Abstract

We develop a theory of the EPR-like effects due to neutrino oscillations in the \( \pi \to \mu \nu \) decays. Its experimental implications are space-time correlations of the neutrino and muon when they are both detected, while the pion decay point is not fixed. However, the more radical possibility of \( \mu \)-oscillations in experiments where only muons are detected (as suggested in hep-ph/9509261), is ruled out. We start by discussing decays of monochromatic pions, and point out a few "paradoxes". Then we consider pion wave packets, solve the "paradoxes", and show that the formulas for \( \mu \nu \) correlations can be transformed into the usual expressions, describing neutrino oscillations, as soon as the pion decay point is fixed. © 1997 Elsevier Science B.V.

PACS: 14.60.Ef; 14.60.Lm; 14.60.Pq

1. Introduction

It was suggested recently \([1,2]\) that the much searched for hypothetical neutrino flavour oscillations can cause space-time oscillations of the observation rate of the beam of muons from the \( \pi \)-meson decay. As emphasized in \([1]\), the same claim applies to other charged leptons, in particular to \( \tau \)-leptons in the decay \( W \to \tau \nu_{\tau} \). If true, this phenomenon could provide a powerful experimental method for the (indirect) observation of neutrino oscillations.

We think that the argumentation of Ref. \([1]\) is unfortunately erroneous, and that there are no oscillations of charged leptons if neutrinos are not observed. However, in coincidence experiments in which both the charged lepton and neutrino, born in the same decay, are detected, specific EPR-like oscillating correlations \([3]\) can show up (see Section 3.2). Once neutrino oscillations \([4-8]\) per se are discovered, this could be of interest for the next generation of experiments.
If the neutrino mass matrix is not flavour diagonal then the “current” neutrinos $\nu_a$ ($a = e, \mu, \tau$) are non-trivial mixtures of the mass eigenstates $\nu_n$ ($n = 1, 2, 3$) with masses $m_1 > m_2 > m_3$,

$$\nu_a = \sum_n U_{an} \nu_n,$$  \hspace{1cm} (1)

i.e. the relevant part of the Standard Model Lagrangian is

$$\mathcal{L}_\nu = g \sum_a \bar{l}_a \hat{W} l_a + \text{H.c.} = g \sum_{a,n} U_{an} \bar{l}_a \hat{W} \nu_n + \text{H.c.}$$  \hspace{1cm} (2)

In what follows we consider a “toy model” with only two charged leptons, a muon and an electron, and two neutrinos, $\nu_1$ and $\nu_2$,

$$\mathcal{L}_\nu = g \left( \bar{\mu} \hat{W} (\nu_1 \cos \theta + \nu_2 \sin \theta) + \bar{e} \hat{W} (-\nu_1 \sin \theta + \nu_2 \cos \theta) \right) + \text{H.c.}$$  \hspace{1cm} (3)

Also, we neglect the width of the emerging muon (as the oscillations discussed in Ref. [1] survive in the limit of stable $\mu$; in any case all the widths are easy to restore in the formulas).

We consider the decay $\pi \rightarrow \mu \nu$, and analyse three types of experiments:

(A) when both neutrino and muon are detected; this is an experiment with two detectors in coincidence;

(B) when only muons are detected;

(C) when only neutrinos are detected.

Evidently, B and C are single-detector experiments, and B is considerably easier than C and A.

Our analysis will demonstrate that the probability to detect both $\mu$ and $\nu$ (case A) oscillates as a function of the distance $d$ between the muon and neutrino detection points and of the time interval $\Delta t$ between “clicks” of the two detectors. Moreover, the oscillation length and oscillation frequency are different from the standard [7] values $L = 2E_\pi/(m_1^2 - m_2^2)$ and $L^{-1}$, respectively.

The oscillations disappear completely in case B, just because of the orthogonality of different neutrino mass eigenstates in the flavour space. However, they can show up in case C. There are two ways in which the neutrino oscillations might manifest themselves: the first may be called global, and the second local.

To see the global effect one does not need to observe the oscillating term: it is sufficient to observe the appearance of $\nu_e$, and/or the disappearance of $\nu_\mu$. This could be done without accurate measurements of the time when the decaying pion was produced in the target, or the position of its decay point. From the theoretical point of view the global effects of appearance and disappearance can be described within the approximation of plane waves (monochromatic pion).

To see the local effect, to observe time and/or space oscillations, one needs an adequate resolution. From the theoretical point of view this also requires the pion to be not exactly monochromatic (which is automatically the case in any realistic experiment). Otherwise the muons born together with $\nu_1$ would be orthogonal to those born with
$\nu_2$, and the oscillation term would drop out from the total probability. Although our conclusions coincide with the usual naive expectations, we feel that it can be useful to present this analysis in a little more detail, to avoid any further confusion in the literature.\(^1\)

In Section 2 we discuss the decay of a monochromatic pion described by a plane wave, derive expression for correlations $P_{\nu\nu}(x, x')$ (Section 2.1), and analyse them in two cases: $p_{\pi} = 0$ (Section 2.2) and $p_{\pi} \neq 0$ (Section 2.3). Then, in Section 2.4, we show that if only one of the particles is detected, either muon (case $B$), or neutrino (case $C$), the oscillating term disappears. We argue that for muons the absence of oscillations is natural, but for neutrinos the absence, or rather non-observability of local oscillations is an artifact of the plane wave approximation for the pion. We also show that the global effects of $\nu_\mu$ appearance and $\nu_e$ disappearance are reproduced even in the plane wave approximation (we start with this non-realistic approximation because it is simple and widely used in the literature).

In Section 3 we use pion wave packet and derive expressions for $\mu\nu$ system, as well as for separate beams of muons and neutrinos (Section 3.1). These expressions are extremely simple, as the oscillating terms depend only on the time of flight of the neutrino from its birth until its detection. In Section 3.2 we rederive some of the results of Section 3.1 by using the technique of plane waves and of the so called “classical a posteriori trajectories” for muons and neutrinos. Section 4 contains a brief summary and a few concluding remarks.

