On the «Spurion» Theory of Strange Particles.

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Summary. — In the nonlinear spinor theory which only involves spinor-isospinor field operators, strange-particle states may be constructed under the assumption of a ground state unsymmetrical with respect to isospin: «spurions» carrying isospin can be detached from the ground state and attached to systems constructed from local field operators to form strange particles. In addition to isospin, further properties have to be attributed to the spurions to be consistent with the requirements of Lorentz- and CPT-invariance of the vacuum. Three such possibilities present themselves: The first possibility which adds a parity property has been considered in an earlier paper. Its group-theoretical implications (e.g., odd $\Delta\Sigma$-parity) are in contradiction to present experimental evidence; therefore this possibility is ruled out. Hence the second possibility, which adds a hypercharge property, is studied in the present paper. The spurion systems then transform according to representations of $U_3$. One finds that the combination $I(I + 1) - \frac{1}{4} Y^2$, known in connection with broken $SU_3$, appears automatically in the factor multiplying the eigenvalue operator for the simplest bosons due to the characteristic form of the field equation and the permutation properties of the spurions. The eigenvalue operators of baryons of spin $\frac{1}{2}$ are multiplied by a factor containing $aBY + b[I(I + 1) - \frac{1}{4} Y^2]$ ($B =$ baryon number) if the simplest bosons are taken into account in the self-interaction. Although the $I, Y$ operators of the broken $SU_3$ appear, the group $SU_3$ is never assumed nor established, and hence the characteristic grouping of particles into 1, 8, 10 etc. cannot be derived group-theoretically. It is interesting, however, to observe that a grouping of 8 for the simplest bosons follows naturally from dynamical considerations.

(*) Part of this work was carried out while the author was a visiting faculty member at the University of California, Berkeley.
**Introduction.**

The strange particles can be represented in the nonlinear spinor theory of elementary particles by making use of a degeneracy of the ground state with respect to isospin. «Spurions» carrying isospin may then be detached from the ground state and attached to the nucleons or bosons to form the strange particles \(^{(1)}\).

Instead of speaking of a degenerate ground state and of «spurions», one may also use a symmetrical ground state and introduce special assumptions concerning the asymptotic behaviour of 4-, or 6-point functions etc., referring to this ground state. One may for instance study the behaviour of

\[
\langle 0 | \chi_{\alpha}(x_1) \chi_{\beta}^{\dagger}(x_2) \gamma_{\mu}(y_1) \chi_{\mu}^{\dagger}(y_2) | 0 \rangle = f_{\alpha \beta \mu \rho}(x_1 x_2, y_1 y_2)
\]

in the asymptotic region, where the two \(x\)-co-ordinates are in a very large spacelike distance from the two \(y\)-co-ordinates; \(\alpha, \beta, \mu, \lambda\) shall indicate here only the isospin indices of the field operators. In this limiting case one would normally expect that \(f_{\alpha \beta \mu \rho}(x_1 x_2, y_1 y_2)\) goes over into a product of the type

\[
f_{\alpha \beta \mu \rho}(x_1 x_2, y_1 y_2) \rightarrow g(x_1 - x_2) \delta_{\alpha \mu} h(y_1 - y_2) \delta_{\beta \rho} .
\]

If however this asymptotic expression contains other terms of the type

\[
g(x_1 - x_2) \tau_{\alpha \beta}^{\mu} \tau_{\rho \delta}^{\mu} h(y_1 - y_2) ,
\]

this would indicate that intermediate states other than the symmetrical ground state (vacuum state) contribute in this limit. These other states must be translation-invariant \((E = p = 0)\) like the vacuum state but must carry isotopic-spin properties, and are formally represented by the notion of the spurion states. These spurions may be «attached» to the field operators and therefore give rise to particles with different spin-isospin combinations as represented by the field operators. More precisely the term \(\sim \tau_{\alpha \beta}^{\mu} \tau_{\rho \delta}^{\mu}\) results from spurion intermediate states with isospin 1 and projections \(i\), i.e.

\[
\langle 0 | \chi_{\alpha}(x_1) \chi_{\beta}^{\dagger}(x_2) \gamma_{\mu}(y_1) \chi_{\mu}^{\dagger}(y_2) | 0 \rangle \rightarrow
\]

\[
\rightarrow \sum_{i} \langle 0 | \chi_{\alpha}(x_1) \chi_{\beta}^{\dagger}(x_2) | i \rangle \langle i | \gamma_{\mu}(y_1) \chi_{\mu}^{\dagger}(y_2) | 0 \rangle \sim \tau_{\alpha \beta}^{\mu} \tau_{\rho \delta}^{\mu} .
\]

We represent this isospin-1 state formally by a configuration of two spurion-

isospinors \((\lambda, \mu = 1, 2)\)

\[
|i\rangle = \tau^i_{\mu} |\tilde{s}^*_\mu s_\mu\rangle.
\]

Here \(\tilde{s}^*\) designates a state which transforms with the complex conjugate spinor representation of the isospin group \(SU_2\). We may also write

\[
\rightarrow \sum_{\lambda, \mu} \langle 0 | \chi_\lambda(x_1) \chi_\lambda^*(x_2) | s_\mu \tilde{s}^*_\lambda\rangle \langle \tilde{s}^*_\mu s_\mu | \chi_\gamma(y_1) \chi_\gamma^*(y_2) | 0 \rangle \sim \\
\sim \frac{1}{2} \sum_{\lambda, \mu} \tau_{\lambda\beta}^i \tau_{\lambda\mu}^j \tau_{\gamma\delta}^f + \cdots = \tau_{\alpha\beta}^i \tau_{\gamma\delta}^f + \cdots.
\]

The other terms contain the antisymmetrical "spurion-antispurion" combinations which are indistinguishable and hence identical with the symmetrical vacuum. It should be noticed that in the separation of 4-point functions only two-spurion configurations have to be taken explicitly into account. In the separation of higher-point functions successively higher configurations have to be introduced. For these higher configurations the introduction of spurion states which transform irreducibly under the isospin group (like (5)) will be more convenient than the use of products of spurion-isospinor states \(s\). However, the latter notation is in many cases more intuitive.

The above expression for the 4-point function can be trivially generalized if we add in the bra and ket identical spurion combinations \(c(s, s^*)\):

\[
\langle c(s^*) | \chi_\lambda(x_1) \chi_\lambda^*(x_2) \chi_\gamma(y_1) \chi_\gamma^*(y_2) | c(s^*) \rangle = \\
= \langle c(s^*) | \chi_\lambda(x_1) \chi_\lambda^*(x_2) | c(s^*) \rangle \langle c(s^*) | \chi_\gamma(y_1) \chi_\gamma^*(y_2) | c(s^*) \rangle = \\
= \langle 0 | \chi_\lambda(x_1) \chi_\lambda^*(x_2) \chi_\gamma(y_1) \chi_\gamma^*(y_2) | 0 \rangle.
\]

These spurions which are not explicitly involved with the localized states may be referred to as "uncoupled" spurions or "not localized" or "free" spurions. The latter expression is justified due to the fact that these spurions cannot be localized because of their translational invariance. The "coupled" spurions however can be considered localized.

As a special case of the above rule (7) we may consider the relation already given by DÜRR and GÉHÉNIAU \(^{2,3}\):

\[
\langle s_\mu | \ldots | \ldots \rangle = \langle s_\mu \tilde{s}^*_\mu | \ldots | \tilde{s}^*_\mu \rangle = \langle \ldots | \ldots \rangle = \langle \ldots | \ldots \rangle.
\]


In particular we also get

\begin{equation}
\langle 0 | \ldots | s_\mu s_\mu^* \rangle = \langle s_\lambda | \ldots | s_\mu \rangle = \langle s_\lambda s_\mu^* | \ldots | 0 \rangle.
\end{equation}

One may consider this relation as a definition of the * prescription of the spurion state, i.e. \( \langle s_\lambda | = \langle s_\lambda^* \rangle \). This expresses the fact that a spurion \( s_\lambda \) on the bra side of a matrix element is always equivalent to a "spurion-hole" on the ket side. The spurion-hole may be considered as an "antispurion", and it is then convenient to introduce the antispurion state as

\begin{equation}
a_\lambda = c_{\lambda\mu} s_\mu^*,
\end{equation}

where \( c_{\lambda\mu} \) is the well-known transformation matrix defined by

\begin{equation}
- \tau_k = \epsilon^{-1} \tau_k^* \epsilon = \epsilon^{-1} \tau_k^T \epsilon.
\end{equation}

\( s_\lambda \) and \( a_\lambda \) have the same transformation properties with respect to isospin-rotation \( (SU_3) \).

Spurion and antispurion may be distinguished by the isospin-gauge group. In the context of the present theory the spurions are always assumed to be invariant under the proper Lorentz group. This statement does not determine their behaviour under the discrete groups of the system, and Dürr and Géhéniaux (3) have shown that essentially three different assumptions can be made concerning the properties of the spurions.

In the first of these possibilities the spurions are assumed to be symmetrical under \( PG \), but to vary under \( P, G \) separately. This case has been studied in detail in a former paper (1,3). It was selected from the other possibilities because it represented at first sight group-theoretically the simplest case. The spurions there transform according to the group \( P \times SU_3 \) with \( P \) the discrete reflection group. This theory allowed predictions on the sequence of the baryon masses simply on group-theoretical grounds. Recent experimental results on the \( \Lambda \Sigma \)-parity and the spin and parity of \( Y_1^+ \) contradict, however, the theoretical results of this paper. A theoretical study of the charge properties of the particles in the framework of this theory also indicated that the hypercharge property could not be incorporated without enlarging the group space of the spurions. Therefore in the present investigation the second possibility will be examined. This second case is simpler than the enlarged first case (which would be equivalent to the third possibility listed) since the spurions are here assumed to be symmetrical under the parity operation \( P \), and hence do not have any connection with space and time. It is therefore sufficient to consider only the four states \( s_1, s_2 \) and \( a_1, a_2 \) which transform according to the 2-dimensional representation of the group \( U_2 = G \times SU_2 \), where \( G \) refers to a spurion-number gauge group, and \( SU_2 \) to the isospin rotations.
In the following we will first study the general properties of a system consisting of particles created by field operators and of spurions, and then apply these rules to the simplest strange baryons and bosons.

1. - Systems of many spurions and antispurions.

For each spurion or antispurion it will be convenient to introduce the Pauli matrices $\tau_k$, by which rotations in isospace may be represented. If the system contains many spurions or antispurions, the question arises, how the wave functions and the operators in the eigenvalue equation should behave under permutation of the spurions or antispurions. The spurions have no Dirac spin; therefore they should obey Bose statistics. In this case the total isospin of $n_s$ spurions must be $\frac{1}{2}n_s$, that of $n_a$ antispurions $\frac{1}{2}n_a$. The wave function of the system is then completely determined by the total isospin and by the numbers $n_s$ and $n_a$. It is questionable, however, whether the isospin connected with the field operator, i.e. the index of the field operator, should in this connection be counted as a spurion, or whether it should be treated separately. We will assume for the moment, that the isospin part of the field operator $\gamma_s$ represents a spurion as well, i.e. $\gamma_s^*$ means the creation, $\gamma_s$ the annihilation of a state with a spurion. This assumption will be discussed in detail later.

Finally we should expect that the addition of a spurion-antispurion pair with zero isospin should not cause any change in the system; that means, that the state of the system should depend, besides isospin, only on the difference $n_s - n_a$ and not on their total number $n = n_s + n_a$. It has been pointed out in earlier papers (*) , that

$$u = \frac{1}{2}(n_s - n_a) = \frac{1}{2}Y,$$

$$Y = n_s - n_a,$$

can be identified with the hypercharge (*) of the system.

An operator $O(n_s, n_a)$ constructed for a system of $n_s$ spurions and $n_a$ antispurions hence shall only explicitly depend on $u = \frac{1}{2}(n_s - n_a)$. Furthermore it shall have the property that under $G$-conjugation of $n_s$ spurions and $n_a$ antispurions it shall transform like

$$O(n_s, n_a) \rightarrow O(n_s - n_a' + n_a', n_a + n_a' - n_a'),$$

i.e. in particular under $G$-conjugation of an equal number of spurions and

(*) The word "hypercharge" is used here in its actual sense whereas commonly $2u = Y$ is called hypercharge.
antispurions

\[ O(n_s, n_a) \rightarrow O(n_s, n_a) . \]

This important property arises from the fact that these operators are related to matrix elements, in which spurions and antispurions can be arbitrarily distributed with appropriate meaning on the bra and ket side, as expressed by eq. (9).

