about the abundance of the Ni isotopes which are not very well known. The assumption of a larger radius \( r_0 \) would produce smaller values of \( \xi_p \) and would give rise to a definite tendency to increase with higher energies. This would be in disagreement with our knowledge about sticking probabilities, insofar as we are allowed to apply evidence taken from neutron experiments to proton reactions and to the extent that we may trust the theoretical reasons for choosing a sticking probability near unity for the energies in question.

These assumptions can be tested by determining the nuclear radius with the aid of other independent methods. The most direct method is the measurement of the cross section for an inelastic neutron-collision \((n,n)\)-reaction), which should be equal to \( \pi R^2 \) for neutron energies of several Mev or more. The existing measurements of this cross section by Graham and Seaborg\(^{18}\) are not very conclusive. Their observed values give \( r_0 = 1.60 \) for C, 1.65 for Al, 1.7 for Zn, 1.5 for Sn and Sb and 1.35 for Pb in \( 10^{-13} \) cm. The values for C and Al\(^{19}\) are in definite disagreement with the very accurate values obtained by Wigner\(^6\).

\( ^{19} \) The experiments on Al have been done by G. Kuerti and S. N. VanVoorhis, Phys. Rev. 56, 614 (1939) and suggest a value of not more than \( r_0 = 1.35 \times 10^{-13} \) cm.

The Ionization Loss of Energy in Gases and in Condensed Materials*

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It is shown that the loss of energy of a fast charged particle due to the ionization of the material through which it is passing is considerably affected by the density of the material. The effect is due to the alteration of the electric field of the passing particle by the electric polarization of the medium. A theory based on classical electrodynamics shows that by equal mass of material traversed, the loss is larger in a rarefied substance than in a condensed one. The application of these results to cosmic radiation problems is discussed especially in view of the possible explanation on this basis of part of the difference in the absorption of mesotrons in air and in condensed materials that is usually interpreted as evidence for a spontaneous decay of the mesotron.

The determination of the energy lost by a fast charged particle by ionization and excitation of the atoms through or near which it is passing has been the object of several theoretical investigations. The essential features of the phenomenon are explained as well known.
in terms of a classical theory due to Bohr; the electrons near which the particle passes are treated as classical oscillators that are set in motion by the electric field of the passing particle. The energy thus absorbed by the electrons is equal to the energy lost by the particle. This theory gives a satisfactory description of the influence of the impacts for which the minimum distance \( b \) between the electron and the passing particle is larger than the atomic dimensions. Quantum mechanical corrections have to be introduced for the very close impacts, when the particle passes through the atom.

Both in the classical and in the quantum mechanical theories the action on every atom has been discussed so far by neglecting the perturbation of the field of the passing particle arising from the electric polarization of the surrounding atoms. A detailed analysis shows however that in some cases these polarization effects are very important.\(^1\)

The influence of the polarization has been calculated in the present paper on the basis of the classical theory. This may be expected to give correct results for at least that part of the energy loss which is due to collisions at distances \( b \) greater than the atomic radius.

**Theory**

The field of a charged particle moving through a medium having dielectric properties is affected by the polarization of the medium. We shall first calculate the field by applying classical electrodynamics. The amount of energy lost by the particle at distances greater than a certain minimum distance \( b \) from the path of the particle will then be calculated as flux of the Poynting vector across a cylindrical surface of radius \( b \) having the path of the particle as axis. We may reasonably hope to get in this way a correct estimate of the losses due to the atoms for which \( b \) is somewhat larger than the interatomic distances. Indeed we might then expect the quantum mechanical correction not to be very important; moreover a description of the dielectric properties of the medium with a continuum theory is permissible.

\(^1\)The possibility that a screening effect due to the polarization of the medium might reduce the ionization loss was suggested by W. F. G. Swann, J. Frank. Inst. 226, 598 (1938).

We write the Maxwell equations in the usual form:

\[
\text{div} (E + 4\pi P) = 4\pi \rho, \\
\text{div} H = 0, \\
c \text{rot} E = -\dot{H}, \\
c \text{rot} H = \dot{E} + 4\pi \dot{P} + 4\pi \rho V,
\]

where \( E, H, P, \rho, V \) are, respectively, the electric and magnetic field strength, the electric polarization vector, the density and velocity of the electric charges. The magnetic polarization of the medium has been neglected.