2. Decay of a monochromatic pion state

2.1. Experiment of the type A: The probability formula

To begin with, let us assume that pion has a definite 4-momentum $p_\pi = (E_\pi, p_\pi)$. Then the 4-momenta $p_{\mu\nu} = (E_{\mu\nu}, p_{\mu\nu})$ and $p_{\nu\nu} = (E_{\nu\nu}, p_{\nu\nu})$ are determined by the conservation law

$$p_\pi = p_{\mu\nu} + p_{\nu\nu}, \quad n = 1, 2,$$

\(^1\)Let us note that in the literature neutrino oscillations are treated usually in an oversimplified way. The superposition of two neutrino mass eigenstates is described by a wave function, not by a density matrix. The two terms of the wave function are usually assumed to be plane waves with the same momentum $p_\nu$ and different energies, $E_{\nu 1} = \sqrt{p_\nu^2 + m_1^2}$, $E_{\nu 2} = \sqrt{p_\nu^2 + m_2^2}$ (see, e.g., Refs. [9-12]), or with the same energy $E_\nu$, and different momenta $p_{\nu 1} = \sqrt{E_\nu^2 - m_1^2}$, $p_{\nu 2} = \sqrt{E_\nu^2 - m_2^2}$ (see Refs. [8,13], and especially Refs. [14-16]). The 4-momentum conservation in the decay $\pi \rightarrow \mu\nu$ is usually ignored. Besides Refs. [1,2], there were only a few papers [17-19] in which possible kinematic manifestations of the 4-momentum conservation were discussed. The neutrino wave packet has been considered only by a few authors [20-22], but in a way different from ours. The energy–momentum conservation and wave packets were discussed in a spirit close to ours in Section 5.2 of Ref. [12] (see, however, footnote 4).
and the direction of, say, \( p_{\nu m} \). In what follows we assume that the momenta of all three particles are known: either measured, or deduced from kinematics. All particles are on the mass shell:

\[
p^2_{\mu} = m^2_{\mu}, \quad p^2_{\mu 1} = p^2_{\mu 2} = m^2_{\mu}, \quad p^2_{\nu 1} = m^2_1, \quad p^2_{\nu 2} = m^2_2. \tag{5}
\]

The wave function of the \( \mu \nu \) system evolves in space-time as

\[
\psi_{p_{\pi}}(x_{\mu}, x_{\nu} | x_i) = |\mu\rangle e^{-ip_{\mu 1}(x_{\mu} - x_i)} |\nu_1\rangle e^{-ip_{\nu 1}(x_{\nu} - x_i)} \cos \theta \\
+ |\mu\rangle e^{-ip_{\mu 2}(x_{\mu} - x_i)} |\nu_2\rangle e^{-ip_{\nu 2}(x_{\nu} - x_i)} \sin \theta \\
= |\mu\rangle (|\nu_1\rangle e^{-ip_1} \cos \theta + |\nu_2\rangle e^{-ip_2} \sin \theta), \tag{6}
\]

where the index \( p_{\pi} \) reminds us that the total momentum of the \( \mu \nu \) system is equal to \( p_{\pi} \), and

\[
\begin{align*}
\varphi_1 &= p_{\nu 1}(x_{\nu} - x_i) + p_{\mu 1}(x_{\mu} - x_i), \\
\varphi_2 &= p_{\nu 2}(x_{\nu} - x_i) + p_{\mu 2}(x_{\mu} - x_i).
\end{align*} \tag{7}
\]

\( |\mu\rangle, |\nu_1\rangle \) and \( |\nu_2\rangle \) are “ket” state vectors of the muon \( \mu \) and neutrino mass eigenstates \( \nu_1 \) and \( \nu_2 \), respectively (inessential normalization factors are ignored here and throughout); \( p_{\nu 1} \) and \( p_{\nu 2} \) are 4-momenta of \( \nu_1 \) and \( \nu_2 \), and \( p_{\mu 1} \) and \( p_{\mu 2} \) are those of the muons emitted together with \( \nu_1 \) and \( \nu_2 \); \( x_{\mu} \) and \( x_{\nu} \) are space-time coordinates of the muon and neutrino, while \( x_i \) is the coordinate of the decay point (i for initial).

The amplitude to detect a muon at a space-time point \( x_{\mu} \) together with a neutrino of flavour \( \alpha \) (\( \alpha = e, \mu \)) at the point \( x_{\nu} \) is

\[
a_{\nu_\alpha}(x_{\mu}, x_{\nu}) = \langle \mu; \nu_\alpha | \psi_{p_{\pi}}(x_{\mu}, x_{\nu}) \rangle = \beta_{1\alpha} e^{-ip_1} + \beta_{2\alpha} e^{-ip_2}, \tag{8}
\]

and the corresponding probability is

\[
P_{\nu_\alpha}^{(A)}(x_{\mu}, x_{\nu}) = |a_{\nu_\alpha}(x_{\mu}, x_{\nu})|^2 = \beta_{1\alpha}^2 + \beta_{2\alpha}^2 + 2\beta_{1\alpha}\beta_{2\alpha}\cos(\varphi_1 - \varphi_2), \tag{9}
\]

where

\[
\begin{align*}
\varphi_1 - \varphi_2 &= p_{\nu 1}(x_{\nu} - x_i) + p_{\mu 1}(x_{\mu} - x_i) - p_{\nu 2}(x_{\nu} - x_i) - p_{\mu 2}(x_{\mu} - x_i) \\
&= (p_{\nu 1} - p_{\nu 2})(x_{\nu} - x_i) + (p_{\mu 1} - p_{\mu 2})(x_{\mu} - x_i) \\
&= (p_{\nu 1} - p_{\nu 2})(x_{\nu} - x_i - x_{\mu} + x_i) = (p_{\nu 1} - p_{\nu 2})(x_{\nu} - x_{\mu}). \tag{10}
\end{align*}
\]

We made use of the equality \( p_{\nu 1} - p_{\nu 2} = -(p_{\mu 1} - p_{\mu 2}) \), which follows from the 4-momentum conservation (4). Note that \( \varphi_1 - \varphi_2 \) is explicitly Lorentz invariant.

It follows from (3) that

\[
\beta_{1\mu} = \cos^2 \theta, \quad \beta_{2\mu} = \sin^2 \theta, \quad \beta_{1\nu} = -\beta_{1\mu} = \sin \theta \cos \theta. \tag{11}
\]

Hence, if the neutrino is detected as \( \nu_\mu \),
If the neutrino is detected as $\nu_e$,

$$P^{(A)}_{\nu_e}(x_\mu, x_\nu) = 2\sin^2 \theta \cos^2 \theta - 2\sin^2 \theta \cos^2 \theta \cos(\varphi_1 - \varphi_2).$$  \hspace{1cm} (13)$$

The probability (9) oscillates in space and time with the change of $x_\mu$ and/or $x_\nu$, presenting a kind of EPR effect. Actually these oscillations depend only on the differences $x_\nu - x_\mu$ and $t_\nu - t_\mu$, and can be observed in an experiment which detects both muon and neutrino from the same decay. 2

The expression (9) was in fact derived but misinterpreted in Ref. [11]. An extra $e^{-\imath t_{\nu}}$ in [11], which takes into account the decay of muon, is not essential for our analysis.

