QUARK MIXING IN WEAK INTERACTIONS

Ling-Lie CHAU

Physics Department, Brookhaven National Laboratory, Upton, New York, 11973, U.S.A.
QUARK MIXING IN WEAK INTERACTIONS

Ling-Lie CHAU

Physics Department, Brookhaven National Laboratory, Upton, New York, 11973, U.S.A.

Received 9 July 1982

Contents:

1. The quark mixing matrix and its determination
   1.1. Introductory discussion
   1.2. Determination of $V_{ud}$ and $V_{ub}$
   1.3. The mixing and $CP$ violation in the $K^0\bar{K}^0$ system: A discussion of the formalism
   1.4. Determination of $V_{ud}$ from the box-graph calculation of $\Delta m$ and $\epsilon$ in the $K^0\bar{K}^0$ system
   1.5. Determination of $V_{ub}$ from other experimental information
   1.6. Concluding remarks

2. Nonleptonic decays
   2.1. Introductory remarks
   2.2. Quark diagram approach
   2.3. Symmetry approach
   2.4. Concluding remarks

3. $CP$ violation
   3.1. The neutral particle–antiparticle mixing and its $CP$ violation effects

3.2. $CP$ violation in partial decay rates
3.3. The estimate of $\epsilon'$
3.4. The estimate of neutron electric dipole moment $d_n$
3.5. $CP$ violation from other sources
3.6. Concluding remarks

4. Quark mixing matrix and quark masses
   4.1. Introduction and formulation
   4.2. Parameter counting, calculability of quark mixing matrix in terms of quark masses, and some examples
   4.3. Relating quark mixing matrix to quark masses necessarily leads to flavor changing neutral couplings
   4.4. Concluding remarks

5. Alternative to the three left-handed doublet model
   5.1. Alternative models
   5.2. Experimental status of the $b$-particle decay
   5.3. Concluding remarks

6. Summary and outlook

Acknowledgments
References

Abstract:
The status of the quark mixing in weak interaction is reviewed. The $3 \times 3$ quark mixing matrix for the three left-handed doublet model is analyzed using various experimental information involving strange, charmed, and $b$-flavored particles. Its interplay with nonleptonic decays, implication on neutral particle–antiparticle mixing and $CP$ violation in heavy quark systems, and the possible origin of the quark mixing from quark mass matrix are discussed. Finally we briefly review the status of alternative sources for $CP$ violation, and alternative models to the three left-hand quark doublet model.

Single orders for this issue

PHYSICS REPORTS (Review Section of Physics Letters) 95, No. 1 (1983) 1–94.

Copies of this issue may be obtained at the price given below. All orders should be sent directly to the Publisher. Orders must be accompanied by check.

Single issue price Dfl. 52.00, postage included.
**Introduction**

The discovery of the quark hierarchy and the development of our understanding of weak interactions in recent years have truly made a splendid page in the history of particle physics. Not only does the quark generation repeat itself, even the history of the discovery of the new quark states seems to be repeating. In 1973 Kobayashi and Maskawa observed that in order for CP violation to be related to the coupling of the intermediate boson with quarks, there must exist at least three left-handed quark doublets, one more doublet than the two left-handed doublet model originally postulated in 1970 by Glashow, Iliopoulos and Maiani based upon the sole experimental observation that $K_L \rightarrow \mu^+ \mu^-$ is being extremely small. Amazingly, in an almost duplicative way to the history of the discovery of the charmed particles following $J/\psi$, the b quark states have now manifested themselves following the discovery of their implicit state $Y$ in 1977. Even though the sixth quark, the t quark, has so far not been observed, the impressive detailed studies of the decay properties of the b-particles in the $Y(4s)$ region at CESR have disqualified most of the alternative models without the t quark.

As was observed in the early sixties, the particle states to which the weak intermediate bosons couple, are mixtures of the physical particle states, and the mixing is characterized by one rotation angle $\theta_c$. In the theory with three left-handed quark doublets, the mixing of the quark states is given by a $3 \times 3$ unitary matrix characterized by three angles and one phase. This phase, as originally noted by Kobayashi and Maskawa, can give the CP-violation effects. In this framework there is very rich interplay between the weak decay properties of the particles and the CP-violation effects.

In this review, we present the status of our current knowledge of the quark mixing matrix, its relation with the weak decay properties of particles, implications for neutral particle–antiparticle mixing and CP-violation effects in heavy quark systems. Discussions are also given on the attempts to understand the origin of the quark mixing from symmetry properties in the quark mass matrix. Finally the status of alternative sources for CP violation, and the status of alternative models without the t quark are briefly reviewed. (Many of the formulation and discussions given here for the quarks can also be applied to the leptons.)

1. **The quark mixing matrix and its determination**

1.1. **Introductory discussion**

The history of the development of our understanding about the quark mixing in weak interactions is a fascinating one. In the early sixties, the left-handed quark states in weak interaction were a left-handed doublet, and a lonely strangle singlet [1.1], and all the right-handed quarks are singlets:

$$
\begin{pmatrix}
  u \\
  d'
\end{pmatrix}^L, s^L; u_R, d_R, s_R,
$$

where $d'$ is a mixed state of the down quark $d$ and the strange quark $s$,

$$
d' = V_{ud}d + V_{us}s
$$

when expressed in terms of the Cabibbo angle $\theta_c$, $V_{ud} = \cos \theta_c$, $V_{us} = \sin \theta_c$. Remarkably with this single
angle $\theta_c$, the nuclear $\beta$-decays and all the hyperon decays could be fitted. The determination in ’74 by M. Roos [1.2] was

$$V_{ud} = \cos \theta_c = 0.9737 \pm 0.0025 ,$$

(1.3)

from the $0^+ \rightarrow 0^+$ nuclear $\beta$-decays of $^{14}$O, and the metastable $^{26\text{m}}$Al in comparison with the $\mu$ decay; and

$$|V_{us}| = \sin \theta_c = 0.230 \pm 0.003 ,$$

(1.4)

from the then available data of semileptonic decays of the hyperons. This analysis gave

$$|V_{ud}|^2 + |V_{us}|^2 = 1.004 \pm 0.005 .$$

(1.5)

Thus the Cabibbo theory of $|V_{ud}|^2 + |V_{us}|^2 = 1$ is, within the error, satisfied.

However, in this theory there is the strangeness-changing neutral current $(d\bar{d} + s\bar{s})$ term arising from

$$d\bar{d}' = \cos^2 \theta_c \bar{d} d + \sin^2 \theta_c s\bar{s} + \cos \theta_c \sin \theta_c (d\bar{s} + s\bar{d}).$$

The rate of $K_L \rightarrow \mu^+\mu^-$ given by such a neutral strangeness changing current is much too big for the measured branching ratio of [1.3],

$$\text{Br}(K_L \rightarrow \mu^+\mu^-) = (9.1 \pm 1.8) \times 10^{-9} .$$

(1.6)

This led to the introduction of the charm quark to form another left-handed doublet in the pioneer paper by Glashow–Iliopolou–Maiani (GIM) [1.4], 1970,

$$\begin{pmatrix} u \\ (d', s')_L \end{pmatrix} , \begin{pmatrix} c \\ (t', s')_L \end{pmatrix} ; u_R , d_R , c_R , s_R ,$$

(1.7)

where $(d', s') = (d, s) V^T$, and $V^T$ is the transposed matrix of

$$V = \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix} = \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix} ,$$

(1.8)

and all the right-handed quarks are still singlets. The orthogonality of the mixing matrix, $(d\bar{d}' + s\bar{s}') = d\bar{d} + s\bar{s}$, guarantees the absence of strangeness-flavor changing neutral current $d\bar{s}$ term. It is now history that after a long wait since the original discoveries of $J$ [1.5] and $\psi$ [1.6], the charm particles were found in 1975 [1.7], fig. 1.1. The prediction of the GIM theory that charm particles would predominantly decay into strange particles has now been confirmed [1.8]. However the precise values of $V_{cs}$ and $V_{cd}$ are yet to be determined (see discussions later in this section).

The evolution of ideas kept pace. It was noticed in ’73 by Kobayashi and Maskawa [1.9] that the reality of the matrix in eq. (1.8) would not allow $CP$ violation via the intermediate-boson $W^\pm$ coupling in the standard SU(2)$_L \times U(1)$ theory. Based upon this purely theoretical observation they boldly introduced two more quarks, the $t$ and the $b$, which form the third pair of left-hand quark doublets. Now the left-handed doublets become

$$\begin{pmatrix} u \\ (d', t', s')_L \end{pmatrix} ; u_R , d_R , c_R , s_R , t_R , b_R$$

(1.9)
where \((d', s', b') = (d, s, b)V^\dagger\),

\[
V = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}.
\]

(1.10)

Again all the right-handed quarks are singlets. The \(V_{ij}\)'s characterize the coupling of the quarks \(q_i, q_j\) to the weak intermediate boson \(W^\pm\). The unitarity of the \(V\) matrix, \(VV^\dagger = I\) insures the absence of flavor-changing neutral current.

In general for \(n\) doublets, the number of physically significant parameters in the quark mixing matrix \(V\) is equal to the number of parameters for an \(n \times n\) unitary matrix, \(n^2\), minus the arbitrary phases of the quark field \((2n - 1)\), that can be removed, i.e., \(n^2 - (2n - 1)\), \([1.10]\). An orthogonal matrix can be characterized by \(n(n - 1)/2\) angles, thus the rest of the parameters \([n^2 - (2n - 1)] - n(n - 1)/2 = (n - 1)(n - 2)/2\) have to be characterized by phases. For example, for a \(2 \times 2\) unitary matrix, there are \(2^2 = 4\) parameters, \(\theta, \alpha, \beta, \gamma\), in the following most general parametrization

\[
A = \begin{pmatrix}
\cos \theta \exp(i\alpha) & \sin \theta \exp(-i\beta) \\
-\sin \theta \exp(i\gamma) & \cos \theta \exp(i(\beta + \gamma - \alpha))
\end{pmatrix}.
\]

(1.11a)

The \((2 \cdot 2 - 1) = 3\) phase parameters \(\alpha, \beta, \gamma\) can be removed by the bi-similarity transformation of two diagonal unitary matrices,

\[
\begin{pmatrix}1 & 0 \\0 & \exp(i(\gamma - \alpha))\end{pmatrix}A\begin{pmatrix}\exp(-i\alpha) & 0 \\0 & \exp(-i\beta)\end{pmatrix} = \begin{pmatrix}\cos \theta & \sin \theta \\-\sin \theta & \cos \theta\end{pmatrix}.
\]

(1.11b)

These transformations by the matrices \(\text{diag}[1, \exp i(\gamma - \alpha)]\) and \(\text{diag}[\exp(i\alpha), \exp(i\beta)]\) correspond to changing the phases of the up and down quark fields respectively (see more discussions in section 4). What is left then is just the single angle parameter \(\theta\) (sometimes we call it the magnitude parameter in contrast to phase parameter) corresponding to the \(2(2 - 1)/2 = 1\) angle parameter of a \(2 \times 2\) orthogonal matrix. Therefore, \(V_{ij}\) can be characterized by an angle \(\theta\) and no phase. This is the observation made by Kobayashi and Maskawa that in the case of two quark doublets there is no complexity in the mixing matrix, thus no \(CP\) violation from the \(W^\pm\) coupling (see more elaboration on this point in sections 1.3 and 1.4).

For the Kobayashi–Maskawa (K–M) theory \(n = 3\), following the same reasoning, one can see that \(V_{ij}\)'s are characterized by three angles and one phase \([1.9]\). One way to parametrize the K–M matrix is

\[
V = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix} = \begin{pmatrix}
c_1 & s_1c_3 & s_1s_3 \\
-s_1c_2 & c_1c_2c_3 + s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\
-s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta}
\end{pmatrix}.
\]

(1.12)

It is this complexity in \(V\) that provides the \(CP\) violation. Thus, the salient feature of the K–M model is that the \(CP\) violation effect is tied with the nonvanishing of some of the matrix elements in the third row or third column, which means that the \(b\) and the \(t\) flavored particles must have pure hadronic decays. Therefore, if the \(CP\) violation is indeed coming from this complexity of \(V\), or the phase \(\delta\), the unitarity of the \(2 \times 2\) matrix \(V\) in eq. (1.8) cannot be true.

It is interesting to note that this essential and elegant generalization by Kobayashi and Maskawa did
not gain much attention until several years later [1.11, 1.12]. Eventually the $b\bar{b}$ particle $Y$ was found in 1977 [1.13], and the $b$ flavored particles have just recently been found [1.14, 1.15]. However, so far the $t$ quark has not showed up in any of the searches for it. Current experiments at PETRA [1.16] have set a mass limit $m_t > 18.5$ GeV. On the other hand, as we shall discuss in more detail in section 5, all the alternative models built by theorists with a left-handed $b$ quark without the $t$ quark have failed the experimental tests. We are now living in an interesting time of waiting. In this review we shall discuss the phenomenology of quark mixing assuming that the $t$ quark exists [1.17].

Discussions presented in this section can be easily used for the mixing matrix for the lepton doublets.

1.2. Determination of $V_{ud}$ and $V_{us}$

Going back to our historical discussion, the theory of Glashow–Weinberg–Salam [1.18] for electroweak interactions in the meantime has gained much success. Its renormalizability was proven [1.19, 1.20]. The weak neutral current which is a natural part of the theory was observed [1.21]. With its single parameter $\sin^2 \theta_W = 0.23$, a wide class of electro and weak interactions can be explained [1.22]. In the light of this new possibility and inspired by the first indication that a heavier flavored particle cannot be stable [1.23], plus the fact that since Roos' work of '74 the data on hyperon semileptonic decay had been improved, with R. Shrock, we decided to re-analyze the mixing matrix [1.24]. Comparing the $0^+ \rightarrow 0^+$ nuclear $\beta$ decay of $^{14}$O and $^{26}$Al with the $\mu$ decay as shown in figs. 1.2a, 1.2b, assuming the absence of lepton mixing, $|V_{ud}|$ was determined,

$$|V_{ud}| = c_1 = 0.9737 \pm 0.0025,$$

which implies

$$s_1 = 0.2270^{+0.0104}_{-0.0110}.$$

Here the important point is to include the weak radiative correction effects as shown in fig. 1.3, [1.25]. This effect amounted to about 10% of the value of $V_{ud}$.

$|V_{us}|$ can be determined from strange particle decays. One way is from the $K_{e3}$ decay and the other is from the semileptonic decays of the hyperons. The average value of $|V_{us}|$ obtained was

$$|V_{us}| = 0.219 \pm 0.002.$$

To compare decay rates among hyperons with that of neutron, and to estimate the $K-\pi$ form factor $f_{K-\pi}^{\ast}(0)$, SU(3) symmetry must be used. The error in $|V_{us}|$ does not include the uncertainties due

Fig. 1.2a. Diagram for $\mu^+ \rightarrow \nu_\mu e^+ \nu_e$.

Fig. 1.2b. Diagram for nuclear $\beta$ decay $N_1 \rightarrow N_2 e^- \bar{\nu}_e$. 
to SU(3) breaking. The reader is referred to [1.24] for detailed discussions of errors and uncertainties involved.

Combining the results of eqs. (1.13, 1.14) we have,

$$|V_{ud}|^2 + |V_{us}|^2 = 0.996 \pm 0.005.$$  \hspace{1cm} (1.15)

The central value of $|V_{ud}|^2 + |V_{us}|^2$ is now determined to be smaller than that of Roos, eq. (1.5), though it is still consistent with one. From unitarity of $V$, eq. (1.15) gives

$$|V_{ub}|^2 = (s_1s_3)^2 = 0.004 \pm 0.005.$$  \hspace{1cm} (1.16)

From $V_{ud} = c_1$, $V_{us} = s_1s_3$, and the determined values of $V_{ud}$, $V_{us}$ of eqs. (1.13, 1.14), the $s_3$ can be determined [1.12, 1.24],

$$|s_3| = 0.28^{+0.21}_{-0.28}.$$  \hspace{1cm} (1.17)

Aside from the improvement from future more accurate data of hyperon semileptonic decays, it is hard to improve the determination of these two numbers, mainly due to theoretical uncertainties. For recent developments of hyperon decay data see ref. [1.26]. Therefore it will be best to obtain $V_{ub} = s_1s_3$ from a more direct source of the $b$ decay. That will be discussed later in section 1.5.

1.3. The mixing and CP violation in the $K^0\bar{K}^0$ system: A discussion of the formalism

Since the model is designed to provide $CP$ violation, some of the parameters must be determined from the $CP$ violation. The $K_L, K_S$ system is still the only experimentally established system having $CP$ violation since its observation in 1964 [1.27]. To constrain the other two parameters $V_{cs}, V_{cd}$ we use the
two sets of experimental information, i.e., the $K_L$, $K_S$ mass difference $\Delta m = m_{L} - m_{S}$ and the $CP$ violation parameter $\varepsilon$. Since we shall later in section 3 discuss the mixing and $CP$ violation in other charge zero meson systems, $D^0\bar{D}^0$, $B^0\bar{B}^0$ and $T^0\bar{T}^0$, we shall discuss the formulation in some detail here.

