VECTORLIKE WEAK CURRENTS AND NEW ELEMENTARY FERMIONS

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Received 15 September 1975

Recent experimental discoveries encourage speculation that there may be more than four “flavors” of tricolored quarks and/or new flavors of leptons. A vectorlike gauge theory of the weak, electromagnetic, and associated neutral current interactions is then an attractive possibility. We present as an illustrative example a minimal theory based on six quark flavors and six lepton flavors and on the group SU$_2$ × U$_1$ (presumably a subgroup of a larger gauge group).

In this note we suggest that the weak, electromagnetic, and neutral current interactions are described by a “vectorlike” theory involving the spontaneously broken gauge group SU$_2^{\text{weak}}$ × U$_1$, which was used in a different way by Salam and Ward [1] and Weinberg [2]. The strong interactions are described by an unbroken gauge theory based on tricolored quarks and color octet vector gluons, with color supposed to be entirely confined, so that quarks are fractionally charged $\pm \frac{1}{3}$. With such an approach it is natural to conjecture that there is a larger gauge group $G = SU_3^{\text{color}} × SU_2^{\text{weak}} × U_1$ that does not contain a U$_1$ factor and that unifies all the interactions [4, 5] except gravitation. We suppose that the overall gauge theory based on $G$ is vectorlike as well. If all elementary fermions and antifermions occur together in an irreducible representation of $G$, then there are difficulties with baryon number conservation. We shall suppose here that there is an irreducible representation for elementary fermions alone, in which case we have the condition $\Sigma Q = 0$, where $Q$ is the electric charge in units of the proton charge.

It has been remarked that a satisfactory unified theory can only be constructed if one introduces new “flavors” of quark besides charm and new lepton flavors as well [5, 6]. Recent evidence [7] on the ratio $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ at energies of many GeV suggests that speculation about such new flavors may not be idle. If there are at least six flavors of tricolored quarks, then a vectorlike theory is not in contradiction with experimental facts, and it is an attractive way of accommodating in a unified theory the purely vectorial strong and electromagnetic interactions.

In a vectorlike theory, which has no divergences arising from anomalies [8], it is assumed that in the absence of masses for the elementary fermions (leptons and quarks), each fermion can be described by a Dirac spinor (left- and right-handed, with particle and antiparticle distinct), the gauge currents are vectorial, and parity is conserved. When the fermion mass matrix is turned on, it has scalar and pseudoscalar pieces and parity is thus violated. The unitary transformation that diagonalizes the fermion mass matrix and eliminates its pseudoscalar part introduces apparent axial vector currents. (This is in the absence of $CP$ violation; if the small observed violation of $CP$ is to be attributed to the mass matrix, then that matrix still contains, after diagonalization, a small pseudoscalar part proportional to the scalar part.)

In this note we restrict ourselves to the case in which all the elementary fermions are doublets under $SU_2^{\text{weak}}$ and have charges $+\frac{2}{3}$ and $-\frac{1}{3}$ for quarks and 0 and $-1$ for leptons. The eigenstates of $SU_2^{\text{weak}}$ are, in general mixtures of the eigenstates of the fermion mass matrix, and the corresponding transformation gives rise not only to axial vector currents but also to the Cabibbo angle and any other weak interaction angles. The assumption that all elementary fermions are doublets implies that the neutral current interaction in $SU_2^{\text{weak}} × U_1$ is not only vectorlike but purely vectorial (with a technical exception for the neutrinos that becomes obvious below). To preserve $\Sigma Q = 0$, the number of lepton and quark flavors must be equal. We have the

* Work supported in part by the Energy Research and Development Administration under contract AT(11-1)-68.

† A good name for this theory is quantum chromodynamics (QCD). See ref. [3].

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relation \( Q = \frac{1}{2}(A - n^e) \), baryon number minus lepton number.

If baryon and lepton number were both conserved, along with electric charge, then in the usual theory of broken gauge symmetry we would be unable to give a mass to either of the neutral gauge bosons in SU\(^2\)\(^{\text{weak}} \times U_1\). However, there is a natural term in the fermion mass matrix that violates lepton number \( n^e \) and fermion number \( 3A + n^e \) but preserves baryon number \( A \). It can occur only in the mass terms for certain neutral leptons are thus Majorana spinors, and their masses are, in general, arbitrary.