The relationship between \( E \) and \( P \) can be simply expressed on the assumption that the electrons are elastically bound to equilibrium positions and are furthermore subjected to a friction force. It is then:

\[
E = \frac{m}{n e^2} (\dot{P} + 2\dot{P} + \nu_0^2 P),
\]

where \( m, e, n \) are the mass, the charge and the number of electrons per unit volume; \( \nu_0/2\pi \) is the frequency of the electronic oscillators when \( E = 0; \ 2\dot{P} \) is the coefficient of the friction force.

Let \( E_\nu \) and \( P_\nu \) be the components of frequency \( \nu/2\pi \) (with the time dependence: \( \exp(-ivt) \)) in the harmonic analysis of \( E \) and \( P \); we have then:

\[
E_\nu = \frac{m}{n e^2} (\nu_0^2 - \nu^2 - 2i\nu \nu_0) P_\nu.
\]

In general we may assume between \( E_\nu \) and \( P_\nu \) a relationship of the type:

\[
E_\nu = 4\pi \gamma(\nu) P_\nu
\]

\( \gamma(\nu) \) is in general a complex function of \( \nu \).

In the special case (5) that the dispersion law can be described in terms of one kind of dispersion oscillators only we have:

\[
\gamma(\nu) = \frac{m}{4\pi n e^2} (\nu_0^2 - \nu^2 - 2i\nu \nu_0).
\]

Our problem is to find the field produced by a concentrated charge \( e \) moving with the constant velocity \( v \). We take the path of the particle as \( x \) axis and the position occupied by the particle at \( t = 0 \) as origin of the coordinates.

If we know the field at \( t = 0 \) we can obtain the field at any time by translating it with the
uniform velocity \( v \). This enables us to eliminate the time from the Maxwell equations by using the relationship:

\[
\frac{\partial}{\partial t} = -v \frac{\partial}{\partial x}.
\] (9)

Since at \( t=0 \) the charge is at the origin we have further \( \rho = e\delta(x)\delta(y)\delta(z) \); and the Maxwell equations become

\[
\text{div} \ (E + 4\pi P) = 4\pi \delta(x)\delta(y)\delta(z),
\] (10)
\[
\text{div} H = 0,
\] (11)
\[
c \text{rot} E = vH',
\] (12)
\[
c(\text{rot} H)_x = -vE_x' - 4\pi vP_x' + 4\pi v\delta(x)\delta(y)\delta(z),
\] (13)
\[
c(\text{rot} H)_y = -vE_y' - 4\pi vP_y' \quad ; \quad c(\text{rot} H)_z = -vE_z' - 4\pi vP_z',
\] (14)

where a dash means the derivative with respect to \( x \). Since the field moves with the uniform velocity \( v \) we may develop the field vectors in Fourier components with respect to \( x \) instead of \( t \) and interpret \( E_x \) and \( P_x \) in (7) as those components whose dependence on \( x \) is represented by the factor \( \exp(ivx/v) \).

The integration of the equations (7) (10) (11) (12) (13) (14) with the boundary condition that the fields must vanish at infinite distance can be performed as follows. From (7) and (10) we first calculate \( \text{div} P \) and \( \text{div} E \) as:

\[
\text{div} P = \frac{e}{2\pi v} \int_{-\infty}^{\infty} \frac{\delta(y)\delta(z)}{1+\gamma} dv;
\]
\[
\text{div} E = \frac{2e}{v} \int_{-\infty}^{\infty} \gamma \exp \left(\frac{ivx}{v}\right) dv
\] (15)

we can then eliminate \( H \) from (13) and (14) in the usual way and eliminate also \( E \) with (7). Taking into account (15) we find an equation for each component of \( P \). Developing these components in Fourier integrals with respect to \( x \) one easily finds the solutions in terms of the Bessel functions \( K_\theta \) and \( K_1 \):

\[
P_x = -\frac{e}{4\pi^2 v^2} \int_{-\infty}^{\infty} \left( \frac{1}{1+\gamma} - \frac{v^2}{c^2\gamma} \right) K_\theta(kb) \exp \left(\frac{ivx}{v}\right) dv,
\] (16)
\[
P_y = \frac{e}{4\pi^2 v} \int_{-\infty}^{\infty} \frac{kK_1(kb)}{1+\gamma} \exp \left(\frac{ivx}{v}\right) dv,
\] (17)

where \( P_x \) is the component perpendicular to the \( x \) axis and

\[
k^2 = \frac{v^2}{c^2} \left( 1 - \frac{v^2}{c^2} \right) - \frac{v^2}{c^2\gamma(v)}.
\]

