New, accurate measurements\textsuperscript{1-7} of the magnetic moments of the charged Σ and Ξ hyperons, along with earlier measurements of baryon moments,\textsuperscript{8-11} make possible sensitive tests of quark-model wave functions. In this paper, sum rules\textsuperscript{12-14} and other parametrizations are used to test various quark-model assumptions against the measured baryon magnetic moments. The tests generally fail and the conclusion is that a good understanding of baryon magnetic moments will require a detailed model with a large baryon-dependent nonstatic contribution.

The measured values of the baryon magnetic moments including the \((\Lambda, \Sigma)\) transition moment are listed in Table I. For the \(\Sigma^\pm\) moments, the statistical average of the values in the listed references is used. Of particular note is the new measurement\textsuperscript{5} of \(\Sigma^- = -1.11 \pm 0.03\) (folding the statistical and systematic errors of Ref. 5) obtained from fine-structure splitting in \(\Sigma^-\) exotic nuclei. It is the accuracy of this result that makes the sum rules particularly effective. The new measurement also differs considerably from a slightly earlier measurement\textsuperscript{6} by the more standard beam-polarization-precession technique of \(\Sigma^- = -0.89 \pm 0.14\). The increase in magnitude for \(\Sigma^-\) and the sharp improvement in accuracy of Ref. 5 has a significant effect on the conclusions to be drawn from the sum rules.

The statistical average of the two new published measurements\textsuperscript{4,5} is \(\Sigma^- = -1.10 \pm 0.03\) with a \(\chi^2\) of 2.2, and the statistical deviation for Table I has been increased by a factor \(S = (\chi^2/1)^{1/2} = 1.5\) to account for this large \(\chi^2\). One might be concerned by the existence of two different results by two such different methods, but another newly reported preliminary result\textsuperscript{6} using the beam-polarization-precession method \(\Sigma^- = -1.18 \pm 0.03\) statistical only) tends to confirm the higher magnitude of Ref. 5.

The experimental results are compared in Table I to static-quark-model\textsuperscript{16} predictions. The experimental accuracy for most baryon moments is now good enough that the earlier procedure of fixing the \(u\) and \(d\)-quark moments by the nucleon magnetic moments can be extended to a full \(\chi^2\) fit to all baryon moments. In this fit, the experimental error has been kept to at least 0.05 nuclear magnetons for each moment so as to weight most of them equally. This seems to be a reasonable expectation for the predictability of the static quark model. In the fits, the \(\Lambda^0\) moment is adjusted for isospin-breaking \(\Lambda-\Sigma\) mixing.\textsuperscript{17-19} This mixing was noted a long time ago,\textsuperscript{17} but was not generally included in magnetic-moment considerations because of the large experimental uncertainties at that time. However, with accurate measurements, it is inconsistent not to include the mixing when discussing the interior members of the stable baryon octet. That is, the physical \(\Lambda^0\) particle has a rather large admixture of the neutral \(I = 1\) state, which should be included before making comparison with quark-model predictions. The mixing correction used for Table I is that determined in Ref. 19, where the corrected \(\Lambda^0\) moment is given as

\[
\Lambda^0 = s + (0.022 \pm 0.004)(\Lambda, \Sigma) = s - 0.040 \pm 0.009
\]  

TABLE I. Baryon magnetic moments (in nuclear magnetons).