We consider here the most economical theory of the type we have discussed that is not in direct disagreement with observation, namely, one with six flavors of quark and six flavors of lepton. The SU\(^2\)\(^{\text{weak}} \) doublets are as follows, ignoring angles that must be relatively small:

Quarks:
\[
\begin{pmatrix}
  u & t & c \\
  d' & b' & s'
\end{pmatrix}_L \quad \begin{pmatrix}
  t & u & c \\
  d'' & b'' & s''
\end{pmatrix}_R
\]

Leptons:
\[
\begin{pmatrix}
  \nu_e & \nu_M \\
  e^- & M^-
\end{pmatrix}_L \quad \begin{pmatrix}
  \mu^+ & \mu^- \\
  M^+ & M^-
\end{pmatrix}_R
\]

Here, \( u, d, \) and \( s \) are the usual "up", "down", and "strange" quarks, and \( c \) is the usual hypothetical charged quark, while \( t \) and \( b \) are new flavors ("top" and "bottom"), with charges \( +2/3 \) and \( -1/3 \) respectively, as suggested, for example, by Harari [9]. The left-handed quarks are rotated by the angle \( \theta \) into \( d' = d \cos \theta + s \sin \theta \) and \( s' = s \cos \theta - d \sin \theta \). There can be no significant mixing with \( b \) because of measured universality. Among the right-handed quarks, experimental facts eliminate any appreciable coupling of \( u \) to either of the familiar flavors \( d \) or \( s \), and consequently its partner must be nearly pure \( b \). The linear combinations \( s'' = s \cos \theta'' - d \sin \theta'' \) and \( d'' = d \cos \theta'' + s \sin \theta'' \) are, for the moment, arbitrary, but we argue that probably \( s'' \approx s \) and \( d'' \approx d \) for two reasons. One is discussed below in connection with the \( |\Delta I| = 1/2 \) nonleptonic rule and the other is as follows: Charm was originally introduced in order to obviate the \( |\Delta S| = 1 \) neutral current and also to reduce the \( K_1^0 - K_2^0 \) mass difference to its observed value with an effective charm quark mass \( m_c \) in excess of 1 GeV. But the right-handed currents, contribute to the mass difference, estimated in the usual way by using the vacuum as the only intermediate state, a contribution [11] of order \( \sin^2(\theta - \theta'') \)

The new charged lepton \( M^- \) should be produced in pairs in \( e^+ e^- \) collisions; experiments performed so far do not require its mass to be greatly in excess of 1 GeV.

We assume that the neutral lepton \( N_M \) is a Dirac particle, with a mass term that conserves lepton number.\(^{+3}\) Violation of lepton number is assumed to occur only through the mass term of the Majorana spinor \( N_\mu \), composed of the right-handed muon neutrino and its antiparticle. The remaining two component spinors \( (\nu_e)_L, (\nu_e)_R \) and \( (\nu_\mu)_L \) are assumed to be massless in the absence of interactions.\(^{+4}\)

The new charged lepton \( M^- \) decays weakly by emitting \( (\nu_e)_R \) plus hadrons or a lepton pair. It can also decay by emitting \( N_M \), provided \( m_M > m_{NM} \). The neutral lepton \( N_M \) decays into an electron plus hadrons or a lepton pair. If \( m_M < m_{NM} \), it can decay by emitting \( M^- \). The neutral Majorana lepton \( N_\mu \) decays into muons of either sign, accompanied by hadrons or

\[^{+2}\] See for example, ref. [10]. There is also a term proportional to \( \sin^2(\theta + \theta'') \) that does not contribute in the intermediate vacuum approximation but may be significant anyway.

\[^{+3}\] If \( N_M \) had a lepton number violating mass term, then this term would induce neutrinoless double \( \beta \)-decay at an unacceptable rate (see ref. [12]), unless \( M_{NM} > M_X \). This would violate the spirit of our scheme, which is to consider a closed subsystem of elementary particles with boson masses \( \lesssim M_X \) and fermion masses much smaller.

\[^{+4}\] In the present state of symmetry-breaking theory, we do not understand how, in the absence of cumbersome machinery, to calculate finite mass corrections arising from weak and electromagnetic interactions. Nevertheless, we can attempt to estimate such effects by cutting off logarithmic infinities at a mass \( \Lambda \) of order \( M_X \). Let us do so, for example, in case of the muon neutrino. The off-diagonal element of the mass matrix between \( \nu_\mu \) and \( N_\mu \) is of order \( \alpha^{-1} m_\mu \ln(\Lambda^2 m_X^2) \) and the muon neutrino acquires a mass (besides the intrinsic fourth order term) of order \( \alpha^{-2} m_{NM} \left[ \alpha^{-1} \ln(\Lambda^2 m_X^2) \right]^2 \), which might be around 20 eV for \( m_{NM} \sim 2 \) GeV. The electron neutrino acquires a tiny mass \( \lesssim 1 \) eV through mixing with \( N_M \). These masses are far below experimental limits, but there is supposed to be a cosmological bound (see ref. [13]) for the masses of stable, or nearly stable, neutrinos, which reduces in our case to \( 4 m_{\nu_e} + 2 m_{\nu_\mu} \leq 33 \) eV, not necessarily in disagreement with our crude estimate.
a lepton pair.