Before proceeding to the discussion of experiments B and C, let us look at Eq. (9) a little closer by substituting the expressions for $p_\mu$ and $p_\nu$ in two cases: $p_\pi = 0$ and $p_\pi \neq 0$, the former being a limiting case of the latter. We start with the absolutely non-realistic case of vanishing $p_\pi$, because it had been discussed in [11], because it has its own subtleties, and because the pion rest-frame values of the muon and neutrino energies enter some of the expressions for the more general case.

### 2.2. Experiment of the type A: pion with momentum strictly equal to zero

In this situation all four particles ($\pi, \mu, \nu_1, \nu_2$) are described by plane waves and are fully non-localized. The wave function of the $\mu\nu$ system evolves in space-time according to Eqs. (6) and (7).

In the rest frame of the pion

$$E_{\nu\mu}^0 = \frac{m_\pi^2 - m_\mu^2 + m_\nu^2}{2m_\pi},$$

$$E_{\mu\nu}^0 = \frac{m_\pi^2 + m_\mu^2 - m_\nu^2}{2m_\pi},$$

$$p_{\mu\nu}^0 = -p_{\nu\mu}^0.$$  \hspace{1cm} (14)\hspace{1cm} (15)\hspace{1cm} (16)

After the neutrino and the muon are detected we may conclude that the pion has decayed somewhere on the line connecting the two detection points, but a priori we are in principle unable to indicate the position of the pion decay point on this line.

As for the frequency of oscillations and the oscillation length, they are determined from Eqs. (14)-(16):

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2 In fact, an experiment of this kind has been started at IHEP (Protvino) by Denisov et al. [23,24]. They are looking for (semi)leptonic decays of kaons: $K \rightarrow \mu \nu$, $K \rightarrow \mu \nu \tau$ and $K \rightarrow e \nu \tau$. The energy of the kaons is $E_K = 35 \pm 1$ GeV. The experiment has two detectors. The first detector measures muons and electrons coming directly from the kaon decay; the second detector measures muons and electrons produced by neutrinos. Thus, by detecting a neutrino interaction they are able to determine the momentum of the electron or muon, which was born together with that neutrino in kaon decay. In this way neutrinos are "tagged". By analysing the kinematics they are able to reconstruct the position of the kaon decay point with an accuracy of 3 m. The tagging provides accurate information not only on the position of the origin of the neutrino, but also on its original flavour. This allows, in principle, a more accurate determination of the neutrino mixing angle $\theta$. 

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\[
\Delta E^0_\nu = E^0_\nu - E^0_\nu = \frac{m_1^2 - m_2^2}{2m_\pi} = \frac{1}{L^0} \frac{E^0_\nu}{m_\pi},
\]
\[
\Delta p^0_\nu = p^0_\nu - p^0_\nu,
\]
\[
u^0_\nu \Delta p^0_\nu \approx \frac{m_1^2 - m_2^2 E^0_\nu}{2E^0_\nu} m_\pi = \frac{1}{L^0} \frac{E^0_\mu}{m_\pi} = \Delta E^0_\nu - \frac{1}{L^0},
\]

where \( L^0 = 2E^0_\nu/(m_1^2 - m_2^2) \) is the standard oscillation length. Note that \( \Delta E^0_\nu \) and \( |\Delta p^0_\nu| \) are not equal to each other and to \( 1/L^0 \). Thus, we have in this case in Eq. (9)
\[
\varphi_1 - \varphi_2 = \frac{E^0_\nu}{L^0 m_\pi} \Delta t + \frac{E^0_\mu}{L^0 m_\pi} d,
\]

where \( d \) is the distance between the two detectors (detection points). Taking \( d \) to be fixed, and by measuring \( \Delta t = t_\nu - t_\mu \), one can deduce a posteriori the point on the line connecting the two detectors where pion has decayed (see Section 3.2).

Let us stress again that a pion at rest which is spread over a region larger than the distance between the two detectors is absolutely non-realistic. Therefore the considerations of this section are of purely "gedanken" nature.

2.3. Experiment of the type A: pion in flight with strictly fixed (sharp) momentum

Let us consider now a beam of pions moving from left to right along the line which connects the muon and neutrino detectors.\(^3\)

If the velocity of a pion \( u_\pi = p_\pi/E_\pi \) (\( u_\pi = |v_\pi| \)) is low enough, \( u_\pi < u^0_\mu \), where \( u^0_\mu = p^0_\mu/E^0_\mu \), both \( \mu \) and \( \nu \) will be detected if the pion decays "inside", between the two detectors. For \( u_\pi > u^0_\mu \) both \( \mu \) and \( \nu \) will be detected if pion decays "outside", to the left of the muon detector. For \( u_\pi = u^0_\mu \) pion must decay in the muon detector. Let us express \( \varphi_1 - \varphi_2 \) through the time interval \( \Delta t = t_\nu - t_\mu \) and the distance between the detectors \( d = x_\mu - x_\nu \):
\[
\varphi_1 - \varphi_2 = \Delta E_\nu \Delta t - d \Delta p_\nu = (E^0_\nu - E^0_\nu) \Delta t - (p^0_\nu - p^0_\nu) d. \tag{20}
\]

Lorentz transformations give
\[
E_{\nu\mu} = (E^0_{\nu\mu} + \nu_{\mu}^0 p^0_{\nu\mu}) \gamma_{\nu\mu}, \quad p_{\nu\mu} = (p^0_{\nu\mu} + \nu_{\mu} E^0_{\nu\mu}) \gamma_{\nu\mu}, \tag{21}
\]
\[
E_{\mu\nu} = (E^0_{\mu\nu} + \nu_{\nu}^0 p^0_{\mu\nu}) \gamma_{\mu\nu}, \quad p_{\mu\nu} = (p^0_{\mu\nu} + \nu_{\nu} E^0_{\mu\nu}) \gamma_{\mu\nu}, \tag{22}
\]

where \( \gamma_{\nu\mu} = 1/\sqrt{1 - u^2_{\pi}} = E_{\pi}/m_{\pi} \), and thus\(^4\)
\[
\varphi_1 - \varphi_2 = \frac{\gamma_{\pi}}{L^0 m_\pi} \left\{ (E^0_{\nu} - \nu_{\nu} E^0_{\mu}) \Delta t + (E^0_{\mu} - \nu_{\mu} E^0_{\nu}) d \right\}. \tag{23}
\]

If we take into account that

\(^3\) The muon detector has to be to the left of the neutrino detector because the latter must be shielded.