The sign of \( k \) is determined so as to have its real part \( \geq 0 \). From (16) and (7) we obtain the components of the electric field

\[
E_x = -\frac{e}{\pi v^2} \int_{-\infty}^{\infty} \left( \frac{\gamma}{1+\gamma} - \frac{v^2}{c^2\gamma} \right) K_\theta(kb) \exp \left(\frac{ivx}{v}\right) dv,
\] (18)
\[
E_y = \frac{e}{\pi v} \int_{-\infty}^{\infty} \frac{\gamma k K_1(kb)}{1+\gamma} \exp \left(\frac{ivx}{v}\right) dv.
\]

From these and (12) we finally obtain the magnetic field. This reduces to one component only,
perpendicular to the \(x,b\)-plane. Its magnitude is given by

\[
H = \frac{e}{\pi c} \int_{-\infty}^{\infty} kK_1(kb) \exp \left( i\nu x/v \right) dv. \tag{19}
\]

The amount of energy lost by the particle per unit time at distances larger than \(b\) is given by the flux of the Poynting vector out of a cylinder of radius \(b\). Dividing this flux by \(v\) we obtain the corresponding loss of energy \(W_b\) per unit path. This is

\[
W_b = \frac{c}{4\pi v} \int \left[ E, H \right]_{\nu} d\sigma = \frac{cb}{2v} \int_{-\infty}^{\infty} \left| H \right| E_{\nu} dx. \tag{20}
\]

We substitute in this expression (18) and (19) changing in the last the name of the integration variable to \(\nu'\). \(W_b\) is then expressed by a triple integral over \(x, \nu, \nu'\). The integration over \(x\) gives a \(\delta\)-function. This fact enables us to perform in the usual way also the integral over \(\nu'\). We find at last, taking into account \(k(-\nu) = k^*(\nu)\)

\[
W_b = \frac{e^2 b}{\pi v^2} \int_{-\infty}^{\infty} \left( \frac{\gamma}{1 + \gamma} \right) i\nu k^*K_1(k^*b)K_0(kb) dv. \tag{21}
\]

One easily recognizes that the integrand takes complex conjugate values for the values \(\nu\) and \(-\nu\) of the integration variable; we have therefore:

\[
W_b = \frac{2e^2 b}{\pi v} R \int_{0}^{\infty} \left( \frac{\gamma}{1 + \gamma} \right) i\nu k^*K_1(k^*b)K_0(kb) dv. \tag{22}
\]

where we have indicated the real part of \(a\) by \(Ra\).

In order to calculate this integral we specialize our assumptions as to the dielectric properties of the medium by using (8). We find

\[
W_b = \frac{8ne^2}{mv^2(\epsilon - 1)} R \int_{0}^{\infty} \left( \frac{1 - x^2 - 2i\eta x - v^2}{1 - x^2 - 2i\eta x - c^2} \right) i\eta x k^*bK_1(k^*b)K_0(kb), \tag{23}
\]

where

\[
\epsilon = 1 + \frac{4\pi ne^2}{mv^2} \tag{24}
\]

is the dielectric constant for low frequencies.\footnote{It should be noticed that \(\epsilon\) is equal to the dielectric constant for small frequencies only when the description of the dielectric properties with dispersion oscillators of one frequency only is a sufficiently good approximation. When this is not the case \(\epsilon\) might differ considerably from the actual value of the dielectric constant.}

We have further

\[
\eta = \rho/r_0; \quad x = v/r_0; \quad k^2 = g^2 x^2 \frac{a - x^2 - 2i\eta x}{1 - x^2 - 2i\eta x} \tag{25}
\]

with

\[
g^2 = \frac{4\pi ne^2}{mc^2(\epsilon - 1)} \left( \frac{c^2}{v^2} - 1 \right); \quad a = \frac{c^2 - v^2}{c^2 - v^2} \tag{26}
\]