Consider the regeneration wave function for the $K^0\bar{K}^0$ system

$$\phi(t) = a_K(t)|K^0\rangle + a_{\bar{K}}(t)|\bar{K}^0\rangle = \begin{pmatrix} a_K(t) \\ a_{\bar{K}}(t) \end{pmatrix}, \quad (1.18)$$

with the equation of motion

$$i\frac{d\phi(t)}{dt} = \mathbf{H}(t)\phi(t) \equiv (M - i\Gamma/2)\phi(t), \quad (1.19)$$

where

$$\mathbf{H} = \begin{pmatrix} M_{11} - i\Gamma_{11}/2 & M_{12} - i\Gamma_{12}/2 \\ M_{21} - i\Gamma_{21}/2 & M_{22} - i\Gamma_{22}/2 \end{pmatrix}. \quad (1.20a)$$

$M_{ij}$, $\Gamma_{ij}$ are transition matrix elements from virtual and physical intermediate states respectively and can be complex [1.28, 1.29]

$$M_{ij} = m_{K}\delta_{ij} + \langle i|H_{W,\Delta s=2}|j\rangle + P\sum_{\lambda} \frac{\langle i|H_{W,\Delta s=2}|\lambda\rangle \langle \lambda|H_{W,\Delta s=2}|j\rangle}{m_{K} - E_{\lambda}}, \quad (1.20b)$$

and

$$\Gamma_{ij} = 2\pi\sum_{\lambda} \rho_{\lambda}\langle i|H_{W,\Delta s=1}|\lambda\rangle \langle \lambda|H_{W,\Delta s=1}|j\rangle, \quad (1.20c)$$

where $P$ takes the principal value, and $\rho_{\lambda}$ is the density of the $\lambda$ state. From $CPT$,

$$M_{11} = M_{22}, \quad \Gamma_{11} = \Gamma_{22}. \quad (1.20d)$$

From hermiticity,

$$M_{21} = M_{12}^*, \quad \Gamma_{21} = \Gamma_{12}^*. \quad (1.20e)$$

Now eq. (1.20a) becomes

$$\mathbf{H} = \begin{pmatrix} M_0 - i\Gamma_0/2 & M_{12} - i\Gamma_{12}/2 \\ M_{12}^* - i\Gamma_{12}^*/2 & M_0 - i\Gamma_0/2 \end{pmatrix}. \quad (1.20f)$$

where $M_0 = M_{11} = M_{22}$, $\Gamma_0 = \Gamma_{11} = \Gamma_{22}$. Note that since the $|K^0\rangle$ and $|\bar{K}^0\rangle$ states do not communicate through strong interactions their relative phase is not specified. The relation of $|K^0\rangle$ to $|\bar{K}^0\rangle$ is related through $CP$ transformation up to an arbitrary phase $e^{i\varepsilon}$,
\[ |K^0\rangle = e^{2i\epsilon} CP |\bar{K}^0\rangle = -e^{2i\epsilon} C |\bar{K}^0\rangle. \] (1.21)

To find the physical states, we diagonalize \( H \) and find its eigenvalues \( \lambda_\pm \) which are the physical masses and widths of the physical states

\[ \det (H - \lambda_\pm I) = 0 \] (1.22a)

which becomes

\[ \det \begin{pmatrix} (\Delta m - i\Delta f/2)/2 & M_{12} - i\Gamma_{12}/2 \\ M_{12}^{*} - i\Gamma_{12}^{*}/2 & (\Delta m - i\Delta f/2)/2 \end{pmatrix} = 0, \] (1.22b)

where

\[ (\Delta m - i\Delta f/2)/2 \equiv M_0 - i\Gamma_0/2 - \lambda_\pm. \]

From eq. (1.22b), we easily obtain

\[ [(\Delta m - i\Delta f/2)/2]^2 = (M_{12} - i\Gamma_{12}/2)(M_{12}^{*} - i\Gamma_{12}^{*}/2). \] (1.23)

After diagonalizing the matrix \( H \), we denote the \( \lambda_\pm \) eigenstates respectively by

\[ |K_L\rangle = N_e [(1 + \bar{\epsilon}) |K^0\rangle + (1 - \bar{\epsilon}) |\bar{K}^0\rangle], \]

\[ |K_S\rangle = N_e [(1 + \bar{\epsilon}) |K^0\rangle - (1 - \bar{\epsilon}) |\bar{K}^0\rangle], \] (1.24a)

where \( N_e \) is a normalization factor

\[ N_e = [2(1 + |\bar{\epsilon}|^2)]^{-1/2}, \] (1.24b)

then from the eigenvalue equation

\[ (H - \lambda_\pm I)(1 + \bar{\epsilon}) = \begin{pmatrix} (\Delta m - i\Delta f/2)/2 & M_{12} - i\Gamma_{12}/2 \\ M_{12}^{*} - i\Gamma_{12}^{*}/2 & -(\Delta m - i\Delta f/2)/2 \end{pmatrix}(1 + \bar{\epsilon}) = 0, \] (1.25)

we easily obtain

\[ \frac{1 - \bar{\epsilon}}{1 + \bar{\epsilon}} = \frac{(\Delta m - i\Delta f/2)/2}{M_{12} - i\Gamma_{12}/2} = \frac{M_{12}^{*} - i\Gamma_{12}^{*}/2}{(\Delta m - i\Delta f/2)/2}. \] (1.26)

Note that \( \bar{\epsilon} \) is not yet a physical quantity and dependent on the phase convention of \( |K^0\rangle \) and \( |\bar{K}^0\rangle \). For example if \( \bar{\epsilon} \) is pure imaginary, we have \( (1 \pm \bar{\epsilon}) = [1 + |\bar{\epsilon}|^2]^{1/2} e^{\pm i\theta} \), where \( \tan \theta = \bar{\epsilon}/|\bar{\epsilon}|. \) We can then redefine \( |K^0\rangle = e^{i\theta} |K^0\rangle \), \( |K^0\rangle' = e^{-i\theta} |K^0\rangle \). Then \( |K_S\rangle = (|K^0\rangle + |\bar{K}^0\rangle), \) \( |K_L\rangle = (|K^0\rangle' - |\bar{K}^0\rangle' \), which are \( CP \) eigenstates. On the other hand, the magnitude
\[ \eta = \frac{1}{(1 - \bar{\epsilon})/(1 + \bar{\epsilon})} = \left| \frac{M_{12}^* - i\Gamma_{12}/2}{M_{12} - i\Gamma_{12}/2} \right|^{1/2} \]

(1.27)

is a physical quantity. The deviation from \( \eta = 1 \) specifies the amount of CP violation. When \( \eta \neq 1 \) the \( K_S, K_L \) states are not orthogonal, the overlap is given by

\[ \langle K_S|K_L \rangle = 2 \text{ Re } \bar{\epsilon}/(1 + |\bar{\epsilon}|^2) = (1 - \eta^2)/(1 + \eta^2), \]

(1.28)

which is independent of phase convention. Using eq. (1.26), we obtain

\[ \frac{2 \text{ Re } \bar{\epsilon}}{1 + |\bar{\epsilon}|^2} = \frac{2 \text{ Im}(M_{12}^* \Gamma_{12}/2)}{1 + \eta^2 + |M_{12}|^2 + (\Delta m)^2 + (\Delta \Gamma)^2} \]

(1.29)

Therefore CP violation is indicated by \( \eta \neq 1 \) or \( \text{Im}(\Gamma_{12}^* M_{12}) \neq 0 \), independent of phase convention [1.30].

In order to relate \( \bar{\epsilon} \) to measurable quantities, we need to consider the decay of the \( K^0 \overline{K^0} \) system. \( K^0, \overline{K^0} \) decay mainly into two pions [1.31]. We denote

\[ \langle (\pi \pi)_1 | H_w | K^0 \rangle = a_0 \exp(i\delta_0), \quad \langle (\pi \pi)_2 | H_w | K^0 \rangle = a_2 \exp(i\delta_2), \]

(1.30a)

\[ \langle (\pi \pi)_1 | H_w | \overline{K^0} \rangle = a_0^* \exp(i\delta_0), \quad \langle (\pi \pi)_2 | H_w | \overline{K^0} \rangle = a_2^* \exp(i\delta_2), \]

(1.30b)

where the phases from strong interaction \( \exp(i\delta_0) \) and \( \exp(i\delta_2) \) are factored out explicitly so that the phases of \( a_0, a_2 \) are all from weak interaction,

\[ a_0 = |a_0| \exp(i\theta_0), \quad a_2 = |a_2| \exp(i\theta_2). \]

(1.30c)

Note that the strong-interaction phases are the same for \( K^0 \) and \( \overline{K^0} \) due to \( C, P, T \) invariance of strong interactions. From eq. (1.24) we obtain

\[ a_{0,S(L)} = \frac{\langle (\pi \pi)_1 | H_w | K_{S(L)} \rangle}{\sqrt{3}} = N_s [(1 + \bar{\epsilon}) a_{0} + (1 - \bar{\epsilon}) a_{0}^*] \exp(i\delta_0), \]

(1.31a)

\[ a_{2,S(L)} = \frac{\langle (\pi \pi)_2 | H_w | K_{S(L)} \rangle}{\sqrt{3}} = N_s [(1 + \bar{\epsilon}) a_{2} + (1 - \bar{\epsilon}) a_{2}^*] \exp(i\delta_2). \]

(1.31b)

Further, from

\[ \langle \pi^0 \pi^0 \rangle = \langle (\pi \pi)_1 | (1/\sqrt{3}) - \langle (\pi \pi)_2 | (\sqrt{2}/\sqrt{3}) \rangle, \]

(1.32a)

\[ \langle \pi^+ \pi^- \rangle = \langle (\pi \pi)_1 | (\sqrt{2}/\sqrt{3}) + \langle (\pi \pi)_2 | (1/\sqrt{3}) \rangle, \]

(1.32b)

we have

\[ A_{00,S(L)} = \langle \pi^0 \pi^0 | H_w | K_{S(L)} \rangle = (1/\sqrt{3})a_{0,S(L)} - (\sqrt{2}/\sqrt{3})a_{2,S(L)}, \]

(1.33a)

\[ A_{+,-,S(L)} = \langle \pi^+ \pi^- | H_w | K_{S(L)} \rangle = (\sqrt{2}/\sqrt{3})a_{0,S(L)} + (1/\sqrt{3})a_{2,S(L)}. \]

(1.33b)
One obtains
\[
\eta_{00} = \frac{A_{00,L}}{A_{00,S}} = \frac{(a_{0,1} - \sqrt{2} a_{2,1})/(a_{0,5} - \sqrt{2} a_{2,5})}{a_{0,1}}. \quad (1.34)
\]

Dividing \(a_{0,5}\) both in the numerator and denominator in eq. (1.34) and defining
\[
\varepsilon \equiv a_{0,1}/a_{0,5}, \quad \varepsilon_2 \equiv a_{2,1}/a_{0,5}, \quad \text{and} \quad \omega \equiv a_{2,5}/a_{0,5}, \quad (1.35)
\]
we obtain
\[
\eta_{00} = \left(\varepsilon - \sqrt{2} \varepsilon_2\right)/(1 - \sqrt{2} \omega) \equiv \varepsilon - 2\varepsilon'/(1 - \sqrt{2} \omega), \quad (1.36)
\]
where
\[
\varepsilon' \equiv (1/\sqrt{2})\omega(\varepsilon_2/\omega - \varepsilon). \quad (1.37)
\]

Similarly
\[
\eta_{+-} \equiv \frac{A_{+-,L}}{A_{+-,S}} = \frac{a_{0,1} + (1/\sqrt{2})a_{2,1}}{a_{0,5} + (1/\sqrt{2})a_{2,5}} = \frac{\varepsilon + (1/\sqrt{2})\varepsilon_2}{1 + (1/\sqrt{2})\omega} = \varepsilon + \varepsilon'/(1 + (1/\sqrt{2})\omega). \quad (1.38)
\]

Experimentally we know \(|a_{2}/a_{0}|\) is very small (see discussions in section 2.1)
\[
|a_{2}/a_{0}| \approx 1/20. \quad (1.39)
\]

When \(\omega\) can be neglected comparing to one, eqs. (1.36, 1.38) become
\[
\eta_{00} \approx \varepsilon - 2\varepsilon', \quad (1.40a)
\]
\[
\eta_{+-} \approx \varepsilon + \varepsilon'. \quad (1.40b)
\]

Note here that these expressions for \(\eta_{00}, \eta_{+-}, \varepsilon, \varepsilon'\) are general, with only the last approximation utilizing the experimental information eq. (1.39). Since both \(\eta_{00}\) and \(\eta_{+-}\) are physically measurable quantities, \(\varepsilon, \varepsilon'\) are too and are phase-convention independent,
\[
\varepsilon = (\eta_{00} + 2\eta_{+-})/3, \quad (1.41a)
\]
\[
\varepsilon' = (\eta_{+-} - \eta_{00})/3. \quad (1.41b)
\]

Now coming back to discuss the exact formula, we substitute eqs. (1.31a,b) into eq. (1.35) and obtain
\[
\varepsilon = \frac{\tilde{\varepsilon} \Re a_0 + i \Im a_0}{\Re a_0 + i \tilde{\varepsilon} \Im a_0} = \frac{\tilde{\varepsilon} + i \tilde{\varepsilon}_0}{1 + i \tilde{\varepsilon} \tilde{\varepsilon}_0}, \quad \text{with} \quad \tilde{\varepsilon}_0 \equiv \Im a_0/\Re a_0, \quad (1.42)
\]
(note here that the physically measured $\varepsilon$ equals $\tilde{\varepsilon}$ in defining the $K_S, K_L$ states eq. (1.24a) only when $t_0 = 0$),

$$
\varepsilon_2 = \frac{\tilde{\varepsilon} \text{ Re } a_2 + i \text{ Im } a_2}{\text{ Re } a_0 + i \tilde{\varepsilon} \text{ Im } a_0} \exp i(\delta_2 - \delta_0) = \left( \frac{\text{ Re } a_2}{\text{ Re } a_0} \right) \left( \frac{\tilde{\varepsilon} + i t_2}{1 + i \tilde{\varepsilon} t_0} \right) \exp i(\delta_2 - \delta_0),
$$
(1.43)

and

$$
\omega = \frac{\text{ Re } a_2 + i \tilde{\varepsilon} \text{ Im } a_2}{\text{ Re } a_0 + i \tilde{\varepsilon} \text{ Im } a_0} \exp i(\delta_2 - \delta_0) = \left( \frac{\text{ Re } a_2}{\text{ Re } a_0} \right) \left( \frac{1 + i \tilde{\varepsilon} t_2}{1 + i \tilde{\varepsilon} t_0} \right) \exp i(\delta_2 - \delta_0),
$$
(1.44)

where

$$
t_i = \text{ Im } a_i / \text{ Re } a_i.
$$
(1.45)

Now $\varepsilon'$ of eq. (1.37) can be re-expressed as

$$
\varepsilon' = (1/\sqrt{2}) \omega \left[ \frac{\tilde{\varepsilon} + i t_2}{1 + i \tilde{\varepsilon} t_0} - \frac{\tilde{\varepsilon} + i t_0}{1 + i \tilde{\varepsilon} t_0} \right] = \frac{i}{\sqrt{2}} \omega (1 - \varepsilon^2) \frac{t_2 - t_0}{(1 + i \varepsilon t_2)(1 + i \varepsilon t_0)}
$$
(1.46)

Note that so far no approximation has been made. Note also that in our formulation here the fact that $\varepsilon'$ vanishes as $a_0$ and $a_2$ have the same phases, i.e. $t_2 - t_0 = 0$, is always manifestly obvious [1.32].

If in a phase convention $|\tilde{\varepsilon}| \ll 1$, eqs. (1.42), (1.46) become respectively

$$
\varepsilon = \tilde{\varepsilon} + i t_0,
$$
(1.47)

$$
\varepsilon' = \frac{i}{\sqrt{2}} \left( \frac{\text{ Re } a_2}{\text{ Re } a_0} \right) (t_2 - t_0) \exp i(\delta_2 - \delta_0).
$$
(1.48)

If we choose the convention of $|K^0\rangle, |\bar{K}^0\rangle$ so that $a_0$ is real and i.e. $t_0 = 0$, then eqs. (1.47), (1.48) become

$$
\varepsilon = \tilde{\varepsilon},
$$
(1.49)

$$
\varepsilon' = \frac{i}{\sqrt{2}} \left( \frac{\text{ Re } a_2}{\text{ Re } a_0} \right) t_2 \exp i(\delta_2 - \delta_0).
$$
(1.50)

This is the result first obtained in the elegant work of Wu and Yang [1.31].

In the literature, however some other phase convention is used, for example the K–M phase convention

$$
|K^0\rangle = CP|\bar{K}^0\rangle,
$$
(1.51)

which has the advantage that in calculating weak interaction amplitudes, all the complexities are from
the K–M phase $\delta$. So to make a comparison, we shall consider the cases that in certain phase convention $a_0$ is complex. Since in $K^+$ decay

$$\varepsilon \ll 1,$$

with very small imaginary part of the amplitude $a_0$, i.e.

$$\text{Im} \ a_0 / \text{Re} \ a_0 = t_0 \ll 1.$$

Under these conditions, relations among $M_{ij}, \Gamma_{ij}$ and physical quantities are simplified. From eq. (1.42) we then obtain

$$\varepsilon = \bar{\varepsilon} + i t_0,$$

with

$$\bar{\varepsilon} \ll 1.$$

From eq. (1.20c), $\Gamma_{12}$ comes from the physical decays of $K^0$ and $\bar{K}^0$, which is dominated by the $I = 0$ states of the $2\pi$ system,

$$\Gamma_{12} \propto \langle K^0|H_W|(\pi\pi)_{I=0}(\pi\pi)_{I=0}|H_W|\bar{K}^0 \rangle,$$

where the $(\pi\pi)_{I=2}$ and three-pion contribution are negligible. So

$$\Gamma_{12}^\text{i}/\Gamma_{12}^\text{R} = \text{Im}(a_0^*)^2/\text{Re}(a_0^*)^2 = -2\text{Im} a_0 / \text{Re} a_0 = -2t_0.$$

From $\varepsilon, \bar{\varepsilon} \ll 1$, we can also conclude from eq. (1.26) that

$$M_{12}^1/M_{12}^R \ll 1.$$

Then in this approximation,

$$\Delta \Gamma \approx 2\Gamma_{12}^\text{R}, \quad \Delta m \approx 2M_{12}^R.$$

Further, using the experimental information

$$\Delta \Gamma \approx -2 \Delta m$$

from eqs. (1.26, 1.57), we obtain

$$\bar{\varepsilon} \approx \frac{1}{1 + i \left( \frac{\text{Im} a_0}{\text{Re} a_0} \right)}.$$

Substituting eq. (1.61) into (1.54), we obtain
\[ e = \frac{1}{1 - i} \left[ M_{12}^l/2M_{12}^R + \text{Im} a_0/\text{Re} a_0 \right] \]
\[ \approx \frac{1}{2} \left[ t_{sd}/2 + t_0 \right], \tag{1.62a} \]

where
\[ t_M \equiv M_{12}^l/M_{12}^R. \tag{1.62b} \]

These are the approximate formula used often in the literature. However for heavy quark systems, the approximations may not apply.

