PION–NUCLEUS SCATTERING AND SYSTEMATICS OF THE DELTA–NUCLEUS POTENTIAL

Roger A. FREEDMAN, Gerald A. MILLER and Ernest M. HENLEY

Institute for Nuclear Theory and Department of Physics, University of Washington, Seattle, WA 98195, USA

Received 16 December 1980

A new calculational technique for the isobar-hole model is used to study pion elastic scattering from $^{16}\text{O}$, $^{28}\text{Si}$, and $^{40}\text{Ca}$. The central value of the $\Delta$–nucleus interaction potential is found to be independent of nuclear mass number.

A proper description of pion–nucleus scattering at energies near the (3,3) resonance requires that account be taken of the dynamics of the resonant pion–nucleon system. In particular the motion of this system through the nucleus and its strong interaction with the nuclear medium cannot be neglected [1,2]. Recently several groups [2–6] have incorporated these effects into calculations of pion scattering from light nuclei based on the isobar–hole model. Published calculations of this kind which deal with the fully quantum mechanical problem of $\pi$ propagation in the presence of the $\Delta$–nucleus interaction [3–6] are unfortunately quite complicated. Consequently no such calculations have been performed for nuclei of mass number $A > 16$. We present here a new technique of calculation within the isobar–hole model which we have used to consider pion elastic scattering from closed-shell nuclei up to $A = 40$. An important result of these calculations is that the $\Delta$–nucleus interaction is well described by a local potential whose central value is independent of $A$. This potential is analogous to the nuclear shell model potential for nucleons [7].

Our technique of calculation differs from that of previous workers in that we do not calculate the pion–nucleus $T$-matrix directly. Instead, an approach based on the pion–nucleon optical potential is used. This approach gives a significant reduction in computational labor and interfaces well with more conventional approaches to pion–nucleus scattering. The resonant part of the optical potential may be written as

$$U(k_i, k_f, \omega) \propto \sum_N \int d^3r \int d^3r' \exp(-i k_i \cdot r) \times \phi_N^*(r) G_\Delta(r, r', \omega) \phi_N(r') \exp(i k_f \cdot r').$$

(1)

Here $\omega$ is the pion energy in the pion–nucleus center of mass (ACM) and $k_i$ and $k_f$ the initial and final pion momenta in the ACM. The quantity $G_\Delta$ is the non-local propagator function for the $\Delta$, and $\phi_N$ is the wave function of the struck nucleon. The sum over all nucleon states $N$ in the target is displayed explicitly in eq. (1), while factors involving spin, isospin, and gradient operators, the $\pi N\Delta$ coupling constant and form factor, and the “angle transformation” from the pion–nucleon center of mass (2CM) to the ACM [8] are suppressed. The effects of these factors are, however, included in the actual calculation. The nucleon wave functions in eq. (1) are assumed to be known, so that the quantity to be determined is the $\Delta$ propagator function $G_\Delta$. This function embodies the dynamics of the $\Delta$ isobar.

In a fixed-scatterer calculation, $G_\Delta$ would satisfy the following Schrödinger equation:

$$[E - M_\Delta + \frac{i}{2} \Gamma_\Delta(E)] G_\Delta(r, r', \omega) = \delta^3(r - r'),$$

(2)

in which $E$ is the energy of the $\Delta$ in the 2CM and $M_\Delta$ and $\Gamma_\Delta(E)$ its mass and energy-dependent width. In order to incorporate the dynamics of the $\Delta$ into eq. (2), we must account for both its quantum mechanical motion and for its interaction with the nucleus. To provide for the first of these we replace the c-number...
$E$ in eq. (2) as follows:

$$E \rightarrow M + \omega - \tilde{T}_\Delta,$$

(3)

where $\tilde{T}_\Delta$ is the $\Delta$ kinetic-energy operator and $M$ is the mass of the nucleon (including the nucleon binding energy). Since $M + \omega$ is the energy of the pion–nucleon system in the ACM (in which the nucleon is assumed at rest), $\tilde{T}_\Delta$ describes the motion of the $\Delta$ through the nucleus. The $\Delta$–nucleus interaction is included in eq. (2) by adding to the quantity in square brackets the operator $-\Sigma_\Delta$, where $\Sigma_\Delta$ represents the $\Delta$ self-energy. Upon implementing these changes we obtain

$$[\tilde{T}_\Delta (1 + \frac{1}{2} i \Gamma'_\Delta (M + \omega)) + M + \omega - M_\Delta$$

$$+ \frac{1}{2} i \Gamma'_\Delta (M + \omega) - \tilde{\Sigma}_\Delta] G_\Delta (r, r', \omega) = \delta^3 (r - r').$$

