The Emission of Radiation in the Disintegration of Mesons*

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The probability has been calculated that a meson of integral spin disintegrates into an electron, a neutrino, and a photon. This is essentially a classical process, the radiation being regarded as caused by the acceleration of charge in the disintegration. Because of the classical nature of the problem, the result is independent of the quantum mechanical properties (spin, coupling) of the meson field. For definiteness, the non-radiative decay is taken as being a two-body process in accordance with the original Yukawa theory, but several different types of couplings are used. The quantum mechanical probability for emission of one photon diverges at the low frequency end of the spectrum; this difficulty is avoided by using as a measure of the process the ratio of mean energy emitted to mean energy available for the process per unit time. This is of order $e^2/\hbar c$; the energy spectrum also is in agreement with the classical result.

I. INTRODUCTION

The nature of the products of decay of the ordinary cosmic-ray mesons ($\mu$-mesons) is still unsettled. Recent experiments, however, have established that the decay does not result in the emission of a 50-Mev photon, as would be expected if the process involves simply the emission of two bodies, and the $\mu$-meson has a spin of one-half. From the theoretical point of view, there is another possibility for the emission of electromagnetic radiation in the decay, which is of interest since it can occur regardless of the nature of the other particles emitted and is one of the few processes involving mesons which is essentially classical in nature. If the meson is at rest before disintegrating, and the decay electron has a high velocity, the sudden acceleration of charge should produce radiation. Quantum mechanically, this corresponds to a higher order process, in which a photon is emitted as well as the ordinary decay products. Similar calculations for radiative beta-decay of nuclei have been performed by Bloch and by Knipp and Uhlenbeck.

Because of the classical nature of the problem, the quantum mechanical details of meson spin, and type of interaction between the meson and electron fields, which are not known from experimental evidence, should not affect the results. Some calculations for different types of interactions will be indicated, however, since they afford an interesting example of the way in

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1 E. Hincks and R. Pontecorvo, Phys. Rev. 73, 257 (1948); R. Sard and E. Althaus, Phys. Rev. 73, 1251 (1948); O. Piccioni, Phys. Rev. 74, 1236 (1948).

which the classical nature of the problem can be used to simplify the quantum mechanical results. Since our primary purpose is merely to indicate this relation between classical and quantum mechanical results, for definiteness the original idea of Yukawa, that the decay products are an electron, and a neutrino will be used, although there is evidence that the process involves the emission of three bodies.  

II. CLASSICAL TREATMENT

The qualitative classical ideas given above can be extended to give a quantitative estimate of the probability of the process. The acceleration of the charged particle can be taken as caused by a finite pulse, say of form,

\[ a(t) = \frac{ca}{(\alpha^2 + t^2)}. \]

(Since this argument is concerned only with the order of magnitude of the effect, we omit the factor \( 2/\pi \) needed to make this approach \( c \) times a delta-function.) The "width" \( \alpha \) can be determined by the uncertainty relation,

\[ \alpha \Delta \omega \sim 1, \]

\( \Delta \omega \) being the frequency band-width of the emitted spectrum. In order of magnitude, \( \Delta \omega \) is roughly the same as the frequency of the quantum of maximum energy which can be emitted,

\[ \hbar \omega_{\text{max}} = \frac{1}{2} \mu c^2, \]

\( \mu \) being the meson mass. Thus,

\[ \Delta \omega \approx \mu c^2 / \hbar. \]

This mixing of quantum properties with a classical theory is essential, since the basic process of meson decay is non-classical in nature.

The classical total energy emitted during such an acceleration is then,

\[ E \sim \mu c^2 \left( \frac{e^2}{\hbar c} \right). \]

The energy available for the process is of order \( \mu c^2 \), so less than one percent of the energy should be emitted in the form of radiation. The fact that the effect is of order of the fine-structure constant of course is a quantum mechanical result also.

The form of the energy spectrum can be found by taking the fourier transform of the acceleration pulse. This spectrum remains finite at the low frequency end, and must be cut off at the upper limit determined by energy and momentum requirements. The first fact corresponds quantum mechanically to the so-called "infra-red catastrophe" which results from an unjustified calculation of the probability of emission of a finite number of photons, and the second fact means that the spectrum must fall rapidly to zero near its upper limit.

III. QUANTUM MECHANICAL TREATMENT

For simplicity, we consider the meson initially at rest, and later indicate the results if the decay occurs while it is in motion. Only a negative meson will be considered, since the results are identical for both signs of charge. Further, the process will be described in the usual manner as involving the creation of an anti-neutrino, rather than a neutrino, both electron and neutrino then being regarded as two states of the same particle with different isotopic spin.

For the fields of vector and pseudoscalar types, there exist matrix elements for the direct transition from the initial state to the final state in which the electron and photon are present. However, such first-order processes do not give rise to low frequency divergences characteristic of the radiative corrections, and hence are negligible in comparison with the second-order terms at the low frequency end. Since the contributions to the cross section from all terms vanish at the high frequency end of the spectrum, these first-order terms do not give appreciable contributions to the cross section at any part of the spectrum, and hence may be neglected. For the pseudoscalar field, this neglect is further justified by the fact that the direct interaction vanishes if the coupling is of the pseudoscalar type. Because of the relation between the pseudoscalar and pseudovector couplings in the weak coupling approximation, this further justifies the neglect of the direct term for the general pseudoscalar field.

There then exist the following types of possible second-order transitions:

\[ E. C. Nelson, \text{Phys. Rev.} \text{ 60}, 830 (1941); F. J. Dyson, \text{Phys. Rev.} \text{ 73}, 929 (1948). \]
1. The meson is virtually annihilated, creating an electron and an anti-neutrino; the electron then virtually emits the photon.