The integral (23) can be calculated when \(b\) is very small. In this case we may use for the Bessel functions the following expressions

\[
K_0(kb) = \frac{1}{2} \log \frac{4}{3.17 k^2 b^2}; \quad K_1(k^*b) = \frac{1}{k^* b}, \tag{27}
\]

in which \(3.17 = \exp (2\times\text{Euler's constant})\).
I ONIZATION LOSS OF ENERGY

(23) becomes now:

\[ W_b = \frac{4n\epsilon^4}{mv^2(e-1)} R \int_0^\infty \left( \frac{1-x^2-2ix\eta}{\epsilon-x^2-2ix\eta - c^2} \right) ixdx \log \frac{4(1-x^2-2ix\eta)}{3.17 g^b \beta^b x^2(a-x^2-2ix\eta)}. \]

The integral can be reduced to the following integrals, that can easily be calculated by complex integration.

\[ R \int_0^\infty \frac{ixdx}{e-x^2-2ix\eta} = \frac{\pi}{2} \; R \int_0^\infty \frac{ixlogx \; dx}{e-x^2-2ix\eta} = \frac{\pi}{4} \log \frac{\pi \eta}{2(\epsilon-\eta^2)} \; \text{artg} \left( \frac{\epsilon-\eta^2}{\eta} \right); \]

\[ R \int_0^\infty \frac{ixdx}{e-x^2-2ix\eta} = \begin{cases} \pi - a & \text{for } a > 0; \\ \pi \log \frac{e-a}{\epsilon-\eta^2} \; \text{artg} \left( \frac{\epsilon-\eta^2}{\eta} \right) \frac{\eta}{2(\epsilon-\eta^2)} & \text{for } a < 0; \\ \frac{1}{2}(1-a) & \text{for } a > 0; \\ \frac{1}{2} \pi(1-a) - \frac{1}{2} \pi \left[ \eta^2 + (\eta^2-a) \right] & \text{for } a < 0. \end{cases} \]

The integration path of these integrals is along the positive real axis. They can be calculated by deforming the integration path moving first from the origin in the positive imaginary direction to a very large distance from the origin and then coming back on the real axis along a quarter of a circle of very large radius with center in the origin and by taking proper account of the singularities. Notice further that only the real part of the above integrals is convergent. The divergence of the imaginary part has been introduced when we have used the approximate expressions (27) for the Bessel functions. This divergence is of course immaterial to us, since in our formulae only the real part of the integrals is used.

We obtain now:

\[ W_b = \frac{2\pi n\epsilon^4}{mv^2} \left\{ \log \frac{mv^2}{3.17 n\epsilon^6 b^2} + \log \frac{e-1}{\epsilon(1-v^2/c^2)} - \frac{\epsilon}{c^2} - \frac{2\eta}{\epsilon} \text{artg} \left( \frac{\epsilon-\eta^2}{\eta} \right) \right\} \quad \text{for } v < \frac{c}{\sqrt{\epsilon}}, \tag{28} \]

\[ W_b = \frac{2\pi n\epsilon^4}{mv^2} \left\{ \log \frac{mv^2}{3.17 n\epsilon^6 b^2} - \frac{\epsilon}{c^2} - \frac{2\eta}{\epsilon} \text{artg} \left( \frac{\epsilon-\eta^2}{\eta} \right) \right\} \quad \text{for } v > \frac{c}{\sqrt{\epsilon}}. \tag{29} \]

When the damping is negligible (lim \( \eta = 0 \)) the above formulae become:

\[ W_b = \frac{2\pi n\epsilon^4}{mv^2} \left\{ \log \frac{mv^2}{3.17 n\epsilon^6 b^2} + \log \frac{e-1}{\epsilon(1-v^2/c^2)} - \frac{\epsilon}{c^2} \right\} \quad \text{for } v < \frac{c}{\sqrt{\epsilon}}, \tag{30} \]

\[ W_b = \frac{2\pi n\epsilon^4}{mv^2} \left\{ \log \frac{mv^2}{3.17 n\epsilon^6 b^2} - \frac{\epsilon}{c^2} \right\} \quad \text{for } v > \frac{c}{\sqrt{\epsilon}}. \tag{31} \]
These results should be compared with the corresponding energy loss calculated with the usual theory by neglecting the polarization effects:

\[
W_b \text{ (usual theory)} = \frac{2\pi ne^4}{mv^2} \left\{ \log \frac{mv^2}{3.17\pi ne^4 b^2} + \log \left( \frac{\epsilon - 1}{\epsilon c^2} \right) \right\}.
\]  