If we restrict ourselves to operators which only take 2-spurion correlations into account, it can be seen that the operator

\[ O(n_s, n_a) = \left( \sum_{s=1}^{n_s} \sum_{a=1}^{n_a} \tau_s - \sum_{a=1}^{n_a} \tau_a \right)^2 - f(n) = 2 \sum_{i<j} \epsilon_{ij} \tau_i \tau_j + [3n - f(n)] , \]

with

\[ \epsilon_{ij} = \begin{cases} +1 & \text{like spurions,} \\ -1 & \text{unlike spurions,} \end{cases} \]

is the only one which fulfills the above requirements, if the function \( f(n_s + n_a) \) can be chosen in such a manner that the eigenvalues of \( O \) depend only on the total isospin \( I \) and the hypercharge \( u \), but not on \( n \). The first sum runs over all \( \tau \)-matrices connected with spurions, the second sum over all antispurion matrices. From (15) one gets

\[ O(n_s, n_a) = - \left( \sum_{s=1}^{n_s} \tau_s + \sum_{a=1}^{n_a} \tau_a \right)^2 + 2n_a(n_s + 2) + 2n_s(n_a + 2) - f(n) = -4[I(I+1)] + (n_s - n_a)^2 + (n_s + n_a)^2 + 4(n_s + n_a) - f(n) = -4[I(I+1) - w^2] , \]

if one puts

\[ f(n) = n(n + 1) . \]

The operator \( O \) is also invariant under charge conjugation. Besides containing the operator \( O \), an eigenvalue equation may also depend on \( u \). But \( u \) changes sign under charge conjugation. Hence it may occur only in the product \( u \cdot B \), where \( B \) is the baryonic number of the system. Therefore an eigenvalue equation may finally contain any function of \( O \) and \( uB \). It is remarkable that these results are very similar to those obtained from the particular breakdown of the \( SU_3 \) symmetry discussed in the literature. It should be emphasized, however, that our result, at this point, by no means indicates any immediate connection to \( SU_3 \) symmetry, but rather states that the particular connection between \( I \) and \( u \), which occurs in the broken \( SU_3 \) group, is here
due to a permutation property. Characteristic for SU, is only the particular grouping of states into 1, 3, 8 etc., multiplets which reflect the various irreducible representations of SU, and produce limitations on the possible values of \( I \) and \( u \) in the operators for the various representations. Such limitations cannot be given in our case on purely group-theoretical grounds, except \( I > |u| \), but should arise from dynamical reasons. Even the latter rule will not be valid, in general, for more complicated systems, as will be seen later.

We have to come back now to the problem, whether the isospin connected with a field operator can be treated on the same footing as a «spurion». One can see at once that two field operators acting at different points in space-time may produce a state of isospin zero, i.e. a state in which the two isospin variables are connected antisymmetrically. Therefore it would have no meaning to introduce the permutation of these isospin variables as a possible operation and to postulate Bose statistics for them. We get however a different result, if we consider field operators at the same point in space and time. In a theory starting from the fundamental field equation

\[
-i\sigma^\mu \frac{\partial}{\partial x^\mu} \chi + V_{\sigma^\mu} \chi^* \sigma_\mu \chi = 0,
\]

interaction takes place only by means of the expression \( \sigma^\mu \chi(x)(\chi^*(x)\sigma_\mu \chi(x)) \), which may be considered as a contact interaction of bare particles. In this term the isospins of the two operators \( \chi(x) \) are connected only symmetrically, they are necessarily «parallel». This can be seen from the fact, that the operator in Dirac spin space

\[
\sigma^\mu \sigma_\mu = -11 + \sigma_k \sigma_k
\]

has the eigenvalues 0 and \(-4\); it vanishes for the total Dirac spin 1, and is \(-4\) for Dirac spin 0. Therefore the interaction term is different from zero only if the Dirac spin variables are connected antisymmetrically. Then it follows from Fermi statistics (anticommutation relations for field operators), that the iso-spin variables must be connected symmetrically. This can also be immediately deduced from the «Fierz-symmetrical» form of the interaction

\[
\sigma^\mu \sigma_\mu \rightarrow \frac{1}{4}(311 + \tau \tau)\sigma^\mu \sigma_\mu,
\]

which vanishes for \( \tau \tau = -311 \) (isosinglet). The physical aspect of this situation is the fact, that two «bare» (2-component) particles can collide only if in the centre-of-mass system their spins are opposite to each other; i.e. if they are connected antisymmetrically. Because in the c.m. system the momenta of the two particles are opposite, in each bare particle the spin has the direction of the momentum, therefore the two spins are opposite. The two isospins
must be parallel. The mathematical reason for this result is the fact, that from
two Weyl spinors $a_1$ and $b_1$ with equal transformation properties in Lorentz
space one can construct a scalar $a_1 b_2 - a_2 b_1$, but no vector. From $a^*$ and $b$
one can construct the well-known vector $a^* \sigma \mu b$, but no scalar.

Therefore the fundamental field equation (18) allows the definition that all
iso-spins (isospin in field operators or spurions and antispurions) connected with
the same point in space-time are of the same type, in each of the two groups
they obey Bose statistics and are connected symmetrically, as indicated in
(12)-(17). If we start with a state of the system, in which this is true at a
certain time, eq. (18) will ensure that it remains true for all times.

This assumption connects the spurions with definite points in space-time
and suggests the following way of writing. Let now $\chi^*_{\alpha}$ be the hermitian con-
jugate of $\chi_\alpha$, the dot over the isospin index in $\chi^*_{\alpha}$ indicating, that $\chi^*_{\alpha}$ has
different transformation properties in isospace (rotations and gauge transforma-
tions) from $\chi_\alpha$. We then define (*)

\begin{equation}
\langle A | \chi_\alpha(x) | B, s_\alpha^* s_\alpha \rangle = \langle A s_\alpha^* \chi_\alpha(x) | B \rangle = \langle A | \chi_{a\beta\mu}(x) | B \rangle,
\end{equation}

with

\begin{equation}
\chi_{a\beta\mu} = \chi_{a\beta\mu} = \chi_{\mu a\beta} = \cdots,
\end{equation}

or

\begin{equation}
\langle A | \chi_\alpha(x) \chi^*_\beta(x) \chi_\gamma(x) | B s_\alpha^* s_\beta \rangle = \langle A | \chi_{a\beta\mu}(x) \chi^*_\beta(x) \chi_\gamma(x) | B \rangle = \langle A | \chi_\alpha(x) \chi^*_\beta(x) \chi_\gamma(x) | B \rangle
\end{equation}

again with complete permutation symmetry in $\alpha \beta \gamma \mu$. The operators $\chi_{a\beta\gamma\mu}(x)$ etc.,
deﬁned in this way may be considered as secondary ﬁeld operators producing
bare $\Lambda$- or $\Sigma$-particles etc. In expressions that contain only ﬁeld operators
at the same point, a pair of indices $\alpha \beta \gamma \mu$ may be added without change, following
the rules laid down in (12)-(17), since it is only the total isospin and hyper-
charge carried by the operator that counts. These general assumptions should
be sufﬁcient to calculate mass eigenvalues, scattering amplitudes etc., for
strange particles.

2. – Strange bosons.

The integral equation which determines the mass eigenvalues of the system
will, according to (12)-(17), generally depend on the two invariants $I(I+1) - u^2$

(*) Note that a transformation on the operator has the same effect as the inverse
transformation on the state vector. The undotted one-spurion state $\left| s_\alpha \alpha \right> = (S^*)_\alpha \left| 0 \right>$, with $S^*$ the spurion creation operator, transforms like a dotted operator index.
and \(Bu\), if we consider only systems which are constructed from field operators at the same space-time point. For the bosons of baryonic number zero the second invariant vanishes and only the first term remains. Therefore the calculation of the mass eigenvalues is especially simple for these bosons. In the former papers the eigenvalue equation for \(\pi\)- and \(\gamma\)-mesons had been derived in a first approximation. The two equations can be combined in the form

\[
[1 - K_I(X^2)] \varphi_I = 0
\]

for the one-meson matrix element

\[
\varphi_I = \langle 0 | x(x) x^*(x) | I, p \rangle, \quad X^2 = \lambda = -\frac{p^2}{\kappa^2}, \quad \text{with} \quad I = \begin{cases} 0 & \text{for } \eta \\ 1 & \text{for } \pi \end{cases}
\]

and

\[
K_I(\lambda) = -2Z_I Q(\lambda) = -\frac{Z_I}{\kappa^2} \left( \frac{\kappa}{2\pi} \right)^2 q_0(\lambda),
\]

where \(Z_I\) contains the isospin dependence

\[
Z_I = 3 - I(I + 1)
\]

and \(q_0(\lambda)\) is a function characteristic for the spin = 0 solutions

\[
q_0(\lambda) = q_0 \left( -\frac{p^2}{\kappa^2} \right) = \frac{2i\kappa^2}{\pi^2 p^2} \int dq \frac{p^2 - qp}{(p - q)^2(q^2 + \kappa^2)q^2} = \left[ \ln |\lambda| - \frac{1}{\lambda} - \frac{(1 - \lambda)^2}{\lambda^2} \ln |1 - \lambda| \right],
\]

which only depends on \(\lambda = -(p^2/\kappa^2)\) with \(\kappa\) the average baryon mass.

Equation (23) refers only to the normal bosons of hypercharge \(u = 0\). The form of \(Z_I\) arises directly from the particular isospin dependence \(3 + \tau\tau\) of the original interaction. It is also given by

\[
Z_I = (I_I - I_u)^2 = \frac{1}{4}(\tau_\mu - \tau_\mu)^2 = \frac{1}{4}n^2 + n - I(I + 1) \quad \text{with} \quad n = 2.
\]

One may now include the K-mesons by formally generalizing the \(I(I + 1)\) dependence to the characteristic dependence \(I(I + 1) - u^2\) established in (16) and hence write

\[
Z_{I,u} = 3 - [I(I + 1) - u^2].
\]

This \(Z_{I,u}\) has to be used in the kernel function \(K_{I,u}\) of a generalized eigenvalue
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equation:

\[ [1 - K_{I,u}(\lambda)] \varphi_{I,u} = 0 \]

with

\[ K_{I,u}(\lambda) = - \frac{1}{2} Z_{I,u} \left( \frac{x_I}{2 \pi} \right)^2 \varphi_0(\lambda) . \]

The matrix element \( \varphi_{I,u} \) then will involve, in general, spurion states.

In particular we get the following \( Z \)-factors:

\[
\begin{align*}
Z_\eta &= 3 \quad \text{for } \eta\text{-meson } (I = 0, u = 0), \\
Z_\pi &= 1 \quad \text{for } \pi\text{-meson } (I = 1, u = 0), \\
Z_K &= \frac{3}{2} \quad \text{for } K\text{-meson } (I = \frac{1}{2}, u = \frac{1}{2}).
\end{align*}
\]

The resulting numerical values for the corresponding masses of the particles will be discussed in the fourth Section of the present paper.

We have not included in the above discussion states with higher values of \( I \) and \( u \) (satisfying the restriction \( I > u \)). E.g., we also could use the above formula for a \( K \)-quartet state \( I = \frac{3}{2}, u = \pm \frac{1}{2} \) and would get \( Z_\eta = - \frac{1}{2} \), i.e. a negative number for which no solution of the eigenvalue equation can be found. However, application of the formula with higher values of \( u \) would lead to possible solutions which have no experimental counterpart.

Therefore the question arises whether above formal generalization is a valid procedure in the general case, or has to be restricted to some, e.g. the lowest \( I \) and \( u \) values. To answer this question we should try to establish a procedure to derive rather than formally assume the eigenvalue equation for strange particles. This can be done in the following way:

The eigenfunction of \( K \)-mesons can be represented by the one-particle matrix element:

\[
\varphi_{\alpha \beta} = \langle s_\gamma^* | \chi_\alpha(x) \chi_\beta^*(x) | K \rangle = \langle 0 | \chi_\alpha(x) \chi_\beta^*(x) | K \rangle = \langle 0 | \chi_\alpha(x) \chi_\beta^*(x) | K \rangle,
\]

which by definition of the \( K \)-meson state is symmetrical under the exchange of \( \alpha \) and \( \gamma \).