The photon \( A_\mu \) and the neutral intermediate boson \( Z_\mu \) are, of course, supposed to be linear combinations of the gauge boson \( X_{3\mu} \) coupled to \( I_{3\text{weak}} \) and the gauge boson \( Y_\mu \) coupled to \( Y_{\text{weak}} \):

\[
X_{3\mu} = Z_\mu \cos \theta_V + A_\mu \sin \theta_V ,
Y_\mu = A_\mu \cos \theta_V - Z_\mu \sin \theta_V .
\] (1)

With a coupling of the form

\[
g g'_{\text{weak}} X_{3\mu} + (\sqrt{3}/2) g'_{\text{weak}} Y_\mu ,
\]

we have \( \tan \theta_V = \sqrt{3} g' / g, \quad e = g \sin \theta_V \), and \( 4 \sqrt{2} G = g^2 m_X^2 = e^2 m_X^2 \sin^2 \theta_V \). A simple overall symmetry group \( G \) would lead to the condition \( g' = g \) or \( \theta_V = 60^\circ \), and we may speculate that the actual value of \( \theta_V \) after renormalization, might lie in the vicinity of \( 60^\circ \). If the charged boson \( X^\pm \) were to acquire its mass only from the lepton-number-violating vacuum asymmetries that supply the \( Z^0 \) mass, then it would have the relation \( 2 m_X^2 = m_Z^2 \cos^2 \theta_V \); but in general there are many other vacuum asymmetries that must contribute to \( m_X^2 \). We therefore define \( \rho = 2 m_X^2 m_Z^2 \cos^{-2} \theta_V \) and expect that \( \rho > 1 \).

Let us now summarize detailed predictions of the theory.

Since the neutral current is vectorial, it causes no parity violation in atomic, molecular, or nuclear physics. It leads to \( \bar{\nu}_e \) and \( \nu_e \) differential cross-sections that are equal and are proportional to \( 1 + (1 - y)^2 \). The neutral current cross-section for \( \nu_e \) or \( \bar{\nu}_e \) on \( e^- \) is

\[
G^2 \rho^2 (3\pi)^{-1} (s - m_e^2) \sin^2 \theta_V - 1/2)^2 .
\]

For matter containing equal numbers of up and down quarks, if we ignore the contributions of quark pairs and the effects of the Cabibbo angle \( \theta \), we can express the total neutral current cross-section in units of the charged-current cross-section for neutrinos without the production of new quark flavors. This ratio \( R_\nu \) equals \( \rho^2 [1/6 - (1/3)\sin^2 \theta_V + (5/27)\sin^4 \theta_V] \). For elastic and quasi-elastic processes initiated by \( \nu \) and \( \bar{\nu} \), it is important that the neutral current contains an \( I = 0 \) part, additive for nucleons, and an \( I = 1 \) part that should produce \( \Delta \) resonances copiously.

The charged current inclusive \( \nu \) and \( \bar{\nu} \) cross-sections are expected to change above the production thresholds for hadrons containing new quark flavors. Charmed hadrons can be produced by \( \nu \) to order \( \sin^2 \theta \) or \( \sin^2 \theta'' \) for all values of the Bjorken parameter \( \xi = -q^2 / 2p \cdot q \), or they can be produced without the \( \sin^2 \theta \) or \( \sin^2 \theta'' \) factor from strange quarks in the target nuclei, but such events should be restricted to small values of \( \xi \), mostly less than 0.1. Hadrons containing \( t \) quarks can be produced by \( \nu \) in full strength and without restriction to small \( \xi \) values, once the relevant threshold factor approaches unity; the \( y \) distribution should approach \( (1 - y)^2 \), ignoring quark pairs in the target. Likewise, \( \bar{\nu} \) bombardment can lead to full strength production of hadrons containing \( b \) quarks, with a flat distribution in \( y \), as the corresponding threshold factor approaches unity. High above all thresholds, the \( \nu \) and \( \bar{\nu} \) cross-sections become equal, and proportional to \( 1 + (1 - y)^2 \).