(32)

The comparison is shown in Fig. 1. The three curves represent the energy loss per unit path due to impacts at distances greater than $10^{-3}$ cm, measured taking $2\pi ne^4/mv^2$ as unit, according to the ordinary formula (32) and according to the present theory with formulae (30) and (31) for standard air and for water. In both cases the difference from the ordinary result is very small at low energies. For high energies, however, the loss calculated with the ordinary theory keeps increasing logarithmically with increasing energy, whereas the effect of the polarization produces a flattening out of the curves for air and water in such a way that $W_b$ remains finite even when the energy of the particle becomes infinite.

It can easily be seen from (28) and (29) that the limiting value of $W_b$ for $v=c$ is independent on both the binding frequency $\nu_0$ of the dispersion oscillators and the damping constant $\eta$. It depends only on the number $n$ of electrons per unit volume, and is given by:

\[
W_b(v=c) = \frac{2\pi ne^4}{mc^2} \log \frac{mc^2}{3.17\pi ne^4 b^2}.
\]  

(33)

This shows that the energy loss of very high energy particles due to various materials in layers of such thicknesses as to contain always the same number of electrons per cm$^2$, is smaller for larger electronic densities $n$ of the material (see on Fig. 1 the difference between air and water).

When the damping $\eta$ of the dispersion oscillators is very small it is possible to calculate the integral (23) for arbitrary values of $b$ by using the exact expressions for the Bessel functions instead of the approximate expressions (27). One finds:

\[
W_b = \frac{2\pi ne^4}{mv^2} \left\{ \frac{2b\nu_0}{v} K_0 \left( \frac{b\nu_0}{v} \right) K_1 \left( \frac{b\nu_0}{v} \right) \frac{v^2}{c^2} - \log \left( \frac{1 - \frac{v^2}{c^2}}{\epsilon} \right) \right\} \quad \text{for } v < c/\epsilon; \tag{34}
\]

\[
W_b = \frac{2\pi ne^4}{mv^2} \left\{ \frac{2b\nu_0}{v} K_0 \left( \frac{b\nu_0}{v} \right) K_1 \left( \frac{b\nu_0}{v} \right) \frac{1 - \frac{v^2}{c^2}}{\epsilon - 1} + \log \frac{\epsilon}{\epsilon - 1} \right\} \quad \text{for } v > c/\epsilon. \tag{35}
\]

For very small $b$ these expressions go over into (30) and (31). It is interesting to note, however, that $W_b$ in this case does not vanish, as one might expect, for very large $b$. Indeed it follows from (34) and (35) that for $b = \infty$ it is:

\[
W_\infty = \frac{2\pi ne^4}{mv^2} \left\{ -\frac{v^2}{c^2} - \log \left( \frac{1 - \frac{v^2}{c^2}}{\epsilon} \right) \right\} \quad \text{for } v < c/\epsilon; \tag{36}
\]

\[
W_\infty = \frac{2\pi ne^4}{mv^2} \left\{ -\frac{1 - \frac{v^2}{c^2}}{\epsilon - 1} + \log \frac{\epsilon}{\epsilon - 1} \right\} \quad \text{for } v > c/\epsilon. \tag{37}
\]

$W_\infty$ represents that part of the energy lost by the particle that is emitted in the form of radiation. Such an emission of radiation has actually been observed by Cerenkov,\(^8\) and can easily be seen to occur in those ranges of frequency for which the phase velocity of light in the given medium is smaller than the velocity of the particle. Its theory has been developed by Frank and Tamm\(^4\) with methods very similar to those used here and with similar results. It is noteworthy that the Cerenkov radiation, as results from the preceding formulæ, does not represent a loss of energy to be added to that calculated with the Bohr theory; but it forms instead part of the loss of the Bohr theory, as is


seen from the fact that (30) (31), which include the Cerenkov radiation, give the same result (32) as the Bohr theory in the limit of very low densities (ε = 1) when the polarization effects become negligible.