The experimental bound
\[ |e'/e| < 1/50 \tag{1.63} \]

can put a limit on the phase ratios, using eqs. (1.38), (1.62a). Using \(|a_2/a_0| \approx 1/20\), we obtain
\[ |e'/e| = \frac{1}{200} \left[ (t_0 - t_2)/(t_{sd}/2 + t_0) \right] < 1/50, \]
or
\[ \left| \frac{t_0 - t_2}{t_{sd}/2 + t_0} \right| < 2/5. \tag{1.64} \]

This bound must be satisfied for those models in which a phase convention can be chosen so that all phases are small.

Further for those models \( t_2 = 0 \) and all other phases small, e.g. in the K–M model using the phase convention \(|K^0\rangle = CP|K^0\rangle\) and the “Penguin”-diagram explanation of the \( \Delta I = \frac{1}{2} \) rule, eq. (1.64) then takes the form
\[ |1 + t_{sd}/(2t_0)| > 5/2 \tag{1.65} \]

which gives
\[ t_{sd}/(2t_0) > 3/2, \quad \text{if } t_{sd}/(2t_0) > 0 \tag{1.66} \]
\[ |t_{sd}/(2t_0)| > 7/2, \quad \text{if } t_{sd}/(2t_0) < 0. \tag{1.67} \]

Another measurable quantity is the lepton asymmetry from \( K_L \) decay. Due to \( CP \) violation, \( K_L \) has unequal mixture of \( K^0 \) and \( \bar{K}^0 \), eq. (1.24). The \( K^0, \bar{K}^0 \) have very distinct semileptonic decays, \( K^0 \to \pi^+ \ell^- \bar{\nu}, \bar{K}^0 \to \pi^- \ell^+ \nu \). Due to this unequal mixture of \( K^0, \bar{K}^0 \) in \( K_L \), the decay rates of \( K_L \) into \( \pi^+ \ell^- \bar{\nu} \) and \( \pi^- \ell^+ \nu \) are different, and can be calculated to be \[ |x| = A(K^0 \to \pi^+ \ell^+ \nu)/A(K^0 \to \pi^- \ell^+ \nu). \]

where \[ x = A(K^0 \to \pi^- \ell^+ \nu)/A(K^0 \to \pi^- \ell^- \nu). \]

The phase-independent measurable quantity is just the one given in eq. (1.29).
1.4. Determination of $V_{ud}$ from the box-graph calculation of $\Delta m$ and $\epsilon$ in the $K^0\bar{K}^0$ system

In this section we shall discuss the contribution to neutral particle–antiparticle transition matrix $M_{12}$ in the standard electroweak theory. The main contribution is from the box-graph shown in fig. 1.4 [1.33]. Here we shall adopt the phase convention

$$|K^0\rangle = CP|\bar{K}^0\rangle,$$  \hspace{1cm} (1.51)

so that the complexity in $M_{12}$ is solely from that of $V_{ud}$.

In comparing the calculated $M_{12}^\ast /M_{12}^R$ with measured $\epsilon$, we assume $t_o/t_M \ll 1$ in eq. (1.62a), so that

$$\epsilon = [(1 + i)/2] M_{12}^\ast /2 M_{12}^R.$$ \hspace{1cm} (1.69)

After calculating $M_{12}$, we fit the mixing matrix so that

$$M_{12}^R = \Delta m/2,$$ \hspace{1cm} (1.70)

$$\frac{1}{2} M_{12}^\ast /\Delta m = \text{Re} \, \epsilon.$$ \hspace{1cm} (1.71)

Note that Im $\epsilon$ does not provide further information.

The experimental measurements for the $K^0$ systems are [1.34, 1.35]

$$\Delta \Gamma = \Gamma_S - \Gamma_L = 0.738 \times 10^{-14} \text{ GeV},$$ \hspace{1cm} (1.72a)

$$\Delta m = m_S - m_L = -0.352 \times 10^{-14} \text{ GeV}.$$ \hspace{1cm} (1.72b)

Note that the experimental measurements give $\Delta \Gamma \approx -2 \Delta m$, which we have used in eq. (1.60). From $K_{L,S} \rightarrow 2\pi,$

$$|\eta_{+-}| = (2.274 \pm 0.022) \times 10^{-3}, \quad \phi_{+-} = (44.6 \pm 1.2)^\circ,$$ \hspace{1cm} (1.73a)

$$|\eta_{00}| = (2.33 \pm 0.08) \times 10^{-3}, \quad \phi_{00} = (54.0 \pm 5.0)^\circ,$$ \hspace{1cm} (1.73b)

which gives

$$\text{Re} \, \epsilon = (1.536 \pm 0.062) \times 10^{-3};$$ \hspace{1cm} (1.73c)

Fig. 1.4. The box diagrams for $K^0 \leftrightarrow \bar{K}$. Our convention is such that the mixing matrix element at a $qq'W$ vertex is $V_{q'}$ if the outgoing quark $q'$ is a $u$-like quark; is $V_s^\ast$ if the outgoing quark $q$ is a $d$-like quark.
from $K_L \rightarrow \pi^+ \ell^- \nu$ asymmetry,

\begin{align}
\Delta_L &= (3.30 \pm 0.12) \times 10^{-3}, \\
\text{Re } x &= (9 \pm 20) \times 10^{-3} \\
\text{Im } x &= (-4 \pm 26) \times 10^{-3},
\end{align}

which gives

\[ \text{Re } \varepsilon = (1.620 \pm 0.088) \times 10^{-3}. \] (1.74d)

Comparing with eq. (1.73c), the two experimental results are consistent. From these experimental information we obtain a complex number for $M_{12}$, using eqs. (1.70, 1.71),

\[ M_{12} = (-0.176 - i 0.114 \times 10^{-2}) \times 10^{-14} \text{ GeV}. \] (1.75)

Now we shall calculate $M_{12}^\uparrow$, $M_{12}^\uparrow$ from the box-graph of fig. 1.4 and compare them to data eq. (1.75) to determine all $V_{\ell i}$. Ever since the original analysis [1.38], though no basic change in conclusions, there has been refinement of formula and extended analysis to heavier quark systems [1.39–1.47]. In a recent work [1.48] a thorough analysis of $V_{\ell i}$ and its phenomenological implication are done using well organized formula and all currently available experimental information. Many results and formula used in this section are from ref. [1.48].

The $K^0 \leftrightarrow \bar{K}^0$ transition matrix $M_{12}$ given by the box graph as shown in fig. 1.4 is

\[ M_{12} = -\frac{G_F^2 m_w^2}{16\pi^2} \left( \sum_{i,j=u,c,t} \lambda_i \lambda_j A_{ij} \right) M_{12} \] (1.76)

where

\begin{align}
\lambda_i &= V_{\ell i} V_{\ell i}^*, \\
A_{ij} &= \frac{x_i + x_j - (11/4)x_i x_j}{(1-x_i)(1-x_j)} + \frac{1}{(x_i - x_j)} \left[ x_i^2 \left[ 1 - 2x_i + (1/4)x_i x_j \right] \ln x_i - (i \leftrightarrow j) \right], \\
A_{ii} &= \frac{x_i}{(1-x_i)^3} \left[ 3 - \frac{19}{4} x_i + \frac{1}{4} x_i^2 \right] + 2 \left[ 1 - \frac{3x_i^2}{4(1-x_i)} \right] x_i \ln x_i,
\end{align}

with the definition

\[ x_i = m_i^2/m_w^2, \quad m_w = \left[ \pi\alpha/(\sqrt{2} \ G_F \sin^2 \theta_w) \right]^{1/2}. \]

In this formula [1.41, 1.42] all the external momenta associated with the external quark lines are neglected in comparison with the W-mass. For heavy quark states with mass scale comparable to that of $m_w$, exact formula including the external momenta must be used, see section 3.1. The factors in $M_{12}$ multiplying $M_{12}$ are the quark diagram contributions of the box diagram. The matrix

\[ M_{12} = \langle K^0 | [\bar{d} \gamma_\mu (1 - \gamma_5)s] [\bar{d} \gamma^\mu (1 - \gamma_5)s] | \bar{K}^0 \rangle \] (1.78)
contains the strong interaction contributions of the making of $K^0$ from $\bar{s}d$ quarks. To calculate this quantity accurately requires our fundamental understanding of the nonperturbative part of the strong interaction or the long distance effect, in the current QCD terminology [1.44]. We certainly do not have this knowledge. The estimate based upon the vacuum intermediate state insertion gives

$$M_{12,\text{vac}} = -\frac{\pi f_K^2}{2V_{12}} m_K,$$

(1.79a)

where $f_K = 1.23 m_n$. The value of $M_{12}$ varies as other methods are used. We use $M_{12,\text{vac}}$ as a norm for comparison, and define

$$M_{12} = B M_{12,\text{vac}},$$

(1.79b)

where $B$ is a constant characterizing the difference from the vacuum-insertion calculation with its deviation from one. The bag model [1.44] calculation is very uncertain depending upon the parameters used. Recently $B$ was estimated using PCAC and SU(3) and a positive valued $B$ with a value smaller than one was obtained. We shall use

$$B = 1 \text{ or } 0.4.$$

(1.80)

The range of the results should give some indication of the uncertainty involved in the analysis.

Now we have two constraints on $M_{12}$, eqs. (1.70, 1.71). Given $s_1$, $s_3$, from the results of section 1.2 the two experimental information $\Delta m$ and $\epsilon$ should give unique solution for $s_2$ and $s_8$ in given quadrants of the angles. Due to the large uncertainty in $s_3 = 0.28_{-0.28}^{+0.44}$ determined from strange-particle decays, it is appropriate to consider in the discussion

$$s_3 < 0.5.$$  

(1.81)

It was noted in ref. [1.48] that the two constraints on $M_{12}$ of eq. (1.76) can be solved algebraically for two parameters as function of the third. First we use the unitarity condition

$$\sum_{i=u,c,t} \lambda_i = 0$$

(1.82)

in eq. (1.76) to obtain

$$M_{12} = -\frac{G_F^2 m_n^2 B f_K^2 m_K}{12 \pi^2} \cdot \frac{1}{3}[\lambda_e + (A_{uu} - 2A_{ct} + A_{tt})(\lambda_e)^2 + 2(A_{uc} - A_{ut} - A_{ct} + A_{tt})\lambda_u \lambda_c + (A_{uu} - 2A_{ct} + A_{tt})(\lambda_u)^2],$$

(1.83)

which is a quadratic equation of $\lambda_e$. Given $\lambda_u = s_1 c_1 c_3$, we obtain two solutions for $\lambda_e$ corresponding to $s_8 > 0$, i.e. $\text{Im} \lambda_e > 0$; or $s_8 < 0$, i.e. $\text{Im} \lambda_e < 0$. Once $\lambda_e$ is solved as a function of $s_3$, $c_2$ then can be obtained from the quadratic equation

$$c_2^2 (s_1^2 c_1^2 c_3^2 + s_2^2 s_3^2) + c_2^2 (2s_1 c_1 c_3 \text{ Re } \lambda_2 - s_1^2 s_3^2) + |\lambda_2|^2 = 0,$$

(1.84)

which is derived from
Re $\lambda_2 = -s_1c_1c_2^2 - s_1s_3s_2c_2c_8$, \hspace{1cm} (1.85a)

Im $\lambda_2 = -s_1s_3s_2c_2s_8$, \hspace{1cm} (1.85b)

by eliminating $\delta$. The two solutions of $c_2^2$ give the two branches in $\delta$:

$$c_8 = (s_1c_1c_2^2 + \text{Re } \lambda_2)/(s_1s_3c_2s_2),$$

and

$$s_8 = \text{Im } \lambda_2/(s_1s_3c_2s_2).$$

In the literature perturbative-QCD modifications on the naive diagram have been considered [1.46, 1.47]. They modify eq. (1.83) in the following way

$$A_{ee} - 2A_{ct} + A_{tt} \rightarrow \eta_1(A_{ee} - 2A_{uc} + A_{uu}) - 2\eta_3(A_{ct} - A_{uc} - A_{ut} + A_{uu}) + \eta_2(A_{tt} - 2A_{ut} + A_{uu}),$$

$$A_{uc} - A_{ut} - A_{ct} + A_{tt} \rightarrow -\eta_3(A_{ct} - A_{uc} - A_{ut} + A_{uu}) + \eta_2(A_{tt} - 2A_{ut} + A_{uu}),$$

$$A_{tt} - 2A_{ut} + A_{uu} \rightarrow \eta_2(A_{tt} - 2A_{ut} + A_{uu}).$$

Note that as $\eta_1 = \eta_2 = \eta_3 = 1$, the equations reduce to the original form of eq. (1.83). These $\eta_i$'s are rather insensitive to $\mu$, but sensitive to the mass scale $\Lambda_{QCD}$ in the perturbative-QCD calculation. For $m_\tau < 100 \text{ GeV}$, $\Lambda_{QCD}^2 = 0.1 \text{ GeV}^2$, $\eta_1 \approx 0.91$, $\eta_2 \approx 0.63$, $\eta_3 \approx 0.31$. These changes turn out not to effect the final results significantly ($\approx 10\%$), compared to the variation given by the change of $B = 0.4 - 1.0$. Besides, the validity of such perturbative-QCD calculation is not established [1.47].

The results of this analysis from refs. [1.38–1.42] and the recent up-to-date analysis [1.48] are the following. Because of the redundancy in the contribution of the mixing angles to the K-M matrix, one can pick the convention that $\theta_1$, $\theta_2$, $\theta_3$ are all in the first quadrant, and let $\delta$ vary in all four quadrants. (Note that even after selecting $\theta_1$, $\theta_2$, $\theta_3$ to be in the first quadrant, there are effectively still two possible conventions for the $\delta$, which are related by $\pm \pi$. Almost all authors, except Barger et al. of ref. [1.38] and the current Particle Data Book, use the convention chosen here.)

It happens that $\Delta m$ and $\varepsilon$ do not allow $\delta$ to be in the fourth quadrant, i.e. $-\pi/2 < \delta < 0$, but $\delta$ is allowed in the whole region of $0 < \delta < \pi$, especially including $\delta \approx 90^\circ$. The agreement of the model with the sign of Re $\varepsilon$ is important. If Re $\varepsilon$ were negative, the forbidden region for $\delta$ would be the first quadrant in the phase convention here for $\delta$. For $s_8 = 1$, in order to explain the smallness of the CP violation effect eq. (1.33), we must have $s_2s_3 \approx 10^{-4}$. The solution for $s_8 \approx 1$ is $s_2 = 0.2$, $s_3 = 2 \times 10^{-3}$. We shall see in section 3 that this choice may give large CP violation effects in the partial-decay rate difference in the b and t systems.

In fig. 1.5 the dependence of $s_2$ and $s_3$ on $\delta$ are given for the allowed region of $0.35^\circ < \delta < 179.9^\circ$. $s_3$ decreases drastically below 0.1 as $\delta$ moves 1° from the extremities. $s_2$ stays constant, about $s_2 \approx 0.25$ in the $B = 0.4$ calculation and $s_2 \approx 0.12$ in the vacuum-insertion $B = 1$ calculation, for most values of $\delta$; and
begins to part as $\delta$ moves $2^\circ$ toward the extremities. We shall see that these two features are common to many mixing-matrix elements. In figs. 1.6—1.8 various $V_{ij}$'s and their ratios are given as function of $\delta$. The results are also summarized in table 1.1. Like $s_2$ and $s_3$ these $|V_{ij}|'$s stay rather constant for a large range of $2^\circ < \delta < 178^\circ$, and some $|V_{ij}|$ begin to vary drastically only as $\delta$ moves toward $\delta < 2^\circ$ and $\delta > 178^\circ$. This implies that the value of $\delta$ cannot be determined uniquely from the values of $|V_{ij}|$ unless $\delta < 2^\circ$ or $\delta > 178^\circ$. In the region of $2^\circ < \delta < 178^\circ$, we need to rely upon the phase-dependent phenomenon of $CP$ violation to determine the angle regions.

Besides the region of $0.35^\circ < \delta < 179.92^\circ$, there is a small region in the third quadrant of $\delta$ that is allowed, $180.1^\circ < \delta < 180.25^\circ$, $0.14 < s_2 < 0.38$, $0.45 < s_3 < 0.5$, which was first found in ref. [1.39]. In fig. 1.9, $s_2$, $s_3$ and various $V_{ij}$'s are given as function of $\delta$ in this region. The results are also summarized in table 1.1. It is interesting to note that with all these variations in $\delta$, for $s_3 < 0.5$, we always have,

$$|s_2| < 0.5.$$  (1.87)

From this box-graph fit to $\Delta m$ and $\epsilon$, as shown in figs. 1.5—1.9 and table 1.1, we have learned the following general features. The following quantities stay constant, $|V_{cd}| \approx 0.22$, $|V_{cd}/V_{cs}| \approx 0.22$, $|V_{ub}| < 0.1$ for the entire allowed ranges of $\delta$. Therefore precise experimental measurements of these quantities provide a test on the validity of the box-graph calculation. Since $|V_{cb}|$, $|V_{ub}/V_{cb}|$ assume quite different
Fig. 1.7. Plots of $|V_{ub}|$, $|V_{cb}|$, $|V_{ub}/V_{cb}|$ as functions of $0^\circ < \delta < 180^\circ$, obtained from the box-graph fit to $\Delta m$, $\epsilon$ of the $K^0\bar{K}^0$ system.

