac{1}{\sqrt{2}} s_2c_2s_3 \sin \delta \cdot F(x, \theta_2).
\]  

(6.37)
Since \(|\varepsilon|\) is observed to be about $2 \times 10^{-3}$, at least one of the angles $\theta_2$, $\theta_3$ or $\delta$ must be relatively small.

In deriving eq. (6.37) we have neglected the possible imaginary contribution in the transition $K_{0L} \to 2\pi$. On the other hand we have argued previously that the dominant weak decay amplitude in the $|\Delta I| = 1/2$ channel for the $K$ decay may come from the gluon radiation amplitude.

If this is the case, one finds another contribution to $\varepsilon$ as it follows from [6.24]. In the presence of the $(\frac{1}{b})$ doublet the induced weak Hamiltonian has the form

$$H_w = \frac{2G}{\sqrt{2}12\pi} \left\{ \ln\left(\frac{m_{\pi}^2}{\mu^2}\right) \left(s_1 c_2^2 + i s_1 s_2 c_2 s_3 \sin \delta\right) + \ln\left(\frac{m_{\pi}^2}{\mu^2}\right) \left(s_1 s_2^2 - i s_1 s_2 c_2 s_3 \sin \delta\right) \right\} (\bar{s}d)_L (\bar{u}u + \bar{d}d + \cdots ) + \text{h.c.} \quad (6.38)$$

where $\mu$ is a typical hadronic mass (we have neglected the color indices in the current–current product). We expand the $K_{0L}$ decay amplitude $A_0$ at the point $\delta = 0$:

$$A_0 = A_0^{(\delta = 0)} (1 - i f R) \quad (6.39)$$

where $A_0^{(\delta = 0)}$ is $A_0$ in the limit $\delta = 0$, $f$ is the fraction of the amplitude $A_0$ which is due to the induced Hamiltonian (6.38) [one may expect $f \approx 1$], and $R$ is the ratio of the imaginary part of the coefficient in eq. (6.38), divided by its real part:

$$R = \left( \frac{\ln(m_{\pi}^2/m_s^2)}{c_2^2 \ln(m_{\pi}^2/\mu^2) + s_2^2 \ln(m_{\pi}^2/\mu^2)} \right) \cdot s_2 c_2 s_3 \sin \delta. \quad (6.40)$$

According to eq. (6.38) a new phase $e^{i\xi} (\xi = f \cdot R)$ is induced in $A^0$. On the other hand in analyzing $CP$ violation one normalizes the decay amplitudes such that $A_0$ is taken to be real. In order to adopt this convention we have to redefine the $K$-states

$$|K^0\rangle \to e^{-i\xi}|K^0\rangle, \quad |\bar{K}^0\rangle \to e^{i\xi}|\bar{K}^0\rangle. \quad (6.41)$$

Thus we find

$$\text{Im} \langle K^0|H_w|\bar{K}^0\rangle \to \text{Im} \ e^{2i\xi} \langle K^0|H_w|\bar{K}^0\rangle \approx \text{Im} \ \langle K^0|H_w|\bar{K}^0\rangle + 2\xi \text{ Re} \ \langle K^0|H_w|\bar{K}^0\rangle. \quad (6.42)$$

The $CP$ violation parameter $\varepsilon$ is defined by

$$\varepsilon = i \frac{\text{Im} \ \Gamma_{12} + \text{Im} \ M_{12}}{(\Gamma_s - \Gamma_L)/2 + i(m_s - m_L)}. \quad (6.43)$$

Experimentally one has [6.25]:

$$\frac{1}{3}(\Gamma_s - \Gamma_L) \approx m_L - m_s. \quad (6.44)$$
If we take \( A_0 \) real, we can neglect \( \text{Im} \Gamma_{12}/\text{Im} M_{12} \). Using \( 2\text{Re} M_{12} = m_s - m_L \), one finds

\[
\varepsilon = \frac{1}{2\sqrt{2}} e^{i\pi/4}(\varepsilon_0 + 2\delta)
\]  

(we have approximated the phase angle by \( \pi/4 \); the precise value is \( 37 \pm 6^\circ \) [6.26]).

The \( CP \) violation parameter \( \varepsilon' \) is given by

\[
\varepsilon' = \frac{i}{2} e^{i(\delta_0 - \delta_0)} \frac{\text{Im} A_2}{A_0},
\]

where \( \delta_0, \delta_2 \) are the \( \pi\pi \) phase shifts in the \( I = 0, I = 2 \) channel respectively. The \( CP \) violation induced by the penguin diagram contributions enters only the \( |\Delta I| = 1/2 \) channel, i.e. it cannot enter the amplitude \( A_2 \) which involves a \( |\Delta I| = 3/2 \) transition. However our redefinition of the phases of \( K^0 \) and \( \bar{K}^0 \) to make \( A_0 \) real introduces a phase \( e^{-i\epsilon} \) in \( A_2 \). The experimental \( \pi\pi \) phase shifts together with \( |A_2/A_0| \approx 1/20 \) yield

\[
\varepsilon' \approx \frac{1}{20\sqrt{2}} e^{i\pi/4}(-\xi).
\]  

Combining eq. (6.47) and eq. (6.45), one finds

\[
\frac{\varepsilon'}{\varepsilon} \approx \frac{1}{20} \left( \frac{-2\xi}{\varepsilon_0 + 2\xi} \right).
\]

Thus \( \varepsilon'/\varepsilon \) differs from zero to the extent that the penguin diagrams contribute to the \( K^0 \)-decay. In the ratio \( \varepsilon'/\varepsilon \) the factor \( s_2c_2s_3 \sin \delta \) cancels out, and it depends only on \( 2\xi/\varepsilon_0 \), which in turn depends on \( f, \alpha_s, m, \) and the angle \( \theta_2 \). If \( f \) is of order unity, typical values are \( \varepsilon'/\varepsilon \approx 0.5\% - 2\% \) [6.24]. The sign of \( (\varepsilon'/\varepsilon) \) is positive.

The present experimental value is \( (\varepsilon'/\varepsilon) = -0.003 \pm 0.014 \) [6.25]. It seems possible to measure \( (\varepsilon'/\varepsilon) \) up to a level of 0.1\%. This would be an important check. If one finds \( (\varepsilon'/\varepsilon) \neq 0 \), the superweak model [6.27] would be ruled out, and one would obtain useful additional information about the origin of \( CP \) violation.

**Dipole moment of the neutron.** If we disregard possible nonperturbative effects in QCD, a dipole moment of the neutron is generated due to exchanges of virtual W bosons in the vertex of a d-quark and a virtual photon. It is of the order of [6.23]:

\[
(D/e)_{\text{neutron}} \approx \frac{G\alpha}{\pi^2} \sin \delta_s s_1^2 s_2 c_2 s_3 \frac{(m_t^2 - m_c^2)(m_0^2 - m_s^2)}{M_w^4} m_d,
\]

and one expects

\[
(D/e)_{\text{neutron}} \sim 10^{-30} \text{ cm},
\]

a value, which is more than five orders of magnitude below the present experimental limit [6.28]:
\[(D/e)_{\text{neutron}} = (0.4 \pm 1.5) \times 10^{-24} \text{ cm};\]

and which is about as small as the dipole moment expected within the superweak theory [6.27].

7. Extended schemes of QFD

7.1. Extended flavor gauge groups

The minimal SU(2) × U(1) theory is able to describe all observed properties of the weak and electromagnetic interactions, and one is inclined to believe that the SU(2) × U(1) theory is either the correct theory of the electromagnetic and weak interactions or at least a rather good approximation to the correct theory. There are two possible ways to extend the minimal SU(2) × U(1) theory.