We have considered so far only the phenomena occurring at distances greater than the interatomic distance, for which it is legitimate to apply the continuum electrodynamics. A description of the effects of impacts at close distances requires a quantum mechanical description of the impact process as well as of the field of the passing particle and of its change due to the polarization of the other atoms. If we assume, however, that the effect of the polarization on the close distance impacts is not large we may use the ordinary theory, as developed by Bethe and Bloch for the calculation of the energy loss due to such impacts.

Under this assumption we may take the difference between the formulae (30) (31) of the present theory and formula (32) giving the result of the Bohr theory as the correction representing the polarization effects. We find thus that the energy loss per unit path is less than that given by the ordinary formulae by the following amount:

\[
\frac{2\pi ne^4}{mc^2} \log \frac{\varepsilon}{v} \quad \text{for } v < c/\sqrt{\varepsilon}; \quad (38)
\]

\[
\frac{2\pi ne^4}{mc^2} \left[ \log \frac{\varepsilon - 1}{1 - v^2/c^2} + \frac{1 - \varepsilon v^2/c^2}{\varepsilon - 1} \right] \quad \text{for } v > c/\sqrt{\varepsilon}. \quad (39)
\]

The correction is negligible for low velocities; for very large energies instead it reduces the energy loss to less than 50 percent of the loss calculated with the ordinary theory.

For very large energies we have from the ordinary theory the following energy loss:

\[
\frac{2\pi ne^4}{mc^2} \frac{mc^2 W}{(1 - v^2/c^2) \hbar^2 \nu_0^2}. \quad (40)
\]

We obtain thus from (39) the following formula valid asymptotically for very large energies:

\[
\frac{2\pi ne^4}{mc^2} \left[ \log \frac{\pi mc^2 W}{mc^2} - 1 \right]. \quad (41)
\]

We note that this asymptotic formula does not contain the binding frequency of the electrons but only their number per unit volume.

**Applications**

Only a very small polarization effect should be expected according to the present theory in the stopping power of different materials for α-particles, protons or deuterons having energies up to the order of magnitude of some Mev. In all these cases the velocity is rather small compared with c, so that we must use (38). This formula gives an entirely negligible correction in the case of air, since in this case \( \varepsilon \) is very close to unity. Corrections of the order of several percent might be expected in the case of the stopping power in solid or liquids. It is doubtful whether such differences are large enough to be observable. Moreover, when the velocity of the particle is small, as in the present case, the ionization produced by the particle does not reach far enough from the trajectory as to make the description of the field in terms of continuum electrodynamics a good approximation.

Greater effects are to be expected in the case of β-particles and especially for particles of...
several millions of ev. The correction for air is again negligible but an appreciable correction is found for condensed substances especially for high energy \(\beta\)-particles. The energy loss per cm of water calculated with the usual theories for \(\beta\)-particles of \(10^4, 10^5, 10^6\) ev is, respectively, 1.93, 2.15, 2.72 Mev. The correction \(38\) \(39\) reduces these losses to 1.83, 1.75, 1.94 Mev if we take \(\epsilon=1.7\); since, however, the effective \(\epsilon\) may be as low as 1.1, the corrected energy losses may be 1.92, 1.91, 2.09 Mev. In spite of the fact that the differences are in some cases rather large, it is very difficult to compare these results with the available experimental material. Indeed the data for relatively low energies are largely affected by the scattering \(37\) while those for large energies are perturbed by the radiation losses of energy.

The results for mesotrons are represented in Fig. 2. The diagrams have been calculated for a rest energy of the mesotron of 80 Mev. The energy of the mesotron is plotted in a logarithmic scale on the abscissae; and the energy loss in units \(2\pi \rho e^4/mv^2\) is plotted on the ordinates for standard air, water and lead. The curves \(A\) are calculated with Bloch’s formula; the curves \(B\) are corrected for the polarization effects according to the present theory. It should be noticed that the incomplete knowledge of the dispersion law makes the shape of the curves \(B\) in the neighborhood of the point at which they begin to deviate appreciably from the corresponding curves \(A\) rather uncertain. For somewhat greater energies instead the curves \(B\) become practically independent of the dispersion law, being given by \(41\).