If we indicate symmetrization by \( \sim \) and antisymmetrization by \( \widetilde{\sim} \), we can immediately show by Clebsch-Gordan algebra

\[
\varphi_{\alpha \beta} |_{I=\frac{1}{2}} = \frac{\sqrt{3}}{2} \varphi_{\gamma \alpha \beta} + \frac{1}{2} \varphi_{\gamma \alpha \beta}.
\]

This indicates that the \( K \)-meson wave function can be decomposed into two
parts, the one of which means the combination spurion–γ-meson, the other spurion–π-meson. In the norm of the K-meson state the weight of the «γ» is three times larger than that of the «π». As a consequence the eigenvalue operator will be multiplied by a Z-factor which is constructed from the γ and π factors as

\[ Z_K = \frac{3}{4} Z_γ + \frac{1}{4} Z_π = \frac{3}{4} \cdot 3 + \frac{1}{4} \cdot 1 = \frac{9}{4}, \]

since the spurion does not «participate» in the interaction. Hence we get exactly the value we expected.

In the derivation of this result we have assumed that in a first approximation of the 2-point function, which occurs as contraction function in the construction of the eigenvalue operator, any binding terms between «nucleons» and spurions can be neglected, i.e. that the masses of the Λ, Σ, Ξ particles can be considered as approximately equal. The mass \( z \) occurring in \( K_{\gamma K} \) then refers to some average baryon mass. This seems justified, since in the approximation of the former calculations, using graphs of the type \( \gamma \gamma \) for the fermions, the assumption of exact equality of the fermion masses would give consistent results. For the bosons, however, already the first approximation leads to a strong dependence of the masses on the isospin, the mass of the γ-meson comes out roughly four times larger than the π-mass. There is no reasonable approximation in which the boson masses could be considered as nearly equal.

If we apply the procedure outlined above to a K-quartet state \((I=\frac{3}{2}, u=\frac{1}{2})\) we obtain the same Z-factor as for the π-meson, since

\[ \mathcal{P}_{\gamma\gamma K} \big|_{u=\frac{1}{2}} = \mathcal{P}_{\gamma\gamma K} \]

contains only spurion π-contributions. The spurion in this case is only formally attached, it is not bound and hence not localized. A physically meaningful K-quartet could only be obtained if \( I = 2, u = 0 \) states would be included in the original (zero spurion) discussion. The K-quartet then could appear as a combination of a spurion-π and a spurion-(I=2) meson state. However, a \( I = 2 \) eigenfunction has to involve from the outset matrix elements of at least 4 field operators, and hence poses a different dynamical problem.

This can be immediately generalized: For the consideration of a system with isospin \( I \) and strangeness \( S=2u-B \) at least matrix elements with \( n=2I+|S| \) field operators have to be taken into account. Or, expressed in a different way: In an approximate calculation of the eigenvalue operators for bosons of baryon number \( B \) which explicitly considers only matrix elements of \( \chi, \chi^* \) operator products up to \( n \), only eigenvalues for states with isospin \( I \) and strangeness \( S \) with \( 2I+|S| \ll n \) can be discussed.
On the other hand states which can be built up from field operators at the same space-time point ("$S$"-states) may be of principal importance, since they can take advantage of the original contact interaction. Such states we may call "primary states" or "primary particles". For such states an approximation of the $NTD$-type may be reasonable. For the other states, which we may term "secondary states" the original interaction will be ineffective in their formation but the secondary finite-range interactions which arise through the exchange of "primary particles" will be most important. Such states may be better approximated by methods which consider the primary particles as "elementary".

Due to the particular form of the differential equation the system of coupled matrix elements will never involve matrix elements with more than three field operators at the same point. For bosons therefore only $n=2$ operators are at equal points. For baryon number $B = 0$ we then have the limitation for "primary bosons".

\begin{equation}
I + \frac{1}{2} |\mathbf{s}| = I + |u| \text{ integer } \leq 1.
\end{equation}

Together with the condition $|u| < I$ this allows only the eight meson states $u = 0; I = 0, 1$ ($\eta, \pi$) and $u = \pm \frac{1}{2}, I = \frac{3}{2}$ ($K, \bar{K}$) to be considered "primary" in the above sense. Similarly the $\phi, \omega$ and $K^*$—which we have not discussed up to now—may be labelled "primary states", at least to the extent that they can be considered $^8S$ rather than $^3D$ states of the $\chi(x)\chi^*(x)$ combination.

The spurion concept therefore as used above permits us to construct new states beyond the normal (zero-spurion) states. These new states appear as "mixtures" of two (or more) normal states. The K-meson is a "mixture" of the primary particles $\pi$ and $\eta$.

Using this derivation of the eigenvalue equations for strange particles we may understand the fundamental octet grouping of the pseudoscalar (and vector) bosons without implying $SU_3$-symmetry. The bosons within our "octet" are not degenerate in mass. In fact, the mass difference between $\pi$ and $\eta$, which results from the original interaction, is even necessary for the "binding" of the spurion, and hence for the existence of the K's. The well-known $SU_3$-violating operators occur in first approximation as factors of the eigenvalue operators in the eigenvalue equations of the particles, and hence will lead to the Gell-Mann–Okubo mass formulas if the mass dependence of these operators is sufficiently smooth.

The superficial similarity to the $SU_3$ description of the bosons can be more clearly understood if we consider the operators which allow us to transform the components of the boson octet into each other which should be compared with the eight operators of the $SU_3$ Lie algebra. We introduce the spurion creation and annihilation operators $s_\alpha^*$, $s_\alpha$ and the corresponding antispurion
operators $a^\alpha_\xi$, $a^\alpha_\eta$ where the index refers to the isospin projection, and define the operators indicated in the figure ($I_\pm = I_x \pm iI_y$, $I_0 = I_z$)

\begin{align}
\begin{cases}
  u &= \frac{1}{2} \left( s_1^* s_1 + s_2^* s_2 - a_1^* a_1 - a_2^* a_2 \right), \\
  I_0 &= \frac{1}{2} \left( s_1^* s_1 - s_2^* s_2 + a_1^* a_1 - a_2^* a_2 \right), \\
  I_+ &= s_1^* s_2 + a_1^* a_2, \\
  I_- &= s_2^* a_1 + a_2^* a_2^*.
\end{cases}
\end{align}

(37)

This is unproblematic. According to our construction of the K-mesons from the $\pi$ and $\eta$ we may be inclined to define the strangeness changing operators simply in the following way:

\begin{align}
\begin{cases}
  U_+ &= s_2^* + a_1, \\
  U_- &= s_2 + a_1^*, \\
  V_+ &= s_1 + a_2^*, \\
  V_- &= s_1^* + a_2.
\end{cases}
\end{align}

(38)

With the help of the boson commutation relations for the spurion operators $[s_\alpha, s_\beta^*] = \delta_{\alpha\beta}, [a_\alpha, a_\beta^*] = \delta_{\alpha\beta}$ we immediately establish the commutation relations:

\begin{align}
\begin{cases}
  [I_0, I_\pm] = \pm I_\pm, & [I_0, U_\pm] = \mp \frac{1}{2} U_\pm, & [I_0, V_\pm] = \mp \frac{1}{2} V_\pm, \\
  [u, I_\pm] = 0, & [u, U_\pm] = \pm \frac{1}{2} U_\pm, & [u, V_\pm] = \mp \frac{1}{2} V_\pm,
\end{cases}
\end{align}

(39)

and further

\begin{align}
\begin{cases}
  [I_\pm, U_\mp] = [I_\pm, V_\mp] = 0, & [I_\pm, U_\pm] = \pm V_\mp, \\
  [I_\pm, I_\pm] = 2I_0, & [I_\pm, V_\pm] = \mp U_\mp,
\end{cases}
\end{align}

(40)

which agree with the corresponding relations of the $SU_3$-generators.
The commutation relations

\[
\begin{align*}
[U_-, U_+] &= -[V_-, V_+] = 1, \\
[V_-, V_+] &= [U_+, V_+] = 0,
\end{align*}
\]

however are different. The corresponding \(SU_5\)-generators fulfill the relations:

\[
\begin{align*}
[U_-, U_+] &= I_0 - 3u, \\
[V_-, V_+] &= I_0 + 3u, \\
[U_-, V_-] &= -I_+, \\
[U_+, V_+] &= I_+.
\end{align*}
\]

This difference is due to the fact that our definition (38) of the operators changes the number of spurious or antispurions. But the connection of even and odd spurion number states is rather queer. We could formally overcome this difference if we connect with the creation of a spurion or an antispurion of a certain type an annihilation of a spurion of any kind «far away». We may remember in this connection that the transition from a \(\pi, \gamma\) to a \(K\)-system is always connected with a compensating transition, e.g. from an \(N\) to a \(\Lambda\) system. Hence we may redefine

\[
\begin{align*}
U_+ &= s^*_1 s + a^*_1 a, \\
U_- &= s^*_2 s + a^*_2 a, \\
V_+ &= s^*_1 s + a^*_2 a, \\
V_- &= s^*_2 s + a^*_1 a,
\end{align*}
\]

where \(s^*, s, a^*, a\) are operators of spurious and antispurions with polarization 1 or 2 which are not connected with the system under consideration but indicate the compensating transition of the «vacuum» (or «far away»). Then \(N_s = s^*_1 s_1 + s^*_2 s_2 + s^* s\) and \(N_a = a^*_1 a_1 + a^*_2 a_2 + a^* a\) will commute with all operators. For the commutation relations (38) we now get

\[
\begin{align*}
[U_-, U_+] &= (N_s - N_a) + I_0 - 3u, \\
[V_-, V_+] &= -(N_s - N_a) + I_0 + 3u, \\
[U_-, V_-] &= -I_+, \\
[U_+, V_+] &= I_+.
\end{align*}
\]
For systems with \( N_s = N_b \) the operators then behave exactly like the \( SU_2 \) generators.

Although the above derivation of the generalized eigenvalue equation for the bosons leads for the relevant configurations to a form we expected from the more general considerations of Sect. 1, it is nevertheless hard to see how the above result can be interpreted as an immediate consequence of these general considerations. The \([I(I + 1) - \mu^2]\)-dependence seems to arise here in a very different way from the symmetry property of the spurions. Since the second interpretation allows very naturally a restriction of the possible \( \mu, I \)-values which agrees with our empirical knowledge, it may suggest that the general considerations of Sect. 1 give only a limited insight into the intrinsic reason for the mass-formula.

3. – Strange baryons.

If in the 2-point functions the mass differences between \( N, \Lambda, \Sigma, \Xi \) are neglected, the procedure used in the derivation of a generalized boson eigenvalue equation does not give anything new in case of the baryons since no explicit isospin-dependence occurs here in lowest approximation. According to earlier calculations (4) we have the baryon eigenvalue equation

\[
\sigma_s J^p \left[ 1 + \frac{3}{2} \left( \frac{\mu^2}{2\pi} \right)^4 L \left( -\frac{J^p}{\mu^2} \right) \right] \varphi(J) = 0
\]

with \( \mu \) the average baryon mass. A splitting of the mass eigenvalues is produced first by terms which correspond to virtual creation and annihilation of a boson in the well-known fermion self-energy graph \( \overline{\Phi} \); in our theory such terms are special 4-point contraction terms which have poles at the boson masses. The terms corresponding to \( \gamma \)- and \( \pi \)-meson contributions, however, will not cause a splitting of the masses. This is obvious for the \( \gamma \)-meson which has isospin 0, and will be created and annihilated by all baryons concerned in the same manner. It is also true for the \( \pi \), since the \( \pi \) cannot take up any spurions from the strange baryons. Finally it is only the K-meson which is responsible for the splitting, like in some older, more conventional theories, e.g. global theory.

Graphically the baryon eigenvalue equation is now represented by the fol-
following equation:

\[(46) \quad \mathcal{T} = \mathcal{T}_1 + \mathcal{T}_2\]

where \(\mathcal{T}_1 = \langle 0 \mid T\chi(x)\chi^*(x') \mid 0 \rangle\) represent 2-point contractions,

\[\mathcal{T}_2 = \langle 0 \mid T\chi(x)\chi^*(x)\chi(x')\chi^*(x') \mid 0 \rangle\]

special 4-point contractions. If the second graph is neglected, and the first approximated by 2-point contractions corresponding to the dipole ghost regularized baryon pole we obtain the eigenvalue eq. (45) which establishes a self-consistent average baryon mass (« baryon-bootstrap »). In the present calculation we consider contributions from the second graph as well, however only in so far as they arise from virtual mesons which occur as solutions in the lowest boson approximation, i.e. in particular from \(\eta, \pi\) and \(K\), and also later from \(\omega, \varphi, K^*\). The latter will, however, not be considered in the present paper, since as stated in earlier papers, the lowest approximation would be still too primitive for vector mesons. The suggested approximation scheme will be valid only if the contribution of the second graph will be smaller than that of the first one. The calculation of the second graph can be divided into an algebraic and an analytical part.