<table>
<thead>
<tr>
<th>Baryon</th>
<th>Ref.</th>
<th>Experiment</th>
<th>Static quark model ((\chi^2)) ((u = -2d)) ((u,d,s \text{ free}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>8</td>
<td>2.79</td>
<td>2.68 (5)</td>
</tr>
<tr>
<td>n</td>
<td>8</td>
<td>-1.91</td>
<td>-1.79 (6)</td>
</tr>
<tr>
<td>(\Lambda^0)</td>
<td>9</td>
<td>-0.61 ± 0.01</td>
<td>-0.62 (0)</td>
</tr>
<tr>
<td>(\Sigma^+)</td>
<td>1.2</td>
<td>2.38 ± 0.02</td>
<td>2.58 (16)</td>
</tr>
<tr>
<td>(\Sigma^0)</td>
<td></td>
<td>0.83</td>
<td>0.74</td>
</tr>
<tr>
<td>(\Sigma^-)</td>
<td>4.5</td>
<td>-1.10 ± 0.05</td>
<td>-1.00 (4)</td>
</tr>
<tr>
<td>(\Xi^0)</td>
<td>10</td>
<td>-1.25 ± 0.01</td>
<td>-1.38 (6)</td>
</tr>
<tr>
<td>(\Xi^-)</td>
<td>7</td>
<td>-0.69 ± 0.04</td>
<td>-0.48 (17)</td>
</tr>
<tr>
<td>((\Lambda, \Sigma))</td>
<td>11</td>
<td>-1.82 ± 0.13</td>
<td>-1.52 (2)</td>
</tr>
<tr>
<td>u</td>
<td></td>
<td>1.79 ± 0.07</td>
<td>1.73 ± 0.08</td>
</tr>
<tr>
<td>d</td>
<td></td>
<td>-0.89 ± 0.04</td>
<td>-0.99 ± 0.08</td>
</tr>
<tr>
<td>s</td>
<td></td>
<td>-0.59 ± 0.08</td>
<td>-0.61 ± 0.08</td>
</tr>
<tr>
<td>(\chi^2/\text{DF for } \delta \mu \geq 0.05)</td>
<td></td>
<td>57/6</td>
<td>44/5</td>
</tr>
</tbody>
</table>

\(\delta \mu\) is the statistical deviation.
using the measured $(\Lambda, \Sigma)$ transition moment.

The $(\Lambda, \Sigma)$ moment is also affected by mixing, being increased by 0.01 nuclear magnetons, but this is not yet experimentally significant.

Two fits are shown in Table I, one with the constraint $u = -2d$ (as expected for Dirac moments and equal-mass $d$ and $u$ quarks) and the second without this constraint. It can be seen from Table I that relaxing the constraint $u = -2d$ somewhat improves the fit. The quark-model parameters of each fit are shown with errors determined by an increase from $\chi^2_{\text{min}}$ of $\chi^2_{\text{min}}/DF$ (degrees of freedom). If the quark moments are interpreted as Dirac moments with $\mu = q/2m$ then the unconstrained fit to all moments corresponds to $m_u = 362 \pm 17$ and $m_d = 316 \pm 26$ MeV. The nucleon-quark mass difference is quite different than that given from the values $m_u = 338$ MeV and $m_d = 322$ MeV suggested by earlier fits to only the nucleon moments. This means that quark-model magnetic moments no longer suggest the near equality of the two nucleon quark Dirac masses. Alternatively (and more reasonably) equal-mass nucleon quarks would be required to have anomalous magnetic moments.

Although the $\chi^2$ values for each fit are calculated with a somewhat arbitrary lower limit to the error, $\chi^2$ does seem large enough to preclude quantitative success of the static quark model. (The individual contributions to $\chi^2$ are listed in parentheses in Table I to indicate which measurements are most at odds with the static quark model.) The fits of Table I would be reasonable if the error on each baryon moment were increased to $\pm 0.15$ nuclear magnetons, so that this can be taken as the order of magnitude of nonstatic effects.

There are three kinds of nonstatic effects that could affect the baryon magnetic moments: (1) orbital contributions (also called configuration mixing), (2) exchange currents (characterized by either meson exchange or $q \leftrightarrow q'$ pairs), and (3) relativistic effects. Each of these affect magnetic moments to second order so that a 0.15-nuclear-magneton nonstatic contribution (in a typical baryon moment of about 1 nuclear magneton) would require relatively large nonstatic wave-function components of about 15% probability or $(u/c)^2 = 0.15$.

The quark model can be extended to include the possibility of nonstatic effects to the extent that the nonstatic components are the same for each octet baryon. Six of the stable octet baryons are composed of two like and one unlike quark. For these baryons, Fermi statistics (including color, spin, and spatial degrees of freedom) permit a difference in the nonstatic effective quark contributions depending only on whether it is one of the two like or the unlike quark in the baryon. This can be used to generalize the quark-model predictions for baryon moments to

$$p = (4u - d')/3,$$  
$$n = (4d - u')/3,$$  
$$\Sigma^+ = (4u - s')/3,$$  
$$\Sigma^- = (4d - s')/3,$$  
$$\Xi^0 = (4s - u')/3,$$  
$$\Xi^- = (4s - d')/3,$$

where the quark symbols now refer to quark-moment contributions including nonstatic effects, but are normalized to reduce to quark moments in the static limit. The magnetic-moment contribution of the unlike quark in the baryon is primed $(u', d', s')$.