Fig. 1.8. Plots of $|V_{ud}|$, $|V_{td}|$, $|V_{ud}/V_{td}|$ as functions of $0^\circ < \delta < 180^\circ$, obtained from the box-graph fit to $\Delta m$, $\epsilon$ of the $K^0\bar{K}^0$ system.

Table 1.1

Values of $|V_{ud}|$ determined from the box-graph fitting of $\Delta m$, $\epsilon$ of the $K^0\bar{K}^0$ system

| $\delta$ | $0.35^\circ < \delta < 2^\circ$ | $2^\circ < \delta < 178^\circ$ | $178^\circ < \delta < 179.92^\circ$ | $180.1^\circ < \delta < 180.25^\circ$
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_2$</td>
<td>0.04—0.22</td>
<td>0.06—0.27</td>
<td>0.16—0.27</td>
<td>0.14—0.38</td>
</tr>
<tr>
<td>$s_3$</td>
<td>0.03—0.5</td>
<td>0.0—0.14</td>
<td>0.03—0.45</td>
<td>0.46—0.5</td>
</tr>
<tr>
<td>$</td>
<td>V_{ud}</td>
<td>$</td>
<td>0.23</td>
<td>0.23</td>
</tr>
<tr>
<td>$</td>
<td>V_{cd}</td>
<td>$</td>
<td>0.78—0.96</td>
<td>0.94—0.97</td>
</tr>
<tr>
<td>$</td>
<td>V_{cd}/V_{ub}</td>
<td>$</td>
<td>0.22—0.24</td>
<td>0.22</td>
</tr>
<tr>
<td>$</td>
<td>V_{ub}</td>
<td>$</td>
<td>0.01—0.10</td>
<td>0.0—0.04</td>
</tr>
<tr>
<td>$</td>
<td>V_{ub}/V_{cd}</td>
<td>$</td>
<td>0.02—0.2</td>
<td>0.0—0.14</td>
</tr>
<tr>
<td>$</td>
<td>V_{ud}</td>
<td>$</td>
<td>0.01—0.05</td>
<td>0.01—0.06</td>
</tr>
<tr>
<td>$</td>
<td>V_{td}</td>
<td>$</td>
<td>0.22—0.57</td>
<td>0.09—0.27</td>
</tr>
<tr>
<td>$</td>
<td>V_{ub}</td>
<td>$</td>
<td>0.82—0.97</td>
<td>0.97—0.99</td>
</tr>
<tr>
<td>$</td>
<td>V_{ud}/V_{cd}</td>
<td>$</td>
<td>0.24—0.63</td>
<td>0.03—0.24</td>
</tr>
</tbody>
</table>
values for $\delta < 2^\circ$ and $\delta > 178^\circ$, the measurements of these quantities can provide information about the value of $\delta$. The important features for the region $180.1^\circ < \delta < 180.25^\circ$ are that $0.9 < |V_{cb}|$, and $0.3 < |V_{ub}/V_{cb}| < 0.9$. As we shall see in our later discussion, the current experimental bounds on $|V_{ub}/V_{cb}| < 0.5$ have already put some constraint in this region. This $s_8 < 0$ region can be completely ruled out if future improved $b$-decay data can establish $|V_{ub}/V_{cb}| < 0.3$. (See Note added in proof near the end of page 32.)

Some other interesting general results have also emerged from this analysis. The diagonal matrix elements of the mixing matrix are always bigger than the off diagonal elements. With the one exception of $|V_{ub}/V_{cb}| > 1$ for the tiny region of $179^\circ < \delta < 179.82^\circ$, the magnitudes of matrix elements decrease as they move away from the diagonal. This says that quarks like to keep their original generation identity though there is some quark mixing among different generations of quark doublets, i.e. a phenomenon of generation gap. In physical terms, quarks decay in a cascade fashion, e.g. the $b$ particles will prominently decay into charm particles, then charm to strange. This is now supported by experiment from CESR...
This principle has also been utilized for the observation of the bare b states in hadronic reactions \[1.15\]. This analysis also predicts that the t particles will decay mainly into b particles. However, this phenomenon of generation gap also prevents us from knowing whether there are more doublets, just like the Cabibbo angle fits strange particle very well though the third doublet can exist.

To be specific, we also give the mixing matrix for three specific values of $\delta$ and a common $s_1 = 0.23$:

1. $\delta_{\text{minimum}} = 0.34^\circ, s_3 = 0.5$,

$$
V_{B=0.4} = \begin{pmatrix}
0.9737 & 0.197 & 0.114 \\
-0.227 & 0.789 - i0.29 \times 10^{-3} & 0.571 + i0.51 \times 10^{-3} \\
-0.0228 & 0.582 - i0.29 \times 10^{-2} & -0.813 - i0.50 \times 10^{-2}
\end{pmatrix}
$$

$$
\delta_{\text{minimum}} = 0.69^\circ, s_3 = 0.5,$$

2. $\delta = 90^\circ, s_3 = 0.00135$,

$$
V_{B=0.4} = \begin{pmatrix}
0.9737 & 0.228 & 0.308 \times 10^{-3} \\
-0.227 & 0.947 - i0.31 \times 10^{-3} & 0.12 \times 10^{-2} + i0.233 \\
-0.053 & 0.227 + i0.13 \times 10^{-2} & 0.61 \times 10^{-3} - i0.972
\end{pmatrix}
$$

3. $\delta = 90^\circ, s_3 = 0.003$,

$$
V_{B=0.4} = \begin{pmatrix}
0.9737 & 0.228 & 0.683 \times 10^{-3} \\
-0.227 & 0.968 - i0.33 \times 10^{-3} & 0.25 \times 10^{-2} + i0.110 \\
-0.025 & 0.107 + i0.30 \times 10^{-2} & 0.42 \times 10^{-2} - i0.994
\end{pmatrix}
$$

4. $\delta_{\text{maximum}} = 179.93^\circ, s_3 = 0.5$,

$$
V_{B=0.4} = \begin{pmatrix}
0.9737 & 0.197 & 0.114 \\
-0.183 & 0.975 - i0.36 \times 10^{-3} & -0.122 + i0.62 \times 10^{-3} \\
-0.135 & 0.0979 + i0.49 \times 10^{-3} & 0.986 - i0.85 \times 10^{-3}
\end{pmatrix}
$$

5. $\delta_{\text{maximum}} = 179.94^\circ, s_3 = 0.5$,

$$
V_{B=0.4} = \begin{pmatrix}
0.9737 & 0.197 & 0.113 \\
-0.191 & 0.980 - i0.29 \times 10^{-3} & -0.0632 + i0.51 \times 10^{-3} \\
-0.124 & 0.0398 + i0.45 \times 10^{-3} & 0.991 - i0.78 \times 10^{-3}
\end{pmatrix}
$$
Indeed our whole analysis is based upon the existence of the t quark as formulated in the model. In figs. 1.10a, 1.10b the \( s_2 \) vs \( s_3 \) are given for two values of \( m_t = 30 \text{ GeV} \), and 100 GeV. We see that for \( \delta \) in the quadrants I and II the changes in allowed values of \( s_2 \), as a function of \( s_3 \), are mild, but for \( \delta \) in quadrant III the changes are big. However as we shall discuss later, in section 1.5.5, if \( \delta \) indeed is in quadrant III, the t quark mass is bounded \( 20 \text{ GeV} < m_t < 47 \text{ GeV} \) from the additional constraint from \( K_L \to \mu^+ \mu^- \). For detailed discussions see ref. [1.48]; further the quadrant III solution is already severely limited by \( |V_{ub}/V_{cb}| \) from b-particle decay, section 1.5.4. (See Note added in proof near the end of page 32.)

1.5. Determination of \( V_{ii} \) from other experimental information

Besides the \( \Delta m - \epsilon \) determination of the mixing matrix, which is dependent on the box-diagram calculation, it is important to have independent determination of \( V_{cs} \), \( V_{cd} \), \( V_{cb} \), \( V_{ub} \) in a way similarly to those of \( V_{ud} \), \( V_{us} \) [1.24], as emphasized in ref. [1.50]. Recently such detailed analysis has been carried out in refs. [1.51, 1.48].

1.5.1. Determination of \( |V_{cs}| \) from semileptonic decays of charmed particles

For determining \( |V_{cs}| \), the inclusive semileptonic decay of \( D^+ \) or \( D^0 \) can be used [1.49–1.52]

\[
\Gamma(D \to e^+ X) = (2 \pm 1) \times 10^{11} \text{ sec}^{-1}.
\]

Using the naive W-emission quark diagram like fig. 1.2, one obtains the width of the charmed semileptonic decay,
\[ \Gamma(c \rightarrow e^+\nu_e) = \frac{G_F^2 m_e^5}{4\pi m_i} \sum_{i=s,d} |V_{ci}|^2 (1 - 8z_i + 8z_i^2 - z_i^4 - 12z_i^2 \ln z_i), \]  

(1.92)

where \( z_i = m_i^2/m_e^2 \). Using the experimental number eq. (1.91) and \( m_c = 1.75 \text{ GeV}, m_s = 0.45 \text{ GeV} \), we obtain

\[ |V_{cs}| = 0.87^{+0.13}_{-0.23}. \]  

(1.93)

However the calculation is very sensitive to the mass \( m_c \), and also the mass ratio \( m_s^2/m_c^2 \). More seriously we have no way to gauge how reliable this type of calculation is [1.53].

Another way to determine \( |V_{cs}| \) is to consider \( D^+ \rightarrow K^0 e^+\nu_e \). From the study of the energy spectrum \( E_e \) of the electron, it is expected that half of the \( D^+ \rightarrow e^+X \) is from \( D^+ \rightarrow K^0 e^+\nu_e \) [1.52]. Just as for \( K_{e3} \) decay to obtain \( V_{us} \) [1.24], the authors of ref. [1.51] derived the following results:

\[ \Gamma(D^+ \rightarrow K^0 e^+\nu_e) \approx 1.5 \times 10^{11} \text{ sec}^{-1} |f_{D^+K}(0)|^2 |V_{cs}|^2, \]  

(1.94)

where the numerical factor is mainly from the numerical integration of such a decay. This is a vector transition and the Ademollo–Gatto theorem [1.28, 1.29], i.e. \( f_{D^+K}(0) = 1 \), is expected to be operative even though the symmetry breaking can be large here. Using the experimental information

\[ \Gamma(D \rightarrow K e^+X) = \frac{1}{2} \Gamma(D \rightarrow e^+X) = (1 \pm 0.5) \times 10^{11} \text{ sec}^{-1}, \]  

(1.95a)

one obtains

\[ |V_{cs}| = 0.8 \pm 0.2, \]  

(1.95b)

which is in agreement with that of eq. (1.93). However in this analysis the uncertainties from the effect of SU(3) breaking and the effect of the rather large momentum transfer involved are hard to estimate.

1.5.2. Determination of \( |V_{cd}|, |V_{cs}|, and thus other \( V_{ij} \) from \( \nu, \bar{\nu} \)-production of opposite-sign dilepton

For \( |V_{cd}| \) and \( |V_{cs}| \) the charm production in neutrino and antineutrino can provide the information. The charm production in antineutrino scattering will produce opposite sign dileptons through the following lepton–quark reaction

\[ \bar{\nu}_\mu \bar{d} \rightarrow \mu^+ \bar{c} \rightarrow \mu^+ \ell^-X \quad \text{and} \quad \bar{\nu}_\mu \bar{s} \rightarrow \mu^+ \bar{c} \rightarrow \mu^+ \ell^-X. \]  

(1.96)

The former is proportional to \( |V_{cd}| \), and the latter to \( |V_{cs}| \). Assuming SU(3) symmetry for the quark structure functions in the nucleons,

\[ \xi(x) = \bar{u}(x) = \bar{d}(x) = s(x), \]  

(1.97)

the reaction is proportional to \( |V_{cd}|^2 + |V_{cs}|^2 \). For an isoscalar target, the charm production cross section by \( \bar{\nu} \) with energy \( E \) is [1.54]

\[ \sigma(\bar{\nu}N \rightarrow \mu^- \bar{c}X) = (|V_{cs}|^2 + |V_{cd}|^2) \hat{c} \text{sea}, \]  

(1.98)
where
\[
\hat{\sigma}(\text{sea}) = (2/\pi)G_F^2m_NE \int dx \int dy \left[ x' - m_c^2/(2m_NE) \right] \xi(x'),
\]
with
\[
x' = x + m_c^2/(2m_NE)\gamma_y,
\]
where the integration ranges are \(\omega/(1 - x) < y < 1, 0 < x < 1 - \omega, \omega = (m_D^2 + 2m_Dm_N)/(2m_NE)\). The \(m_c, m_D\) are the masses of the charmed quark and the charmed meson D, respectively. Therefore we can obtain
\[
|V_{cs}|^2 + |V_{cd}|^2 = \sigma(\bar{\nu}N \to \mu^+\bar{c}X)/\hat{\sigma}(\text{sea}) = \sigma(\bar{\nu}N \to \mu^+\ell^-X)/[\hat{\sigma}(\text{sea}) \text{Br}(\bar{c} \to \ell^-X)],
\]
(1.99)
where \(\text{Br}(c \to \ell^-X)\) is the semileptonic-decay branching ratio of charmed decay. Using [1.49, 1.52],
\[
\text{Br}(\bar{c} \to \ell^-X) = (9 \pm 1)\%,
\]
(1.100)
and the known quark structure functions [1.55], and the latest high energy \(\bar{\nu}\)-production of opposite-sign dilepton [1.56, 1.57] are obtained [1.48],
\[
|V_{cs}|^2 + |V_{cd}|^2 = 0.49 \sim 1.04,
\]
(1.101)
which gives
\[
|V_{cs}|^2 = 1 - |V_{cs}|^2 - |V_{cd}|^2 = 0.51 \sim 0.0,
\]
(1.102)
or \(|V_{cs}| = 0.71-0.0\). The variations here include the effects of using different quark structure functions. Note that this is consistent with that obtained from \(\Delta m-\epsilon\) analysis in section 1.3, and the small-\(\delta\) solution is preferred. From this we can estimate the lifetime of the b-particles, using the naive W-emission diagram
\[
\tau_b \simeq 10^{-14} \text{ sec}.
\]
(1.103)

In the neutrino reaction the opposite-sign dilepton is produced as follows
\[
\nu_\mu d \to \mu^- c \to \mu^- \ell^+X, \quad \nu_\mu s \to \mu^- c \to \mu^- \ell^+X.
\]
(1.104)

For an isoscalar target, the charm production cross section by \(\nu\) is
\[
\sigma(\nu N \to \mu^- cX) = |V_{cd}|^2 \hat{\sigma}(\text{valence}) + (|V_{cs}|^2 + |V_{cd}|^2) \hat{\sigma}(\text{sea}),
\]
(1.105)
where
\[
\hat{\sigma}(\text{valence}) = (2/\pi)G_F^2m_NE \int dx \int dy \left[ x' - x''^2/(2m_NE) \right] v(x'),
\]
with
\[ x' = x + m^2(2m_NE_\gamma), \]

the valence quark structure function is
\[ v(x) = u(x) + d(x) - 2\xi(x), \]

where \( \xi(x) \) is defined in eq. (1.97).

If we take the difference of reactions (1.96) and (1.104), the \( I_{\text{VC}} \) terms will cancel and what remains is the \( I_{\text{VCd}} \) term, which is proportional to the valence part of the d quark distribution,
\[ |V_{\text{cd}}|^2 = \frac{[\sigma(\nu_L N \rightarrow \mu^- cX) - \sigma(\bar{\nu}_L N \rightarrow \mu^+ \bar{c}X)]/\sigma(\text{valence})}{[\sigma(\nu_L N \rightarrow \mu^- \ell^+ X) - \sigma(\bar{\nu}_L N \rightarrow \mu^+ \ell^- X)]/[\sigma(\text{valence}) Br(c \rightarrow \ell^+ X)]}. \]

It is obtained in refs. [1.48, 1.51, 1.58],
\[ |V_{cd}| = 0.17 \sim 0.23. \]

The variation here includes the effects from using different quark structure functions. Combined with the previous results of \( |V_{cd}|^2 + |V_{cs}|^2 \) we obtain
\[ |V_{cs}| = 0.66 \sim 1.04, \text{ or } |V_{cs}| \geq 0.66. \]

Note that the result for \( |V_{cs}| \) is derived based on the assumption of SU(3) symmetric ocean: \( \bar{s}(x) = s(x) = \bar{u}(x) = \bar{d}(x) \). If the strange ocean is appreciably smaller than the up and down ocean as a result of symmetry breaking, the lower limit value of \( |V_{cs}| \) should be larger. Using eqs. (1.108) and (1.109), we obtain
\[ |V_{cd}|/|V_{cs}| = 0.17 \sim 0.35. \]

All these results are consistent with those obtained from the \( \Delta m - \varepsilon \) analysis in section 1.3. This agreement of \( |V_{cd}|, |V_{cs}| \) from these two independent analyses is not trivial, and quite encouraging.