3.1. Isospin-hypercharge dependence. – The algebraic part consists in the easy—although cumbersome—calculation of the isospin-hypercharge dependence of the graph. This is of principal importance since it will be responsible for the form of the mass splitting. From our general considerations in Sect. 1 we expect this dependence to be of the form \(c_0 + c_1Bu + c_2[I(I + 1) - u^2]\). This task will be accomplished in two different ways: 1) By \(\tau\)-matrix algebra using projection operators for the various states. 2) By using Clebsch-Gordan coefficients and the « change of coupling transformations ».

The former method is simpler for the present problem since only states with small isospin and hypercharge are involved. The second method, however, presents itself more readily to generalization for more complicated situations, and for the calculation of coupling constants. It will be treated in a subsequent paper (5). Here we will indicate shortly the first method with the help of graphs. The details of these calculations will be given in the Appendix.

Algebraically the second graph has the basic structure given by Fig. 2, where the line indicates the sequence of multiplication of $\tau$-matrices. The arrow is connected with the hypercharge. The $i$ contain the isospin-dependence of the vertex in the nonlinear equation which we write in the Fierz-symmetrical form

$$O_i O_i = \frac{1}{\sqrt{24}} (3 \mathbf{11} + \tau \tau) = \sqrt{\frac{2}{3}} \frac{1}{4} (3 \mathbf{11} + \tau \tau)$$

(the factor $\sqrt{\frac{2}{3}}$ is added for computational convenience). Depending whether we are interested in the $\eta$- or $\pi$-contribution, in the rectangular boxes the $\eta$- or $\pi$-meson projection operator has to be inserted, which has the general symbolical form $PP$; e.g. $P_\eta P_\pi = \frac{1}{4} [\mathbf{11} + \tau \tau]$. Above diagram is contained as a part in all the other diagrams we have to calculate. It depends, however, on the special form of the projection operator $PP$. Hence we introduce the following abbreviation for the graph:

$$[PP] = O_i P O_i P O_i .$$

If we sum over all meson projections we simply have $\sum PP = \mathbf{11}$, and get $[\mathbf{11}] = 1$, which proves definition (47) particularly convenient.

To calculate the isospin hypercharge dependence for the various baryons: $\mathcal{N}, \Lambda, \Sigma, \Xi$ due to virtual $\eta$- and $\pi$-meson, we have to consider the graphs of Fig. 3. Here the spurions, which have to be added to define strange baryons, are indicated by thin lines. They «run right through» without being affected by the $\gamma, \pi$ projection operators, and hence we get, as mentioned above, the same contribution for the 0-, 1- and 2-spurion system, i.e. simply $[PP]$ which

---

**Fig. 3.**

yields:

\[
\begin{align*}
C_{\eta,\eta} &= [P_\eta P_\eta] = \frac{3}{4}, \\
C_{\pi,\pi} &= [P_\pi P_\pi] = \frac{1}{4}.
\end{align*}
\]

We have to remark here on the identification of the various spurion systems with known baryons. It is by no means clear from the beginning that the zero spurion system, i.e. our original system of fermions of spin \( \frac{1}{2} \) and isospin \( \frac{1}{2} \) which transforms like the field operator \( \chi(x) \), should be identified with the nucleon doublet rather than the \( \Xi \)-doublet, i.e. it is not clear whether our hypercharge \( u \), which by definition is related to the charge number \( Q \) through the relation \( Q = u + I_6 \) — and hence has to be identified with \( u = \frac{1}{2} Y \) in the more conventional notation — should be written as \( u = \frac{1}{2} B \) or \( u = -\frac{1}{2} B \) (\( B \) = baryon number) in case of \( \chi(x) \). The decision between the two possibilities can only be given by dynamics. The system of lower mass has to be called "nucleon", the system of higher mass "\( \Xi \)", independent of which can be represented as a zero- or a two-spurion system. Our dynamical calculation, in fact, will indicate that for \( \chi(x) \) the connection \( u = -\frac{1}{2} B \) should be taken. This means that the zero-spurion system should be identified with the \( \Xi \) rather than the nucleons (\(*\)). This is also indicated in the diagrams. The negative sense (downward) of the arrow of the fat line refers to \( u = -\frac{1}{2} \) for the baryon (\( B = +1 \)) \( \Xi \) (the operator \( \chi(x) \)).

![Fig. 4. - K-contribution for zero-spurion fermion.](image1)

![Fig. 5. - K-contribution for one-spurion fermion.](image2)

To calculate the isospin-hypercharge dependence which results from the virtual emission and absorption of a K meson the graphs given by Fig. 4-6 have to be considered. For the \( \bar{K} \)-meson the graphs are given by Fig. 7-9.

\[ (*) \text{Accidentally it may be remarked that with this identification we have a greater similarity between baryon and lepton states.} \]
The zero spurion graph (Fig. 4), e.g. has to be interpreted in the way that at the first vertex a spurion-antispurion pair is created, where the spurion

![Diagram](image)

Fig. 6. – K-contribution for two-spurion fermion.

combines with the zero spurion boson to form the $K$. The antispurion goes along with the fermion to form some strangeness $S = -3$ configuration. Since

![Diagram](image)

Fig. 7. – $\bar{K}$-contribution for zero-spurion fermion.  
Fig. 8. – $\bar{K}$-contribution for one-spurion fermion.

this intermediate fermion line is a $G$-line it does not distinguish between discrete states and continuum states. This is a peculiarity of the Tamm-Dancoff method. At the second vertex the $\Xi$ is reformed.

![Diagram](image)

Fig. 9. – $\bar{K}$-contribution for two-spurion fermion.
For the one- and two-spurion fermions several possibilities present themselves. In the spirit of the Tamm-Dancoff method we have to sum over all these diagrams, since in a certain sense they represent different contractions of the spurion lines. The K-meson projection operator (as derived in the Appendix ref. (6)) is of the form

$$P_K P_K P_K = \frac{1}{4} [111 + \frac{5}{2} \tau \tau 1 + \frac{5}{2} 1 \tau \tau + \frac{1}{2} \tau 1 \tau].$$

For the graph (Fig. 4) we find, e.g.,

$$C_{\Xi,K}^1 = [P_K P_K](Tr P_K) = O_1 P_K O_1 P_K = \frac{7}{6}.$$

Similarly the other graphs can be evaluated. As shown in the Appendix, eqs. (A.24, 25), the results can be summarized in the following formulas:

$$C_{F,K} = \frac{13}{6} + \frac{31}{18} B u - \frac{5}{12} [I(I + 1) - u^2],$$

$$C_{F,K} = \frac{25}{12} + \frac{29}{18} B u - \frac{2}{9} [I(I + 1) - u^2],$$

or combined

$$C_{F,K} = \frac{17}{4} + \frac{10}{3} B u - \frac{1}{2} [I(I + 1) - u^2].$$

We denote that the isospin and hypercharge again appear exactly in the expected form. Still it seems somewhat surprising that the formal extension of the Tamm-Dancoff method to spurion lines should automatically work. We may, in particular, be interested whether in this case an immediate connection to the general considerations of Sect. 1 can be established. This, indeed, is the case. A closer investigation (see Appendix, eq. (A.23)) reveals that

$$C_{F,K} = \frac{7}{12} (n + 1) - \frac{5}{36} \sum_{i,j} \tau_i \tau_j,$$

i.e. is related to the isospin interaction terms $\sum \tau_i \tau_j$, which only contain spurion-antispurion terms, rather than $\sum_{i<j} \epsilon_{ij} \tau_i \tau_j (\epsilon_{ij} = \pm 1)$ of Sect. 1, eq. (15).

For the partial sums we find

\begin{align}
(55) \quad \sum_{s,s'} \tau_s \tau_{s'} + \sum_{a,a'} \tau_a \tau_{a'} &= \frac{1}{2} n^2 + 2n + 2u^2, \\
(56) \quad 2 \sum_s \tau_s \tau_a &= -\frac{1}{2} n^2 - 2n + 2u^2 + 4[I(I+1) - u^2].
\end{align}

The difference of these equations again leads to (16) of Sect. 1. For the \( C_{\nu,\kappa} \) the second sum is involved, however, only in the limited case that \( n_\ast = 1 \) for the particles \( (B = +1) \) and \( n_\ast = 1 \) for the antiparticles \( (B = -1) \), i.e. \( n = 2(1 + u) \) for \( B = +1 \) and \( n = 2(1 - u) \) for \( B = -1 \), or combined

\begin{equation}
(57) \quad n = 2(1 + Bu).
\end{equation}

Hence

\begin{equation}
(58) \quad \sum_{s,a} \tau_s \tau_a = -3 - 4Bu + 2[I(I+1) - u^2]
\end{equation}

or eq. (52) for \( C_{\nu,\kappa} \).

### 3.2. The modified fermion eigenvalue equation

We have now to deal with the analytical part of the second graph to establish the modified fermion eigenvalue equation. The calculation of the boson graph in eq. (46) proceeds in a similar fashion as the calculation of the first graph, except that the boson propagator has to be inserted as contraction function instead of the double fermion propagators \( FF \). The details of this calculation are given in the Appendix.

The boson contraction function is given, e.g., for the \( \gamma \) as

\begin{equation}
(59) \quad A_\gamma(x-x') = -\frac{i}{(2\pi)^4} \int d^4p \exp[ip(x-x')] \left[ \frac{1}{p^2 + \zeta^2} + \ldots \right],
\end{equation}

where the dots indicate necessary regularization terms and contributions from the continuum.

The 4-point contraction term in (46), and hence also the boson contraction function, should be automatically regularized, even in a theory with definite metric in Hilbert space, since in the most singular term the contribution of the bosons should be just compensated by the corrections of the continuum; this is, at least, true in unrelativistic quantum mechanics. This statement however, does not lead to any definite prescription concerning the regularizing terms. In the (fermion) 2-point functions we have used only contributions of mass zero for regularization. Therefore it looks natural to do the same for the 4-point functions, and hence the boson contraction function. Such an
assumption may involve serious errors in low Tamm-Dancoff approximations. Recent investigations of Stumpf, Wagner and Wahl (7) in connection with the anharmonic oscillator, however, seem to show that in higher Tamm-Dancoff approximations the special form of the contraction function becomes less and less important.

In the case we only consider zero-spurion systems, i.e. $\Xi$-particles with virtual $\gamma$ and $\pi$, we derive for the integral kernel corresponding to the boson graph in eq. (46)

\begin{equation}
\sum_B K_{FB} = \frac{1}{8\pi} \sum_B \left( \frac{f_{FB}^2}{4\pi} \right) r_0 \left( -\frac{J_B^2}{\chi_B^2} \right),
\end{equation}

with $f_{FB}$ ($F = \Xi$) essentially the vector coupling constant of the boson $B$ at the vertex. The $r_0$ is a function characteristic for the spin=0 bosons, which can be easily calculated, and is defined by

\begin{equation}
r_0 \left( -\frac{J_B^2}{\chi_B^2} \right) = i \frac{4\chi_B^2}{\pi^2} \int d^4p \left[ \frac{1}{2} g^{\mu\alpha} - p^{\mu} p^{\alpha}/p^2 \right] J_\rho(J-p)_\lambda
\end{equation}

\begin{equation}
= 1 + (2 - \lambda) \ln |\lambda| + \frac{(1 - \lambda)^2}{\lambda} \ln |1 - \lambda|
\end{equation}

with $\lambda = -J_B^2/\chi_B^2$.