\(^4\) Let us note that the expressions for \( \Delta E_\nu \) and \( \Delta p_\nu \), as given by Eqs. (70) and (73), differ from Eqs. (5.8) and (5.9) derived for the same observables under the same assumptions in Ref. [12] due to a mistake in Ref. [12].
\[ L^0 = \frac{E_{\nu}^0}{E_{\nu}}, \]  

we see that the oscillation frequency is proportional to \( E_{\pi} E_{\nu} / L \), while the oscillation length decreases as \( L / E_{\pi} E_{\nu} \). The explanation of this drastic dependence which looks quite unexpected will be given in Section 3.2.

2.4. Experiments of the types B and C with monochromatic pions

Oscillations described by equations similar to Eqs. (9)-(13) and (19) were referred to in Ref. [1] as oscillations of a muon beam. In order to clarify the situation let us consider first the detection of muons (without detecting neutrinos) in the case \( \nu_{\pi} = 0 \). The beam of muons is described not by a wave function, but by a density matrix (see, e.g., Ref. [25]). The probability of the muon detection is obtained by integrating Eq. (9) over the neutrino position \( x_\nu \) and summing over neutrino flavours. Each of these operations results in the vanishing of the oscillating term in Eq. (9). Integration over \( x_\nu \) simply leaves no dependence on \( x_\mu \). Summing over \( \nu_{\alpha} (\alpha = e, \mu) \) also eliminates oscillations since for the case of final \( \nu_{\mu} \) and for the case of final \( \nu_{e} \) the oscillating term enters with opposite signs, see Eqs. (12), (13). Thus in case B the probability of detecting a muon does not depend on \( x_\mu \). Moreover, it does not depend on the muon momentum \( p_{\mu} \). In the case \( \nu_{\pi} = 0 \) there is no beam of muons: they come to the muon detector isotropically because the decaying pions are fully de-localized due to the uncertainty relation.

Note that the same applies to case C: although the global effects of \( \nu_{e} \) appearance and \( \nu_{\mu} \) disappearance are present, the flux of neutrinos after integration over \( x_\mu \) is isotropic, and the oscillating term is washed out. The latter conclusion may appear to be in contradiction with the generally accepted theory of neutrino oscillations. However, the problem is solved as soon as we take into account the fact that in real experiments we never have a decaying pion with a sharp momentum. Its momentum distribution has a non-vanishing coherent spread. As a result, pions are described by coherent packets of plane waves, and are localized in space-time [26–29].

Let us now repeat the above reasoning in a more formal way, namely, come back to the probability formula (9) and use it for formal description of experiments of the types B and C in the general case of a pion plane wave with sharp momentum \( p_{\pi} \). This is straightforward: one should just sum over all the states of the neutrino and muon, respectively.

We begin with the observation of muons (case B). The states of the neutrino are labeled by the “flavour” index \( \alpha = e, \mu \) and by the “position” \( x_\nu \). Thus the probability of \( \mu \) detection at the space-time point \( x_\mu \) is equal to

\[
P_{\mu}^{(B)}(x_\mu) = \sum_{\alpha} \int P_{\nu_{\alpha}}^{(A)}(x_\mu, x_\nu) \, dx_\nu
= \int \left[ P_{\nu_{\mu}}^{(A)}(x_\mu, x_\nu) + P_{\nu_{e}}^{(A)}(x_\mu, x_\nu) \right] \, dx_\nu.
\]  

(25)
This can be alternatively formulated as a transition from the wave function of the \( \mu \nu \) system to the density matrix \([25]\) for \( \mu \) given by

\[
\rho_{\mu}(x_\mu, x'_\mu) = \text{Tr}_\nu \int \psi(x_\mu, x_\nu)\psi^*(x'_\mu, x_\nu) \, dx_\nu ,
\]

where \( \text{Tr}_\nu \) denotes summation over neutrino flavours. Eq. (25) is identical to (26) for \( x_\mu = x'_\mu \). In accordance with the general principles of quantum mechanics, the r.h.s. of (25) and (26) are automatically independent of \( t_\nu \).

It remains to substitute (9) into (25). Integration over \( x_\nu \) gives rise to \( \delta(p_{\nu_1} - p_{\nu_2}) = 0 \), because \( p_{\nu_1} \neq p_{\nu_2} \). This is already enough to eliminate the oscillation term. However, it vanishes in experiment of type B for a more fundamental reason: because of the summation over neutrino species. Indeed, it is easy to see that the oscillation term cancels in the sum of (12) and (13) even before integration over \( x_\nu \). Thus we get

\[
P_{\mu}^{(B)}(x_\mu) = 1 .
\]

The attentive reader can get suspicious at this point. We identified two reasons for the elimination of oscillations in case B: orthogonality of neutrino states in the flavour and momentum spaces. If we now turn to the observation of neutrinos (case C), the first reason is absent (a muon is always the same particle), but the second reason is still present: \( p_{\mu_1} \neq p_{\mu_2} \) — and this is enough to eliminate the neutrino oscillating term in case C, contrary to any reasonable expectations.

Still, we insist that in the "sharp" case under consideration this is really true: the probability to observe a neutrino \( \nu_\mu \) at point \( x_\nu \),

\[
P_{\nu_\mu}^{(C)}(x_\nu) = \int P_{\nu_\mu}^{(A)}(x_\mu, x_\nu) \, dx_\mu
\]
does not contain an oscillating term if one substitutes (9) into (28). If one uses Eq. (12) for \( P_{\nu_\mu}^{(A)}(x_\mu, x_\nu) \) and Eq. (13) for \( P_{\nu_\mu}^{(A)}(x_\mu, x_\nu) \), one gets

\[
P_{\nu_\mu}^{(C)}(x_\nu) = \beta_{1\mu}^2 + \beta_{2\mu}^2 = \cos^2 \theta + \sin^2 \theta < 1 .
\]

\[
P_{\nu_\mu}^{(C)}(x_\nu) = \beta_{1\nu}^2 + \beta_{2\nu}^2 = 2 \sin^2 \theta \cos^2 \theta > 0 .
\]

Thus, the global manifestations of the oscillations, the appearance of \( \nu_e \) and disappearance of \( \nu_\mu \), are evident, but the oscillating terms themselves are absent. Local oscillations do occur, but we cannot observe them because we do not know (or pretend not to know) the starting point of the neutrino, the position of the neutrino source.

In other words, there is a simple reason for the fact that local oscillations are not observable: our assumption that the decaying pion has a definite momentum, which is never true in experiments.

It is more or less obvious that allowing for a minor coherent dispersion in the momentum distribution of pion, one will always find solutions to the equation

\[
P_{\mu_1}(p_\pi) = p_{\mu_2}(p_\pi) .
\]

As we will see (footnote 8), Eq. (30) has a solution
where \( L = \frac{2E_\nu}{(m_\pi^2 - m_\mu^2)} \).