Using unitarity of the mixing matrix, the mixing-matrix elements involving the t quark can be calculated
\[ |V_{tb}| = 0.67 \sim 1.0, \]
\[ |V_{ts}| = 0.0 \sim 0.72, \]
\[ |V_{tc}| = 0.17 \sim 0.0, \]

and given \( m_t > 20 \text{ GeV} \), we can calculate the lower limit of the t-particle lifetime
\[ \tau_t < 10^{-18} \text{ sec}. \]

We summarize the absolute value of the mixing-matrix elements here, with \( |V_{ud}|, |V_{us}| \) taken from
1.5.3. Restriction of $|V_{cs}|$, $|V_{cd}|$ from nonleptonic decays of charmed particles

The nonleptonic decays of charmed particles are also dependent on the mixing matrix, therefore can provide information on the mixing-matrix elements. Section 2 will be devoted to the discussion of nonleptonic decays. Here we shall pick two nice examples of $D$ decaying to two pseudoscalars, from which limits on $|V_{cd}|, |V_{cs}|$ are obtained.

Assuming SU(3) symmetry [1.59, 1.60], one can show

$$\frac{\Gamma''(D^+ \to \pi^0 \pi^+)}{\Gamma''(D^+ \to K^0 \pi^+)} = \frac{1}{3} |V_{cd}/V_{cs}|^2.$$  

(1.116)

Here $\Gamma''$ denotes reduced decay width which has the phase space factor removed from the decay width $\Gamma$. This is an especially nice relation since both final states are exotic, i.e. nonresonance-forming, and thus free from strong final state interaction [1.61]. From the experimental information

$$\frac{\Gamma(D^+ \to \pi^0 \pi^+)}{\Gamma(D^+ \to K^0 \pi^+)} < 0.30,$$  

(1.117)

we obtain

$$|V_{cd}/V_{cs}|^2 < 0.60, \text{ or } |V_{cd}/V_{cs}| < 0.77.$$  

(1.118)

Another bound on $|V_{cd}/V_{cs}|$ can be derived from the following experimental information [1.8],

$$\frac{\Gamma(D^0 \to K^- K^0)}{\Gamma(D^0 \to K^- \pi^+)} = 0.113 \pm 0.030.$$  

(1.119)

$$\frac{\Gamma(D^0 \to \pi^+ \pi^-)}{\Gamma(D^0 \to K^- \pi^+)} = 0.033 \pm 0.015.$$  

(1.120)

Theoretical calculation based upon SU(3) invariance gives [1.59, 1.60],

$$a(D^0 \to K^- \pi^+) = \sqrt{2} A V_{ud} V_{cs},$$  

(1.121)

$$a(D^0 \to K^- K^0) = \sqrt{2} A V_{us} V_{cs} + \sqrt{2} D(V_{us} V_{cs} + V_{ud} V_{cd}),$$  

(1.122)

$$a(D^0 \to \pi^+ \pi^-) = \sqrt{2} A V_{ud} V_{cd} + \sqrt{2} D(V_{us} V_{cs} + V_{ud} V_{cd}),$$  

(1.123)

where $A, D$ are independent amplitudes. Note that if

$$V_{us} V_{cs} + V_{ud} V_{cd} = 0,$$  

(1.124a)
as in the two-doublet model, we should have

\[ \frac{\Gamma(D^0 \rightarrow K^-K^+)/\Gamma(D^0 \rightarrow K^-\pi^+)}{\Gamma(D^0 \rightarrow \pi^+\pi^-)/\Gamma(D^0 \rightarrow K^-\pi^+)} = |V_{us}/V_{ud}|^2 = 0.056. \tag{1.124b} \]

The fact that the two decay ratios of eqs. (1.119, 1.120) are not equal is an indication within the K–M model that \( |V_{us}/V_{ud}| \neq -|V_{cd}/V_{cs}| \). (For other interpretation see discussion in section 2.2). From eqs. (1.121–1.123), we can derive a bound [1.59, 1.60],

\[ 0.2 \approx \Delta_D \leq |V_{us}/V_{ud} - V_{cd}/V_{cs}| \leq \Sigma_D \approx 0.5 \tag{1.125} \]

where

\[ \Sigma_D \equiv \left| \frac{[\Gamma(D^0 \rightarrow K^+K^-)]^{1/2} - [\Gamma(D^0 \rightarrow \pi^+\pi^-)]^{1/2}}{[\Gamma(D^0 \rightarrow K^-\pi^+)]^{1/2}} \right|, \]

the lower bound is easily satisfied, however the upper bound gives the interesting bound, knowing the imaginary part of \( V_{cd}/V_{cs} \) is very small,

\[ |V_{cd}/V_{cs}| < 0.30 \pm 0.06. \tag{1.126} \]

Note that the \( |V_{us}/V_{ud}| - |V_{cd}/V_{cs}| \) obtained in section 1.4 from the box-graph calculation of \( \Delta m \) and \( \epsilon \) stays within the bound of eq. (1.125). So the results from the two different analyses are consistent. It is very interesting to compare these results, especially as the accuracy of the experimental data improves, because they are determined from very different methods.

1.5.4. Constraints on \( |V_{ub}|, |V_{cb}| \) from the b particle decay

Using the present experimental bound on the b lifetime [1.62],

\[ \tau_b < 1.4 \times 10^{-12} \text{ sec}, \tag{1.127} \]

and using the naive W-emission diagram one obtains a lower bound on \( V_{cb} \),

\[ |V_{cb}| > 0.032. \tag{1.128} \]

It is a rather loose bound. We certainly need more accurate life-time measurement of the b.

Since \( V_{ub} = s_1s_3 \), for \( s_3 < 0.5 \),

\[ |V_{ub}| < 0.12. \tag{1.129} \]

Unlike \( V_{cd} \), so far there is no positive identification of \( V_{ub} \neq 0 \). The same sign dilepton production, \( \bar{\nu}_u \rightarrow \ell^+bX \rightarrow \ell^+c\ell^-X \rightarrow \ell^+\ell^-s\ell^-X \), so far is inconsistent with the model [1.63], i.e. \( V_{ub} \) needs to be of the order of magnitude ten, while the b-decay information from CESR is consistent with \( V_{ub} = 0 \), [1.64].

To estimate \( |V_{ub}/V_{cb}| \), two methods can be used: (1) Study the number of K's in B decay, which is
now estimated to be $1.7 \pm 0.2 \pm 0.3$ per b-particle produced, where the first error is statistical, and the second systematical. The K’s are from the cascade decade of $b \rightarrow c \rightarrow s$. This of course is very sensitive to the details of fragmentation of charm into K’s. Currently, the data is consistent with $|V_{ub}| = 0$ and with an upper bound of $|V_{ub}/V_{cb}| < 50\%$ [1.64].

(2) Study the inclusive lepton distribution from the $b$ decay. The lepton momentum is slower from the semileptonic decay of $b \rightarrow c\ell^-\nu_\ell$ than from $b \rightarrow u\ell^-\nu_\ell$. The subsequent semileptonic decay of $c \rightarrow s\ell^-\nu_\ell$ from $b \rightarrow c\ell^-\nu_\ell$ gives an even slower moving $\ell^-$. Currently the spectrum of $\ell^-$ from $b \rightarrow \ell^-\bar{\nu}_\ell X$ suggests an effective hadron mass $M_h = 2$ GeV. This is consistent with $b$ decaying exclusively to $c$, i.e. $|V_{ub}| = 0$. But at this time a small $b \rightarrow u$ component cannot be excluded. The bound now is [1.64] (see Note added in proof near the end of page 32)

$$|V_{ub}|/|V_{cb}| < 0.5 .$$  \hspace{2cm} (1.130)

This bound combined with that from CP violation analysis, eq. 1.7 limits $\delta$ to be $\delta < 178.85^\circ$. As we mentioned earlier if this bound decreases to $|V_{ub}/V_{cb}| < 0.3$, $\delta$ can be excluded from the third quadrant. Then the allowed region for $\delta$ is $2^\circ < \delta < 178^\circ$. We see that so far, all these bounds are in agreement with $\delta$ determined from CP violation in the $K^0\bar{K}^0$ system.

From the value of the matrix elements we can ask what constraints they put on the mixing angles. In figs. 1.11a,b,c contours of constant $|V_{cs}|$ are plotted in the $s_2-s_3$ plane for $\delta = 0^\circ$, $90^\circ$ and $180^\circ$. In these figures even though the whole ranges of $0 < s_2, s_3 < 1$ are shown, but only $0 < s_2, s_3 < 0.5$ are allowed from the fit to hyperon semileptonic decays, and to $M_1-M_8$. The box-graph-calculation solutions appear as a point on the plot for a given $\delta$, which are shown as circles in the figures. The solid circles are from the $B = 0.4$ calculation, and the dashed circles from the vacuum insertion calculations for $\delta = 179^\circ$, $90^\circ$ and $1^\circ$. The allowed points for minimum $\delta$’s are also shown in triangles, solid triangle is for the $B = 0.4$ calculation at minimum $\delta = 0.3^\circ$ and dashed triangle is for the vacuum insertion calculation at minimum $\delta = 0.9^\circ$. We see that $|V_{cs}|$ can be very limiting on the regions of the angles. For example a value of $|V_{cs}| < 0.83$ can rule out $\delta > 90^\circ$ region. As we shall discuss in section 3, the CP violation effects in the $b$ and $t$ systems are sensitive to the $\delta$ value. Therefore a more precise determination of $V_{cs}$ will be very valuable.

In figs. 1.12a,b,c constant $|V_{cd}/V_{cs}|$ contours are given. The contour topology is very sensitive to the values $|V_{cd}/V_{cs}|$, especially near the demarcation point $|V_{cd}/V_{cs}| = 0.24$. If $|V_{cd}/V_{cs}|$ could be shown to be greater than 0.27, this would imply $\delta < 90^\circ$ as the only solution. In figs. 1.13a,b,c the constant $|V_{cd}|$ contours are given, we see that the current experimental bound $|V_{cd}| > 0.032$ of eq. (2.128) gives very weak limitations on the angles. In figs. 1.14a,b,c constant $|V_{ub}/V_{cb}|$ contours are given. We see that the present limit of $|V_{ub}/V_{cb}| < 0.5$ of eq. (1.130) have no limitations on the angles at all.

In fig. 1.13, the constant $|V_{cb}|$ contours are given, we see that the current experimental bound $|V_{cb}| > 0.032$ of eq. (1.128) gives very weak limitations on the angles. On the other hand, if we take the limitation from eq. (1.38a), $0.70 < |V_{cb}| < 1$, we find $|V_{cb}| < 0.6$, which gives a lower limit of $b$ lifetime $\tau_b > 2.5 \times 10^{-14}$ sec.

From these figures we see that the angles and the phase cannot be well determined from the values of $|V_{ub}|$ alone. The current crudely determined values of $|V_{ub}|$ can hardly give much information about the angles and phase. The box-graph calculation for $\Delta m$ and $\epsilon$ can provide more specific information, see the circles and triangles in these figures. Better measured values of $|V_{ub}|$ and future observation of other CP-violation effects predicted by the model are needed to determine these angles and the phase, see discussion in section 3.2.

In table 1.2 we list all the information obtained so far on mixing angles and matrices and their origins. Comparing these results with those in table 1.1, we see that the $|V_{ub}|$ so far obtained from
Fig. 1.11. Constant $|V_{cd}|$ contours in $s_2-s_3$ for $\delta = 0^\circ$ and $180^\circ$. The solid circles are from the $B = 0.4$ calculation of section 1.3; the dashed circles from the vacuum-insertion $B = 1$ calculation of section 1.3 for $\delta = 1^\circ$, $90^\circ$, $179^\circ$. The solid triangle is for the $B = 0.4$ calculation at minimum $\delta = 0.3^\circ$; the dashed triangle is for the vacuum-insertion $B = 1$ calculation at minimum $\delta = 0.9^\circ$.

Various experimental information are consistent with those obtained from box-graph calculation of the $CP$ violation effects and the mass difference of $K_L$, $K_S$. It is of importance to improve the accuracy of such determination of the values of $V_{ij}$. 
All these analyses discussed in sections 1.5.1–1.5.4 ought to be continued as more data is available for the b and even the t particles [1.65–1.67].

**Note added in proof.** It was reported at the XXI Int. Conf. on High Energy Physics, July 26–31, 1982, Paris; that the CUSB collaboration and CLEO collaboration at CESR, Cornell have narrowed the bound: $|V_{ub}|/|V_{cb}| < 0.2$. This result rules out the $s_0 < 0$ solution given in section 1.4, see fig. 1.9; and from fig. 1.7 we see that the solutions near $\delta \approx 180^\circ$ are also more limited now $\delta < 179.6^\circ$. 
Table 1.2
Summary of $|V_{ij}|$ determination from sources other than box-graph fittings of $\Delta m, \epsilon$ of the $K^0\bar{K}^0$ systems

<table>
<thead>
<tr>
<th>Source</th>
<th>Expression</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>$V_{ud} = c_1 = 0.9737 \pm 0.0025$</td>
<td>$0^- \rightarrow 0^-$ nuclear $\beta$ decay</td>
</tr>
<tr>
<td>(2)</td>
<td>$V_{us} = s_1c_2 = 0.219 - 0.002$</td>
<td>$K_{\alpha 3}$ and semileptonic decay of $K^-$</td>
</tr>
<tr>
<td>(2a)</td>
<td>$s_1 = 0.28^{+0.3}_{-0.2}$</td>
<td>hyperons</td>
</tr>
<tr>
<td>(3)</td>
<td>$0.64 &lt;</td>
<td>V_{ud}</td>
</tr>
<tr>
<td>(3a)</td>
<td>$</td>
<td>V_{ud}</td>
</tr>
<tr>
<td>(4)</td>
<td>$</td>
<td>V_{cd}</td>
</tr>
<tr>
<td>(4a)</td>
<td>$</td>
<td>V_{cd}</td>
</tr>
<tr>
<td>(5)</td>
<td>$</td>
<td>V_{ud}</td>
</tr>
<tr>
<td>(5a)</td>
<td>$</td>
<td>V_{cd}</td>
</tr>
<tr>
<td>(6)</td>
<td>$</td>
<td>V_{cd}</td>
</tr>
<tr>
<td>(7)</td>
<td>$</td>
<td>V_{cd}</td>
</tr>
<tr>
<td>(8)</td>
<td>$</td>
<td>V_{cd}</td>
</tr>
<tr>
<td>(9)</td>
<td>$</td>
<td>V_{cd}</td>
</tr>
<tr>
<td>(10)</td>
<td>$</td>
<td>V_{cd}</td>
</tr>
<tr>
<td>(11)</td>
<td>$</td>
<td>V_{cd}</td>
</tr>
<tr>
<td>(12)</td>
<td>$0.06 &lt;</td>
<td>V_{cb}</td>
</tr>
<tr>
<td>(13)</td>
<td>$</td>
<td>V_{cb}</td>
</tr>
<tr>
<td>(14)</td>
<td>$</td>
<td>V_{cb}</td>
</tr>
</tbody>
</table>

1.5.5. Constraints from $K_L \rightarrow \mu^+ \mu^-$
As we mentioned before the smallness of the rate $K_L \rightarrow \mu^+ \mu^-$ prompted the introduction of the GIM mechanism and the introduction of the charmed quark. Now we can ask if it can still further constrain the parameters in the model, like the mixing-matrix elements and the unobserved $t$ quark masses.

The analysis of $\epsilon$ and $\Delta m$ in section 1.3 has determined all the mixing matrix in terms of one angle, say $s_3$, for a given $m_t$ value. Figs. 1.10a, 1.10b show the dependence of the mixing angles on $m_t$, the dependence is rather weak for $\delta$ in the quadrants I and II, but strong for $\delta$ in the quadrant III. The hope is that by studying one more reaction which also depends on the quark mixing matrix and $m_t$, we can determine the allowed values of $m_t$ and the mixing matrix.

It turns out, as already observed in [1.68], that $K_L \rightarrow \mu^+ \mu^-$ does not put much more constraint on the mixing matrix for $s_5 > 0$. For $\delta$ in the third quadrant, i.e. $s_5 < 0$, it is concluded in ref. [1.48] that $m_t$ is bounded $20 \text{ GeV} < m_t < 47 \text{ GeV}$. This indeed helps to bound the allowed values of mixing angles in this region of $\delta$ where the mixing angles are more sensitive to $m_t$, figs. 1.10a, 1.10b. For detailed discussion of the analysis $K_L \rightarrow \mu^+ \mu^-$ and its implications on the quark mass in the region $s_5 > 0$, see ref. [1.48].

1.6. Concluding remarks
We have come a long way since the original work of Kobayashi–Maskawa. The original determination of the mixing matrix in fitting $\Delta m, \epsilon$ of the $K^0\bar{K}^0$ system with the crude box-graph calculation has surprisingly survived further independent experimental constraints. One of the striking features emerging from these analyses is the generation-gap phenomenon, i.e. quarks of different generations do mix
but the mixing gets weaker as the generation gap increases. Physically this gives the cascade pattern of decay, e.g. the observed decay of $b \to c \to s$, and the predicted decay pattern for the yet-to-be observed $t \to b \to c \to s$. The analysis of $|V_{ij}|$ can be refined as more experimental information about the $c$, $b$, and even the $t$ quarks are available. It will be a continuing study, I am sure, for years to come.