For the coupling constants we can take in case of $\gamma$ and $\pi$ the values calculated by Dhar and Katayama (8) (there the zero-spurion fermion was identified with the nucleon). In the more general case where spurions are involved, however, the situation is somewhat more complicated. There $f_{FB}$ is related only to a certain sum over squares of coupling constants, and hence the explicit knowledge of these individual coupling constants is not necessary for the evaluation of $K_{FB}$. In the Appendix we show how $K_{FB}$ can be directly derived by interpreting the meson lines $\gamma \gamma \gamma$ as an infinite series of fermion-antifermion bubble graphs, i.e.

\begin{equation}
\gamma \gamma \gamma = \includegraphics{bubble1} + \includegraphics{bubble2} + \includegraphics{bubble3} + \cdots = \frac{\includegraphics{bubble1}}{1 - \includegraphics{bubble1}}
\end{equation}

Algebraically the bubble chains will all look like the first link, and hence will lead to the characteristic isospin-hypercharge dependence of Sect. 3'1. As will be shown in the Appendix (eq. (A.55)) we get the result:

\begin{equation}
K_{FB} = \frac{3}{2} \frac{C_{FB} r_0(-J_B^2/\chi_B^2)}{Z_B^2 q_0(\chi_B^2/\chi^2)}.
\end{equation}


with \( q'(\tfrac{x_0^2}{\kappa^2}) = \frac{dq(\lambda)}{d\lambda} \big|_{\lambda = \tfrac{1}{\kappa^2}} \) the derivative of the function (27), which occurs in the zero-spin boson eigenvalue eq. (23), at the corresponding boson mass; \( Z_\phi \) the corresponding factor (29) and \( C_{p, \text{b}} \) the isospin-hypercharge factor (eqs. (49), (52)) calculated in Sect. 3.1. A comparison of eq. (63) with eq. (60) will lead to an effective \( f_{p, \text{b}}^2 \) from which eventually the individual coupling constants can be extracted. This will be shown in detail in a subsequent paper (5).

If we combine the boson graph eq. (63) with the main graph (45) we obtain the modified fermion eigenvalue equation

\[
\sigma_p J^p \left[ 1 + \frac{3}{2} \left( \frac{xl}{2\pi} \right)^4 L \left( \frac{J_0^2}{x^2} \right) - \frac{3}{2} \sum_B \frac{C_{p, \text{b}} r_0 \left( -\frac{J^2}{\kappa_0^2} \right)}{q'(\tfrac{x_0^2}{\kappa^2})} \right] \phi = 0.
\]

The sum runs over all spin-0 boson states, i.e. \( \eta, \pi \) and \( K, \bar{K} \).

If we insert the \( C_{p, \text{b}} \) and \( Z_\phi \) values for \( \eta, \pi \) and \( K \) and approximate \( q'(\tfrac{x_0^2}{\kappa^2}) = \frac{\kappa_0^2}{x_0^2} \) (see Appendix, eq. (A.59)), we can write

\[
\sigma_p J^p \left[ 1 + \frac{3}{2} \left( \frac{xl}{2\pi} \right)^4 L \left( \frac{J_0^2}{x^2} \right) - \frac{1}{8} \frac{x_0^2 \kappa_0^2 r_0 \left( -\frac{J_0^2}{\kappa_0^2} \right)}{x^2 \kappa_0^2 r_0 \left( -\frac{J_0^2}{\kappa_0^2} \right)} - \frac{3}{8} \frac{x_0^2 \kappa_0^2 r_0 \left( -\frac{J_0^2}{\kappa_0^2} \right)}{x^2 \kappa_0^2 r_0 \left( -\frac{J_0^2}{\kappa_0^2} \right)} - \frac{6}{25} \frac{x_0^2}{x^2} \left\{ \frac{17}{4} + \frac{10}{3} Bu - \frac{1}{2} \left[ I(I + 1) - u^2 \right] \right\} r_0 \left( -\frac{J_0^2}{\kappa_0^2} \right) \phi(J) = 0.
\]

If we would neglect the boson-dependent terms, the \( xl \) could be determined from the baryon self-consistency requirement that the above equation is fulfilled for \( J^2 = -\kappa_0^2 \), i.e. (4.9)

\[
\frac{xl}{2\pi} = \left[ -\frac{3}{2} L(1) \right]^{-1} = \left[ \frac{3\pi}{4} \left( \frac{3}{8} - \frac{3\sqrt{3}}{8} \right) \right]^{-1}.
\]

This value will now be modified by the boson-contributions. Since \( r_0 \) is positive in the relevant region, \( xl \) will be smaller than in the former calculations.

We further establish that \( r_0 \) as a function of the argument has roughly the behaviour of \( -L \), and that \( (dL(\lambda)/d\lambda) / L(\lambda) > 0 \) and \( (dr_0(\lambda)/d\lambda) / r_0(\lambda) > 0 \) in the relevant region. As a consequence the masses of the baryons will be the smaller the bigger the isospin-hypercharge factor will be. Since

\[
\frac{10}{3} Bu - \frac{1}{2} \left[ I(I + 1) - u^2 \right] =
\begin{array}{c}
\frac{17}{12} \\
0 \\
-1 \\
-\frac{23}{12}
\end{array}
\]

for \( N^' \),

\( \Lambda \),

\( \Sigma \),

\( \Xi \),

the empirical mass sequence will result. The Gell-Mann–Okubo mass formulas would follow, if roughly $L^{-1}(x^2)$ depends linearly on $x$. This condition is not too well fulfilled in our case due to the fact that the baryon solution lies rather close to a minimum of the $L$-curve. The numerical values will be considered in the next Section. A comparison of the ratio of the constants

$$\frac{m_2}{m_1} \approx \frac{39 \text{ MeV}}{380 \text{ MeV}} \approx \frac{1}{10}$$

in the empirical mass formula

(68) \[ m = m_0 - m_1 B u + m_2 [I(I + 1) - u^2] \]

with the ratio of the corresponding constants in our case

$$\frac{C_2}{C_1} = \frac{1/2}{10/3} = \frac{3}{20}$$

indicates a fair agreement. As shown in the Appendix, eq. (A.24) and Table A.II, small contributions of the continuum states with other symmetries than the K (e.g. K-quartet) will further decrease this ratio.

We have not considered up to now the contributions of the vector mesons to the baryon masses. However, the vector mesons are not very well represented in the lowest Tamm-Dancoff approximation since the important $^3D$ contributions are not included. As a consequence their coupling constants cannot be derived at this stage. Nevertheless one can show that vector mesons with the empirical properties will further increase the tendency of the pseudoscalar mesons in all respects, and will be of similar importance for the baryon eigenvalues. Hence without a consideration of the vector bosons in higher approximations which are carried out by STUMPF and YAMAMOTO (10), the baryon mass problem cannot be treated in a satisfactory manner.

The above derivation of the baryon mass eigenvalues does not automatically present a restriction of the $I, u$-values for the baryons, as it happened in the boson case. The restriction to the eight fermions $N, \Lambda, \Sigma, \Xi$ is not suggested by the above procedure. In fact, we could similarly derive the eigenvalue of an $N$-quartet state which would fulfil the eigenvalue eq. (65) with $u = \frac{1}{3}$, $I = \frac{1}{3}$. We suspect, however, that the restriction to the «octet» arises from the condition, that a single fermion line can only be «loaded» maximally with a single electric charge. The $N$-quartet represented by a single field operator $\chi(x)$ with two spurions would have doubly charged components

which should be excluded. Because of the isospin invariance, hence the whole multiplet has to be omitted. This rule may seem rather artificial at first sight. However, one has to remember, that electrical charge is a localizable conserved quantity, whereas isospin, in general, is not. Local conservation can only be expressed in terms of the field operators. The theoretical justification of this rule has to come from a thorough investigation of the cluster decomposition of matrix elements for large mutual distances which shall not be discussed in the present paper.

4. - Numerical evaluation of the masses.

The calculations in the foregoing Sections are not sufficient for a complete derivation of the mass-eigenvalues. The masses of the fermions are considerably influenced by the virtual emission and reabsorption of vector bosons, and the coupling of these particles has not been studied in the present paper. What shall be done in direction of a numerical evaluation, is to take the results concerning the coupling of the pseudoscalar mesons and to show, that one obtains a reasonable picture explaining approximately the mass spectrum of bosons of spin 0, and giving—on account of the complete omission of the vector bosons—only qualitatively the mass splitting of the fermions.

The eigenvalue eq. (30) for the bosons contains, besides a known analytical function $q_0^2(\lambda)$ of $\lambda = -p^2/\kappa^2$ given by eq. (27), the constant $\kappa \lambda$ ($\kappa$ is the average mass of the fermions appearing in the 2-point function). In the first paper on the nonlinear spinor theory (4) neglecting the strange particles altogether the value $\kappa \lambda = 6.39$ had been derived from the postulate of consistency eq. (66). In a later paper on the spurion model of the strange particles (1) —using however a different and, as we now know, unjustified assumption about the symmetry of the spurions—the value $\kappa \lambda = 5.88$ had been given.

In the present paper it will be seen later, that the correction of the fermion masses by the emission and reabsorption of virtual bosons lowers the $\kappa \lambda$-value considerably. For the following calculations we will use the value $\kappa \lambda = 5.78$ without pretending that this is the best value with respect to the postulate of consistency. But the uncertainty concerning the coupling of the vector bosons and the inaccuracy of the lowest Tamm-Dancoff approximation do not allow a more accurate determination.

Equation (30) then gives for the boson-masses the values shown in Table I.

<table>
<thead>
<tr>
<th></th>
<th>$\pi$</th>
<th>$K$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{\kappa \lambda} = \kappa</td>
<td>\pi</td>
<td>$</td>
<td>0.198</td>
</tr>
</tbody>
</table>
The average fermion mass $z$ should empirically lie between 1 and 1.2 GeV. One sees that all boson masses turn out considerably too high, while their ratio agrees rather well with the observed values. Since the formula (31) for $K_{L, u}$ is the result of the lowest Tamm-Dancoff approximation, one could scarcely expect any better accuracy.

The eigenvalue equation for the fermions (65) contains, besides $z_1$, the analytical function $L$ depending on the ratio of the fermion mass and the average fermion mass $z$, furthermore as contribution from the pseudoscalar mesons the function $r_0$ depending on the ratio of the fermion mass and the boson mass. In eq. (65) we have already omitted the contributions of the vector bosons and of continuous pseudoscalar states having a different symmetry from $\pi$, $K$ or $\eta$ meson (e.g. isoquartet states).

![Fig. 10.](image)

The result of the numerical calculation is given in Fig. 10. The lower intersection of the curves with the abscissa defines the masses. The mass splitting is entirely due to the $K$-meson contribution. For the mass values one obtains the values of Table II. The fermion masses therefore turn out definitely too low as compared with the boson masses and as compared with the self-consistency requirement $(z_f/z)$ average = 1. This is mainly due to
the bad behaviour of \( L \) which does not allow the \( \Xi \) to become much heavier than the average mass without disappearing as a solution. However, a calculation which takes into account the different baryon masses in the \( L \)-function will improve the situation to some extent. The distance between \( \Lambda \) and \( \Sigma \) is somewhat too large, but would be lowered by pseudoscalar contributions from the continuous spectrum of different symmetry (the terms \( K' \) and \( K_0 \) in eq. (A.24) in the Appendix). The contribution of the vector bosons would probably lead to a still lower value of \( \tilde{\alpha} \) and could thereby reduce the boson masses somewhat. But it remains doubtful whether the ratio of boson masses to fermion masses could be sufficiently improved. Probably only the replacement of the analytical functions \( L, \gamma_0 \) and \( r_0 \) by more complicated functions in higher approximations could essentially improve the numerical agreement. In these higher approximations the second root of the fermion eigenvalue equation (at higher masses) would probably not appear, since the continuous spectrum would lead to a cut, corresponding to the simultaneous existence of a fermion and a boson, and would thereby change the analytical behaviour of \( L \) essentially. The order of the mass eigenvalues in Table II shows, as has been discussed already in Sect. 3'1 that the \( \Xi \)-particle should be considered as the particle produced by the field operator without spurious.

5. - Electromagnetic mass-corrections.

The mass differences between \( N \), \( \Lambda \), \( \Sigma \) and \( \Xi \) particles are largely due to the influence of pseudoscalar and vector bosons as virtual intermediate particles. Since this influence has been included in the calculations, it would be natural to include also the influence of the photons, \( i.e. \) the electromagnetic mass-corrections. The theory of the coupling of the vector bosons to the baryons however has not yet been worked out, neither for the heavy vector bosons nor for the photons. Therefore we will confine the considerations based on the present theory to a brief discussion of the group-theoretical properties of the electromagnetic mass terms which are not symmetrical in isospin space.