A: The total flavor gauge group \(G'\) is larger than SU(2) × U(1), but all (or most) of the generators of \(G'\) are broken more strongly than the SU(2) × U(1) generators (i.e. the associated gauge bosons are heavier than the gauge bosons coupled to the SU(2) × U(1) generators, and do not lead to observable effects at low energies).

B: The total flavor group \(G'\) is larger than SU(2) × U(1), but all \(G'\) generators which do not belong to the SU(2) × U(1) subgroup are coupled to heavy, yet unobserved quarks or leptons.

An example of type A are the left–right symmetric gauge theories, based on the group SU(2) × SU(2) × U(1), SU(2) × SU(2) × U(1) × U(1), etc. ([7.1]–[7.8]). Here one assumes that parity violation in the weak interactions is a consequence of the spontaneous symmetry breaking. At very high energies, where all weak boson masses can be disregarded, the weak interactions become parity conserving. Note that this is not the case in the maximal SU(2) × U(1) theory where the lefthanded fermions are doublets under SU(2)\(^L\), and the righthanded ones are singlets.

We suppose that the fermions transform under SU(2)\(_L\) × SU(2)\(_R\) like \((1/2, 0) + (0, 1/2)\), i.e. the lefthanded fermions are doublets under the “lefthanded” gauge group SU(2)\(_L\), and the righthanded fermions are doublets under the “righthanded” gauge group SU(2)\(_R\). In such a theory there exist two charged gauge bosons W\(^+\)\(_L\) coupled to the lefthanded fermions and two charged gauge bosons W\(^+\)\(_R\), coupled to the righthanded fermions. The latter would induce righthanded weak currents, which are not observed. Thus the bosons W\(^+\)\(_R\) must be much heavier than the bosons W\(^+\)\(_L\).

The experimental limits on the presence of righthanded currents are such that we must have \(M_{W_R}/M_{W_L} \geq 4-5\), i.e. \(M_{W_R} > 350\) GeV.

In order to generate a spontaneous breakdown of the left–right symmetric gauge group SU(2)\(_L\) × SU(2)\(_R\) × U(1) . . . , scalar multiplets transforming like (2, 2) and (2, 1) + (1, 2) under SU(2)\(_L\) × SU(2)\(_R\) are needed. It is typical of left–right symmetric gauge theories that the neutral current interaction at low energies is very similar to the minimal SU(2) × U(1) theory, if one chooses symmetry breaking terms using the scalars mentioned above. Nevertheless the pattern of the Z boson spectroscopy can be quite different. If one employs the gauge group SU(2) × SU(2) × U(1)\(^n\), one is dealing with \((n + 1)\) massive neutral gauge bosons. It is possible to generate very high masses \((M \gg M_w)\) for all of these bosons except one. In this case one obtains effectively the minimal SU(2) × U(1) theory, i.e. the mass and the coupling parameters of the “light” Z boson are approximately equal to the ones predicted in the SU(2) × U(1) theory. However it is quite possible that in the real world several relatively light Z bosons exist. In this case at least one boson must be lighter than the Z boson expected in the minimal SU(2) × U(1) theory, e.g. one could have Z bosons as light as 50 GeV. The experiments now in progress in \(e^+e^-\)
annihilation will soon find the effects of such bosons if they exist (e.g. forward–backward asymmetry in the reaction $e^+e^- \to \mu^+\mu^-$) [7.9].

It is also possible that the observed charged current effects are due to the mediation of several (e.g. two) charged bosons (for a corresponding analysis using the gauge group $(SU(2) \times SU(2)' \times U(1))$, see e.g. ref. [7.10]). In such models one of the charged $W$ bosons is lighter than the one obtained in the minimal $SU(2) \times U(1)$ theory. Nevertheless the effective charged and neutral current interaction at low energy is essentially identical to the interaction in the minimal $SU(2) \times U(1)$ theory.

7.2. Predictions for the fermion mass spectrum

Although the lepton and quark mass spectrum observed thus far shows some sort of order (as discussed in section 5), one is far from being able to understand the details of the spectrum and to make predictions. One question which is especially interesting at present is whether there exists a $t$-quark and if yes, what its mass is.

(a) *Flavordynamics without a $t$-quark.* Models without a $t$-quark have been discussed in the literature [7.11, 7.12]. In most of these models one is dealing with the six quarks (u, d, b; c, s, h), where $h$ is a fourth heavy quark of charge $(-1/3)$. In such schemes the GIM-mechanism to suppress the flavor changing neutral currents would be endangered, if there were mixing between the $b$ and $h$ quarks and the $d$ and $s$ quarks. One way to deal with the situation would be to forbid such a mixing entirely, by introducing a new quantum number [7.12]. In such a case the $SU(2) \times U(1)$ gauge interactions can no longer mediate the $b$ decay, and there must exist new gauge interactions. The associated gauge bosons are likely to be heavier than the gauge bosons $W, Z$ in the $SU(2) \times U(1)$ framework, and the lifetime of the $b$-quark is expected to be larger than $\sim 10^{-14}$ sec ($\tau(b) \sim 10^{-10}$–$10^{-14}$ sec). The new gauge interactions may lead to decays like $b \to \tau^+ (e^- \text{ or } \mu^-) \text{ or } (d \text{ or } s)$, or $b \to \nu_\tau \text{ or } \nu_e \text{ or } \nu_\mu \text{ or } (d \text{ or } s)$, or $b \to \tau^+ \bar{u}\bar{u}$ (in the latter case the $B$ meson decays into $\tau^+ \text{ plus an antiproton}$).

Very soon experiments studying $B$ meson decays will establish what the leading decay modes are. If the decays are of the type $b \to (u, c) + \ldots$, mediated by the charged current, one would have to conclude that there exists a $t$-quark. If on the other hand decays of the type $b \to (d, s) + \ldots$ or $b \to (\bar{u}\bar{u}) + \ldots$ dominate, there would be no need to introduce a $t$-quark.

(b) *Predictions for the $t$ quark mass.* Experiments carried out thus far in $e^+e^-$-annihilation have established that the $t$-quark, if it exists, is heavier than about 18 GeV. In the literature there exist several predictions for the $t$ quark mass, all based on special assumptions about the coupling of the Higgs scalars to the quarks.

If we request that the mass matrix given in eq. (6.25) is generated by the minimal scheme of scalars within the flavor gauge group $SU(2)_L \times SU(2)_R \times U(1)$, one finds the relation [7.13]

$$\frac{m_um_d}{m_bm_d} = \frac{m_t - m_c + m_u}{m_b - m_s + m_d}. \tag{7.1}$$

For $m_t \gg m_c \gg m_s$ and $m_b \gg m_s \gg m_d$ one obtains:

$$m_u/m_b \approx \sqrt{m_cm_d/m_bm_d}. \tag{7.2}$$

For the quark mass values discussed previously and $m_b \approx 4.8$ GeV one finds $m_t \approx 12$ GeV, which is already excluded by experiment. However according to eq. (7.2) the ratio $m_u/m_b$ depends critically on
the ratio $m_u/m_d$, which is known only with a rather high uncertainty. For this reason relation (7.1) is not yet ruled out; values of $m$, between 17 and 20 GeV may still be tolerated.

In a different approach one derives the relation [7.14]

$$
\begin{pmatrix}
  m_e & m_d & m_u \\
  m_\mu & m_s & m_c \\
  m_e & m_b & m_t
\end{pmatrix} = 0.
$$
(7.3)

One expects in this case $m_t \approx 15$–20 GeV [7.15].

Another relation has been discussed in ref. [7.16]:

$$
\frac{m_t}{m_c} = \frac{m_\tau}{m_\mu},
$$
(7.4)

which gives $m_t \approx 22$ GeV.