2. An electron-positron pair is created with emission of a photon; the meson is then annihilated by absorbing the positron and emitting an anti-neutrino.

3. The meson first virtually emits the photon and then decays with the creation of an electron and an anti-neutrino.

The necessary matrix elements are all either well-known or obtainable from previous literature, and will not be derived here. The apparent complexity due to the large number of possible intermediate states can be considerably reduced as follows.

The second type of virtual transition is due to the requirements of the hole theory of the Dirac electron, which forbids transitions into filled negative energy intermediate states. However, the same results are obtained if we do not use the hole picture, but use only the first process, and sum over both positive and negative energy intermediate states.

The third process need not be considered at all for the scalar or pseudoscalar fields, for a spinless particles at rest cannot interact with the electromagnetic field. The interaction term is,

\[ H'_{MB} = -\left(\frac{ie\omega}{\hbar}\right) \int \left[ \mathbf{A} \cdot (\mathbf{\phi} \mathbf{\phi}^* - \mathbf{\phi}^* \mathbf{\phi}) \right] d\tau, \]

\( A \) being the transverse vector potential and \( \phi \) the meson wave function. In a momentum space representation of this term, the gradient operator produces terms of the form \( \hat{\mathbf{l}} \cdot \mathbf{k}' \), \( \hat{\mathbf{l}} \) being the photon polarization, and \( \mathbf{k}' \) the momentum of the meson in the intermediate state. However, if the meson is initially at rest,

\[ \mathbf{k}' = -\mathbf{k}, \]

where \( \mathbf{k} \) is the photon wave vector, so \( \hat{\mathbf{l}} \cdot \mathbf{k}' = 0 \).

The meson-radiation interaction does not vanish for particles of non-zero spin at rest (Dirac electron, vector meson). However, a direct calculation of terms resulting from this process shows that they do not give the low frequency divergence, and hence may be neglected. This is again understandable from the classical model. In the process in which the electron emits the radiation, the interacting particle has been accelerated, hence classically is capable of emitting radiation, whereas if the meson interacts with the radiation first it has not been accelerated, and hence classically will not radiate.

With the above simplifications, the transition probability can be calculated in a straightforward manner. One must be careful, however, not to make the approximation of neglecting the terms of order (electron mass/meson mass)\(^2\) which arise in the energy denominators, or the integrals over the directions of the emitted particles will diverge. This again follows from the classical model, since in the decay of an infinitely heavy meson, the electron would be accelerated to the velocity of light instantaneously since \( \Delta \omega \) would be infinite, and hence infinite radiation would be produced. Although the integrals which occur can all be evaluated in terms of elementary functions, it is convenient and of some interest to make use of the relation between the two pseudoscalar theories to avoid having to evaluate the integrals for the vector meson separately.

The probability per unit time of emitting a photon of frequency \( \omega \) can be written as,

\[ \mathcal{W}(\omega) d\omega = \frac{1}{2\pi \hbar \omega} \left[ \frac{e^2}{\mu \omega^2} - \frac{2h}{\omega - \frac{(m\omega)^2}{\mu \omega^2}} \right] d\omega, \]

where \( \mu \) is the reduced mass of the electron and meson.

\[ \begin{align*}
G_1^2 & \cdot I_1(\omega); \text{ scalar meson; pseudoscalar coupling.} \\
\frac{\hbar c}{G_2^2} \cdot I_2(\omega); \text{ pseudoscalar meson with pseudovector coupling. (a)} \\
\frac{1}{3\hbar c} \left( I_1 + 2I_2 \right) + \frac{G_2^2}{\hbar c} \left( 2I_1 + I_2 \right) \text{ vector meson, (c)}
\end{align*} \]

where \( I_1 \) and \( I_2 \) are certain functions of \( \omega \) obtained by integration over the relative angles of emission of the electron and anti-neutrino. Now from the relation between the two pseudoscalar theories, we must get the same result from (a) as from (b) if we make the replacement,

\[ G_1 \rightarrow -\left(\frac{m}{\mu}\right) G_1. \]
mean energy emitted in radiation per unit time to the mean energy available for the process per unit time, which is essentially the non-radiative (first order) lifetime times the meson rest-energy. Because the interaction constants for the various theories of non-radiative decay always occur in the same combination as here given, the result is,

\[ R = \frac{1}{4\pi \hbar c} \frac{8}{3} \ln \left( \frac{\mu}{m} \right) - \frac{31}{9} \]

for all interactions. This result also holds for mixed theories of meson fields. For a meson mass of 200 electron masses, \( R = 0.84e^2/hc = 0.6 \) percent.

**IV. CONCLUSION**

The essential features of the quantum mechanical result all have analogs in the classical treatment. The final result can be transformed to a moving system to consider possible changes that occur if the meson decays while in motion.

The results of such a Lorentz transformation are four in number: the relativistic mass increase, aberration and Doppler shift of the radiation, as well as the time dilatation which alone occurs for non-radiative decay. If the angular distribution of the emitted radiation is isotropic in the rest system, then from a system in motion relative to it the radiation appears bunched along the direction of travel.\(^6\) However, although more energy is emitted by a moving meson because of the mass increase, the time dilatation just compensates this effect, and both the mean energy and the relative mean energy emitted per unit time are independent of the motion.

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\(^6\) Note that in Chang's paper the sign of \( f_1 \) should be reversed in the formula for the lifetime, if the usual choice for the Dirac Hamiltonian,

\[ H = -\alpha \cdot p - \beta mc^2 \]

is used.

\(^\dagger\) This well-known result is derived, for example, by L. Janossy, *Cosmic Rays* (Oxford University Press, London, 1948), Section 152.