Since only the mass differences between components of different \( I_z \)-values can be observed immediately, it will be sufficient to take terms depending on \( I_z \) into account. In a first approximation (omitting terms with higher
powers of $I_3$ than $I_3^2$) one gets from the symmetry under $C$:

$$\langle A\kappa \rangle_{\text{symm}} = \alpha I_3 B + \beta I_3 u + \gamma I_3^2,$$

where $\alpha, \beta, \gamma$ are again functions of the invariants $I(I + 1) - u^2$ and $(uB)^2 = u^2$; odd powers of $uB$ are already included by allowing the two independent terms $\alpha I_3 B$ and $\beta I_3 u$ in (69). Since the term proportional to $I(I + 1) - u^2$ in the main mass formula (68) has only a small influence on the masses of the particles, one would expect that in a first approximation $\alpha, \beta, \gamma$ can be considered as constants, independent of $I$ and $u$. In this approximation one gets from (69) the formula

$$\kappa_{\Sigma^+} - \kappa_{\Sigma^-} = \kappa_{\Xi^+} - \kappa_{\Xi^0} + \kappa_0 - \kappa_a$$

derived earlier by Coleman and Glashow (11) from the assumption of an approximate validity of the $SU_3$ group. Here again one sees that the present spurion model of the strange particles leads to results very similar to those of the $SU_3$ group. The degree of approximation by which the formulas (68) and (69) are expected to hold, is determined in the $SU_3$ group by the level spacing within the octet in comparison with the distance between different representations of the $SU_3$ group; in the present theory the level spacing is to be compared with that range of $\kappa^2$ values, in which the analytical function $L(\kappa^2/\kappa^2)$ changes appreciably. This range may be measured by the ratio $L'/L''$. In both cases the empirical validity of the formulas (68) and (69) seems to be slightly better than one would expect. The mass distance between different representations of the $SU_3$ group (if the levels are interpreted in this way) is known empirically mostly in cases where the other properties, especially the Lorentz-group properties, may also be different. There the mass differences are of the order $\frac{1}{2}$ GeV, certainly not much larger than the level spacing within the single representation. In the present theory the range $L'/L''$ is also of the order $(\frac{1}{2}$ GeV)$^2$ in the critical region. It would however be premature to discuss the reasons for the smallness of the higher-order correction terms.

The empirical values of $\alpha, \beta, \gamma$ are

$$\alpha = -4.1 \text{ MeV}, \quad \beta = 5.4 \text{ MeV}, \quad \gamma = 0.3 \text{ MeV}.$$

They reflect the distribution of charge in the baryons. The charge is connected partly with the field operator, partly with the spurions, and the latter again predominantly with the K-mesons within the baryons. Therefore it can be well understood that the distribution of charge is rather different in the dif-

---

ferent particles of the octet group $N'$, $\Lambda$, $\Sigma$, $\Xi$. The difference between the masses of the charged and the neutral particle is 6.8 MeV for the $\Xi$ and only 1.3 MeV for the nucleon. This result seems to show, that the charge distribution is narrower in the $\Xi$ than in the nucleons in agreement with our general picture which implies that the nucleon consists of a $\Xi$ and two spurious and contains therefore a larger contribution from the virtual K-mesons than the $\Xi$.

**Appendix**

**A.1. – Isospin and hypercharge dependence of the fermion self-interaction due to virtual meson contributions.**

In this Section we derive the explicit isospin and hypercharge dependence of the fermion eigenvalue operator due to the virtual emission and absorption of mesons of the $\gamma$, $\pi$ and K type as indicated in Sect. 3'1 of the main part of this paper.

All the meson contribution graphs in eq. (46) contain as an algebraical substructure the «fat line» graph (Fig. 2), which directly results from the double application of the fundamental field equation and contains the characteristic (Fierz symmetrical) vertex eq. (47). In this graph which is formally expressed by eq. (48) we have not introduced any projection operators in the virtual fermion line. This is valid if all virtual fermion configurations are analytically alike, because then the summation over all configurations can be immediately performed. In the Tamm-Dancoff approximation the virtual fermion line is represented by a $G$-line which is the same for all configurations and hence allows this procedure. The vertex (47) was normalized in such a way that by summation over the various boson virtual state configurations (which, however, in the actual case will be modified by the meson mass differences) $\sum PP = 11$ we simply get

\[(A.1) \quad \sum PP = [11] = O_1O_2O_3O_4 = 1.\]

To calculate the graph for a particular boson we have to know the expression (48) for other values $PP$. As an intermediate step we calculate (48) for the combinations $\tau \tau$, $1\tau$, $\tau 1$:

\[(A.2) \quad [\tau \tau] = 21 , \]
\[(A.3) \quad [1\tau] = \frac{3}{2}\tau , \]
\[(A.4) \quad [\tau 1] = \frac{1}{2}\tau . \]

The projection operators of the various bosons (which are derived in the Appendix of (6)) are given by the following expressions:
Zero spurion systems:

\[
P_{\pi}P_{\pi} = \frac{1}{4}[11 + 2\tau \tau],
\]
\[
P_{\pi}P_{\pi} = \frac{1}{4}[311 - 2\tau \tau].
\]

One-spurion systems:

\[
P_{\pi}P_{\pi}P_{\pi} = \frac{1}{4}[111 + \frac{3}{2} \tau \tau 1 + \frac{3}{2} 1 \tau \tau + \frac{3}{2} \tau 1 \tau],
\]
\[
P_{\pi}P_{\pi}P_{\pi} = \frac{1}{4}[111 - \frac{3}{2} \tau \tau 1 - \frac{3}{2} 1 \tau \tau + \frac{3}{2} \tau 1 \tau],
\]
\[
\text{and}
\]
\[
P_{\pi}P_{\pi}P_{\pi} = \frac{1}{4}[111 + \frac{3}{2} \tau \tau 1 + \frac{3}{2} 1 \tau \tau + \frac{3}{2} \tau 1 \tau],
\]
\[
P_{\pi}P_{\pi}P_{\pi} = \frac{1}{4}[111 - \frac{3}{2} \tau \tau 1 - \frac{3}{2} 1 \tau \tau + \frac{3}{2} \tau 1 \tau],
\]

where we have again indicated \(\tau\)-spurions by fat lines, the vacuum spurions by thin lines. The configuration with antisymmetrical spurions we have called \(K'\), the quartet configuration \(K_q\). Both these states cannot occur as discrete boson states. With these projection operators we immediately get with (A.1-4)

\[
[P_{\pi}P_{\pi}]P_{\pi} = \frac{3}{4} 1,
\]
\[
[P_{\pi}P_{\pi}] = \frac{1}{4} 1,
\]

and

\[
[P_{\pi}P_{\pi}]P_{\pi} = \frac{3}{8}[\frac{5}{2} 11 + \frac{3}{2} \tau \tau],
\]
\[
[P_{\pi}P_{\pi}]P_{\pi} = \frac{3}{8}[\frac{5}{2} 11 - \frac{3}{2} \tau \tau],
\]

and

\[
[P_{\pi}P_{\pi}]P_{\pi} = \frac{3}{8}[\frac{5}{2} 11 + \frac{3}{2} \tau \tau],
\]
\[
[P_{\pi}P_{\pi}]P_{\pi} = \frac{3}{8}[\frac{5}{2} 11 - \tau \tau],
\]

where we have used the obvious abbreviation

\[
[PP]P = (0, 10, P0, PO)(P).
\]

The formulas (A.9, 10) can be summarized in the form

\[
[PP]P = a_0 11 + a_1 \tau \tau
\]

with \(a_0, a_1\) constants as given in Table A.I. With (A.8) and (A.12) we can now easily calculate the various boson contributions.


**Table A.I.**

<table>
<thead>
<tr>
<th></th>
<th>$K$</th>
<th>$K'$</th>
<th>$K_q$</th>
<th>$\bar{K}$</th>
<th>$K'$</th>
<th>$K_q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>$\frac{7}{12}$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{7}{12}$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{6}$</td>
</tr>
<tr>
<td>$a_1$</td>
<td>$\frac{5}{36}$</td>
<td>$\frac{1}{12}$</td>
<td>$\frac{1}{18}$</td>
<td>$\frac{1}{12}$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{18}$</td>
</tr>
</tbody>
</table>

A) $\eta, \pi$ contributions. — The contribution of the strangeness zero bosons is the same for all baryons as can be easily seen from the graphs Fig. 3. The contribution is simply $C_1$, $C(C(\delta_{n}))$, $C1$ respectively, with

\[
\begin{align*}
C_{p,\eta} &= \frac{3}{4} \\
C_{p,\pi} &= \frac{1}{4} \\
C_{p,\eta} + C_{p,\pi} &= 1.
\end{align*}
\]

B) $K$-type contributions. — We consider here the contributions of strangeness $\pm 1$ bosons in the intermediate states. However, only the $K$-configuration will correspond to an actual meson intermediate state, *i.e.* will be connected with a discrete pole. The $K'$ and $K_q$ configurations are not connected with a pole but occur as a part of the 4-point contraction function, and should be interpreted as arising from certain more-particle configurations, *e.g.* $N'\Lambda$ combinations etc. If we are only dealing with the algebraic properties of the graph this important difference is not taken into account. Hence $K$, $K'$, $K_q$ in this context only indicate general state configurations of the symmetry type described by the symbol.

1) Zero-spurion fermion ($\Xi$). — For each $K$- or $\bar{K}$-configuration only one graph contributes, which is given by Fig. 4 and Fig. 7.

We obtain

\[
C_{\Xi,K} = [PP](\text{Tr } P) = 2a_0 1
\]

with $a_0$ of Table A.I. The symbol $K^{(\dagger)}$ stands for all $K$- and $\bar{K}$-type configurations. The sum over all $K$- and $\bar{K}$-type contributions has to give four times the contribution of the $\eta, \pi$ sum, because of the four possible ways to create a virtual spurion-antispurion pair at the vertex.

2) One-spurion fermion ($\Lambda, \Sigma$). — For each $K$- or $\bar{K}$-type there are two graphs as given in Fig. 5 and Fig. 8. They yield

\[
\begin{align*}
C_{\Lambda,K}^{(\dagger)} &= \frac{1}{2}(\text{Tr } [PP])(\text{Tr } P) = 2a_0, \\
C_{\Sigma,K}^{(\dagger)} \delta_{ik} &= \frac{1}{2}(\text{Tr } [PP]\tau_i\tau_k)(\text{Tr } P) = 2a_0 \delta_{ik}; \\
C_{\Lambda,K}^{(\dagger)} &= \frac{1}{2}(\text{Tr } [PP]P) = a_0 + 3a_1, \\
C_{\Sigma,K}^{(\dagger)} \delta_{ik} &= \frac{1}{2}(\text{Tr } [PP]\tau_k P\tau_i) = (a_0 - a_1) \delta_{ik};
\end{align*}
\]

with $a_0, a_1$ of Table A.I.
The graphs may also be interpreted as an effective interaction of the incoming spurion with the \( \chi' \) antispurion which is of the form

\[
\begin{align*}
C^{(1)} &= 2a_0, \\
C^{(2)} &= a_0 - a_1 \tau_s \tau_a;
\end{align*}
\]

or

\[
C^{(1)} + C^{(2)} = 3a_0 - a_1 \tau_s \tau_a
\]

for the first and second graph and their sum, respectively, because \( \tau_s \tau_a \) has the eigenvalues \(-3\) or \(+1\) for singlet and triplet, respectively.

3) Two-spurion fermions \((N')\). Here we have a total of three graphs as seen in Fig. 6 and Fig. 9. The second and third graph are topologically equal, but the rôle of the two spurions in \( N' \) is interchanged. They give

\[
\begin{align*}
C^{(1)}_{N',K'} &= O_4 O_k [PP] O_4 O_k (\text{tr } P) = 2a_0 1, \\
C^{(2)}_{N',K'} &= O_4 O_k [PP] O_4 PO_k = (a_0 + 2a_1) 1,
\end{align*}
\]

with \( a_0, a_1 \) of Table A.1.

Again the graphs may be interpreted as an effective interaction of the two spurions and the \( \chi' \)-antispurion:

\[
\begin{align*}
C^{(1)} &= 2a_0, \\
C^{(2)} &= a_0 - a_1 \tau_s \tau_a, \\
C^{(3)} &= a_0 - a_1 \tau_s \tau_a,
\end{align*}
\]

or

\[
C^{(1)} + C^{(2)} + C^{(3)} = 4a_0 - a_1 \sum_s \tau_s \tau_a
\]

for the various graphs and their sum.

The above contributions from the different boson configurations for the various baryons can be combined in simple formulas involving only \( Bu \) and \([I(I+1)-u^2]\) dependence.