7.3. The family gauge group

There is evidence that there exist at least three different lepton–quark families. The weak interaction mixing angles provide the communications between the various families. It is quite possible that those are the relics of new types of gauge interactions which connect the various lepton–quark families. Such interactions are desirable for the following reasons:

(a) It may be possible to calculate the weak mixing angles in terms of the corresponding gauge coupling.

(b) The family gauge group will involve flavor changing currents like $\bar{e}\mu$, or $\bar{d}s$. It may be possible to calculate the lepton and quark mass spectrum.

(c) Often it is speculated that the spontaneous breaking of the gauge symmetry in QFD is generated dynamically, without the need of scalar bosons (for a recent attempt, see e.g. [7.17]). In such an approach the gauging of the family group is mandatory, for the following reason. Suppose we are dealing with a gauge group like $SU(3)^c \times SU(2) \times U(1)$ where the various lepton–quark families are simply replications of the first family ($\nu_e, e; u d$). If the gauge symmetry is broken dynamically, there is no distinction between the various families, and one would expect a degeneracy in mass between the first and the other families, e.g. $m(e) = m(\mu)$, $m(d) = m(s)$, etc. Since this is not the case, the family symmetry must be broken dynamically at the same time. As a consequence we expect the existence of massless Goldstone bosons (one for each broken generator of the family group). Such Goldstone bosons do not exist. This problem does not arise if the family group is gauged.

If we start out with the gauge group $SU(3)^c \times SU(2) \times U(1)$, and incorporate the three families ($\nu_e, e; u d$), ($\nu_\mu, \mu; c s$) and ($\nu_\tau, \tau; t, b$), there are various ways to gauge the family symmetry.

(a) One may interpret the three families as a vector representation of $O(3)$. In this case the whole gauge group would be $SU(3)^c \times SU(2) \times U(1) \times O(3)$. (See e.g.: [7.18].)

(b) The three families are interpreted as a 3-representation of $SU(3)$, or $(3, 1) + (1, 3)$ of $SU(3) \times SU(3)$. (See e.g.: [7.19, 7.20].)

There are rather stringent limits on the existence of gauge bosons which would mediate between the families. The best one can be derived from the nonobservation of the decay $K^0_L \rightarrow e\mu \ (B < 2 \times 10^{-9})$. The effective Fermi constant for the family gauge interactions should therefore be less than $10^{-4} \cdot G_F$. If we assume that the family gauge coupling constant is of the order of the SU(2) coupling constant, the
masses of the family group gauge bosons must be heavier than $M_{w} \cdot 10^{2} \approx 10000$ GeV. It remains to be seen if better limits on those masses can be derived from other rare processes.

8. Future tests of flavordynamics

(a) Decays of the W and Z bosons. The decisive tests of the current theories on flavordynamics will be to verify the predicted energy dependent behaviour of the weak processes at high energies and, of course, to find the W and Z bosons.

It is easy to work out the decay properties of the W and Z bosons. The coupling of the W boson to the $(\nu_{e}, e^{-})$-doublet is given by the Fermi constant, and one finds [8.1]:

$$\Gamma(W^{-} \rightarrow e^{-} + \bar{\nu}_{e}) = G_{F} M_{w}^{3} / 6\pi\sqrt{2}. \quad (8.1)$$

Neglecting QCD corrections, the branching ratios for the decays $W^{-} \rightarrow l\bar{\nu}_{l}$ ($l = e, \mu, \tau$) and $W^{-} \rightarrow q\bar{q}$ are given simply by the counting of the various channels:

$$\Gamma(W^{-} \rightarrow \bar{u} + d) = 3\Gamma(W^{-} \rightarrow \bar{\nu}_{e} + e^{-}) \quad (8.2)$$

$$\Gamma(W^{-} \rightarrow \bar{u} + d) = \Gamma(W^{-} \rightarrow \bar{c} + s) = (W^{-} \rightarrow \bar{t} + b) = \cdots \quad (8.3)$$

$$\Gamma(W^{-} \rightarrow \bar{\nu}_{e} + e^{-}) = \Gamma(W^{-} \rightarrow \bar{\nu}_{\mu} + \mu^{-}) = \Gamma(W^{-} \rightarrow \bar{\nu}_{\tau} + \tau^{-}) = \cdots \quad (8.3)$$

(Phase space factors have been neglected; it is assumed that $m_{t} + m_{b} \ll M_{w}$. The three dots denote further leptons and quarks, in case they exist.)

In the three family model one finds $B(W^{-} \rightarrow l + \bar{\nu}_{l}) = 1/12 = 8.3\%$ (8.3) ($l = e, \mu, \tau$). Using the constraint $\sin^{2} \theta_{w} = 0.23 \pm 0.02$, one predicts in the minimal model $M_{w} = 80 \pm 3$ GeV and $M_{Z} = 91 \pm 3$ GeV. In the three-family case the width of the W boson is found to be about 2.5 GeV, i.e. the W boson is not a narrow state.

Analogously we can estimate the decay widths of the Z boson. In the limit $\theta_{w} = 0$ ($M_{w} = M_{Z}$) the Z and W couplings to $(\bar{\nu}_{e} e^{-})$ and $(\bar{\nu}_{e} \nu_{e})$ respectively are given by $(g/\sqrt{2}) \cdot \bar{\nu}_{e} \gamma_{\mu} e^{-} \cdot W_{\mu}^{+} - g \cdot \frac{1}{2} \bar{\nu}_{e} \gamma_{\mu} \nu_{e} Z^{\mu}$. Therefore we have

$$\Gamma(Z \rightarrow \bar{u}u) = \frac{1}{2} \Gamma(W^{-} \rightarrow e^{-} \bar{\nu}_{e}) = G_{F} M_{Z}^{3} / 12\pi\sqrt{2}. \quad (8.4)$$

This relation remains true for $\theta_{w} \neq 0$, since $G_{F} M_{Z}^{2} / (g')^{2}$ is independent of $\theta_{w}$ ($g' = g / \cos \theta_{w}$ is the dimensionless neutral current coupling constant).

For the various Z decays one has

$$\Gamma(Z \rightarrow \bar{\nu}_{e} \nu_{e}) = \Gamma(Z \rightarrow \bar{\nu}_{\mu} \nu_{\mu}) = \Gamma(Z \rightarrow \bar{\nu}_{\tau} \nu_{\tau})$$

$$\frac{\Gamma(Z \rightarrow e^{+} e^{-})}{\Gamma(Z \rightarrow \bar{\nu}_{e} \nu_{e})} = \left(1 + (-1 + 4 \sin^{2} \theta_{w})^{2}ight)^{2}$$

$$\frac{\Gamma(Z \rightarrow \bar{u}u)}{(Z \rightarrow \nu^{-} \nu_{e})} = \frac{3(1 + (1 - \frac{1}{2} \sin^{2} \theta_{w}))^{2}}{2} \quad (8.5)$$
\[
\frac{\Gamma(Z \rightarrow \dbar \bar{d})}{\Gamma(Z \rightarrow \bar{\nu}_e \nu_e)} = \frac{3(1 + (-1 + 4 \sin^2 \theta_w)^2)}{2}
\]

\[
\Gamma(Z \rightarrow \bar{u}u) = \Gamma(Z \rightarrow \bar{c}c) = \Gamma(Z \rightarrow \bar{t}t) = \Gamma(Z \rightarrow \bar{s}s) = \Gamma(Z \rightarrow \bar{b}b).
\]

For \(\sin^2 \theta_w = 0.23\) one finds for the various branching ratios:

\[
\bar{\nu}_e \nu_e : \bar{e}e : \bar{u}u : \bar{d}d = 2 : 1.01 : 3.45 : 4.44.
\]