The momentum dispersion is necessarily present in any realistic experiment: pions are usually localized in a region much smaller than \( 1/|\delta p_\pi| \), given by Eq. (31), and then, by the uncertainty relation, their momentum dispersion\(^5\) should be much larger than \( |\delta p_\pi| \).

Thus we need to return to the very beginning and repeat our analysis, allowing for some small (as compared to the masses and energies in the problem), but non-vanishing momentum dispersion in the wave function of the original pion. This is a simple calculation, but still it deserves being done: for example, one should check that such dispersion does not wash out the oscillations in experiments of types A and C. We shall also use this new calculation to represent Eq. (9) in a somewhat different form; this can help one to better understand the "paradoxical" results of Section 2.3.

3. Decay of a pion with small momentum dispersion

3.1. Experiments A, B, and C

Let us now assume that a pion has been created at a space-time point \( x_\pi \) with some momentum distribution \( \phi(p_\pi) \). This means that at the decay point \( x_i \) the pion wave function is

\[
\psi_\pi(x_i - x_\pi) = \int \phi(p_\pi) e^{-i p_\pi (x_i - x_\pi)} dp_\pi, \quad p_\pi^2 = m_\pi^2,
\]

and that of the emerging \( \mu\nu \) system,

\[
\Psi(x_\mu, x_\nu) = \int \phi(p_\pi) e^{-i p_\pi (x_i - x_\pi)} \psi_{p_\pi}(x_\mu, x_\nu | x_i) dp_\pi,
\]

with \( \psi_{p_\pi}(x_\mu, x_\nu | x_i) \) given by Eq. (6).

Actually \( \Psi(x_\mu, x_\nu) \) does not explicitly depend on \( x_i \), because we imposed the conservation law (4) in all our formulas.\(^6\) However, \( x_i \) plays an important role in the physical interpretation of the results. In order to see this let us keep \( x_i \), and introduce the condensed notations \( x_{\pi i} = x_i - x_\pi, \quad x_{\mu i} = x_\mu - x_i \) and \( x_{\nu i} = x_\nu - x_i \).

By using Eq. (33) we define the amplitude \( A_{\nu\mu}(x_\mu, x_\nu) \) (compare with Eq. (8) for \( a_{\nu\mu}(x_\mu, x_\nu) \)).

\(^5\)To avoid confusion, let us emphasize that here we speak about the coherent dispersion of a pion produced in a given act of collision with accompanying particles being in a given state. This should not be mixed up with non-coherent momentum dispersion of pions in the same beam. Non-coherent means that the pion is produced with the same accompanying particles, but being in a different state, or with other accompanying particles, or produced in a different collision act.

\(^6\)The usual logic in quantum field theory is reversed: one includes integration \( \int d^4x_i \) in the definition (33) of \( \Psi(x_\mu, x_\nu) \), and this integration leads to the conservation law (4).
\[ \mathcal{A}_{\nu}(x_{\mu}, x_{\nu}) = \int dp_{\pi} \phi(p_{\pi}) \sum_{n} \beta_{na} \exp \left(-ip_{\pi}x_{\pi} - ip_{\mu n}x_{\mu i} - ip_{\nu n}x_{\nu i} \right) \] (34)

and get for the probability
\[ P_{\nu}(x_{\mu}, x_{\nu}) = |\mathcal{A}_{\nu}(x_{\mu}, x_{\nu})|^2 = \int \int dp_{\pi} d\vec{p}_{\pi} \phi(p_{\pi}) \phi(\vec{p}_{\pi}) \sum_{n, \bar{n}} \beta_{na} \beta_{\bar{n}a} e^{-i(\varphi_{n} - \varphi_{\bar{n}})}, \] (35)

where
\[ \varphi_{n} - \varphi_{\bar{n}} = (p_{\pi} - \vec{p}_{\pi})x_{\pi} + (p_{\mu n} - \vec{p}_{\mu n})x_{\mu i} + (p_{\nu n} - \vec{p}_{\nu n})x_{\nu i}, \] (36)

and \( n(\bar{n}) = 1, 2. \)

Let us now make use of the assumption that the dispersion of the distribution \( \phi(p_{\pi}) \) is small as compared to all the momenta in the problem. This allows one to put
\[ \delta E_{\pi} \equiv E_{\pi} - \bar{E}_{\pi} = \frac{1}{E_{\pi}} p_{\pi}(p_{\pi} - \vec{p}_{\pi}) = v_{\pi}(p_{\pi} - \vec{p}_{\pi}) \equiv v_{\pi} \delta p_{\pi}, \]
\[ \delta E_{\mu n} \equiv E_{\mu n} - \bar{E}_{\mu n} = \frac{1}{E_{\mu}} p_{\mu n}(p_{\mu n} - \vec{p}_{\mu n}) = v_{\mu}(p_{\mu n} - \vec{p}_{\mu n}) \equiv v_{\mu} \delta p_{\mu}, \]
\[ \delta E_{\nu n} \equiv E_{\nu n} - \bar{E}_{\nu n} = \frac{1}{E_{\nu}} \left( p_{\nu}(p_{\nu n} - \vec{p}_{\nu n}) + \frac{m_n^2 - m_{\bar{n}}^2}{2} \right) \]
\[ = v_{\nu}(p_{\nu n} - \vec{p}_{\nu n}) + \frac{m_n^2 - m_{\bar{n}}^2}{2E_{\nu}} \equiv v_{\nu} \delta p_{\nu} + \frac{m_n^2 - m_{\bar{n}}^2}{2E_{\nu}} \] (37)

where the \( v \)'s are velocities of the particles, \( v = p/E. \) (Formulas (37) are derived by subtraction of the on-mass-shell equalities \( E^2 = p^2 + m^2 \) and \( \bar{E}^2 = \bar{p}^2 + m^2. \)) Note also that from energy conservation one gets
\[ \delta E_{\pi} = \delta E_{\mu} + \delta E_{\nu}. \] (38)

Substituting this into (36) we get
\[ \varphi_{n} - \varphi_{\bar{n}} = -(p_{\pi} - \vec{p}_{\pi})(x_{\pi} - v_{\pi} t_{\pi}) - (p_{\mu n} - \vec{p}_{\mu n})(x_{\mu i} - v_{\mu i} t_{\mu i}) \]
\[ -(p_{\nu n} - \vec{p}_{\nu n})(x_{\nu i} - v_{\nu i} t_{\nu i}) + \frac{m_n^2 - m_{\bar{n}}^2}{2E_{\nu}} t_{\nu i}. \] (39)

From Eq. (39) we may derive formulas for the cases A, B, C with the pion being described by a wave packet.