(4)

In eq. (4) we have followed the procedure of ref. [3] and expanded $\Gamma_\Delta$ to first order in $\tilde{T}_\Delta$. Rather than solving eq. (4) directly for the non-local function $G_\Delta$, we define a new function of one spatial variable by

$$\chi_{\Delta N} (r, k_\parallel, \omega) = \int d^3 r' G_\Delta (r, r', \omega) \phi_\Delta (r') \exp (i k_\parallel \cdot r') .$$

(5)

The function $\chi_{\Delta N}$ has the interpretation of an effective wave function for the $\Delta$ formed from a nucleon in state $N$ and a pion of momentum $k_\parallel$. In terms of $\chi_{\Delta N}$ the optical potential (1) becomes

$$U(k_\parallel, k_\parallel, \omega) \propto \sum_N \int d^3 r \exp (-i k_\parallel \cdot r)$$

$$\times \phi_\Delta^* (r) \chi_{\Delta N} (r, k_\parallel, \omega) ,$$

so that a knowledge of the $\chi_{\Delta N}$ allows calculation of the optical potential.

From eqs. (4) and (5) $\chi_{\Delta N}$ obeys

$$[-\tilde{T}_\Delta (1 + \frac{1}{2} i \Gamma'_\Delta (M + \omega)) + M + \omega - M_\Delta$$

$$+ \frac{1}{2} i \Gamma'_\Delta (M + \omega) - \tilde{\Sigma}_\Delta] \chi_{\Delta N} (r, k_\parallel, \omega)$$

$$= \phi_\Delta (r) \exp (i k_\parallel \cdot r).$$

(7)

Eq. (7) is a local equation only if $\tilde{\Sigma}_\Delta$ represents a local interaction. The majority of effects contributing to $\tilde{\Sigma}_\Delta$ are non-local in character (Pauli blocking, pion true absorption, etc.) [3–6] so that it may appear that the non-locality of $\tilde{\Sigma}_\Delta$ is essential. The same situation prevails for the nucleon–nucleus interaction, however, and yet this interaction is well described by a local potential. It may therefore be an equally good approximation to treat $\tilde{\Sigma}_\Delta$ as a local potential. We take this potential to be proportional to the nuclear density, with central value $V + i W$. In our calculations $V$ and $W$ are treated as free parameters whose values are adjusted to fit the angular distribution in pion elastic scattering and the pion total cross section. (Inclusion of a $\Delta$–nucleus spin–orbit potential [6] is important in maintaining the energy independence of $V + i W$, but is neglected in the present calculations which consider only a single pion energy.)

For a given nucleon orbital, $\chi_{\Delta N}$ is found by solving the (local) Schrödinger equation (7). It is then integrated in eq. (6) to yield the resonant pion optical potential. We use this potential in the momentum-space code PIPIT [9] in combination with the Coulomb potential as well as with the PIPIT fixed-scatterer optical potential for the non-resonant pion–nucleon partial waves and for the non-resonant part of the $(3,3)$ channel which is small but nonetheless necessary to describe the pN data.

In fig. 1 we display our results for $\pi^+$ elastic scattering from $^{16}$O at a pion laboratory kinetic energy $T_\pi = 163$ MeV. For this calculation we used the density-dependent Hartree–Fock (DDHF) nucleon wave func-
Table 1
Comparison of theoretical integrated cross sections with experiment. The value of $\sigma_{\text{tot}}$ for $^{16}\text{O}$ is taken from Clough et al. [12]; all other experimental values are taken from Ashery et al. [13].