In case of three families the decays of the Z into a pair of charged leptons constitute about 3\% of all decays:

\[
B(Z^0 \rightarrow e^+ e^-) = B(Z^0 \rightarrow \mu^+ \mu^-) = B(Z^0 \rightarrow \tau^+ \tau^-) \approx 3\%.
\]

Furthermore one has

\[
B(Z^0 \rightarrow \bar{\nu}_e \nu_e) = B(Z^0 \rightarrow \bar{\nu}_\mu \nu_\mu) = B(Z^0 \rightarrow \bar{\nu}_\tau \nu_\tau) \approx 6\%.
\]

The total width of the Z\(^0\) is about equal to the W width, namely about 2.5 GeV. Each neutrino contributes to the Z decay rate 150 MeV, i.e. in case of three families altogether 450 MeV (almost 20\% of \(\Gamma_{\text{tot}}(Z^0)\)). This implies that about 20\% of all Z\(^0\) decays will be “invisible”. By comparing the “visible” decay rate with the actual decay rate, one could determine the number of neutrinos in the world (for details see e.g. ref. [8.1]).

(b) **Producing the weak bosons in hadronic collisions.** The chances are good that the first direct evidence for the existence of the Z and W bosons will be obtained around 1982/3 in the experiments at the CERN proton–antiproton colliding beam facility, followed by similar experiments at the Isabelle machine in the US. The cross section for producing W and Z bosons in hadronic collisions can be estimated by extrapolating the cross sections for lepton pair production. Via CVC, one can relate the isovector part of the virtual photon emission (mass \(M_w\)) to the W emission [8.2]:

\[
p + p \rightarrow W^\pm + X \sim p + p \rightarrow \mu^+ \mu^- + X.
\]

The lepton pair production cross sections at \(q^2 = M_{W,Z}^2\) can be estimated using the observed scaling behaviour:

\[
q^4 \frac{d\sigma}{dq^2} = f(\tau) \quad (\tau = q^2/s)
\]

where \(q^2\) is the virtual photon mass and \(s^{1/2}\) the total c.m. energy of the collision. More specifically the production cross sections can be estimated using the quark–antiquark annihilation model [8.3]. The W\(^+\) is produced by the process \(\bar{d} + u \rightarrow W^+\); analogously the W\(^-\) is produced in \(d + \bar{u} \rightarrow W^-\). One expects \(\sigma(p + p \rightarrow W^+ + X) \sim 3 \times 10^{-33}\) cm\(^2\) at \(\sqrt{s} = 600\) GeV. Since the \(u\) quark distribution function in the proton is about twice the \(d\) quark distribution function, and since \(\bar{u} \approx \bar{d}\), the cross section for producing W\(^+\) and W\(^-\)in pp collisions will not be equal:
The cross section for producing the Z boson is of the same order of magnitude.

In p–p̅ collisions the W and Z production is expected to be larger by a factor 2–10, depending on $M_W/\sqrt{s}$, since here the W or Z can be produced by the annihilation of valence quarks and antiquarks.

The easiest way to observe the W is to look for its leptonic decays ($W^-\rightarrow e^-\bar{\nu}_e, \mu^-\bar{\nu}_\mu$). The $p_\perp$ distribution of the leptons should show a peak at $p_\perp \approx M_W/2$. This peak will, however, be smeared out if transverse motion of the W boson is too large. Recently one has obtained evidence that the transverse momenta of the lepton pairs produced in hadronic collisions increase approximately linearly with energy [8.4], in agreement with QCD (increase of the transverse momenta due to gluon bremsstrahlung processes [8.5, 8.6]). As a consequence one expects that the average transverse momenta of the W and Z bosons produced in hadronic collisions are quite large (of the order of 10 GeV at $\sqrt{s}=600$ GeV), and it remains unclear, if it will be possible to identify the W bosons in p–p̅-collisions by their leptonic decay modes [8.7]. In view of the large hadronic background it seems impossible to identify the W boson by its hadronic decay modes ($W^-\rightarrow \bar{q}+q\rightarrow 2$ jets). Probably it will be much easier to observe the W boson by its leptonic decay modes. The Z boson can be observed by looking for peaks in the invariant mass spectrum of muon pairs or electron pairs.

The coupling of the W boson to the quarks is of the type $V-A$. This implies that the W boson tends to be produced with helicity $-1$ if the c.m. momentum of the quark is much larger than that of the antiquark, and vice versa. The $\mu^+$, coming from the $W^+$ decay, is emitted preferentially along the spin direction of the $W^+$. Correspondingly the $\mu^-$, coming from the $W^-$ decay, is emitted preferentially along the direction opposite to the spin of the $W^-$. That way an asymmetry in the inclusive muon signal is introduced. In pp collision the $\mu^+$ distribution is more sharply peaked at rapidity $y=0$ than the $\mu^-$ distribution.

The consequences of the W polarization are more profound in pp-collisions. Here the spin of the produced W is strongly aligned along the antiproton direction, since only righthanded antiquarks, i.e. antiquarks, whose spin is aligned in the $\bar{p}$ direction, can produce a W boson. For this reason e.g. the $\mu^+$ coming from a $W^+$ boson is emitted preferentially in the direction of the incoming $\bar{p}$.

(c) The Z boson in $e^+e^-$-annihilation. The only reaction, where the Z boson can be produced without any background is $e^+e^-$-annihilation [8.1]. At the project LEP machine it is planned to attain a luminosity of $10^{32}$ cm$^{-2}$ sec$^{-1}$; the $e^+e^-$ energy resolution is expected to be of the order of 0.1% of the beam energy. Therefore the energy resolution of the $e^+$ and $e^-$ ($\sim$100 MeV for $\sqrt{s} \approx 100$ GeV) is small compared to the Z width. At the peak of the resonance one has [8.1]

$$\frac{\sigma(e^+e^-\rightarrow Z^0\rightarrow all)}{\sigma_{pt}} \approx \frac{9}{\alpha^2} B(Z^0\rightarrow e^+e^-)$$

where $\sigma_{pt}$ denotes the pointlike QED cross-section for the production of muon pairs in $e^+e^-$ annihilation:

$$\sigma_{pt} = \frac{4\pi\alpha^2}{3s} = \frac{87}{s[GeV^2]} \text{nb.}$$

Using $B(Z^0\rightarrow e^+e^-) \approx 3\%$, as indicated by our considerations above, one finds $\sigma(e^+e^-\rightarrow Z^0\rightarrow all)/\sigma_{pt} \approx 5000$. At $\sqrt{s} = M(Z) \approx 90$ GeV and with $\sigma_{pt} \approx 10^{-2}$ nb this gives several Z decays/sec.
As an example we consider within the minimal SU(2) x U(1) model the shape of the cross section for the reaction $e^+e^- \rightarrow \mu^+\mu^-$ in the neighborhood of the Z pole. One finds [8.1]

$$R(\mu^+\mu^-) = \frac{\sigma(e^+e^- \rightarrow \gamma, Z \rightarrow \mu^+\mu^-)}{\sigma_{pt}}$$

$$= 1 - \frac{2sK(-1 + 4 \sin^2 \theta_w)^2}{s/M_Z^2 - 1 + \Gamma_Z^2(s - M_Z^2)} + \frac{s^2K^2[1 + (-1 + 4 \sin^2 \theta_w)^2]^2}{(s/M_Z^2 - 1)^2 + \Gamma_Z^2/M_Z^2}$$