A basic simplifying assumption in Eqs. (2)–(7) is that the nonstatic effects for a given quark-moment contribution are independent of which baryon the quark is in. This assumption can be referred to as “baryon independence.” It is true in some models of relativistic effects and configuration mixing (orbital admixtures). While not generally true for exchange currents for individual baryon moments, the sum rules of this paper form linear combinations of baryon moments that cancel out exchange contributions as well, to the extent that they depend only on relative quark charges. The six equations (2)–(7) involve six quark moments, but are not independent and lead to one sum rule that does not depend on quark moments. This can be written

$$\Sigma^+ - \Sigma^- + \Xi^- - \Xi^0 - p - n = 4.70 (4.04 \pm 0.07) \ ,$$  
(8)

where the hyperon-moment sum is given in parentheses. This sum rule is not satisfied and the conclusion must be that baryon independence is broken. It is particularly significant that Eq. (8), which should be better satisfied than individual baryon-moment predictions, is in far worse agreement than any single moment.

Other sum rules can be derived using SU(3) symmetry to characterize the $\Lambda$ wave function. The most accurate of these can be written

$$3\Lambda + \frac{1}{3}(\Sigma^+ + \Sigma^-) - (\Xi^0 + \Xi^-)$$

$$= p + n = 0.88 (0.84 \pm 0.06) \ .$$  
(9)

Another sum rule involving the transition moment $(\Lambda, \Sigma)$ can be written in three different ways, each leaving out one baryon-moment contribution

$$\Xi^0 - \Xi^- - 2\sqrt{3}(\Lambda, \Sigma) = p - n = 4.70 (5.74 \pm 0.62) \ ,$$  
(10a)

$$\Sigma^+ - \Xi^- - 2\sqrt{3}(\Lambda, \Sigma) = 2(p - n) = 9.40 (9.78 \pm 0.63) \ ,$$  
(10b)

$$\Sigma^+ - \Sigma^- + 2(\Xi^- - \Xi^0) + 2\sqrt{3}(\Lambda, \Sigma) = 0 (-1.70 \pm 0.63) \ .$$  
(10c)

We note (without a good explanation) that all the discrepancy in sum rules (8)–(10) would be removed by an increase in the single experimental difference $\Xi^- - \Xi^0=0.56 \pm 0.04$ by about half a nuclear magneton (keeping the sum $\Xi^- + \Xi^0$ fixed). Such an increase would also reduce $\chi^2$ in Table I by about 20 for each model.

Further sum rules can be obtained by solving Eqs. (2)–(7) for effective quark-magnetic-moment contributions using the constraint $u = -2d$. For the nucleon-quark contributions this gives two sum rules

$$d = \frac{1}{3}(\Sigma^+ - \Sigma^-) = -\frac{1}{3}(2p + n) = -0.92 (-0.87 \pm 0.01) \ .$$  
(11)

and

$$d' = \Xi^0 - \Xi^- = p + 2n = -1.03 (0.56 \pm 0.04) \ .$$  
(12)
Equations (11) and (12) still include symmetric, nonstatic effects except for meson components that could produce anomalous quark moments. The difference \( (\Xi^- - \Xi^0) \) is still seen as the only significant symmetry-breaking combination.

There are no sum rules for the strange quark, but the strange-quark-moment contribution can be isolated (using \( u = -2d \)) and is given by

\[
s = -\frac{1}{4}(\Xi^0 + 2\Xi^-) = -0.66 \pm 0.02
\]

or

\[
\frac{s'}{s} - \frac{\Sigma^+ - 2\Sigma^-}{-0.18 \pm 0.10 . \quad (14)
\]

Another measure of the strange-quark moment can also be given from Eq. (1) to be

\[
s'' = \lambda^0 + 0.04 \pm 0.01 - 0.57 \pm 0.01 . \quad (15)
\]

The quark-moment contributions of Eqs. (11)–(15) can be used to test several proposals for the dominant nonstatic effects. A suggestion based on earlier data that relativistic effects were dominant \(^{13,14}\) cannot explain the anomalously small value of \( s' \) in Eq. (14). Another suggestion that nucleon-quark “quenching” in the hyperons could account for baryon magnetic moments \(^{23}\) is refuted by Eq. (11) showing little quenching of the nucleons in \( \Sigma \) and Eq. (14) with large quenching for the strange quark contribution in \( \Sigma \).