When considering case \( A, \) let us note that the first three terms in Eq. (39) vanish on the trajectories of the particles,
\[ x_{\pi} = v_{\pi} t_{\pi}, \quad x_{\mu i} = v_{\mu i} t_{\mu i}, \quad x_{\nu i} = v_{\nu i} t_{\nu i}. \] (40)
Hence
\[ \varphi_n - \varphi_{\tilde{n}} = \frac{t_{vi}}{L} \quad \text{for } n = 1, \tilde{n} = 2. \]  

(41)

Note that in case A the muon detector is used in order to deduce the position in space-time of the pion decay point \( \iota \). Hence the value of \( t_{vi} \) (in case A we may call it \( t_{vi}^A \)) can be determined without the knowledge of the moment of the pion production \( t_{\pi} = (x_{\pi}^C, x_{\pi}^\nu) \), see Eq. (32)). This will be done explicitly in Eqs. (55)-(58). Our new expression for the probability in case A,

\[ P_{\nu_i}^{(A)}(x_{\mu}, x_{\nu}) = \sum_{n, \tilde{n}} \beta_{\mu n} \beta_{\tilde{n} \tilde{n}} \exp \left( i \frac{m_n^2 - m_{\tilde{n}}^2}{2E_{\nu} t_{vi}} \right) \]

(42)

looks absolutely different from the old one (compare (39) and (42) with (9), (10), (23)). In the next section we will show that in fact they are equivalent, and \( \varphi_1 - \varphi_2 = \varphi_1 - \varphi_{\tilde{2}} = t_{vi}/L \).

The same momentum dispersion technique can be applied to the description of realistic experiments of types B and C. In case C (single neutrino detector) by integrating (35) over \( x_{\mu} \), one gets a delta function \( \delta(p_{\mu n} - \tilde{p}_{\mu \tilde{n}}) \), where \( p_{\mu n} \) and \( \tilde{p}_{\mu \tilde{n}} \) are expressed through \( p_{\mu} \) and \( \tilde{p}_{\mu} \), respectively. If \( n \neq \tilde{n} \), the argument of the delta function vanishes for non-vanishing \( p_{\mu} - p_{\tilde{n}} = \delta p_{\mu} \), which for high energy pions must be equal to \( 2\gamma_{\pi} L = \frac{\gamma_{\pi}}{L^0} \) (see also Eq. (23)). The corresponding probability is

\[ P_{\nu_i}^{(C)}(x_{\mu}, x_{\nu}) = \beta_{1a}^2 + \beta_{2a}^2 + 2\xi \beta_{1a} \beta_{2a} \cos \frac{t_{vi}^C}{L}, \]

(43)

where

\[ \xi = \frac{\int dp_{\mu} \phi(p_{\mu} - \delta p_{\mu}/2) \phi(p_{\mu} + \delta p_{\mu}/2)}{\int dp_{\mu} \phi^2(p_{\mu})}, \]

(44)

and \( t_{vi}^C \) can be expressed through \( x_{\nu} \) and \( x_{\pi} \), the space-time point where pion has been created; in the collinear case \( (p_{\nu} || p_{\pi}) \),

\[ t_{vi}^C = \frac{|x_{\nu} - x_{\pi}| - v_{\nu}(t_{\nu} - t_{\pi})}{v_{\nu} - v_{\pi}}. \]

(45)

For monochromatic pions \( \xi = 0 \), and

\[ \frac{7}{\text{Let us check again that the phase difference given by Eqs. (39)-(41) is Lorentz invariant. In the reference frame which moves with the velocity } \mathbf{u}, \mathbf{t}_{vi} - \gamma_u (t_{vi} + \mathbf{ux}_{\mu}) = (1 + \mathbf{w}_{\nu} \gamma_u) \mathbf{l} + \mathbf{u}_{\nu} \mathbf{l} \text{ and } E_{\nu} - \gamma_u (E_{\nu} + w_{\nu} \mathbf{u}) = (1 + \mathbf{w}_{\nu} \gamma_u E_{\nu}), \text{ where } \gamma_u = 1/\sqrt{1 - w_{\nu}}, \text{ so that } t_{vi}/L = (t_{vi}/E_{\nu})(m_1^2 - m_2^2)/2 \text{ remains invariant.}} \]

\[ \frac{8}{\text{To find an exact expression for } p_{\mu} - \tilde{p}_{\mu} = \delta p_{\mu}, \text{ let us consider } p_{\mu} \text{ as a function of } p_{\mu} \text{ and } m_{\nu}, \text{ and calculate the difference } \delta p_{\mu} = \delta p_{\mu} - \delta p_{\nu}. \text{ By using Eqs. (37) and (38) it is easy to find } \delta p_{\mu}, \text{ corresponding to } \delta p_{\mu} = p_{\mu} - \tilde{p}_{\mu} = 0 \text{ (and hence } \delta E_{\mu} = 0). \text{ The condition } \delta p_{\mu} = 0 \text{ means } \delta p_{\mu} = \delta p_{\nu} \text{ and } \delta E_{\mu} = \delta E_{\nu}, \text{ hence } (v_{\nu} - v_{\nu}) \delta p_{\mu} = 1/L, \text{ where we put } \delta m_1^2 = m_1^2 - m_2^2. \text{ For the collinear case, } v_{\nu} \approx v_{\pi}, \text{ we get } |\delta p_{\mu}| = L^{-1} (1 - v_{\nu}) \text{ since } v_{\nu} \approx 1. \text{ For high energy pions } |\delta p_{\mu}| \text{ is much larger than } L^{-1} : |\delta p_{\mu}| \approx 2\gamma_{\pi}/L = \gamma_{\pi}/L^0.}\]
while in realistic experiments \( \xi = 1 \), and

\[
\mathcal{P}_{\nu_e}^{(C)}(x_i|\xi = 1) = \beta_{1a}^2 + \beta_{2a}^2 + 2\beta_{1a}\beta_{2a}\cos \frac{t_{vi}^C}{L}.
\]

An obvious way to fix \( t_{vi}^C \) and thus to observe the oscillating term is to ascertain the position of the pion decay point \( x_i \). If \( x_i \) is not known experimentally, then the value of \( t_{vi} \) in Eq. (43) can be fixed only if the moment of the production of a pion in target, \( t_\pi \), is known with a very high accuracy. For that purpose the time structure of the proton beam should be of the order of a few nanoseconds. Otherwise the oscillating term in Eq. (43) would be washed out, and only global manifestations of neutrino oscillations (\( \nu_e \) appearance and \( \nu_\mu \) disappearance) would remain; compare with Eqs. (29).