While we do have the approximate range of values of the quark mixing elements matrix now, we still have very poor information about the angles $\theta_2$, $\theta_3$ and especially about the phase $\delta$, which can be anywhere in the first and second quadrants. The solutions near $\delta \approx 0^\circ$ or $180^\circ$ may be differentiable from the values of $|V_{ij}|$. But if $\delta$ happens to fall in the middle values away from $0^\circ$ and $180^\circ$, the values of $\delta$ cannot be determined from $|V_{ij}|$, since $|V_{ij}|$ is calculated in fitting $\Delta m$ and $\varepsilon$ to have quite constant values in that region. Since $\delta$ is closely related to the $CP$ violation phenomenology, the confirmation of the K–M model must come from the observation of the many interesting $CP$ violation effects given by the K–M model other than the $\varepsilon$ in the $K^0$ system, e.g. $\varepsilon'$, partial decay rates differences in $\Lambda \to \pi^- p$, $\bar{\Lambda} \to \pi^+ \bar{p}$, and other strange particle decays, see section 3. Unfortunately the prediction of $CP$-violation effects in the heavy quark $D^0\bar{D}^0$, $B^0\bar{B}^0$, $T^0\bar{T}^0$ systems from the particle–antiparticle transition are extremely small. Except for the purpose of checking the validity of the model by the non-observation of particle–antiparticle-transition type of $CP$ violation, we cannot use the information to determine the value of $\delta$. It seems that the only possibility to determine the value of $\delta$ is the measurement of partial decay-rate difference between particle and antiparticle into their $CP$-conjugated final particle states, especially in the heavy quark systems of $b$ and $t$, see section 3.2. It is a strong point of the model that many interesting $CP$-violation phenomena are predicted. The observation of any such $CP$-violation effects is extremely interesting.

One naturally raises the question whether there are more quark doublets than three. Due to the generation-gap phenomenon of the mixing-matrix $V$, we cannot predict whether higher quark doublets exist from the deviation from unitarity of $V$, for the same reason that the Cabibbo angle worked so well that the existence of the third quark doublet was not suspected until a good physical reason was found by Kobayashi and Maskawa. Then the question is whether there is a compelling reason for the existence of quark doublets beyond three. Recently, $CP$ violation with the instability of the proton have been reasoned to be the cause of matter–antimatter asymmetry in our universe, developed during its thermal unstable state shortly after the big bang. There have been inklings of hints that the three doublet model with the K–M $CP$-violation may not be enough to accomplish the matter–antimatter asymmetry. More heavy quark doublets may be needed. It will be interesting to see if such reasoning can be substantiated in the future [1.69, 1.70].

2. Nonleptonic decays

2.1. Introductory remarks

The main reasons to discuss nonleptonic decays in the context of this review are two-fold. Firstly the analysis of nonleptonic decay amplitudes needs to be carried out in conjunction with mixing matrix element analysis, thus providing information about the mixing matrix elements. Secondly, the $CP$-violation effects can appear in nonleptonic decays from the interference between amplitudes which have phases both from the weak and strong interactions. Therefore the quark diagram formulation discussed in this section will be useful for the discussion in the next section about $CP$ violation in nonleptonic decays. In this section I shall give a very qualitative description of the current status of our knowledge about nonleptonic decays.
Let us first briefly review the history of the K nonleptonic decays. About 1954, it was observed that the $K^+ \rightarrow \pi^+ \pi^0$ decay rate was greatly suppressed compared to that of $K^0 \rightarrow \pi^+ \pi^-$, i.e. $\Gamma(K^+ \rightarrow \pi^+ \pi^0)/\Gamma(K^0 \rightarrow \pi^+ \pi^-) = 1/670$, [2.1]. Since only the isospin $I = 2$ component contributes in $K^+ \rightarrow \pi^+ \pi^0$, i.e. $\Delta I = 3/2$, it was proposed [2.2] that the weak Hamiltonian $H_w$ was dominated by $I = 3/2$; or in SU(3) classification, $H_w$ transforms like an $[8]$ (more detailed discussions in section 2.3). However, no good dynamical reason was found for this $\Delta I = 3/2$ rule. Soon after the weak interactions were realized to be renormalizable [1.19] and presenting the possibility for doing perturbative QCD calculation [2.3], the W-emission quark diagrams with the leading-order gluon corrections (fig. 2.1) were shown to enhance the $\Delta I = 3/2$ contribution [2.4]. But the enhancement factor is much smaller than the experimental number 670. Later it was suggested [2.5] that the so-called “Penguin” diagram of fig. 2.2 may give significant enough enhancement [2.6].

Since it has taken more than 20 years to search for the explanation of the mechanism of the K decay, it will probably take some time before we really understand the nonleptonic decays of the charmed and new flavored particles. It is therefore fruitful also to discuss the nonleptonic decays in a phenomenological framework. I shall begin with the quark-diagram approach for the nonleptonic decays. Afterwards the experimental data and the development of the more ambitious dynamical interpretation of the importance of certain diagrams will be discussed. Later in section 2.3, the charmed particle decays will also be discussed in the traditional symmetry approach.

2.2. Quark diagram approach

First we find all distinct quark diagrams for heavy-quark decays. Then, by a recombination of various final state quarks the final hadron states are formed. The hope is that a description consistent with experimental data emerges. One may find that one or a subset of diagrams dominate so that the description of the nonleptonic decay becomes simple and has further predictive power. This should provide a first step toward the understanding of the mechanism of the nonleptonic decays. The ultimate goal is of course to find a dynamical scheme that such graphs can be calculated and provide a description of the nonleptonic decays. In the following we shall discuss what has been learned from the strange and charmed meson decays, and the interesting effects to look for in the future.

For the meson decay there are six distinct diagrams [2.7], as shown in fig. 2.3. Diagram (a) corresponds to what is the usually called external W-emission diagram, while diagram (c) corresponds to the internal W-emission diagram. Note that diagrams (a) and (c) are treated differently. In diagram (a) the W produced quark–antiquark pair must combine with each other to produce hadrons while in diagram (c) they must not combine with each other but with the other available quarks to produce hadrons. Diagram (e) is the well-known W-exchange diagram, it contributes only to neutral mesons, as in $K^0$, $D^0$ decays. Diagram (d) is the W-annihilation diagram in which the initial quark and antiquark
annihilate to make a W-boson which subsequently converts into a $q\bar{q}$ pair involving only light quarks. Diagram (e) is the “Penguin” diagram while diagram (f) is the side-ways “Penguin” diagram. In the diagrams of fig. 2.3 the quark lines are not joined to make specific hadrons. These diagrams give inclusive total decay amplitudes for heavy quark systems which can decay into many hadronic channels.

For exclusive decays we need to combine $q\bar{q}$ into specific hadron states. In some graphs quark–antiquark pairs need to be created. For example in graphs (e)–(f) we need to produce an extra quark–antiquark pair in order to produce a pair of light mesons in the final state (which we assume to be taken care of by the strongly interacting gluons). In our discussions here it is assumed that $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ pairs are produced with equal probability from the vacuum, as implied by SU(3) symmetry. The final state quarks are then allowed to hadronize and arrange themselves in all possible ways. For pseudoscalar mesons we use the well-known SU(3) particle contents of the various $q\bar{q}$ system:

\[
\begin{align*}
    u\bar{u} & : (1/\sqrt{2})\pi^0 + (1/\sqrt{6})\eta_0 + (1/\sqrt{3})\chi^0, \\
    d\bar{d} & : -(1/\sqrt{2})\pi^0 + (1/\sqrt{6})\eta_0 + (1/\sqrt{3})\chi^0, \\
    s\bar{s} & : -(\sqrt{2}/\sqrt{3})\eta_0 + (1/\sqrt{3})\chi^0, \\
    u\bar{d} & : \pi^+, \quad d\bar{u} : \pi^-, \\
    u\bar{s} & : K^+, \quad d\bar{s} : K^0, \\
    s\bar{d} & : K^0, \quad s\bar{u} : K^-.
\end{align*}
\]

Fig. 2.4 gives the six graphs of a meson with heavy quark decaying into two lighter mesons.

We would like to emphasize that these graphs are symbolic and meant to have all the strong interactions included, i.e. gluon lines are included in all possible ways. Only in specific model calculations will we specify the strong interaction involved. For example in the leading-order QCD calculation one gluon exchange will be specifically included. In the following our discussion on the leading-order QCD calculation will be very qualitative. For more quantitative discussion see the excellent reviews in refs. [2.8, 2.9].

Fig. 2.3. The six quark diagrams for the inclusive nonleptonic decay of a meson.
2.2.1. Strange meson decays

In Table 2.1a,b we list the quark diagram amplitudes for K⁺, K⁰ decay into two pions and three pions. Replacing Vᵣ by Vᵣ* in Table 2.1a,b, we obtain the corresponding decay amplitudes for K⁻, K⁰ into charge conjugated particles respectively. We shall discuss in section 3 the implication in CP violation due to this change of Vᵣ to Vᵣ* in the corresponding antiparticle decay amplitudes.

Note that the absence of the W-annihilation diagram (d) and the “Penguin” diagram (e) in A(K⁺ —→ π⁺π⁰) is special to the two-pion π⁺π⁰ final states of the K⁺ decay. Amplitudes (d) and (e) do contribute to the 3π final states of the K⁺ decay, as indicated by Table 2.1a, which however is inhibited by phase space. Amplitudes (e) and (f) intrinsically do not contribute to the K⁺ decay. To the K⁰ decays, into 2π as well as 3π final states, all the amplitudes except (d) contribute. The one gluon approximation for the amplitude (e) is the “Penguin” diagram fig. 2.2. As mentioned in the introduction of this section, it has been pointed out in ref. [2.5] that the relative enhancement of A(K⁰ —→ π⁺π⁻) comparing to A(K⁺ —→ π⁺π⁰) is due to the dominance of diagram (e). We shall see in the next section that because of the complexity in Vₑs amplitude (e) provides a weak interaction phase in A(K⁰ —→ π⁺π⁻) and gives the CP violation effects of a sizeable e'.

Table 2.1

| Quark Diagram Amplitudes for K Decays into Two and Three Pions† |
|-----------------|-----------------|
| **a. K⁺ decays** |                  |
| π⁺π⁰            | (1/2√3) VₑdVₑs(a+e) |
| π⁺π⁺π⁰         | (1/3) [VₑdVₑs(a+e)-VₑdVₑs(e²)] |
| π⁺π⁻           | (1/2√3)[VₑdVₑs(a+e)+VₑdVₑs(e)] |
| **b. K⁰ decays** |                  |
| π⁺π⁰            | VₑdVₑs(a+e) |
| π⁺π⁺π⁰         | (1/3) [VₑdVₑs(a+e)-VₑdVₑs(e²)] |
| π⁺π⁻           | (1/3) [VₑdVₑs(a+e)+VₑdVₑs(e)] |
| π⁺π⁻π⁻         | (1/3) [VₑdVₑs(a+e)+VₑdVₑs(e)] |

†Replacing Vᵣ by Vᵣ* one obtains the charge conjugated decays.

††The factors in the curly brackets are Bose-statistics factors for identical particles such that the phase space is always 1/[(d²p²)/(2π²)] irrespective of whether the particles are identical or not.
However I would like to point out an important alternative: the enhancement of \( A(K^0 \to \pi^+ \pi^-) \) over \( A(K^+ \to \pi^+ \pi^0) \) can also be the consequence of the dominance of the W-exchange diagram (\( \epsilon \)) [2.10]. If indeed diagram (\( \epsilon \)) dominates over diagrams (\( \delta \)), (\( \epsilon ' \)), (\( \varphi \)), due to the fact that \( \Gamma(K^+ \to 3\pi) \approx \Gamma(K^0 \to 3\pi) \), we also need the dominance of the (\( \delta \)) amplitude for \( K^+ \to 3\pi \) decay, see table 2.1. However such dominance by diagrams (\( \epsilon \)) and (\( \delta \)) was considered not possible in the leading-order QCD calculation. In the perturbative QCD scheme, diagrams in fig. 2.4 are considered to be the lowest order Feynman diagrams. In such a scheme, decay diagrams (\( \epsilon \)) and (\( \delta \)) for pseudoscalar meson are negligible due to the smallness of the u, d quark masses, just like \( \Gamma(\pi^+ \to e^+ \nu_e) \) is extremely small. As we shall see later in the discussion of the charmed meson decays, there are indications from the data that the W-exchange diagram (\( \epsilon \)), and the W-annihilation diagram (\( \delta \)) need not be small. If the enhancement of \( \Gamma(K^0 \to 2\pi) \) is indeed due to the dominance of the diagram (\( \epsilon \)), not diagram (\( \delta \)), the CP violation effect \( e' \) will be much smaller than that estimated from considering diagram (\( \epsilon \)) alone.

2.2.2. Charmed meson decays

In tables 2.2 and 2.3 we list the charm meson lifetimes [2.11, 2.12] and decay rates into two pseudoscalar bosons [1.8]. In table 2.4 we list the quark-diagram amplitudes for \( D^0, D^+, F^+ \) inclusive decays, and table 2.5 for exclusive decays into two pseudoscalar mesons [2.13]. Replacing \( V_{ud} \) by \( V_{ud}^* \) in tables 2.4–2.6, we obtain the corresponding amplitudes for \( D^0, D^+, F^- \) decays. Since this is still a very rapidly developing field we shall discuss the experimental data of charmed decay following a historical sequence.

In April 1979 came the first surprise [2.14]

\[
R_1 = \frac{\Gamma(D^0 \to K^- K^+)/\Gamma(D^0 \to K^- \pi^+)}{0.113 \pm 0.030},
\]

\[
R_2 = \frac{\Gamma(D^0 \to \pi^- \pi^+)/\Gamma(D^0 \to K^- \pi^+)}{0.033 \pm 0.015}.
\]

(Though the errors are still large and the possibility that \( R_1 = R_2 \) may still turn out to be true, we proceed in the context that \( R_1, R_2 \) are unequal.) This was a surprise because calculations were made only in the two left-handed doublet model assuming SU(3) symmetry [2.15, 2.16], and the prediction was \( R_1 = R_2 = \tan^2 \theta_c = 0.05 \). This can easily be seen from the \( D^0 \) decay listed in table 2.5a, with \( V_{us} V_{cs} = -V_{ud} V_{cd} \). The historically interesting thing is that although the Kobayashi–Maskawa three left-handed doublet model was proposed in 1973, no such calculation was done in that framework before the data of eqs. (2.2a,b). Several explanations have so far been offered:

1. Rather than the four-quark model, one ought to generalize the SU(3) symmetric calculations to the six-quark model. Then if \( V_{us}/V_{ud} \neq -V_{cd}/V_{cs} \), \( R_1 \) and \( R_2 \) do not have to be the same and the data shown in eqs. (2.2a,b) can be easily accommodated [2.17]. Actually they provide lower and upper bounds on the value of \( V_{us} V_{cs} + V_{ud} V_{cd} \), as discussed in section 1.5.3. See also the discussions at the end of this section. If \( V_{us}/V_{ud} = -V_{cd}/V_{cs} \), one must resort to other explanations.

2. The inequality of \( R_1 \) and \( R_2 \) may be from SU(3) breaking [2.18], but it is difficult to give a quantitative estimate.

3. It was suggested that maybe a charged Higgs [2.19] is involved since Higgs particle prefers to couple to heavier quarks than the lighter ones, so as to make \( R_1 > R_2 \). Such charged Higgs boson can give CP violation effects in \( K^0 \) system via the double Higgs-exchange box diagram, therefore their coupling and masses may be quite constrained [2.19].

4. The “Penguin” diagram [2.20] may be important for the charmed decay too, however there is no definite conclusion as to its magnitude.
Table 2.2
Lifetime measurements for charmed particles

<table>
<thead>
<tr>
<th>Particle</th>
<th>Data</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>D⁺</td>
<td></td>
<td>E531, WA58, BC72/73, NA16, NA18, Mark II</td>
</tr>
<tr>
<td>D⁻</td>
<td></td>
<td>E531, WA58, BC72/73, NA16, NA18, NA1</td>
</tr>
<tr>
<td>𝔼ᶻ</td>
<td></td>
<td>E531, NA16, NA18, NA1</td>
</tr>
<tr>
<td>𝔼ᶜ</td>
<td></td>
<td>E531, WA58, NA16</td>
</tr>
</tbody>
</table>

Mean Lifetime (10⁻¹³) sec

E531: Fermilab, 𝜈, hybrid emulsion spectrometer experiment.
NA16: CERN, 𝜈, p LEBC bubble chamber and EHS spectrometer experiment.
NA28: CERN, 𝜈, BIBC bubble chamber and streamer chamber experiment.
NA1: CERN, 𝜈, emulsion-Ο experiment.
BC72/73: SLAC, 𝜈, hybrid facility bubble chamber experiment.
Mark II: SLAC, 𝜉⁺, Mark II detector experiment.