Baryons or mesons that differ from conventional ones in having a \( u \) or \( d \) quark replaced by a \( t \) quark can decay nonleptonically or with the emission of a positive lepton pair, mainly into hadron states with \( S = 0 \). If there is a \( b \) quark instead of the \( t \), then the situation is similar except that leptonic decay involves a negative lepton pair. If we are dealing with a \( c \) quark, the lepton pair is again positive, and leptonic and non-leptonic decays should be primarily into hadrons with total \( S = -1 \). For antiquarks, all the signs are, of course, reversed.

In the reaction \( \nu_\mu + N \rightarrow \mu^- + \text{hadrons} \), there seems to be some slight evidence for the production of hadrons containing \( t \) quarks, decaying with the emission of \( \mu^+ + \nu_\mu \); there appear to be di-muon events [14], with general values of \( \xi \), and with no sign of a \( \sin^2 \theta \) factor in the cross-section.

In \( e^+ e^- \)-annihilation, the ratio \( R \) should approach five above the thresholds for hadron states containing pairs of \( c, t, \) and \( b \) quarks. If we include the pair production of the charged lepton \( M^\pm \), which should decay mostly into hadrons plus \( \nu_\mu \) or \( \nu_e \), then \( R \) should eventually reach six for six flavors of quark and lepton. (In general, for equal numbers of conventional quark and lepton doublets, \( R + 2 \rightarrow 4 n_f / 3 \), where \( n_f \) is the number of flavors.)

The \( \psi, \psi' \), and related resonances are interpreted as bound quark pairs, linear combinations of \( tt, bb \), and \( cc \). What linear combinations are involved depends on the mass matrix of the new quarks. Further clarification of the experimental situation is required in order to establish how many and which of the new quark flavors are relevant for the \( \psi \)-phenomenology.

The new right-handed weak currents, in conjunction with the old left-handed weak currents, give a new kind
of contribution to the \(|\Delta I| = 1/2\) nonleptonic weak amplitudes for hadron decay. Recently it was proposed [15] that a right-handed \(\bar{c}d\) current without a \(\sin \theta\) factor be used to explain the bulk of the \(|\Delta I| = 1/2\) amplitude and its predominance over \(|\Delta I| = 3/2\). In the theory described here, that would correspond to taking \(\theta'' \approx 90^\circ\). The resulting \("(\bar{c}d)_R(\bar{s}c)_L\" interaction cannot, however, play a major role in the \(|\Delta I| = 1/2\) nonleptonic amplitudes because, as Golowich and Holstein have shown [16], the successful comparison of \(K \to 2\pi\) and \(K \to 3\pi\) amplitudes using PCAC and current algebra requires the \(d\) quark in the interaction to have the same helicity in the \(|\Delta I| = 1/2\) and \(|\Delta I| = 3/2\) amplitudes. Also, a fairly successful approach to baryon decays based on PCAC and current algebra for s-wave pion emission and on pole dominance for p-wave pion emission leads to the result that the helicity of the \(d\) quark in both cases is primarily \(L\).

In the theory we have discussed, with \(\theta''\) small, there are \(|\Delta I| = 1/2\) terms written schematically as 
\[ (\bar{c}d)_L \sin \theta (\bar{s}c)_L \cos \theta, \quad (\bar{c}d)_L \sin \theta (\bar{s}c)_R \cos \theta'', \]
\[ (\bar{c}d)_R \sin \theta'' (\bar{s}c)_L \cos \theta, \quad (\bar{c}d)_R \sin \theta'' (\bar{s}c)_R \cos \theta'', \]
none of them full strength and all further inhibited by their involving a \(c\bar{c}\) pair. We shall mention certain properties of the term \((\bar{c}d)_L \sin \theta (\bar{s}c)_R \cos \theta''\) that might have the effect of compensating these inhibitions and allowing the term to make a large contribution to the \(|\Delta I| = 1/2\) nonleptonic amplitudes. If that happens, then the argument of Golowich and Holstein requires that \(\theta''\) be not only much less than \(90^\circ\) but much smaller than \(\theta\), perhaps zero, in order to suppress the corresponding \((\bar{c}d)_R \sin \theta'' (\bar{s}c)_L \cos \theta\) term.

The term \((\bar{c}d)_L \sin \theta (\bar{s}c)_R \cos \theta''\) is enhanced, compared to the \(|\Delta I| = 3/2\) part of the usual term 
\[ (\bar{u}d)_L \cos \theta (\bar{s}u)_L \sin \theta, \]
by a factor \(F\) coming from strong interaction effects in quantum chromodynamics [17] that can be estimated\(^{+5}\). For \(m_X \approx 50\) GeV, the estimate of \(F\) ranges from around 2 to around 5, not an enormous enhancement.