According to \(41\) the loss of energy in a layer containing \(N\) electrons per cm\(^2\) depends on the electronic density \(n\) in the layer. For two layers having the same \(N\) and different electronic densities \(n_1\) and \(n_2\) the difference in energy loss is:

\[
W_1 - W_2 = \frac{2\pi \rho e^4}{m c^2} \log \frac{n_2}{n_1},
\]

Fig. 2. Energy loss in units of \(2\pi \rho e^4/mv^2\) for mesotrons of various energies in air, water, and lead. Curves \(A\) are calculated with Bloch’s formula and curves \(B\) are corrected for the polarization effects according to the present theory.

the energy loss being smaller for the substance of greater density. \(42\) is valid only for rather

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\(4\) For a critical discussion of the literature on this subject see: B. Rossi, Rev. Mod. Phys. 11, 296 (1939).


direction. The absorption curve in air can be thus observed for thicknesses of air up to many atmospheres. This absorption curve is compared with that obtained with the counters under water or under condensed materials. Experiments of this kind show that under large and equivalent thicknesses of air and of condensed materials the intensity of the mesotron component of the cosmic radiation is different. The intensity under condensed absorbers is about twice as large as under air.

In order to estimate what part of this difference can be accounted for by the present theory we take as average density of air along the path of the mesotron 1/e of the density at sea level, namely 0.00045, corresponding to an electronic density \( n_{\text{air}} = 1.36 \times 10^{5} \). As a typical condensed substance we take water, for which the electronic density is \( n_{\text{water}} = 3.35 \times 10^{5} \). According to (42) this corresponds to a difference of 0.60 Mev in the energy loss in one g/cm\(^2\) of air and in a thickness of water containing the same number of electrons. For mesotrons of 10\(^4\) Mev the energy loss is about 2.8 Mev \times \text{cm}^2/\text{g}. The loss in the equivalent amount of water is then only 2.2. The energies required for a mesotron to traverse a thick layer of air or an equivalent layer of water are therefore in the ratio 2.8/2.2 = 1.27. Assuming the energy distribution of the mesotrons to be such that the number of mesotrons with energy greater than \( W \) is proportional to \( W^{-1.9} \) we conclude that the number of mesotrons observed under equivalent and very thick layers of condensed matter and of air should be in the ratio 1.27\(^{1.9} \) = 1.38. The effect to be expected according to the present theory is therefore of the order of magnitude of the effect observed experimentally. The present theory, however, does not appear sufficient to explain all the effect but only about one-half of it. If we interpret the residual effect as due to the decay of the mesotron the lifetime of this particle ought to be taken about twice as large as according to the usual estimates.

In a second type of experiment, like the one recently described by Rossi, Hilberry and Hoag,\(^8\) the vertical intensity of the mesotronic component of the cosmic radiation is measured at different heights with and without a graphite absorber; the absorptions in air and in graphite are thus directly compared. Graphite is used in order to eliminate as much as possible the effects due to the differences in atomic number. This arrangement has the advantage of being independent of the assumption that the primary radiation entering the atmosphere is isotropic. Furthermore the absorbers used are relatively thin (82 g/cm\(^2\) graphite and 12.7 cm lead) and consequently the effect of a possible decay of the mesotron is measured for mesotrons of a relatively low energy, so that the apparent lengthening of the lifetime due to relativity effects is not very large. On the other hand, the observed absorptions are rather small (10 to 20 percent) and are therefore more sensitive to perturbations due to possible geometrical transition effects when the graphite is placed above the counters. Professor Rossi, however, informs me that he has very carefully excluded that his results might be considerably affected by such effects. Since this experiment involves mesotrons having energies of only a few hundreds of Mev the polarization effects discussed in this paper should affect its results only very little (see Fig. 2). The effect observed in this experiment has therefore apparently to be attributed to a decay of the mesotron.

We notice finally that the estimates of the energy of the mesotrons penetrating to very great depths should be somewhat changed in order to take into account the polarization effects. For example the energy of mesotrons capable of traversing \( 1.5 \times 10^5 \) g/cm\(^2\) of matter should be reduced from \( 5.6 \times 10^8 \) Mev to about \( 3.9 \times 10^4 \) Mev.

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\(^8\) B. Rossi, H. V. N. Hilberry and J. B. Hoag, Phys. Rev. 56, 837 (1939).