One may establish this result by appropriate combination of the numbers calculated above. However, we may immediately derive it from the observation that the sum over the \( C \)-numbers of the graphs for each K-type meson according to \((A.14, 18, 22)\) can be written as

\[
C^{(1)}_{K'} = \sum_{(0)} C^{(0)}_{F,K'} = a_0(n + 1) - a_1 \sum_s \tau_s \tau_a,
\]

where \( n \) is the total number of spurions and antispurions (including the \( \chi' \)-spurions) which in our cases is simply \( n = 2(1 + Bu) \) as given by eq. \((57)\).
Using eq. (58) we can rewrite (A.23)

\[(A.24) \quad C_{F,K}^{(1)} = 3(a_0 + a_1) + (4a_0 + 2a_1)Bu - 2a_1[I(I+1) - u^2]\]

with \(a_0, a_1\) of Table A.I.

The numerical values of the isospin-hypercharge independent part, and the coefficients multiplying the characteristic combinations \(Bu\) and \([I(I+1) - u^2]\) are given in Table A.II. Of particular interest will be the contributions from

**Table A.II.**

<table>
<thead>
<tr>
<th>(C_{F,K})</th>
<th>(3(a_0 + a_1))</th>
<th>(4a_0 + 2a_1)</th>
<th>(-2a_1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{13}{6})</td>
<td>(\frac{31}{18})</td>
<td>(\frac{5}{18})</td>
<td></td>
</tr>
<tr>
<td>(\frac{1}{2})</td>
<td>(\frac{1}{6})</td>
<td>(\frac{1}{6})</td>
<td></td>
</tr>
<tr>
<td>(\frac{1}{3})</td>
<td>(\frac{1}{9})</td>
<td>(\frac{1}{9})</td>
<td></td>
</tr>
<tr>
<td>(\sum K C_{F,K}^{(1)})</td>
<td>3</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>(\frac{25}{12})</td>
<td>(\frac{29}{18})</td>
<td>(-\frac{2}{9})</td>
<td></td>
</tr>
<tr>
<td>(\frac{1}{4})</td>
<td>(-\frac{1}{6})</td>
<td>(+\frac{1}{3})</td>
<td></td>
</tr>
<tr>
<td>(\frac{2}{3})</td>
<td>(\frac{5}{9})</td>
<td>(-\frac{1}{9})</td>
<td></td>
</tr>
<tr>
<td>(\sum K C_{F,K}^{(1)})</td>
<td>3</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

the \(K\)- and \(\bar{K}\)-mesons which arise in connection with pole terms (the other \(C\)-factors multiply contributions from the continuum which we do not want to consider):

\[(A.25) \quad C_{F,K} = C_{F,K} + C_{F,K} = \frac{17}{4} + \frac{10}{3} Bu - \frac{1}{2}[I(I+1) - u^2].\]

If we allow small contributions from the \(K'\) and \(K_0\) continuum states, the dependence on \([I(I+1) - u^2]\) will be reduced, because of the opposite sign of the \(a_1\) values of these configurations.
A.2. – Fermion eigenvalue equation.

We are now well prepared to derive the Fermion eigenvalue eq. (61), in which the contribution of the virtual bosons are taken into account.

The eigenvalue equation is characterized by the graphical eq. (46), where we have only indicated the $\chi$-lines. The spurion lines have to be added appropriately. The first graph contains only the 2-point contraction functions $\Gamma$ in the context of the NTD-approximation, in which we only explicitly consider a single baryon pole and regularizing terms. The second graph contains a sum over all meson pole contributions of the 4-point function. To explicitly calculate these terms we have to insert boson propagation functions, e.g., of the type (59) with the masses as calculated from (30) and corresponding expressions and coupling constants as, e.g., (in case of $\eta$ and $\pi$) calculated by Dhar and Katayama (6). However, we do not want to use this procedure, but prefer to apply to above boson graphs directly the method which was employed by Dhar and Katayama (6) and earlier, although in a less transparent form, by Yamazaki (4) for the evaluation of the coupling constants.

For this purpose we rewrite the eigenvalue eq. (46) in a more refined graphical form in momentum space:

\[
(A.26)
\]

which for $J^2 \neq 0$ corresponds to the equation

\[
(A.27) \quad \varphi(J) = K_F \left( -\frac{J^2}{\lambda^2} \right) + \sum_n K_{F_B} \left( -\frac{J^2}{\lambda^2_R}, \lambda^2 \right) \varphi(J)
\]

with $\varphi(J) = \langle 0 | \chi(x) | J \rangle$ the baryon eigenfunction of 4-momentum $J$. The arrows on the lines indicate the baryon number. Since it will turn out that the $\Xi$ corresponds to the zero-spurion system, the $u$-arrow which was used in the isospin-hypercharge graphs Fig. 3-9 runs oppositely to the $B$-arrow. For simplicity and generality we again have not drawn the spurion lines, which, of course, are important in the boson graphs. In the first graph the vertex $\bigodot$ has the form of the Fierz-symmetrical nonlinear term in the differential eq. (12):

\[
(A.28) \quad + i\sigma^\nu \frac{\partial}{\partial x^\nu} \chi(x) = t^2 : V_i \chi(x) [\chi^*(x) V_i \chi(x)] :
\]
in the 2-component version, or

$$i \phi = (i \hbar^2) V_i V_i = (i \hbar^2) \sqrt{3} O_i O_i \sigma_\mu \sigma^\mu = (i \hbar^2) \frac{1}{2} (311 + \tau \tau) \sigma_\mu \sigma^\mu$$

in the parity symmetrical version \((12)\). All calculations will be carried out in the 2-component version for practical simplicity. However, all results of this paper can be equally applied to the parity symmetrical version \((12)\), which is more practical if questions concerning parity arise. For the Green's function \(G(p)\) we insert

$$G(p) = -i \frac{\bar{\sigma}_{x} p^\mu}{p^2}$$

and for the contraction function \(F(p)\)

$$F(p) = -i \frac{x^\mu \bar{\sigma}_x p^\mu}{(p^2)(p^2 + x^2)}.$$

All poles are treated in the Feynman sense.

Hence we have

$$K_\tau = \left(-i \frac{\bar{\sigma}_x \xi_\tau}{J^2}\right) 2(i \hbar^2)^2 V_i \bar{\sigma}_\tau V_i \bar{\sigma}_x V_i \bar{\sigma}_x V_i \bar{\sigma}_x V_i \frac{1}{4} \int \frac{d^4 q}{(2\pi)^4} \frac{d^4 r}{(2\pi)^4} \frac{x^\mu p^\mu}{(r^2 + x^2)^2} \frac{x^\mu q^\mu}{(q^2 + x^2)^2} \frac{(J + q - r)^4}{(J + q - r)^2}.$$  

(The factor 2 in eq. \((A.33)\) arises from the fact that there are two possible ways of contraction. Due to the Fierz-symmetrical form of our interaction both contractions give the same result.) Using

$$\begin{aligned}
\left\{ \begin{array}{l}
\sigma_\mu \bar{\sigma}_x \sigma_\nu \sigma_\tau \sigma_\mu = 2 \sigma_\nu \bar{\sigma}_x \sigma_\mu \\
\bar{\sigma}_x \sigma_\nu \bar{\sigma}_x \sigma_\mu = 4 g_{x \lambda}
\end{array} \right.
\end{aligned}$$

and eq. \((A.3)\) we get for the algebraic factor:

$$2 V_i \bar{\sigma}_x V_i \bar{\sigma}_x V_i \bar{\sigma}_x V_i = 24 (O_i O_i O_i) \sigma_\kappa g_{x \lambda} = 24 \sigma_\kappa g_{x \lambda}.$$  

Since the remaining double integral \(J_\kappa\) only depends on \(J\) one can write (if

\[ J^2 \neq 0 \):

\[ \sigma_\kappa I^\kappa(J) = \sigma_\kappa J^\kappa \frac{P \sigma/J}{J^2} \]

and finally obtain the earlier result (4)

\[ K_r \left( -\frac{J^2}{\kappa^2} \right) = -24 \left( \frac{\chi}{4\pi} \right)^4 L \left( -\frac{J^2}{\kappa^2} \right) = -\frac{3}{2} \left( \frac{\chi}{2\pi} \right)^4 L \left( -\frac{J^2}{\kappa^2} \right) \]

with the double integral

\[ L \left( -\frac{J^2}{\kappa^2} \right) = -\frac{\chi^4}{\pi^4} \int d^4q \int d^4r \frac{q \cdot J [r \cdot (J + q - r)]}{J^2 (r^2)^2 (r^2 + \kappa^2)(q^2)^2 (q^2 + \kappa^2)(J + q - r)^2} . \]

We have sketched the old calculation of the first graph in (A.26) in order to use it as a check for the calculation of the boson graphs in the same equation. According to the method of Dhar-Katayama-Yamazaki these boson graphs can be formally composed of only the G- and F-lines (A.31, 32) and the original vertices (A.29), as indicated. For strange bosons also spurion lines should be included in an appropriate manner. However, in our present situation we have not to consider the complications which arose in the calculation of Dhar and Katayama (4) in connection with the dressing of the fermion lines (e.g. the introduction of the \( F^0 \) line), because in the second graph of (A.26) only G-lines appear. Since in (4) it was eventually assumed that the fermion renormalization factor is effectively cancelled by the vertex renormalization (which holds exactly only in quantum electrodynamics due to Ward's identity) our results, which do not include both, will nevertheless agree with results based on (4).

The bubble line in (A.26) represents essentially the boson propagator (without spurion lines, however, only strangeness zero bosons!) and is given by the following formal expression:

\[ \frac{1}{1 - K_B} = \frac{1}{1 - K_B} = \left( \begin{array}{c} \begin{array}{c} \text{bubble} \\ \text{line} \end{array} \end{array} \right) = \left( \begin{array}{c} \begin{array}{c} \text{bubble} \\ \text{line} \end{array} \end{array} \right) + \left( \begin{array}{c} \begin{array}{c} \text{bubble} \\ \text{line} \end{array} \end{array} \right) + \cdots \]

which has poles at the solutions of the (strangeness zero) boson eigenvalue equation

\[ < - \left( \begin{array}{c} \begin{array}{c} \text{bubble} \\ \text{line} \end{array} \end{array} \right) \left( \begin{array}{c} \begin{array}{c} \text{bubble} \\ \text{line} \end{array} \end{array} \right) = 0 \; , \quad \text{or} \quad [1 - K_B] \psi = 0 \; , \]

with \( K_B \) the boson eigenvalue function, which for spin 0 agrees with \( K_r \) of eq. (23). By inclusion of spurion lines, as discussed in the last Section, this
can be immediately generalized to characterize strange poles \( u \neq 0 \), as given by eq. (30). The bar in (A.39) only indicates that the two lines start at the same space-time point; \( \bigcirc \) represents the 4-fermion vertex (A.29). It should be noticed that the numerator function does not contain the vertex part.

The decomposition of the boson propagator into its isospin-hypercharge components was already accomplished in the last Section. It remains its decomposition into the spin zero and spin one components. (In the parity-symmetrical version we have also to use a decomposition with respect to the plus and minus parity configurations. However, only the minus parity bosons can occur on the basis of their construction as baryon-antibaryon \( S \)-states \( \left(_{12} \right) \).