(8.13)

where the strength parameter $\kappa$ is defined by

$$\kappa = \frac{G_F}{8\sqrt{2}\pi\alpha} = 1.4130 \times 10^{-4} \text{ GeV}^{-2}. \quad (8.14)$$

If we neglect $\Gamma_Z$, we find

$$R(\mu^+\mu^-) \approx 1 + 2(-1 + 4 \sin^2 \theta_w)^2\chi + [(-1 + 4 \sin^2 \theta_w)^2 + 1]\chi^2$$

$$\chi = \kappa M_Z^2 \cdot s/(s - M_Z^2). \quad (8.15)$$

The cross section ratio $R(\mu^+\mu^-)$ has a minimum at

$$\frac{s}{M_Z^2} = \frac{1}{M_Z^2\kappa} \cdot \frac{(-1 + 4 \sin^2 \theta_w)^2}{[(-1 + 4 \sin^2 \theta_w)^2 + 1]^2}. \quad (8.16)$$

At the minimum one has

$$R(\mu^+\mu^-) = 1 - \frac{(-1 + 4 \sin^2 \theta_w)}{[(-1 + 4 \sin^2 \theta_w)^2 + 1]^2}. \quad (8.17)$$

The shape of the cross section for $e^+e^- \rightarrow \mu^+\mu^-$ near the Z pole for $\sin^2 \theta_w = 0.25$ can be seen in fig. 13.

Another quantity which is of substantial interest is the forward-backward asymmetry $d\sigma(e^+e^- \rightarrow \mu^+\mu^-)/d(cos \theta)$ where $\theta$ is the angle between the incident $e^-$ and the outgoing $\mu^-$. The integrated asymmetry is defined by

$$A_\mu = \int_0^1 d\sigma_\mu - \int_{-1}^0 d\sigma_\mu) / \int_0^1 d\sigma_\mu. \quad (8.18)$$

In the minimal SU(2) x U(1) theory it is given by

$$A_\mu = \frac{3\chi[1 + 2(-1 + 4 \sin^2 \theta_w)^2\chi]}{1 + 2\chi(-1 + 4 \sin^2 \theta_w)^2 + \chi^2[1 + (-1 + 4 \sin^2 \theta_w)^2]^2}. \quad (8.19)$$

At low energies ($\sqrt{s} \ll M_Z$) it is sufficient to include only the term linear in $\chi$, and one obtains:

$$A_\mu \approx +\frac{3}{2}\chi. \quad (8.20)$$
For e.g. $\sqrt{s} \approx 40$ GeV, one finds $A_\mu \approx 10\%$.

We can work out in a similar way the forward–backward asymmetries for the quarks, by replacing the factor $(-1 + 4 \sin^2 \theta_w)$ in the expressions above by

(a) $(1 - \frac{3}{4} \sin^2 \theta_w)$ for u, c, t, ...

(b) $(-1 + \frac{3}{4} \sin^2 \theta_w)$ for d, s, b, ...

For example, at $\sqrt{s} = 40$ GeV the asymmetry $A_{u,c,t}$ is of the order of 7%, while the asymmetry $A_{d,s,b}$ is twice as large: $A_{d,s,b} \approx 14\%$. The measurements of the forward–backward asymmetries of the quark–antiquark pairs produced in $e^+e^-$-annihilation offer a unique opportunity to measure the neutral current couplings of the s, b ... and c, t ... quarks.

In fig. 14 we have plotted the asymmetry $A_\mu$ in the minimal SU(2) x U(1) model ($\sin^2 \theta_w = 0.25$). It reaches its minimum at

$$s/M_Z^2 = \left[ 1 + \kappa M_Z^2 (1 + 3(-1 + 4 \sin^2 \theta_w)^2) \right]^{-1}$$

(8.22)

where it takes the value

$$A_\mu = -\frac{3}{4} \frac{1}{1 + 2(-1 + 4 \sin^2 \theta_w)^2}$$

($= -0.75$ for $\sin^2 \theta_w = 0.25$).

The asymmetry at the peak of the resonance is given by

$$A_\mu \approx \frac{3(-1 + 4 \sin^2 \theta_w)^2}{\left[ (-1 + 4 \sin^2 \theta_w)^2 + 1 \right]^2}$$

($A_\mu = 0$ for $\sin^2 \theta_w = 0.25$).
The maximum of the asymmetry is at

\[ \frac{s}{M_Z^2} = 1 - \kappa M_Z^2 [1 - (-1 + 4 \sin^2 \theta_W)^2] \]  

where one has

\[ A_\mu = \pm \frac{3}{4}. \]

Probably the forward–backward asymmetry of the produced muon pairs in e^+e^--annihilation will be seen in the near future at the PETRA and PEP experiments. It will not be possible to determine the mass of the Z boson from the forward–backward asymmetry measured at the highest PETRA/PEP energies in a model-independent way. In order to do so, one would have to measure the onset of the nonlinearity in the asymmetry at high energies, which probably is impossible at PETRA or PEP. Nevertheless the measurement of the asymmetry will be an important check of the current theory of the weak interactions.

**W boson production in e^+e^--annihilation.** The W bosons can be produced in pairs in e^+e^- annihilation, either via a virtual γ or Z in the s channel, or via a virtual electron neutrino in the crossed channel (see the diagrams in fig. 15, ref. [8.8]).

The W pair production is especially interesting, since it will be possible to observe whether the interplay between the neutrino exchange and the direct channel exchanges is such that the cancellations occur which are required for a renormalizable theory [8.9]. Furthermore it will be possible for the first time to receive evidence for the existence of the three boson vertex, by observing the γW^+W^- or ZW^+W^- interaction.
The cross section for the W pair production is given by

\[
\sigma(e^+e^- \to W^+W^-) = \frac{\pi a^2 \beta}{2\sin^4 \theta_W} \left\{ (1 + 2a + 2a^2) \frac{L}{\beta} - \frac{5}{4} + \frac{M_W^2 (1 - 2\sin^2 \theta_W)}{s - M_Z^2} \right\}
\times \left[ 2a^2 \left( 1 + \frac{2}{a} \right) \frac{L}{\beta} - \frac{1}{12a} \frac{5}{3} - a \right] \tag{8.24}
\]

where \( a = M_W^2/s, \ \beta = \sqrt{1 - 4a}, \ L = \ln[(1 + \beta)/(1 - \beta)] \). In fig. 16 \( \sigma(e^+e^- \to W^+W^-) \) is plotted for \( \sin^2 \theta_W = 0.25 \).

The cross section rises sharply after threshold and reaches its peak at \( \sqrt{s} = 180 \text{ GeV} \), where it attains a value well above \( 10^{-35} \text{ cm}^2 \). The neutrino exchanges cause the \( W^+W^- \) angular distribution to be sharply peaked forward.

The W bosons can be observed either by looking for their leptonic decays \( (W^+ \to e^+\nu_e, \mu^+\nu_\mu, \tau^+\nu_\tau) \), or by looking for the production of quark jets via the decays \( W^+ \to \bar{u}d, \bar{s}c, \) etc. The first possibility is hampered by the relatively small leptonic branching ratios; only 0.7% of all \( W^+W^- \) pairs decay e.g. into \( e^+\mu^- + \) neutrinos. For this reason it looks more promising to observe the quark jets in the hadronic decays of the W bosons. The mass of the bosons can be determined with an accuracy of a few % by observing the steep threshold of the production of the W pairs. Given precise data, it would be possible to separate the different processes and to determine especially the \( gW^+W^- \) and \( ZW^+W^- \) vertices. This will be crucial for answering the question whether the theoretical ideas about gauge theories are correct or not.