<table>
<thead>
<tr>
<th>Scattering process</th>
<th>$T_\pi$ (MeV)</th>
<th>$\sigma_{\text{tot}}$ (mb)</th>
<th>$\sigma_{\text{eq}}$ (mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experiment</td>
<td>This work</td>
<td>PIPIT</td>
</tr>
<tr>
<td>$\pi^+ - ^{16}\text{O}$</td>
<td>163</td>
<td>808.6</td>
<td>856.5</td>
</tr>
<tr>
<td>$\pi^+ - ^{28}\text{Si}$</td>
<td>162</td>
<td>1142.4</td>
<td>1193.0</td>
</tr>
<tr>
<td>$\pi^+ - ^{40}\text{Ca}$</td>
<td>163</td>
<td>1418.1</td>
<td>1476.6</td>
</tr>
</tbody>
</table>

The solid curve is our best fit to the data using a value of $V + iW$ of $(-55 \pm 5) + i(-5 \pm 5)$ MeV. We obtain a good fit to the data at all angles and in particular to the angular positions of the minima. As is shown in table 1, the calculated values of the integrated cross sections are also consistent with experiment. We note that the calculated angular distribution and integrated cross sections are rather more sensitive to changes in $W$ than to changes in $V$, although the positions of the minima are insensitive to either parameter. Also shown in fig. 1 is a conventional PIPIT [9] calculation in which the dynamics of the $\Delta$ are ignored. The shift of the minima in the present calculation relative to those of PIPIT arises from the motion of the $\Delta$ through the nucleus before reemission of the pion. Because the $\Delta$ moves a finite distance, the pion sees the nucleus as a slightly smaller target than it would if the production and decay of the $\Delta$ occurred at the same point.

We also display in fig. 1 the results of the isobar--hole calculation of Hirata et al. [3]. In that work the nonlocality of the Pauli blocking contribution to the $\Delta$--nucleus interaction is retained, while all other contributions are described by a local complex potential. If this Pauli term is also treated as a local potential and added to the remaining complex potential (shell model potential plus spreading potential), the numerical results for the central value of the total potential inferred from ref. [3] is completely consistent with our best-fit value of $V + iW$. The similarity of the two calculations can be seen from the corresponding angular distributions in fig. 1, which agree very well with each other for angles less than 90$^\circ$.

We note that if the $\Delta$--nucleus potential were equal to the shell model potential for a nucleon, we would expect $V + iW$ to be approximately $-55$ MeV. From our best-fit value of $(-55 \pm 5) + i(-5 \pm 5)$ MeV we conclude that the effects of Pauli blocking, pion true absorption, and reflection cancel almost entirely. If this value is truly analogous to the central value of the nucleon shell model potential, calculations for other nuclei using the same $V + iW$ should produce equally good fits to the data. The results of such a calculation for $\pi^+$ elastic scattering from $^{40}\text{Ca}$, again at $T_\pi = 163$ MeV, are shown in fig. 2. We again used the DDHF nucleon wave functions of Negele [11]. Our calculation again gives a very good fit to the data at all angles and to the angular positions of the minima, as well as to

![Fig. 2. Differential cross section for $\pi^+$ elastic scattering from $^{40}\text{Ca}$](image-url)
the integrated cross sections (see table 1). Thus the description of the Δ–nucleus interaction by a shell-model-like potential appears quite valid.

We have also performed a calculation of π⁺ elastic scattering from ²⁸Si at \( T_\pi = 162 \text{ MeV} \), again using the values of \( V + iW \) determined from the ¹⁶O calculation. This is shown in fig. 3. No DDHF calculations exist for this nucleus, and so harmonic-oscillator nucleon wave functions were used with an oscillator parameter fit to the root-mean-square charge radius of ²⁸Si [15]. While the calculated integrated cross sections (shown in table 1) are consistent with experiment, the fit to the angular distribution is less impressive than in the cases of ¹⁶O and ⁴⁰Ca. However, apart from a small angular shift, the data are quite well described for angles up to about 90°. We believe that the angular shift is caused by our use of an inadequate nuclear wavefunction to describe the deformed nucleus ²⁸Si. Indeed, the positions of the minima cannot be changed by variations of \( V \) or of \( W \). The data also show a third minimum which does not appear in our calculation.

Nonetheless we have found that our model describes very well the angular regions of the first two minima for all target nuclei studied.

In conclusion, we have seen that the use of a Δ–nucleus potential of central strength \( V + iW = (-55 - i5) \text{ MeV} \) leads to a good description of pion–nucleus elastic scattering over a broad range of angles for the three target nuclei considered. This independence of the mass number of the target strongly reinforces the analogy between the Δ–nucleus interaction and the shell model potential for nucleons, and substantiates the belief that pion–nucleus elastic scattering can provide information about this interaction. Calculations of pion elastic scattering from additional nuclei and at other energies, as well as further details of our calculational technique, will be published elsewhere.

It is a pleasure to thank J.H. Koch, F. Lenz, and E.J. Moniz for useful discussions. This work was supported in part by the US Department of Energy.

References