As for case \( B \), it has already been mentioned that the oscillating term vanishes after summation over neutrino flavours (see Section 2.4). This statement is obviously true also in the situation when the pion is described by a packet of plane waves: summation over \( a = e, \mu \) using Eq. (42) gives \( \mathcal{P}^{(B)}(x_\mu) = 1 \).

### 3.2. On the equivalence of the two representations for \( P^{(A)} \)

There are two main ingredients in the previous section, namely: (1) the momentum representation of the pion wave packet (Eq. (33)), and (2) the classical trajectories for fast particles, Eq. (40). Actually we could use only the second one in order to get Eq. (42), and to solve the “paradoxes” of Sections 2.2 and 2.3.

In this section we will work with plane waves and at the same time with classical trajectories for \( \mu \) and \( \nu \) (see, e.g., Ref. [26]). After \( \mu \) and \( \nu \) are detected at certain space-time “points” with given momenta, we have, so to speak, “a posteriori packets” of these particles, for which \( x_{\nu\mu} = v_\mu t_{\mu i}, \quad x_{vi} = v_\nu t_{vi} \), where \( v_\mu = p_\mu / E_\mu \), and \( v_\nu = p_\nu / E_\nu \). Note that the wavelength of \( \mu \) or \( \nu \) does not exceed \( 10^{-13} \) cm, while the characteristic distances in neutrino experiments are larger than hundred meters.

For \( \nu \) and \( \mu \) on the mass shell,

\[
\Delta E_\nu = E_{\nu 1} - E_{\nu 2} = v_\nu \Delta p_\nu + \frac{m_1^2 - m_2^2}{2E_\nu},
\]

\[
\Delta E_\mu = E_{\mu 1} - E_{\mu 2} = v_\mu \Delta p_\mu,
\]

where

\[
\Delta p_\nu = p_{\nu 1} - p_{\nu 2}, \quad \Delta p_\mu = p_{\mu 1} - p_{\mu 2}.
\]

Note that if the pion momentum is sharp,

\[
\Delta E_\pi = 0, \quad \Delta p_\pi = 0.
\]
\[ \Delta E_\mu = -\Delta E_\nu, \quad \Delta p_\mu = -\Delta p_\nu, \] (52)

and hence

\[ \nu_\nu \Delta p_\nu + \nu_\mu \Delta p_\mu = (\nu_\nu - \nu_\mu) \Delta p_\nu = \frac{m_1^2 - m_2^2}{2E_\nu} = \frac{1}{L}. \] (53)

In Eq. (41) we have shown that \( \varphi_1 - \varphi_2 = t_{vi}/L. \) Consider now \( \varphi_1 - \varphi_2 \) as given by Eq. (10), and use Eqs. (48), (49), and (52):

\[ \varphi_1 - \varphi_2 = (p_{v1} - p_{v2})(x_{vi} - x_{\mu i}) \]
\[ = \Delta E_\nu (t_{vi} - t_{\mu i}) - \Delta p_\nu (x_{vi} - x_{\mu i}) \]
\[ = t_{vi}(\Delta E_\nu - \nu_\nu \Delta p_\nu) + t_{\mu i}(\Delta E_\nu - \nu_\mu \Delta p_\mu) = \frac{t_{vi}}{L}. \] (54)

Thus \( \varphi_1 - \varphi_2 = \varphi_1 - \varphi_2, \) as was promised after Eq. (42). Eq. (54) directly brings us from Eq. (9) to Eq. (42). We would get the same result in the cases of a pion with \( p_\pi = 0 \) (Eq. (19)), and of a relativistic pion (Eqs. (20)-(23)). At fixed distance between the detectors \( d \) the measurement of \( \Delta t \) (time difference between "clicks" of the two detectors) allows one to find the space-time point \( x_i \) of the pion decay. Thus for a pion with \( v_\pi > v_\mu^0 \) decaying to the left of the muon detector we have in the collinear case discussed in Section 2.3

\[ |x_\mu - x_i| = \nu_\mu (t_\mu - t_i), \quad |x_\nu - x_i| = \nu_\nu (t_\nu - t_i), \] (55)

hence

\[ t_i = \frac{-|x_\nu - x_\mu|}{\nu_\nu - \nu_\mu} - \frac{\nu_\mu t_\mu + \nu_\nu t_\nu}{\nu_\nu - \nu_\mu}, \] (56)

and

\[ t_{vi}^A = t_{vi} = t_\nu - t_i = \frac{\nu_\mu t_\mu - \nu_\nu t_\nu}{\nu_\nu - \nu_\mu} + \frac{|x_\nu - x_\mu|}{\nu_\nu - \nu_\mu} = -\nu_\mu \Delta t + \frac{d}{\nu_\nu - \nu_\mu}. \] (57)

Taking into account that \( \nu_\nu \approx 1 \), we see that in the last equation the denominator is extremely small for ultrarelativistic muons. That means that the decay of the pion discussed in Section 2.3 must take place at a very large distance from the detectors, unless the numerator in Eq. (57) is also very small.

For a pion with \( v_\pi < v_\mu^0 \), decaying between the two detectors

\[ t_{vi}^A = t_{vi} = \frac{\nu_\mu \Delta t + d}{\nu_\mu + \nu_\nu}. \] (58)

It is important to stress that in Eq. (39) or (54) the phase \( \varphi_1 - \varphi_2 \) depends only on \( t_{vi} \) and does not depend on \( t_{\mu i} \). Thus these formulas simply describe the standard neutrino oscillations from the point of creation of the neutrino until the point of its detection. Accordingly they do not depend on the position of the muon detector. Therefore EPR-like correlations between muon and neutrino detection appear only when we express \( t_{vi} \).
in terms of $\Delta t = t_{\mu} - t_\nu$ and $d = x_{\nu} - x_\mu$. If we assume that a special detector measures the decay point of the pion $x_i$, then the situation becomes absolutely trivial: the muon is not involved at all.

4. Concluding remarks

Let us summarize the main results.

In considering the decay $\pi \rightarrow \mu \nu$ and subsequent neutrino oscillations we assumed that the 4-momentum is conserved and all particles ($\pi, \mu, \nu$'s) are on the mass shell.

We have used two different approximations:

1) Gedanken approximation. The pion is monochromatic (has a definite, sharp momentum). Then the same is true for $\mu$'s and $\nu$'s (up to an ambiguity in rotation of the $\mu\nu$ plane, which is inessential for our purposes). Thus all particles are described by plane waves. Non-trivial phenomena, associated with neutrino oscillations, are due to a slight difference between the 4-momenta of neutrinos with different masses. As a corollary, the muon, created in the decay of the same pion together with different neutrinos, also possesses slightly different 4-momenta.