(d) **Search for Higgs scalars.** In the conventional theories of flavordynamics the masses of the intermediate bosons are generated by the mechanism of spontaneous symmetry breaking employing a number of scalar fields. In the minimal \( SU(2) \times U(1) \) model one \( SU(2) \) doublet of scalars is used, which together with antiparticles consists of four particles, three of which are “eaten up” providing a mass for

![Fig. 16. The total cross section for \( e^+e^- \to W^+W^- \) as a function of the energy in case \( \sin^2 \theta_W = 0.25 \).](image)
the W\(^+\), W\(^-\) and Z. One is left with only one physical particle, which is electrically neutral. If several doublets are used instead (this, for example, is necessary for generating CP violation via scalar couplings), one is left not only with neutral, but also with charged scalars. The same is true for more complicated theories of flavordynamics, e.g. for left-right symmetric theories. There is a great deal of arbitrariness in generating the masses for the scalar particles. Even in the minimal SU(2) \(\times\) U(1) theory, which is the most restrictive theory, the mass of the scalar particle is almost a free parameter; it should lie in a range from a few GeV [8.10] to about one TeV [8.11]. Since the Higgs scalar self-coupling in the minimal model is proportional to the (mass)\(^2\) of the scalar particle, large masses imply large self-couplings. For \(m(\text{scalar}) > 1\) TeV the coupling becomes strong, causing typical strong interaction phenomena (resonances etc.) to occur in weak interaction physics at energies of the order of \(1\) TeV.

The best way to produce and observe a neutral Higgs particle \(H^0\) is in \(e^+e^-\) annihilation at energies in the LEP range. One way to look for \(H^0\) is to investigate the reactions \(e^+e^- \rightarrow Z^0 \rightarrow H^0 + l^+l^-\) \((l = e, \mu)\), and \(e^+e^- \rightarrow Z^0 \rightarrow H^0 + \gamma\) [8.12].

The branching ratio \(\Gamma(Z \rightarrow H^0 + X)/\Gamma(Z \rightarrow \text{all})\) is expected to be of the order of \(10^{-3}\) for \(m(H^0)/m(Z) \sim 0.1-0.2\) [8.13].

Another possibility is to look for the reaction \(e^+e^- \rightarrow Z^0 + H\). As in the previous case, here one makes use of the relatively large coupling of the \(H^0\) to the Z boson. The \(Z^0\) can be used as a trigger to look for the \(H^0\) by searching for a bump in the invariant mass spectrum of the system recoiling against the \(Z^0\). At the proposed LEP machine one can look for H bosons with masses well above 100 GeV.

A similar way to observe the \(H^0\) would be to look for rare decay modes of heavy vector mesons \((Y(9.46), \text{an eventually existing } \bar{t}t \text{ state, ...})\) [8.14].

The decay patterns for the \(H^0\) depend strongly on the relevant mass, since the Yukawa coupling of a lepton or quark to the \(H^0\) is proportional to the mass of the corresponding fermion. For this reason the \(H^0\) decays predominantly into the heaviest fermion pair it can decay into (see e.g. [8.15]). In case \(m(H_0) \sim 4-10.6\) GeV the \(H^0\) will decay mainly into \(\tau\) pairs and pairs of charmed particles. For \(m(H_0) > 10.6\) GeV the decays into b flavored particles will dominate, etc.

In the minimal theory the couplings of the neutral scalar boson to the fermions are given by

\[
g_{\text{flf}} = (2^{1/4}\sqrt{G})m_t
\]

\((f = u, d, \ldots, e, \mu, \ldots)\). On the other hand the coupling of the \(H^0\) to the W bosons is

\[
g_{W^*W^-H^0} = (2^{1/4}\sqrt{G})2M_W^2.
\]

The decay rate of the \(H^0\) into a \(\bar{f}f\) pair is given by

\[
\Gamma(H^0 \rightarrow \bar{f}f) = c \frac{Gm_t^2m_{1f^2}}{4\sqrt{2}\pi} \left(1 - \frac{4m_t^2}{m_{1f^2}}\right)^{3/2}
\]

(8.27)

where \(c\) is a color factor \((c = 1\) for leptons, \(c = 3\) for color triplet quarks). As an example we consider the case of \(m(H^0) \approx 20\) GeV. In this case we have

\[
\Gamma(H^0 \rightarrow e^+e^-) : \Gamma(H^0 \rightarrow \mu^+\mu^-) : \Gamma(H^0 \rightarrow \tau^+\tau^-) : \Gamma(H^0 \rightarrow \bar{u}u) : \Gamma(H^0 \rightarrow d\bar{d}) : \Gamma(H^0 \rightarrow s\bar{s}) : \Gamma(H^0 \rightarrow c\bar{c}) : \Gamma(H^0 \rightarrow \bar{b}b) = m_e^2 : m_\mu^2 : m_\tau^2 : 3m_u^2 : 3m_d^2 : 3m_s^2 : 3m_c^2 : 3m_b^2.
\]

(8.28)

This gives a branching ratio \(B(H^0 \rightarrow \mu^+\mu^-) \sim 10^{-5}\).
The final states are typically quite complicated, and it seems very difficult to observe the $H^0$ in any specific process.

It is typical for the minimal SU(2) $\times$ U(1) model that the $H^0$–fermion couplings are flavor conserving. Flavor changing transitions like $H^0$–$s$ do not occur, since the $H^0$ coupling matrix is by definition proportional to the mass matrix. However this is no longer true if several neutral Higgs scalars exist. In this case the diagonalization of the fermion mass matrix will not in general lead to diagonal coupling matrices. On the other hand nondiagonal couplings, e.g. $H^0$–$d$, may lead to flavor-changing weak transitions, for example to the decay $K_L \rightarrow \mu^+ \mu^-$, or a relatively large $K_L - K_S$ mass difference. Typically the $H$ mesons have to be heavier than a few hundred GeV, in order not to cause flavor-changing transitions with magnitudes larger than observed. For this reason it is desirable to have additional symmetries present, which prevent off-diagonal couplings from occurring [8.16].

In many models one is dealing not only with neutral, but also with charged scalar particles. The latter can be produced in $e^+ e^-$-annihilation; their masses and decay patterns are highly model-dependent (for a discussion see e.g.: [8.17]).

9. Outlook

During the past ten years a relatively clear picture of the world of elementary particles has emerged. The strong interactions are described by QCD; the quarks are color triplets, the leptons are color singlets. The electroweak interactions are described by the standard SU(2) $\times$ U(1) theory; the lefthanded quarks and leptons are flavor doublets, the righthanded ones flavor singlets. Nevertheless it seems that one has found only a partial answer to the set of questions one has asked many years ago, before the present standard scheme of particle interactions has emerged. Many problems are still unsolved. Among them are:

Why are the strong interactions as strong as observed? What determines the numerical values of the QCD coupling constant, the fine structure constant, and the SU(2) $\times$ U(1) mixing angle? Why does nature have a preference for SU(2) doublets of scalars? Why do only color singlets, triplets and octets exist, and not e.g. color sextets? Why are the electric charges quantized? What determines the mass spectrum of the leptons and quarks?

These and other questions can only be answered within a theoretical scheme which contains further interactions. There are several possible scenarios:

(a) Nothing happens in the energy region between $\sim 100$ GeV and the energy region of $\sim 10^{15}$ GeV where the strong and electroweak interactions are (perhaps) unified in a single scheme [9.1].

(b) New flavor interactions exist in the region between 100 GeV and $\sim 10^{15}$ GeV. The effects of those interactions could be seen by the observation of rare processes, such as $\mu \rightarrow e \gamma$, $K^0_L \rightarrow e\mu$, etc., and more directly in the $e^+ e^-$-annihilation experiments at energies in the LEP region or above.