Another effect that might help to overcome the smallness of the term \((\bar{c}d)_L \sin \theta (\bar{s}c)_R \cos \theta''\) in nonleptonic \(K\) decays is the fact that it is not suppressed by \(SU_3\) forbiddenness, as is the conventional term \([18]\)
\[ (\bar{u}d)_L \cos \theta (\bar{s}u)_L \sin \theta.\]

In baryon decays, the ordinary term 
\[ (\bar{u}d)_L \cos \theta (\bar{s}u)_L \sin \theta \]
gives only a \(|\Delta I| = 1/2\) contribution when quark pairs in the baryon are ignored, as pointed out by Pati and Woo [19]. This effect may give a considerable edge to the \(|\Delta I| = 1/2\) piece of the baryon decay amplitude, even in the absence of the new terms.

All told, the new type of term may make a substantial contribution to the \(|\Delta I| = 1/2\) amplitude in various nonleptonic decays, especially if \(\mu\) is large\(^{+5,6}\), but it is probably not dominant.

The minimal vectorlike theory we have discussed here has a number of attractive features and a variety of predictions that are not in disagreement with experiment, including a possible explanation of the observed di-muon events in high energy \(p_\mu\)–nucleus reactions\(^{+6}\).

For all we know, it could be the correct theory of the weak, electromagnetic, and neutral current interactions, ignoring others, presumably mediated by superheavy bosons. However, we consider it only as an interesting example. Many other vectorlike schemes are possible, in particular ones containing fermions that are \(SU_2\) singlets, or ones in which the group \(SU_2\)\(^{weak}\) \(\times U_1\) is en-

\(^{+5}\) Let \(\kappa_M\) be the strong interaction coupling constant renormalized at mass \(M\); let \(b = 11 - 2/3n_f\), which is 7 in the theory we are discussing. Then for sufficiently large \(M\) (already in the asymptotic region, \(\kappa_M \leq 1\)) the mass \(M\exp (-2n_b^{-1}\kappa_M^{-1})\) is renormalization-group invariant; let us call it \(\mu\). We suppose that \(M_0\) (say 2 GeV) is "sufficiently large" and also such that the \(|\Delta I| = 1/2\) and \(|\Delta I| = 3/2\) four-quark operators have comparable matrix elements. The logarithmic dimensions of the four-fermion operators are as follows:

- \(|\Delta I| = 1/2\), LL and RR current-current operators: \(d_{3/2} = 4/b\), LR and RL current-current operators: \(d_{3/2} = 8/b\); \(|\Delta I| = 3/2\), LL and RR current-current operators: \(d_{3/2} = -2/b\). The enhancement factor for a given momentum transfer \(p > M_0\) is roughly \([\ln(p^2/M_0^2) - 1/2]^{1/4}/b\) for the \((\bar{c}d)_L(\bar{s}c)_R\) term and roughly \([\ln(p^2/M_0^2) - 1/2]^{1/4}/b\) for the usual \(|\Delta I| = 3/2\) term. Integrating, together with the \(X^2\) propagator, over the range \(M_0 \to \infty\), we obtain a crude estimate of \(F\) as \(3 - 1 \times 3 - 1\) \((1 + r^{5/7} + r^{10/7})\), where \(r \equiv (\ln M_0^2/\mu^2) \ln(M_0^2/\mu^2)^{-1}\), \(M_X \approx 50\) GeV, the estimate of \(F\) ranges from around 2 to around 5.

Similar calculations have been done by the authors of ref. [11]. It should also be noted that they have treated briefly the quark portion of our scheme as case C in ref. [11]. See also an earlier version of this article written by two of us [20], and similar work for the quarks alone in ref. [21].

\(^{+6}\) The authors of ref. [15] were the first to give an explanation of the observed dimuon events as being due to the production of a new flavor (which they interpreted as charm) off the right-handed \(d\) quark in deep inelastic neutrino-nucleon scattering. The \((\bar{c}d)_R\) currents was first proposed by Mohapatra [22].
larged. Such theories would contain more fermions than our minimal example, and it remains to be seen if they are needed.

We would like to thank B. Barish, S. Frautschi, M. Perl, and F. Sciulli for interesting discussions. Two of us would like to acknowledge the hospitality of the Aspen Center for Physics and the great value of conversations with our colleagues there, including W.A. Bardeen, R.M. Barnett, M.A.B. Bég, T.P. Cheng, E. Golowich, A. Halprin, B.R. Holstein, R. Kingsley, P. M"{u}Namee, S.P. Rosen, K. Sebastian, F. Wilczek, A. Zee, and others too numerous to mention.

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