The boson eigenvalue kernel is of the form

\[
(A.41) \quad p \begin{pmatrix} \hline \hline \end{pmatrix} = \bar{\sigma}_\alpha \sigma_\alpha K'(p)
\]

and can be decomposed into its spin zero and spin one components in the following way:

\[
(A.42) \quad K'(p) = K_0 \left( -\frac{p^2}{\kappa^2} \right) P_{0\alpha} + K_1 \left( -\frac{p^2}{\kappa^2} \right) P_{1\alpha}
\]

with

\[
(A.43) \quad P_{0\alpha} = \frac{1}{2} \gamma_\alpha - \frac{\gamma^\alpha p^\alpha}{p^2}, \quad P_{1\alpha} = \frac{1}{2} \gamma_\alpha + \frac{\gamma^\alpha p^\alpha}{p^2},
\]

where \( K_0 \) and \( K_1 \) correspond to the eigenvalue kernel of the spin zero and spin one configuration, respectively. \( K_0 \) agrees with the kernel function in eq. (23). The operators (A.78) are related to the common projection operators

\[
(A.44) \quad P_0 = \frac{\gamma^\mu p_\mu}{p^2}, \quad P_1 = \frac{\gamma^\mu - p_\mu p^\mu}{p^2},
\]

by a Fierz transformation, which can be expressed in the following relations:

\[
(A.45) \quad \sigma_\mu \sigma_\nu \bar{\sigma}_\gamma P_{0\nu} = 2\sigma_{0\nu}, \quad \sigma_\mu \sigma_\nu \bar{\sigma}_\gamma P_{1\nu} = 2\sigma_{1\nu}.
\]

It can be immediately checked that a decomposition of the kernel with the operators (A.43) is equivalent to a projection on the states with the operators (A.44). The bubble line (A.38) can now be similarly decomposed in the
following way:

\[
(A.46) \quad \equiv \bar{\sigma}_\alpha \sigma_\mu \frac{1}{(4Z_B)i\ell^2} \left[ \frac{P^{\prime \alpha \nu}_0}{1-K_0} + \frac{P^{\prime \alpha \nu}_1}{1-K_1} \right],
\]

where the factor in front of the bracket arises from the fact that the numerator function in (A.38) does not include the vertex expression which is contained in the kernel functions \(K_0\) and \(K_1\). The \(Z_B = Z_{\mu \nu}\) is the isospin-hypercharge dependent number defined in the general case by eq. (29).

The actual boson propagator for zero spurion systems can be obtained from (A.46) by adding at the top and the bottom essentially «half» the vertex \(i\ell^2V_iV_i\):

\[
(A.47) \quad \frac{\mu}{\nu} \sim -(i\ell^2) \frac{Z_B}{2} \text{Tr} (\sigma_\mu \bar{\sigma}_\alpha \sigma_\nu \bar{\sigma}_\mu) \frac{1}{(4Z_B)(i\ell^2)} \sum_B P^{\prime \alpha \nu}_{B} \frac{K_B}{1-K_B} =
\]

\[
= \frac{1}{2} \sum_B P^{\nu \nu}_{B} \frac{K_B}{1-K_B} \sim \sum_B \frac{P^{\nu \nu}_{B}}{p^2 + \chi^2_B} + \ldots.
\]

In close analogy to (A.33) we can now construct with the help of the bubble line (A.46) the integral expression for \(K_{FB}\) in (A.27) which represents the contribution of a particular boson configuration \(B\) to the fermion eigenvalue kernel:

\[
(A.48) \quad K_{FB} = \left( -i \frac{\bar{\sigma}_\alpha J^\nu}{J^2} \right) (i\ell^2)^2 2A \sigma_{\nu \alpha} C_{F,B}:
\]

\[
(\cdots - i) \int \frac{d^4p}{(2\pi)^4} \frac{P^{\prime \alpha \nu}_{B}}{(4Z_B)i\ell^2} \frac{K_B(-p^2/\chi^2) (J-p)^3}{1-K_B(-p^2/\chi^2) (J-p)^2 + \ldots}.
\]

Here we have immediately made use of formulas (A.34, 35). However, instead of \(O_{\mu}O_{\nu}O_{\sigma}O_{\sigma} = 1\) which occurs in (A.33), we have now to insert the isospin-hypercharge coefficients \(C_{F,B}\) as calculated in Sect. A.1, which depend on the isospin-hypercharge properties of the fermion (F) as well as the virtual boson (B). The dots \(\ldots\) shall indicate some additional contributions of the 4-point function specified below which are used to regularize the integral in (A.48).

Of the integrand in (A.48) we only take the pole term explicitly into account which corresponds to the propagation of a stable boson of mass \(K_B\). Since

\[
(A.49) \quad K_B \left( - \frac{p^2}{\chi^2} \right) = 1 - \frac{K_B(-p^2/\chi^2)}{\chi^2} \bigg|_{p^2 = -\chi^2_B} (p^2 + \chi^2_B) + \ldots
\]

in the neighborhood of the pole \(p^2 = -\chi^2_B\) we have

\[
(A.50) \quad \frac{K_B(-p^2/\chi^2_B)}{1-K_B(-p^2/\chi^2_B)} = \frac{\chi^2}{K_B^2(\chi^2_B/\chi^2)} \frac{1}{p^2 + \chi^2_B} + \ldots.
\]
We now regularize this pole similarly as in case of the baryon pole by adding only contributions from the light cone

\[(A.51)\quad \frac{1}{p^2 + \lambda^2} \to \frac{1}{p^2 + \lambda^2 - \frac{1}{p^2} = -\frac{\lambda^2}{(p^2)(p^2 + \lambda^2)}\]

and introduce the functions \(q_\lambda\) defined by

\[(A.52)\quad K_\lambda(\lambda) = -\frac{1}{4} Z_\lambda \left(\frac{\lambda}{2\pi}\right)^2 q_\lambda(\lambda)\]

with \(Z_\lambda = Z_{t,n}\) of eq. (18). Explicitly \(q_\lambda\) (with \(B\) corresponding here to spin 0 or 1) is given by the following relation:

\[(A.53)\quad q_0 P_0^{\lambda \lambda} + q_1 P_1^{\lambda \lambda} = -i \frac{d\lambda}{\lambda} \int d^4q \frac{(p - q)q}{(q^2 + \lambda^2)(p - q)^2}\]

which leads for spin zero to expression (27), for spin one to

\[(A.54)\quad q_1 \left(\frac{p^2}{\lambda^2}\right) = -i \frac{2\lambda^2}{3\pi^2} \int d^4q \frac{2p^2(q \cdot p) - q^2(q \cdot p) - q^2p^2}{p^2(q^2 + \lambda^2)(p - q)^2} = \left(1 - \frac{2}{3}\right) \ln |\lambda| + \frac{1}{3} + \frac{2}{3} \frac{(2\lambda + 1)(1 - \lambda)^2}{3\lambda^2} \ln |1 - \lambda|\]

with \(\lambda = -\frac{p^2}{\lambda^2}\). Equation (A.48) can now finally be written

\[(A.55)\quad K_{\lambda B} = i \frac{24l^2\lambda^2 \lambda^2 C_{\lambda B}}{(2\pi)^4 Z_\lambda^2(\lambda/2\pi)^2 q_\lambda(\lambda^2/\lambda^2)} \int d^4p \frac{P^{\lambda \lambda}(J - p)\lambda J_\lambda}{J^2p^2(p^2 + \lambda^2)(J - p)^2} = \frac{3}{2} \frac{C_{\lambda B}}{Z_\lambda^2} \frac{\lambda^2}{q_\lambda(\lambda^2/\lambda^2)}\]

if we define

\[(A.56)\quad r_\lambda \left(-\frac{J^2}{\lambda^2}\right) = i \frac{4\lambda^2}{\tau^2} \int d^4p \frac{P^{\lambda \lambda}(J - p)\lambda J_\lambda}{J^2p^2(p^2 + \lambda^2)(J - p)^2}\]

These integrals are closely related to the integrals eq. (27) and eq. (A.54), and are explicitly given by:

**Spin = 0:**

\[(A.57)\quad r_0 \left(-\frac{J^2}{\lambda^2}\right) = \frac{1}{2} \left[ q_0 \left(-\frac{J^2}{\lambda^2}\right) + 3q_1 \left(-\frac{J^2}{\lambda^2}\right) \right] = -i \frac{2\lambda^2}{\tau^2} \int d^4p \frac{(J - p)\cdot p}{(p^2)^2(p^2 + \lambda^2)(J - p)^2} = (2 - \lambda) \ln |\lambda| + 1 + \frac{(1 - \lambda)^2}{\lambda} \ln |1 - \lambda|\]
Spin = 1:

\[ r_1 \left( \frac{-J^z}{\lambda^2} \right) = \frac{3}{2} \left( q_0 \left( \frac{-J^z}{\lambda^2} \right) - q_1 \left( \frac{-J^z}{\lambda^2} \right) \right) = \]

\[ = -\frac{i}{3\pi^2} \int d^4p \frac{2p^2(p \cdot J) - J^z(p \cdot J) - p^2J^z}{J^z(p^2)(p^2 + \lambda^2)(J - p)^2} = \]

\[ = \lambda \ln |\lambda| \frac{2}{\lambda} - 1 \frac{(2 + \lambda)(1 - \lambda)^2}{\lambda^2} \ln |1 - \lambda| \quad \text{with} \quad \lambda = -\frac{J^z}{\lambda^2}. \]

For \( \lambda \ll 1 \) we have the behaviour:

\[ \begin{cases} 
q_0(\lambda) = & \ln \lambda - \frac{3}{2} + \frac{1}{\lambda} + O(\lambda^2), \\
q_1(\lambda) = & \ln \lambda + \frac{3}{2} + \frac{3}{8} \lambda \ln \lambda + O(\lambda^2), \\
r_0(\lambda) = & 2 \ln \lambda + \frac{3}{2} \lambda - \lambda \ln \lambda + O(\lambda^2), \\
r_1(\lambda) = & -3 - \frac{5}{8} \lambda + \lambda \ln \lambda + O(\lambda^2). 
\end{cases} \]

For \( \lambda \gg 1 \)

\[ \begin{cases} 
q_0(\lambda) = & \frac{2}{\lambda} \ln \lambda - \frac{1}{\lambda^2} \ln \lambda - \frac{3}{2} \frac{1}{\lambda^2} + O \left( \frac{1}{\lambda^3} \right), \\
q_1(\lambda) = & \frac{1}{\lambda} + \frac{1}{3} \frac{1}{\lambda^2} \ln \lambda + \frac{5}{18} \frac{1}{\lambda^2} + O \left( \frac{1}{\lambda^3} \right), \\
r_0(\lambda) = & \frac{1}{\lambda} \ln \lambda + \frac{3}{2} \frac{1}{\lambda} - \frac{1}{3} \frac{1}{\lambda^2} + O \left( \frac{1}{\lambda^3} \right), \\
r_1(\lambda) = & \frac{3}{\lambda} \ln \lambda - \frac{3}{2} \frac{1}{\lambda} - \frac{2}{\lambda^2} \ln \lambda - \frac{8}{3} \frac{1}{\lambda^2} + O \left( \frac{1}{\lambda^3} \right). 
\end{cases} \]

If we insert eqs. (A.36, 55) into (A.27) we obtain eqs. (64), (65) of the main part of this paper, if we restrict ourselves to the contributions of the \( \eta, \pi \) and K mesons.

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**RIASSUNTO (*)**

Nella teoria spinoriale non lineare che coinvolge solo operatori di campo spinoriali e isospinoriali, si possono costruire gli stati delle particelle strane nell'ipotesi di uno stato fondamentale asimmetrico rispetto all'isospin; si possono distaccare dallo stato fondamentale gli *spurioni* che portano l'isospin ed attaccarli a sistemi costruiti con gli operatori di campo locali in modo da formare particelle strane. Oltre all'isospin, si devono attribuire altre proprietà agli spurioni perché siano in accordo con le condizioni

(*) Traduzione a cura della Redazione.
La prima possibilità che aggiunge una proprietà di parità è stata studiata in un articolo precedente. Le sue conseguenze nella teoria dei gruppi (p. es. parità $\Lambda\Sigma$ dispari) sono in contraddizione con le attuali prove sperimentali; quindi questa possibilità è esclusa. Pertanto, in questo articolo si studia la seconda possibilità, che aggiunge una proprietà di ipercarica. Il sistema di spurioni allora si trasforma secondo le rappresentazioni dell'$U_2$. Si trova che la combinazione $I(I+1)-\frac{1}{4}Y^2$, nota in relazione alla $SU_3$ interrotta, appare automaticamente nel fattore che moltiplica l'operatore di autovalore dei più semplici bosoni, a causa della forma caratteristica dell'equazione di campo e delle proprietà di permutazione degli spurioni. Gli operatori di autovalore dei barioni di spin $\frac{1}{2}$ sono moltiplicati per un fattore che contiene $aBY + b[I(I+1)-\frac{1}{4}Y^2]$ ($B =$ numero barionico), se nell'autointerazione si tiene conto dei bosoni più semplici. Sebbene appaiano gli operatori $I$, $Y$ della $SU_3$ interrotta, non si presuppone né si introduce mai il gruppo $SU_3$, e quindi non si può dedurre in base alle teorie dei gruppi il caratteristico raggruppamento di particelle in $1, 8, 10$ ecc; tuttavia è interessante osservare che un raggruppamento di $8$ dei bosoni più semplici segue naturalmente da considerazioni dinamiche.