(c) New types of color interactions emerge at high energies (hypercolor, etc.) [9.2]; some or all of the particles which are considered elementary in the standard scheme (Higgs scalars ...) are bound states of the basic fermions.

(d) A new layer of elementarity remains yet to be discovered. The quarks, leptons, gauge bosons etc. are bound states of yet smaller constituents (ure, preons, prequarks, ...). In this case one expects that the leptons and quarks have a finite size. Since the charged leptons are observed to be pointlike down to a distance of $10^{-16} - 10^{-17}$ cm, the size of all fermions should be less than that. It is unclear what kind of interaction might give rise to the binding of the ure constituents such that the observed
spectrum of leptons and quarks arises. We note that the hypothetical size of the fermions must be much less than their Compton wavelengths. For example, the size of the muon must be much less ($\ll 10^{-17}$ cm) than its Compton wavelength ($\sim 10^{-13}$ cm). This is a unique situation in physics, since the sizes of atoms, nuclei, nucleons, pions etc. are typically larger or of the same order as the Compton wavelengths of these objects.

The experiments to be carried out during this decade will provide answers to at least some of the questions which we have asked above. One can look forward to a rich harvest.

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Appendix 1. The Bouchiat–Meyer sumrule [4.3]

The amplitudes (s- and p-waves) of the $K \rightarrow 3\pi$ decays can be decomposed according to Bose-symmetry and the isospin of the transition operator ($\Delta I = \frac{1}{2}$ and $\frac{3}{2}$):

\[
K^+ \rightarrow \pi^+ \pi^0 \pi^0
\]

\[
A^{+00} = a(\bar{2}) - \frac{1}{2} a(\bar{3}) + [b(\bar{2}) - \frac{1}{2} b(\bar{3}) + \frac{3}{2} c(\bar{3})] \left( \frac{E_{\pi^+} - \frac{1}{3}}{m_K} \right).
\]

\[
K^+ \rightarrow \pi^+ \pi^+ \pi^-
\]

\[
A^{-+0} = -2[a(\bar{2}) - \frac{1}{2} a(\bar{3})] + [b(\bar{2}) - \frac{1}{2} b(\bar{3}) - \frac{3}{2} c(\bar{3})] \left( \frac{E_{\pi^+} - \frac{1}{3}}{m_K} \right).
\]

\[
K^0 \rightarrow \pi^+ \pi^- \pi^0
\]

\[
A^{+00} = \frac{1}{\sqrt{2}} \left\{ -[a(\bar{2}) - a(\bar{3})] - (b(\bar{2}) + b(\bar{3})) \left( \frac{E_{\pi^0} - \frac{1}{3}}{m_K} \right) + c(\bar{3}) \left| \frac{E_{\pi^+} - E_{\pi^-}}{m_K} \right| \right\}.
\]

\[
K^0 \rightarrow \pi^0 \pi^0 \pi^0
\]

\[
A^{000} = \frac{3}{\sqrt{2}} [a(\bar{3}) + a(\bar{3})].
\]

The independent amplitudes (s- and p-waves) are:

\[
\Delta I = \frac{1}{2}: \quad a(\bar{2}), \quad b(\bar{2})
\]

\[
\Delta I = \frac{3}{2}: \quad a(\bar{2}), \quad b(\bar{2}), \quad c(\bar{3}).
\]
The slope parameters as defined by the particle Data group [A1.1]: $g^{+00}$, $g^{++}$, ... are related to the amplitudes in eq. (A1.1) in the following way

\[-g^{+00} \frac{2m_K^2}{m^2} = \lambda^{+00} = \frac{b(\frac{1}{3}) - \frac{1}{2} b(\frac{2}{3}) + \frac{3}{2} c(\frac{2}{3})}{a(\frac{1}{3}) - \frac{1}{2} a(\frac{2}{3})}\]

\[-g^{++} \frac{2m_K^2}{m^2} = \lambda^{++} = \frac{b(\frac{1}{3}) - \frac{1}{2} b(\frac{2}{3}) - \frac{3}{2} c(\frac{2}{3})}{(-2)(a(\frac{1}{3}) - \frac{1}{2} a(\frac{2}{3}))}\]

\[\frac{E_\pi'}{m_K} - \frac{1}{3} = \frac{s_0 - s_1 m^2}{m^2 m_K^2}; \quad s_i = (p_K - p_\pi)^2\]

\[g^{+00} = 0.607 \pm 0.030\]

\[g^{++} = -0.215 \pm 0.004. \quad (A1.2)\]

From eqs. (A1.1, 2) we infer

\[r_0^- = -\frac{\lambda^{+00}}{\lambda^{++}} = -\frac{g^{+00}}{g^{++}} = \frac{1 + X}{1 - X} = 1.41 \pm 0.07 \quad (A1.3)\]

\[X = \frac{3}{2} \frac{c(\frac{1}{3})}{b(\frac{1}{3}) - \frac{1}{2} b(\frac{2}{3})} = \frac{r_0^- - 1}{r_0^- + 1} = 0.17 \pm 0.02.\]

The amplitudes $a(\frac{1}{3}, \frac{2}{3})$, $b(\frac{1}{3}, \frac{2}{3})$, $c(\frac{1}{3})$ are related in the PCAC limit to the $K \to 2\pi$ decay amplitudes (neglecting final state interactions):

\[K^+ \to \pi^+ \pi^0: \quad \frac{3}{\sqrt{2}} t(\frac{1}{3})\]

\[K^0 \to \pi^+ \pi^-: \quad t(\frac{1}{3}) + t(\frac{2}{3})\]

\[K^0 \to \pi^0 \pi^0: \quad \frac{1}{\sqrt{2}} (-t(\frac{1}{3}) + 2t(\frac{2}{3})). \quad (A1.4)\]

We have e.g. for the $(\pi^+ \pi^0 \pi^0)$ system

\[A^{+00} \frac{F_\pi}{f_\pi} \to -i \langle \pi^+ \pi^0 | [Q_{S0}, H_{\text{eff}}] | K^+ \rangle \quad (A1.5)\]

In eq. (A1.5) $F_\pi = \sqrt{2} f_\pi = 0.945 m_{\pi\pi}$ denotes the pion decay constant and

\[Q_{S0} = \int_{t=\text{const.}} d^3 x \frac{1}{3} [u^+ c \gamma_5 u - d^+ c \gamma_5 d] \quad (A1.6)\]

is the axial charge with the quantum numbers of $\pi^0$. 

Since \( H_{w_{\text{eff}}} = (H_{w_{\Delta I = 1/2}} + H_{w_{\Delta I = 3/2}})_{\text{eff}} \) transforms like a field with \( I_3 = \frac{1}{2} \), we have for chirally pure composition of \( H_{w_{\text{eff}}} \) with respect to nonstrange quarks

\[
[Q_5^0, H_{w_{\Delta I = 1/2, 3/2}}] = \frac{1}{2} \sigma_{1/2, 3/2} H_{w_{\Delta I = 1/2, 3/2}}
\]

\[(\sigma_i = \pm 1) \tag{A1.7}\]

(terms like \( \bar{u}\gamma_\mu u, \bar{u}\gamma_\mu \gamma^5 u, \bar{d}\gamma_\mu d, \bar{d}\gamma_\mu \gamma_5 \gamma^5 d \) ... diagonal in flavor do not count).

The sign of \( F_\pi \) is undetermined so we can choose \( \sigma_{3/2} = 1 \) without loss of generality.