2) Realistic approximation. The pion is localized in space and is described by a (coherent) wave packet. Then the $\mu\nu$ wave function is a linear superposition of wave packets, resulting from decays of the pion at different times. It is reduced to a product of packets for $\mu$ and $\nu$, when any of the particles is detected at a definite space-time point – this gives rise to what we call “classical a posteriori trajectories”.

We analyzed the following three experimental situations.

(A) The muon and neutrino are detected in coincidence. The probability of detecting $\mu$ at a space-time point $x_\mu$ and $\nu_a$ ($a = e, \mu$) at a point $x_\nu$ is given by Eq. (42), where $t_\nu^a$ is defined by Eq. (57) or (58).

(B) The neutrino is ignored, while $\mu$ is detected at the space-time point $x_\mu$. The corresponding probability does not depend on $x_\mu$ at all, $\mathcal{P}^{(B)}(x_\mu) = 1$, and no traces of neutrino oscillations are seen.

(C) The muon is ignored, while $\nu_a$ is detected at the space-time point $x_\nu$. This time the probability depends essentially on the spread of the (unobserved) muon wave packet and thus on that of the original pion. For a monochromatic pion or, more precisely, for $|\delta p_\pi| \ll \gamma^2_\pi/L$ in a collinear case, the parameter $\xi$ in Eq. (43) vanishes, and there is no oscillating $x_\nu$ dependence in the probability, see Eq. (46).

The only consequence of neutrino oscillations in this case is the global effect: $\mathcal{P}_{\nu_a}^{(C)} > 0$, while $\mathcal{P}_{\nu_a}^{(C)} < 1$. For large enough dispersion, $|\delta p_\pi| \gg \gamma^2_\pi/L$, we have $\xi = 1$, and the standard expression $\mathcal{P}_{\nu_a}^{(C)}(x_\nu)$ given by Eq. (47). If $t_\pi$ is not specified (not known in a given experiment), the probability should be averaged over $t_\pi$. This eliminates the oscillation term, but preserves the global

\footnote{Note that for ultrarelativistic pions, and in the collinear approximation, $\delta p_\pi$ has to be many orders of magnitude larger than the naive estimate, $|\delta p_\pi| \gg 1/L$.}
dependence on the neutrino flavour \( \nu_a \). Note, however, that \( t_\pi \) was used by us to
deduce the position of the pion decay point \( x_\pi \). If this position is known from other
considerations (e.g. a short decay pipe), then \( t_\pi \) is fixed, and the oscillating term
is of course retained.

As follows from our discussion, oscillations of the correlation probability may be
observed in the two-detector experiments which measure both charged leptons and
neutrinos in coincidence.\(^{10}\) These oscillations would look like an EPR effect, though
this is a simple consequence of standard neutrino oscillations and relativistic kinematic
relations. Muons do not oscillate.\(^{11}\)

Acknowledgements

We are grateful to Mario Greco, who drew the attention of one of us to Ref. [2].
We thank S.P. Denisov and S.S. Gershtein for the discussion of the IHEP neutrino
experiment, T. Goldman for informing us about some references which were missing
in the preliminary version of this paper, A.N. Rozanov for the discussion of the CERN
neutrino experiment, M.I. Vysotsky for his role of the devil’s advocate, and V.L. Telegdi
for critically reading the manuscript and many helpful suggestions.

This work was partially supported by several grants. The work by A.D. was supported
in part by the Danish National Science Research Council through its support of the
Theoretical Astrophysical Center. The work of A.M. is supported by the RFBR Grant
96-15-96939. He also acknowledges the support of DFG and hospitality of Humboldt
University, Berlin, and IFH, Zeuthen, during the work on this paper. L.O. acknowledges

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\(^{10}\) One is tempted to apply arguments developed in this paper not only to hypothetical neutrino oscillations,
but also to the already observed neutral kaon oscillations and to similar oscillations of neutral heavy mesons
D\(^0\), D\(^*\), \( K\) and \( B\). The obvious analogs of the \( \pi \rightarrow \mu \nu \) decay would be the decays \( \Lambda \rightarrow pK\),
\( \Xi \rightarrow K^0 \bar{K}, \) or \( D \rightarrow K^0 \bar{K}, D^+ \rightarrow \bar{K}^0 \nu, D^+ \rightarrow K^+ \bar{K}, \) or the decays of \( B \) mesons:
\( B^+ \rightarrow D^0 \pi^+, B^0 \rightarrow D^0 \pi^0, B^+ \rightarrow D^0 D^+, B^+ \rightarrow D^0 D^0\). The corresponding decay widths in the above examples are larger than the
oscillation frequencies, and therefore the dispersion of momenta is provided by these decay widths. The
description of the EPR effect in \( \phi \) and \( T \) decays in terms of amplitudes has been advocated in Ref. [30].

\(^{11}\) When this paper was near its final stage, the authors of Ref. [1] have replaced the original paper (E. Sassaroli, Y.N. Srivastava and A. Widom, Charged Lepton Oscillations, hep-ph/9509261; September 1995, 15
pages) by another one: Y.N. Srivastava, A. Widom and E. Sassaroli, Lepton Oscillations, hep-ph/9509261 v2
(November 24, 1996; 2 pages) with the same figures and the same claim formulated in a new abstract: “A
simple but general proof is presented to show that Lorentz covariance and 4-momentum conservation alone are
sufficient to obtain muon oscillations in pion decay if the recoiling neutrino oscillate.” On December 13 a new
paper [36] by A. Widom and Y.N. Srivastava has appeared in the hep-ph archive. In this paper they claim that
muon oscillations associated with mixed neutrino mass matrices should manifest themselves in the experiment
measuring \( g - 2 \) – the anomalous magnetic moment of the muon. As is known, \( g - 2 \) experiments are
not done in coincidence with detection of neutrinos which accompany the production of the muons. Therefore
we believe that no effect associated with neutrino mixing should be seen in \( g - 2 \) experiments.

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the RFBR Grants 96-02-18010 and 96-15-96578 and a Humboldt Award. The work of M.S. is supported by a Grant of the NFR and INTAS-RFBR Grant 95-605.

References

[28] V.I. Kogan and V.M. Galitskiy, Problems in Quantum Mechanics, Ch. 5, Problem 2 (Prentice Hall, 1963);
    V.M. Galitskiy, B.M. Karnakov and V.I. Kogan, Zadachi po Kvantovoi Mekhanike 2nd Ed. (Nauka, Moscow, 1992) Ch. 6, Problem 6.2.