Another suggestion has been “flavor symmetry,” \(^{23}\) which proposes that the \( \Sigma \) and \( \Xi \) hyperons differ only in the quark interchange \( u,d \leftrightarrow s \), have similar orbital configurations and that these orbital contributions are the main nonstatic effect. This would require the constraint \( d/s = d'/s' \), which, along with Eqs. (11)–(14), results in a new sum rule

\[
(\Xi^0 + 2\Xi^-)(\Xi^0 - \Xi^-) = (\Sigma^+ + 2\Sigma^-)(\Sigma^+ - \Sigma^-) . \quad (16)
\]

which is not satisfied \((1.47 \pm 0.07 \neq 0.63 \pm 0.36)\).

At this point, the importance of the new measurement for \( \Sigma^- \) can be seen. If the earlier value \( s' = -0.89 \pm 0.14 \) is used, then Eq. (14) becomes \( s' = -0.60 \pm 0.28 \) and the right-hand (\( \Sigma \)) side of Eq. (16) is changed to \( 1.89 \pm 0.92 \). In each case, the earlier value \( s' \) leads to agreement, but with errors that are too large to make the result conclusive.

Another combination of baryon moments that is difficult to reconcile with any simple model is

\[
s' - d' = 3(p - \Sigma^+) = 1.23 \pm 0.06 , \quad (17)
\]

which cancels out the similar \( u \)-quark contribution in the \( p \) and \( \Sigma^+ \). The difference in \( s' \) and \( d \)-quark contributions is so large here, even more than implied by Eq. (14), that extremely large nonsymmetric, nonstatic effects would be required to produce it. Equation (17) also shows that the difficulty with the \( \Sigma^- \) moments is as much with the \( \Sigma^+ \) as the \( \Sigma^- \). A corresponding difference can be formed by canceling out the \( u \)-quark contributions in \( n \) and \( \Xi^0 \), so that

\[
s - d = \frac{1}{4}(\Xi^0 - n) = 0.50 \pm 0.01 , \quad (18)
\]

which is more reasonable.

The relatively large differences in \( s' \) and \( d' \) of Eq. (17) and between \( s \) and \( s' \) in Eqs. (13) and (14) almost certainly require large meson components since relativistic effects could not affect \( s' \) so much and orbital-wave-function admixtures are unlikely to be large enough to produce a discrepancy this large. An explicit calculation \(^{25}\) of magnetic moments in the bag model including pion contributions and relativistic quark moments has achieved some improvement over the static quark model. The predictions of that model lead to a \( x^2 \) of 26 for the seven pure baryon magnetic moments of Table I.

An interesting regularity that can be seen in the quark-moment contributions of Eqs. (11)–(18) is that the difficulties are only with the odd quark in each baryon. The odd quarks are characterized by being in predominantly spin-0 states (when coupled to other quark spins) and the lack of symmetry for this spin state has also been noted in baryon mass differences \(^{21}\). The strange quark in the \( \Lambda^0 \) is predominantly in a spin-1 state and does give a symmetric magnetic-moment contribution as seen in Eq. (15). The conclusions to be reached from the baryon-moment sum rules of Eqs. (8)–(12) and the quark-moment contributions of Eqs. (11)–(18) are that nonstatic magnetic-moment effects are large and very nonsymmetric (baryon dependent). This is evidenced chiefly by the baryon-moment combinations \((\Xi^- - \Xi^0), (\Sigma^- - \Sigma^0), \) and \((p - \Sigma^+) \). No simple nonstatic effect seems capable of explaining these three baryon-moment combinations, each of which isolates the odd quark in the baryon.

I would like to thank the Temple University for a research leave and the Lady Davis Fellowship Foundation for a fellowship at the Technion, where this work was completed.

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4R. Handler et al., in High Energy Spin Physics—1982; proceedings of the Fifth International Symposium, Upton, New York, edited by G. M. Buncic (AIP, New York, 1983) (\( \Sigma^+ = 2.51 \pm 0.03 \)).  
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11G. Buncic et al., Phys. Lett. 86B, 386 (1979); Cox et al. (Ref. 9).  
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24Equation (16) can also be derived in the explicit formalism of Ref. 23.