Table 2.3a
Charm D decay branching ratios

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>Mark II</th>
<th>LGW</th>
</tr>
</thead>
<tbody>
<tr>
<td>D⁺→K⁺π⁺</td>
<td>3.0 ± 0.6</td>
<td>2.2 ± 0.6</td>
</tr>
<tr>
<td>D⁰→K⁰π⁰</td>
<td>2.2 ± 1.1</td>
<td>-</td>
</tr>
<tr>
<td>D⁺→K⁺π⁺</td>
<td>2.3 ± 0.7</td>
<td>1.5 ± 0.6</td>
</tr>
</tbody>
</table>

Table 2.3b
Measurements on mixing-matrix suppressed D decays. Upper limits are at the 90% confidence level

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>Branching ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>𝐃⁺(π⁻π⁻⁺)/antino D⁺(π⁻π⁻⁺)</td>
<td>0.033 ± 0.015</td>
</tr>
<tr>
<td>𝐃⁺(K⁻K⁻⁺)/antino D⁺(K⁻K⁻⁺)</td>
<td>0.113 ± 0.030</td>
</tr>
<tr>
<td>𝐃⁺(π⁻π⁻⁺)/antino D⁺(K⁻K⁻⁺)</td>
<td>&lt;0.30</td>
</tr>
<tr>
<td>𝐃⁺(K⁻K⁻⁺)/antino D⁺(K⁻K⁻⁺)</td>
<td>0.25 ± 0.15</td>
</tr>
</tbody>
</table>

Table 2.3c
Inclusive strange particle branching ratios for D decays

<table>
<thead>
<tr>
<th>Mode</th>
<th>Mark II (%)</th>
<th>D⁰ (%)</th>
<th>Mark II (%)</th>
<th>D⁺ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D⁻→K⁻X</td>
<td>55 ± 11</td>
<td>36 ± 10</td>
<td>19 ± 5</td>
<td>10 ± 7</td>
</tr>
<tr>
<td>D⁻→K⁻X</td>
<td>8 ± 3</td>
<td>-</td>
<td>6 ± 4</td>
<td>6 ± 6</td>
</tr>
<tr>
<td>D⁻→K⁰X</td>
<td>29 ± 11</td>
<td>57 ± 18</td>
<td>52 ± 18</td>
<td>39 ± 39</td>
</tr>
</tbody>
</table>

Table 2.4*
Quark diagram amplitudes for charmed particle inclusive decays†

<table>
<thead>
<tr>
<th>Mode</th>
<th>𝑉_{u}V_{u}(a+\delta+e) + 𝑉_{u}V_{u}(\delta+\epsilon) + 𝑉_{u}V_{u}(e+f) + 𝑉_{u}V_{u}(e+f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D⁺→hadrons</td>
<td>𝑉_{u}V_{u}(a+\delta+\epsilon) + 𝑉_{u}V_{u}(d+\epsilon) + 𝑉_{u}V_{u}(e)</td>
</tr>
<tr>
<td>D⁺→hadrons</td>
<td>𝑉_{u}V_{u}(a+\delta+\epsilon) + 𝑉_{u}V_{u}(d+\epsilon) + 𝑉_{u}V_{u}(e)</td>
</tr>
<tr>
<td>F⁺→hadrons</td>
<td>𝑉_{u}V_{u}(a+\delta+\epsilon) + 𝑉_{u}V_{u}(d+\epsilon) + 𝑉_{u}V_{u}(e)</td>
</tr>
</tbody>
</table>

†Replacing 𝑉_{u} by 𝑉_{u}' one obtains the charge conjugated decays.
This comment applies to all these tables 2.4-2.7.
* See remarks in ref. [2.13] for reading tables 2.4-2.7 of quark diagrams.
Table 2.5

Quark-diagram amplitudes for charmed meson → two pseudoscalar mesons† (†† same as table 2.1)

<table>
<thead>
<tr>
<th>a. Dº decay</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Kºπº</td>
<td>VudVd*(a+e)</td>
</tr>
<tr>
<td>Kºπº</td>
<td>VudVd*(e+2f)</td>
</tr>
<tr>
<td>Kºπº</td>
<td>VudVd*(a+c)</td>
</tr>
<tr>
<td>Kºηº</td>
<td>VudVd*(c+e)</td>
</tr>
<tr>
<td>Kºηº</td>
<td>VudVd*(a+e+2f)</td>
</tr>
<tr>
<td>Kºηº</td>
<td>VudVd*(c+e)</td>
</tr>
<tr>
<td>Kºχº</td>
<td>VudVd*(e+2f)</td>
</tr>
<tr>
<td>Kºχº</td>
<td>VudVd*(a+c)</td>
</tr>
<tr>
<td>KºKº</td>
<td>VudVd*(e+2f)</td>
</tr>
<tr>
<td>KºKº</td>
<td>VudVd*(a+c+e+2f)</td>
</tr>
<tr>
<td>KºKº</td>
<td>VudVd*(c+e)</td>
</tr>
<tr>
<td>KºKº</td>
<td>VudVd*(a+c+e+2f)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>b. Dº decay</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Kºπº</td>
<td>VudVd*(a+e)</td>
</tr>
<tr>
<td>Kºπº</td>
<td>VudVd*(e+2f)</td>
</tr>
<tr>
<td>Kºπº</td>
<td>VudVd*(a+c)</td>
</tr>
<tr>
<td>Kºηº</td>
<td>VudVd*(c+e)</td>
</tr>
<tr>
<td>Kºηº</td>
<td>VudVd*(a+e+2f)</td>
</tr>
<tr>
<td>Kºηº</td>
<td>VudVd*(c+e)</td>
</tr>
<tr>
<td>Kºχº</td>
<td>VudVd*(e+2f)</td>
</tr>
<tr>
<td>Kºχº</td>
<td>VudVd*(a+c)</td>
</tr>
<tr>
<td>KºKº</td>
<td>VudVd*(e+2f)</td>
</tr>
<tr>
<td>KºKº</td>
<td>VudVd*(a+c+e+2f)</td>
</tr>
<tr>
<td>KºKº</td>
<td>VudVd*(c+e)</td>
</tr>
<tr>
<td>KºKº</td>
<td>VudVd*(a+c+e+2f)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>c. F⁺ decay</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>KºKº</td>
<td>VudVd*(d+e)</td>
</tr>
<tr>
<td>KºKº</td>
<td>VudVd*(e+2f)</td>
</tr>
<tr>
<td>KºKº</td>
<td>VudVd*(a+e)</td>
</tr>
<tr>
<td>KºKº</td>
<td>VudVd*(e+2f)</td>
</tr>
<tr>
<td>KºKº</td>
<td>VudVd*(a+c)</td>
</tr>
<tr>
<td>KºKº</td>
<td>VudVd*(c+e)</td>
</tr>
<tr>
<td>KºKº</td>
<td>VudVd*(a+c+e+2f)</td>
</tr>
<tr>
<td>KºKº</td>
<td>VudVd*(c+e)</td>
</tr>
<tr>
<td>KºKº</td>
<td>VudVd*(a+c+e+2f)</td>
</tr>
</tbody>
</table>

† See footnote at bottom of table 2.4.
Table 2.6
Quark-diagram amplitudes for charmed meson→pseudoscalar and vector mesons†

<table>
<thead>
<tr>
<th>Decay</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. D° decay</td>
<td></td>
</tr>
<tr>
<td>( K^0 p^0 )</td>
<td>( V_{ud} V_{cs} (\alpha + \epsilon + \ell + \beta') ) + ( V_{ud} V_{cd} (\epsilon + \beta') )</td>
</tr>
<tr>
<td>( K^0 \pi^0 )</td>
<td>( (1/\sqrt{2}) V_{ud} V_{cs} (\ell - \epsilon + \beta') )</td>
</tr>
<tr>
<td>( K^+ \pi^- )</td>
<td>( V_{ud} V_{cs} (\alpha + \beta') )</td>
</tr>
<tr>
<td>( K^0 p^0 )</td>
<td>( (1/\sqrt{2}) V_{ud} V_{cs} (\ell - \epsilon) )</td>
</tr>
<tr>
<td>( K^0 \eta^0 )</td>
<td>( (1/\sqrt{6}) V_{ud} V_{cs} (\ell - 2 \epsilon + \beta') )</td>
</tr>
</tbody>
</table>

| b. D° decay | |
| \( \bar{K}^0 p^0 \) | \( V_{ud} V_{cs} (\alpha + \epsilon + \ell + \beta') \) |
| \( \bar{K}^0 \pi^0 \) | \( V_{ud} V_{cs} (\alpha + \epsilon + \ell + \beta') \) |

| c. F° decay | |
| \( \bar{K}^0 K^0 \) | \( V_{ud} V_{cs} (\alpha + \epsilon + \ell + \beta') \) |
| \( \bar{K}^0 K^0 \) | \( V_{ud} V_{cs} (\alpha + \epsilon + \ell + \beta') \) |

† See footnote at bottom of table 2.4.

(5) It has been pointed out that the final state interaction effects [2.21, 2.22] may be important. It is difficult to know how to incorporate such effects.

(6) It was also pointed out that this may be an effect due to right-hand coupling [2.23].

The second surprise to the leading-order QCD argument is that [1.8, 1.52],

\[
\frac{\Gamma(D^0 \rightarrow K^- \pi^+) / \Gamma(D^0 \rightarrow \bar{K}^0 \pi^0)}{\Gamma(D^0 \rightarrow K^- \pi^0) / \Gamma(D^0 \rightarrow \bar{K}^0 \pi^0)} = 1.6 \pm 0.9. \tag{2.3}
\]

This came as a surprise because it was reasoned that in the leading-order QCD calculation, fig. 2.1, only
diagrams \((a)\) and \((\ell)\) of fig. 2.4 are important [2.24–2.28]. All other graphs are small because of helicity conservation, as in the suppression of \(\pi^+ \rightarrow e^+ \nu_e\).

From table 2.5a

\[
\Gamma(D^0 \rightarrow K^- \pi^+) = |V_{ud}V_{cs}^\ast(a + \epsilon)|^2, \quad (2.4)
\]

\[
\Gamma(D^0 \rightarrow \bar{K}^0 \pi^0) = \frac{1}{2} |V_{ud}V_{cs}^\ast(b - \epsilon)|^2. \quad (2.5)
\]

The leading order QCD implies that \((\epsilon)\) can be neglected. The correspondence between our amplitudes in table 2.5 and those used in the leading-order QCD calculations are

\[
a = 2X = (2f_+ - f_-)/3, \quad (2.6a)
\]

\[
b = 2X = (2f_+ - f_-)/3, \quad (2.6b)
\]

where \(f_+\) and \(f_-\) are defined in ref. [2.28]. Without gluon corrections

\[
f_+ = f_- = 1 \quad \Rightarrow \quad a = 3 \quad , (2.7)
\]

which is the naive result of SU(3) color counting. Leading-order QCD calculations typically give [2.24–2.28]

\[
f_- \approx 2.19 \quad \text{and} \quad f_+ = (f_-)^{-1/2} = 0.68 \quad . (2.8)
\]

From eqs. (2.4–2.6, 2.8) one obtains

\[
\Gamma(D^0 \rightarrow K^- \pi^+) / \Gamma(D^0 \rightarrow \bar{K}^0 \pi^0) \approx 36.6 \quad , (2.9)
\]

which is in violent disagreement with data, eq. (2.3). Notice that this is worse than the naive color counting result of eq. (2.7).

Barring the possible effects due to final state interaction [2.21], two types of suggestions have been given to “explain” eq. (2.3). First suggestion is, ref. [2.29, 2.30], that one abandons the actual leading order QCD result eq. (2.8), but assumes

\[
f_- \gg f_+ \quad , (2.10a)
\]

thus

\[
a = -b \quad (2.10b)
\]

which, as we shall see in section 2.3, can be a consequence of \([b^+]\) dominance. Then one obtains \(\Gamma(D^0 \rightarrow K^- \pi^+) / \Gamma(D^0 \rightarrow \bar{K}^0 \pi^0) \approx 2\), in agreement with data eq. (2.3). Another way is to give some reason why amplitude \((\epsilon)\) in eqs. (2.4, 2.5) may be important. If one gluon emission is included, fig. 2.5, then there is no helicity conservation argument [2.31]. In this calculation two more parameters are introduced. Educated guesses on these parameters give reasonable numbers compared to the data.
As we see, we can from the very beginning take a naive phenomenological view that the exchange graph (e) is important [2.32, 1.50].

The third surprise came during the summer of 1979. The lifetimes of \(D^+\) and \(D^0\) were different [1.52]:

\[
\frac{\tau(D^+)}{\tau(D^0)} > 5.8 \pm 1.5; 3.1^{+4.1}_{-1.3}. \tag{2.11a}
\]

For more recent data see table 2.2 and refs. [2.11, 2.12]. [Note that the \(\tau(D^0)\) measured from the recent SLAC Hybrid Facility bubble chamber measurement [2.12] is considerably longer than previous experiments and

\[
\frac{\tau(D^0)}{\tau(D^+)} = 1.2^{+0.9}_{-0.5}, \tag{2.11b}
\]

consistent with unity.] This is again a surprise because the leading-order QCD calculations neglect amplitudes (c) and (d), thus in table 2.4 only inclusive amplitudes (a) and (f) contribute and \(\tau(D^+) \approx \tau(D^0)\). The two types of reasoning offered before can explain this surprise. If, \(a = -6\) as given in the type one argument, eq. (2.10),

\[
\Gamma(D^+ \to \bar{K}^0 \pi^+) \approx \Gamma(D^0 \to K^- \pi^+), \tag{2.12}
\]

which is also a prediction of [6*] dominance (section 2.3). However data in table 2.3a tells us

\[
\text{Br}(D^+ \to \bar{K}^0 \pi^+) \approx \text{Br}(D^0 \to K^- \pi^+), \tag{2.13}
\]

so we obtain

\[
\frac{\tau(D^+)}{\tau(D^0)} \frac{\text{Br}(D^+ \to \bar{K}^0 \pi^+)}{\text{Br}(D^0 \to K^- \pi^+)} \frac{\Gamma(D^0 \to K^- \pi^+)}{\Gamma(D^+ \to \bar{K}^0 \pi^+)} \tag{2.14}
\]

consistent with eq. (2.11a).

Again, if we take a phenomenological view, we would say eq. (2.11a) implies the importance of diagram (e), from table 2.4. It was pointed out in ref. [2.33] that the full dominance of amplitude (e) has also the interesting implication that, using table 2.4, \(\Gamma(D^0 \to \bar{K}^0 \eta^0)/\Gamma(D^0 \to \bar{K}^0 \chi^0) \approx 1/8\), while the W-emission amplitude (d) dominance only gives \(\Gamma(D^0 \to K^0 \eta^0)/\Gamma(D^0 \to K^0 \chi^0) \approx 1/2\). Therefore by studying the relative rate of these two decays we can obtain the relative strength of amplitudes (e) and (d). Of course the analysis will always be complicated by the relative phases of the two amplitudes. Other results of W-exchange dominance can be worked out from table 2.5, [2.34].

The amplitude (e) does not contribute to the decays of charged meson \(D^+\) and \(F^+\). The corresponding leading-order QCD helicity-suppressed amplitude for \(D^+\) and \(F^+\) is the W-annihilation diagram (d). Unlike for diagram (e), in order to use gluon emission from the initial quark line, at least two gluons

---

Fig. 2.5. The W-exchange diagram with one gluon emission.
must be emitted due to color conservation, fig. 2.6. It is unclear whether this amplitude is enhanced or not. No numerical estimates have been made. There are interesting implications if the amplitude \((\Delta)\) is enhanced to a similar amount as amplitude \((\epsilon)\). The lifetime of \(F^+\) ought to be close to that of \(D^0\),

\[
\tau(D^+) \approx \tau(F^+) \approx \tau(D^0).
\]  

(2.15)

In the limit of complete\((\Delta)\) dominance we find from table 2.5

\[
\Gamma(F^+ \rightarrow \bar{K}^0K^+) \approx (3/2)\Gamma(F^+ \rightarrow \eta^0\pi^+) \approx \Gamma(F^+ \rightarrow K^0\pi^+) \approx 3\Gamma(F^+ \rightarrow \chi^0\pi^+).
\]  

(2.16)

See table 2.5 for many more such relations.

It is interesting to notice that neither \((\epsilon)\) nor \((\Delta)\) contribute to the mixing-matrix nonsuppressed decay of \(D^+\). The amplitude \((\Delta)\) contributes to \(D^+\) decay only with the \(V_{cd}V_{ud}\), which is suppressed. So that even if the amplitude \((\Delta)\) is as large as the amplitude \((\epsilon)\), the lifetime of \(D^+\) is still longer than that of \(D^0\). The dominance of \((\Delta)\) amplitude, which has \(cd\) first annihilated then \(ud\) created, will also have the interesting prediction that the \(\bar{K}X\) mode inclusive is smaller in the \(D^+\) decay than in \(D^0\), where the mixing-matrix nonsuppressed amplitude \((\epsilon)\) always produces a strangle particle in the final state. This is interestingly in agreement with experimental indications, table 2.3. Also, from table 2.5b the \((\Delta)\) amplitude dominance predicts

\[
\Gamma(D^+ \rightarrow \bar{K}^0K^+)/\Gamma(D^+ \rightarrow \bar{K}^0\pi^+) > |V_{cd}/V_{cs}|^2.
\]  

(2.17a)

This is in agreement with data from table 2.3b,

\[
\Gamma(D^+ \rightarrow \bar{K}^0K^+)/\Gamma(D^+ \rightarrow \bar{K}^0\pi^+) = 0.25 \pm 0.15,
\]  

(2.17b)

while

\[
|V_{cd}/V_{cs}|^2 < 0.08
\]  

(2.17c)

as determined in section 1. The reader can readily see many such relations using table 2.5. All these discussions are very qualitative. So we see that we need much more data to check the importance of amplitudes \((\epsilon)\) and \((\Delta)\). Even more urgently, after so much earnest effort, is our need to have a reliable dynamical calculation scheme.

Besides phenomenological studies of amplitudes, charm decays can also provide information about the mixing matrix [2.17]. For example,

\[
\Gamma(D^+ \rightarrow \pi^0\pi^+)/\Gamma(D^+ \rightarrow \bar{K}^0\pi^+) = \frac{3}{2}|V_{cd}/V_{cs}|^2.
\]  

(2.18a)

Fig. 2.6. The W-annihilation diagram with two gluon emission.
which is a good relation since both final states are exotic and thus free of final state interaction [1.49]. From table 2.3b we obtain

\[ |V_{cd}/V_{cs}|^2 < 0.30. \]  

(2.18b)

This we have already used in Section 1.