After some algebra we obtain [4.3, A1.2]

\[
a(3) = \frac{i}{3F_\pi} \sigma_{1/2} t(3)
\]

\[
b(3) = -6a(3) = -\frac{2i}{F_\pi} \sigma(3) t(3)
\]

\[
a(3) = -\frac{2i}{F_\pi} t(3); \quad b(3) = -\frac{5i}{F_\pi} t(3)
\]

\[
c(3) = -\frac{9i}{F_\pi} t(3).
\]

(A1.8)

Inserting eq. (A1.8) in eq. (A1.3) we obtain the Bouchiat–Meyer sumrule

\[
X = \frac{r_0 - \frac{1}{2}}{r_0 - 1} = \frac{27}{4} Y \frac{1}{1 - \frac{3}{2} Y}
\]

\[
Y = \sigma(3) \Re \frac{t(3)}{t(3)} \tag{A1.9}
\]

The phases of the amplitudes \( t(3, 3) \) arise from the final state \( \pi\pi \)-interaction. \( t(3)/t(3) \) can be evaluated from the \( 2\pi \)-decay widths of \( K^0, \bar{K}^0, K^\pm \):

\[
\frac{T_{K^+\to\pi^+\pi^-}}{2T_{K^+\to\pi^+\pi^-}} = \left| \frac{1 + t(3)/t(3)}{1 - 2t(3)/t(3)} \right|^2 \approx 1 + 6\Re \frac{t(3)}{t(3)} \to \Re \frac{t(3)}{t(3)} = 0.015 \pm 0.002 \tag{A1.10}
\]

We note for comparison

\[
\left( \frac{T_{K^+\to\pi^+\pi^-}}{T_{K^\pm\to\pi^+\pi^-}} \right)^{1/2} : \frac{2}{3} = \left| \frac{t(3)}{t(3)} \right| = 0.031 \tag{A1.11}
\]

Since \( X \) and \( \Re(t(3)/t(3)) \) are both positive

\[
X = 0.17 \pm 0.02, \quad \Re(t(3)/t(3)) = 0.016 \pm 0.002 \tag{A1.12}
\]

eq. (A1.9) implies
\[ \frac{\sigma(1/2)}{\sigma(1)} = +1 \Rightarrow Y = \text{Re}(t(1/2)/t(1)). \]  

(A1.13)

This corresponds to the verification of the sum rule in the following way:

\[ X = \frac{r_0 - 1}{r_0 + 1} = 0.17 \pm 0.02 \equiv \frac{27}{4} Y \frac{1}{1 - \frac{3}{4} Y} = 0.11 \pm 0.02. \]  

(A1.14)

If the argument within the errors and the PCAC extrapolation performed of eq. (A1.14) is not fortuitous we have to conclude that at least to a good approximation both the \( \Delta I = \frac{1}{2} \) and \( \Delta I = \frac{3}{2} \) effective Hamiltonians are chirally pure and equal i.e. left-handed as far as the nonstrange quarks (u, d) are concerned. We note that using previous data [A1.3] \( X \) in eq. (A1.14) was determined to be \( 0.13 \pm 0.015 \). Agreement of experiment with the sum rule was much better.

**Appendix 2. Penguin diagram**

In straight away perturbation theory to order \( g_{SU2}^3 g_{\text{color}}^2 \) one finds [4.6, A2.1]

\[
H_w^{\text{eff}}(\text{Penguin}) = 2 \sqrt{2} G_F \sin \theta_c \cos \theta_c \sum_q \left( \bar{q} \gamma^\mu \frac{X^a}{2} \gamma^\nu + \bar{q} \gamma^\nu \frac{X^a}{2} \gamma^\mu \cdot q \right)
\]

(A2.1)

\[
c_1 = -\left( \frac{\alpha_s}{\pi} \right)^{\frac{3}{2}} \log(m_c^2/\mu^2) \approx -(0.1) \log(m_c/\mu) \alpha_s.
\]

The scale \( \mu \) is a hadronic scale which overtakes the mass of the up quark if the latter is nearly zero. Thus \( \log(m_c/\mu) = O(1) \).

This estimate of \( c_1 \) thus yields

\[
|c_1| \approx (0.1) \alpha_s.
\]

(A2.2)

If furthermore \( \alpha_s \) is to be related to the region where perturbation theory is valid we have

\[
\alpha_s \ll \frac{1}{3}
\]

\[
|c_1| \approx [0.03 \div 0.04]
\]

(A2.3)

to be compared with the value \( |c_1| = 0.28 \) (see eq. (4.22)). The expression for \( c_1 \) in eq. (A2.1) corresponds to an average (over euclidean momenta) of the following term

\[
-c_1 = \left( \frac{\alpha_s}{\pi} \right)^{\frac{3}{2}} \log(m_c^2/\mu^2)
\]

\[
\left[ \frac{1}{\pi} \int_0^1 dx x(1-x) \log \frac{m_c^2 + x(1-x) q_{E_2}^2}{m_c^2 + x(1-x) q_{E_2}^2} \right]
\]

(A2.4)

\[
q_{E_2}^2 = C \mu^2
\]

for an appropriate numerical constant \( C \).
The further rescaling properties of the operators \( P_1, P_2 \) in eq. (4.22) essentially allow \( \alpha_s^{\text{eff}} \) in eq. (A2.2) to be of order 1:

\[
|c_1| = (0.1)\alpha_s^{\text{eff}} = (0.1) \quad \text{within a factor of 2 say.} \tag{A2.5}
\]

The rescaling of operators \( P_1, P_2 \) in eq. (4.22) is extensively discussed in refs. [4.6, A2.1, A2.2]. The following remarks are in order: If all quark mass effects are neglected \( P_1, P_2 \) only mix upon rescaling among themselves and we obtain

\[
-H_{\text{eff}}(\text{Penguin}) = \left( \alpha_s(m_w^2)/\pi \right)^{\frac{1}{6}} \log(m_c^2/\mu^2) 2\sqrt{2}C_F \sin \theta_c \cos \theta_c \times \left\{ \left[ r^{h_1} + \frac{1}{6}r^{h_2} \right] P_1 + \frac{4}{27}(r^{h_1} - r^{h_2}) P_2 \right\}
\]

\[
h_1 = \frac{8}{b}, \quad h_2 = -\frac{2}{b}
\]

\[
r = 1 + b(\alpha_s/4\pi(\mu_0^2)) \log(m_w^2/\mu_0^2) = 2.4
\]

\[
\mu_0 = 3 \text{ GeV}, \quad m_w = 78 \text{ GeV}, \quad \alpha_s(\mu_0) = \frac{4}{3}.
\]

\( \mu_0 \) in eq. (A2.6) does not have to coincide with \( \mu \).

For the above values of \( \alpha_s(\mu_0), \mu_0, m_w \) we find [A2.2, 3]

\[
-H_{\text{eff}}(\text{Penguin}) = 2\sqrt{2}G_F \sin \theta_c \cos \theta_c \frac{\alpha_s(m_w^2)}{\pi} \frac{1}{6} \log \left( \frac{m_c^2}{\mu^2} \right) (2.2P_1 + 0.2P_2);
\]

\[
\alpha_s(m_w^2) = \alpha_s(\mu_0)/r \approx 0.14.
\]

For \( \log(m_c/\mu) = 2 \) we then obtain

\[
-H_{\text{eff}}(\text{Penguin}) = 2\sqrt{2}G_F \sin \theta_c \cos \theta_c (c_1 P_1 + c_2 P_2)
\]

\[
-c_1 = 0.07 \quad -c_2 = 5.9 \times 10^{-3}
\]

\[
-c_1 = 0.034 \quad -c_2 = 3.8 \times 10^{-3} \quad \text{(in ref. [A2.3]).}
\]

For a detailed and controversial discussion of the inclusion of mass effects in the spirit of the heavy quark mass expansion we refer to refs. [A2.2, 3].

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