From table 2.5a we can also construct the inequality [2.17]

\[
\Delta_D \leq \left| \frac{V_{us} V_{cs}^* - V_{ud} V_{cd}^*}{V_{us} V_{cs}} \right| \leq \Sigma_D, 
\]

(2.19a)

\[
\Delta_D \leq \left| 1 + \frac{D}{A} \right| \left| \frac{V_{us} V_{cs}^* + V_{ud} V_{cd}^*}{V_{us} V_{cs}} \right| \leq \Sigma_D, 
\]

(2.19b)

where

\[
\Sigma_D = \left| \frac{[\Gamma(D^0 \to K^+ K^-)]^{1/2} \pm [\Gamma(D^0 \to \pi^- \pi^0)]^{1/2}}{[\Gamma(D^0 \to K^- \pi^0)]^{1/2}} \right|, 
\]

\[ A \equiv a + e, \quad D = e + f. \]

Using the experimental number from table 2.3b we obtain

\[
0.2 \leq \left| \frac{V_{us} V_{cs}^* - V_{ud} V_{cd}^*}{V_{us} V_{cs}} \right| \leq 0.5, 
\]

(2.20a)

\[ \beta = \left| \frac{V_{ud} V_{cs}}{V_{us} V_{cs} + V_{ud} V_{cd}} \right|, \]

\[ 0.2 \beta \leq |1 + D/A| \leq 0.54 \beta, \]

\( \beta = 0.4 \)

\( M_t = 30 \text{ GeV} \)

Fig. 2.7. The range of the magnitude of the amplitude \(|1 + D/A|\) as function of \(\delta\).
and

\[ 0.2 \leq \left| 1 + \frac{D}{A} \right| \cdot \left| \frac{V_{us} V_{cs}^* + V_{ud} V_{cd}^*}{V_{us} V_{cs}^*} \right| \leq 0.5. \] (2.20b)

Note that whenever phase-space factors are different, all the ratios of the widths are meant for the reduced widths, which has phase-space factor removed. As we discussed in section 1.5.3, eq. (2.20a) gives the interesting bound

\[ |V_{cd}/V_{cs}| < 0.3. \] (2.20c)

From eq. (2.20b), given the \( V_0 \) calculated in the \( \Delta m - \epsilon \) analysis in section 1.4, we can give the range required of \( |1 + D/A| \) as shown in fig. 2.7. The enhancement of amplitude \( D \) over amplitude \( A \) is required to be more than \( 10^2 \) as \( \delta \) becomes greater than a few degrees. Here we see the important interplay between the determination of mixing matrices and amplitudes.

In table 2.6, quark diagram amplitudes for the charmed mesons decaying into a pseudoscalar and a vector meson are given. They are taken from ref. [2.35]. Similar discussions like those given in this section for charmed meson decaying into two pseudoscalar mesons can be made.

2.3. Symmetry approach

Traditionally the nonleptonic decays have been analysed in terms of the isospin SU(2), or SU(3) symmetry properties of the weak interaction Hamiltonian \( H_w \). The current and Hamiltonian of weak interactions are:

\[ J_\mu = (\bar{u}, \bar{c}, \bar{t}) \Gamma_\mu \begin{pmatrix} d' \\ s' \\ c' \end{pmatrix} = (\bar{u}, \bar{c}, \bar{t}) \Gamma_\mu V \begin{pmatrix} d \\ s \\ c \end{pmatrix} \] (2.21)

where \( V \) is the mixing matrix and \( \Gamma_\mu = (1 + \gamma_5)\gamma_\mu \), which shall be dropped in future equations since it is not relevant for the discussion.

\[ H_w = J_\mu J_\mu^* + \text{H.C.}; \] (2.22)

here H.C. stands for Hamiltonian conjugate. The SU(2) properties of \( \Delta s = 1 \) part of \( J_\mu \) are

\[ J_{\mu, \Delta s = -1} = V_{us}\bar{u}s \propto (2^*) + (1) = (2^*), \] (2.23a)

\[ J_{\mu, \Delta s = 1} = V_{ud}\bar{u}d \propto (2^*) \times (1) = (1) \Rightarrow (3), \] (2.23b)

\[ H_{w, \Delta s = -1} = J_{\mu, \Delta s = -1} J_{\mu, \Delta s = -1}^* + \text{H.C.} \propto (2^*) \times (3^*) + \text{H.C.} = (2^*) + (4) + \text{H.C.}. \] (2.23c)

So the isospin of \( H_{w, \Delta s = -1} \) can either be \( I = \frac{1}{2}, (2^*) \); or \( I = \frac{3}{2}, (4) \); the former gives \( \Delta I = \frac{1}{2} \), and the latter \( \Delta I = \frac{3}{2} \) in the strangeness changing weak decays. As we have mentioned before \( \Gamma(K^0 \rightarrow \pi^+ \pi^-)/\Gamma(K^0 \rightarrow \)
\( \pi^+ \pi^- = 1/670 \) is a strong support of \( H_{W \Delta S = 1} \propto I = \frac{1}{2} \), or \( \Delta I = \frac{1}{2} \). This has had many successes in K and hyperon decays. For a nice review, see ref. [1.28].

This symmetry property consideration \( H_W \) was later extended to SU(3):

\[
J_\mu \propto [3^*] \times [3] = [1] + [8] = [1]_{\Delta S = 0} + [3]_{\Delta S = 0} + [2]_{\Delta S = +1} + [2]_{\Delta S = -1} + [1]_{\Delta S = 0},
\]

\[
H_W = J_\mu J_\mu^* + \text{H.C.} \propto [8] \times [8] + \text{H.C.}
\]

\[
= [1] + [8_s] + [27] + ([10] + [10] + [8])_A + \text{H.C.}.
\]

The representations in the curly brackets are antisymmetry. They do not contribute to \( H_W \), which is symmetric. The representations in SU(3) can further be decomposed into representations in SU(2) according to their isospin and strangeness changing properties

\[
[8] = (3)_{\Delta S = 0} + (2)_{\Delta S = \pm 1} + (1)_{\Delta S = 0},
\]

\[
[27] = ((5) + (3) + (1))_{\Delta S = 0} + ((4) + (2))_{\Delta S = \pm 1} + (3)_{\Delta S = \pm 2}.
\]

Since the \( \Delta S = \pm 1 \) part of [27] has both representations (4), \( \Delta I = \frac{3}{2} \), and (2), \( \Delta I = \frac{1}{2} \); however the \( \Delta S = \pm 1 \) part of [8] has only the phenomenologically needed (2), \( \Delta I = \frac{1}{2} \), it was postulated that [2.36]

\[
H_W \propto [8],
\]

which had many successes [2.37–2.39]. However the lack of a consistent picture for the p-wave hyperon decays is still a major difficulty.

Similar internal symmetry structures can be analyzed for charm decays. The SU(2) properties of the charm changing currents:

\[
J_{\mu, \Delta c = \Delta s = 1} = V_{cs} \bar{c}s \propto (1),
\]

\[
J_{\mu, \Delta c = 1, \Delta s = 0} = V_{cd} \bar{c}d \propto (2),
\]

\[
J_{\mu, \Delta c = -1, \Delta s = 0} = V_{ud} \bar{u}d \propto (2^*) \times (2) = (1) + (3) \Rightarrow (3)
\]

\[
H_{W, \Delta c = \Delta s = 1} = J_{\mu, \Delta c = \Delta s = 1} J_{\mu, \Delta c = \Delta s = 0} + \text{H.C.} \propto (1) \times (3) + \text{H.C.}
\]

\[
\Rightarrow (3) + \text{H.C.},
\]

\[
H_{W, \Delta c = 1, \Delta s = 0} = J_{\mu, \Delta c = 1, \Delta s = 0} J_{\mu, \Delta c = \Delta s = 0} + \text{H.C.} \propto (2) \times (3) + \text{H.C.}
\]

\[
= (2) + (4) + \text{H.C.}
\]

So the mixing-matrix nonsuppressed \( \Delta c = \Delta s = 1 \) decay of charm has \( \Delta I = 1 \). This has the consequence of [2.15–2.17, 2.40–2.41]

\[
A(D^* \rightarrow \bar{K}^0 \pi^+) - A(D^0 \rightarrow K^- \pi^+) + \sqrt{2} A(D^0 \rightarrow \bar{K}^0 \pi^0) = 0
\]
and
\[ A(F^+ \to \pi^+ \pi^0) = 0 , \] (2.29b)

which can also be easily seen from table 2.5.

The SU(3) properties of the charm changing currents:

\[ J_{\mu, \Delta c=1, \Delta s=0} = V_{cs} \bar{c}s \propto [3] , \] (2.30a)
\[ J_{\mu, \Delta c=1, \Delta s=0} = V_{cd} \bar{c}d \propto [3] , \] (2.30b)
\[ J_{\mu, \Delta c=1, \Delta s=0} = V_{ud} \bar{u}d \propto [8] , \] (2.30c)
\[ H_{\Delta c=1, \Delta s=0} = J_{\mu, \Delta c=1, \Delta s=0}^* + \text{H.C.} \] (2.30d)

Note that only [6*] and [15] have the SU(2), \( I = 1, (3) \) representation. Similarly for the mixing matrix suppressed charm decay Hamiltonian

\[ H_{\Delta c=1, \Delta s=0} = J_{\mu, \Delta c=1, \Delta s=0}^* + \text{H.C.} \] (2.30f)
\[ \propto [3] \times [8] + \text{H.C.} = [3] + [15] + [6^*] + \text{H.C.} \] (2.30g)

where [3] contains the SU(2), \( I = \frac{1}{2}, (2) \) representation and [15] contains the SU(2), \( I = \frac{3}{2}, (4) \) representation in eq. (2.28e).

The SU(3) property of \( H_w \) gives the following simple relations among the \( \Delta c = \Delta s = 1 \) decays

\[ \Gamma(D^0 \to \bar{K}^0 \eta)/\Gamma(D^0 \to \bar{K}^0 \pi^0) = \frac{1}{3} . \] (2.31a)

and

\[ \Gamma(D^+ \to \pi^0 \pi^+)\Gamma(D^+ \to \bar{K}^0 \pi^+) = \frac{1}{2} |V_{cd}/V_{cs}| . \] (2.31b)

These and many other relations can be obtained from table 2.5.

As the dominance of the SU(3) octet [8] in \( H_{\Delta c=1, \Delta s=0} \) implies the dominance of the SU(2) representation (2) or \( \Delta I = \frac{1}{2} \), it would be nice to see whether the higher groups imply an preference between [6*] and [15]. With the additional charm quark, the symmetry group can be extended to SU(4) with quarks u, d, c, s as the fundamental representation

\[ J_{\mu} \propto 4^* \times 4 = 1 + 5 \quad \Rightarrow 15 . \] (2.32a)

The SU(3) decomposition of 15 according to \( \Delta c = 0, \pm 1 \), is

\[ 15 = [8]_{\Delta c=0} + [3]_{\Delta c=1} + [3]_{\Delta c=-1} + [1]_{\Delta c=0} , \] (2.32b)
\[ H_w = J_\mu J_\mu^* + \text{H.C.} \propto 15 \times 15 + \text{H.C.} = 1 + 15 + 15_\Delta + 20 + 45 + 45^* + 84 + \text{H.C.} \]  \hfill (2.32c)

Among these only the representations 20 and 84 can have \( \Delta_c = \pm 1 \), and their SU(3) decompositions are:

\[
20 = \{6\}_{\Delta_c = -1} + \{8\}_{\Delta_c = 0} + \{6^*\}_{\Delta_c = +1},
\]

\[
84 = \{6^*\}_{\Delta_c = -2} + \{6\}_{\Delta_c = -1} + \{15\}_{\Delta_c = 0} + \{8\}_{\Delta_c = 1} + \{27\}_{\Delta_c = 0}.
\]

We see that the \( \Delta_c = 0 \) part of 84 has both \{8\} and \{27\}, but the \( \Delta_c = 0 \) part of 20 has the phenomenologically needed \{8\} only. So if we proceed as in the extension of SU(2) to SU(3), it is natural to pick 20 as the dominant component for \( H_w \). Then the charm changing part of \( H_{w,\Delta_c = +1} \) to transform like \{6^*\}. Assuming \{6^*\} dominance, the charm decay was analyzed in ref. [2.15].

However right now we are not sure if \{6^*\} is the only dominant representation. We shall analyze the hadronic decay according to the general symmetry properties of \( H_w \), which has \{3\}, \{6\}, \{15\} for the \( \Delta_c = \pm 1 \) transition. Since charm particles always decay into ordinary quark states which are classified according to SU(3), the symmetry used in describing these decays is still SU(3) even though SU(4) has been used for the discussions of the transformation properties of \( H_w \). We consider the decay amplitude, \( \langle \bar{\mu} | H_{w,\Delta_c = +1} | \mu \rangle \), of charmed pseudoscalar meson into two pseudoscalars. Here the SU(3) transformation properties are

\[
\langle \bar{\mu} | \propto \langle \{3^*\} | , \]

\[
H_{w,\Delta_c = +1} \propto [3]_{\Delta_c = -1} \times [8]_{\Delta_c = 0} = [3]_{\Delta_c = +1} + [\bar{6}^*]_{\Delta_c = +1} + [15]_{\Delta_c = +1} \]

\[
|\bar{\mu} \rangle \propto |(8) \times (8)_c\rangle = |(1)\rangle + |(8)\rangle + |(27)\rangle. \]

Then there are five independent amplitudes that can be constructed for \( \langle \bar{\mu} | H_{w,\Delta_c = +1} | \mu \rangle \), which should be a SU(3) singlet,

\[
S = \langle \bar{\mu} | [9][8] \rangle, \quad E = \langle \bar{\mu} | [15^*][8] \rangle, \]

\[
T = \langle \bar{\mu} | [15^*][27] \rangle, \quad F = \langle \bar{\mu} | [3^*][8] \rangle, \]

\[
G = \langle \bar{\mu} | [3^*][1] \rangle. \]

In table 2.7 the two pseudoscalar decays of \( D^+ \), \( F^+ \), \( D^0 \) are listed in terms of these five amplitudes, as given in ref. [2.42].

Comparing tables 2.5 and 2.7, we note the six to five correspondence between the quark diagram amplitudes and the symmetry amplitudes. Recently the correspondence between quark diagram and the tensor decomposition of the symmetry amplitudes have been worked out by M. Gorn [2.35] for the mixing matrix nonsuppressed \( \mu_c \rightarrow \mu \bar{\mu} \). The strict SU(3) classification gives

\[ \alpha = \delta. \]  \hfill (2.37)

However in general there should be six amplitudes.
Table 2.7
SU(3) amplitudes for charmed meson decays†

a. $D^0$ decays

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^+\pi^-$</td>
<td>$(2T + E - S)$</td>
</tr>
<tr>
<td>$K^0\pi^0$</td>
<td>$(1/\sqrt{2})(3T - E + S)$</td>
</tr>
<tr>
<td>$K^0\eta^0$</td>
<td>$(1/\sqrt{6})(3T - E + S)$</td>
</tr>
<tr>
<td>$K^0\chi^0$</td>
<td>$(2/\sqrt{3})(E - S)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^0\pi^+$</td>
<td>$\Sigma(2T + E - S) + (1/2)\Delta(3T + 2G + F - E)$</td>
</tr>
<tr>
<td>$K^0\pi^0$</td>
<td>$-(2T + E - S) + (1/2)\Delta(3T + 2G + F - E)$</td>
</tr>
<tr>
<td>$K^0\pi^-$</td>
<td>$(1/\sqrt{2})(3T - E + S) + (1/4)\Delta(-7T + 2G + F - E)$</td>
</tr>
<tr>
<td>$K^0\eta^0$</td>
<td>$(1/\sqrt{6})(3T - E + S) + (1/2)\Delta(-12T + 2G + F - E)$</td>
</tr>
<tr>
<td>$K^0\chi^0$</td>
<td>$(1/\sqrt{3})(3T - E + S) + (1/4)\Delta(-18T + 2G + F - E)$</td>
</tr>
</tbody>
</table>

$\Sigma = (V_{ud}V_{us}^* + V_{ud}V_{ud}^*)/2$

† The factors in the curly brackets are Bose-statistics factors for identical particles such that the phase space is always $d^3p_1(2\pi)^3$ irrespective of whether the particles are identical or not.

b. $D^+$ decays

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^0\pi^+$</td>
<td>$\frac{5TV_{ud}V_{us}^*}{\sqrt{2}}$</td>
</tr>
<tr>
<td>$K^0\rho^0$</td>
<td>$(5/\sqrt{2})(3T + E + S)$</td>
</tr>
<tr>
<td>$K^0\eta^0$</td>
<td>$(1/\sqrt{6})(3T - E + S)$</td>
</tr>
<tr>
<td>$K^0\chi^0$</td>
<td>$(1/\sqrt{3})(3T + E + S)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^0\pi^+$</td>
<td>$\frac{5TV_{ud}V_{us}^*}{\sqrt{2}}$</td>
</tr>
<tr>
<td>$K^0\rho^0$</td>
<td>$(5/\sqrt{2})(3T + E + S)$</td>
</tr>
<tr>
<td>$K^0\eta^0$</td>
<td>$(1/\sqrt{6})(3T - E + S)$</td>
</tr>
<tr>
<td>$K^0\chi^0$</td>
<td>$(1/\sqrt{3})(3T + E + S)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Amplitude</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^0\pi^+$</td>
<td>$\frac{5TV_{ud}V_{us}^*}{\sqrt{2}}$</td>
</tr>
<tr>
<td>$K^0\rho^0$</td>
<td>$(5/\sqrt{2})(3T + E + S)$</td>
</tr>
<tr>
<td>$K^0\eta^0$</td>
<td>$(1/\sqrt{6})(3T - E + S)$</td>
</tr>
<tr>
<td>$K^0\chi^0$</td>
<td>$(1/\sqrt{3})(3T + E + S)$</td>
</tr>
</tbody>
</table>

† See footnote at bottom of table 2.4.
If the $[6^*]$ is dominant to the same degree for charmed decay as the $[8]$ is for strange particle decay, only the amplitude $S$ will contribute. Certainly the data of $\Gamma(D^0 \to K^+ K^-)/\Gamma(D^0 \to \pi^+ \pi^-) \neq 1$ does not support the dominance to such a degree. The information $\text{Br}(D^0 \to K^- \pi^+) = (3.0 \pm 0.6)\%$ and $\text{Br}(D^+- \to K^0 \pi^+) = (2.3 \pm 0.7)\%$ from table 2.2 can give an estimate of $S/T$, assuming the dominance of $S$: from tables 2.1 and 2.5

$$\frac{\text{Br}(D^0 \to K^- \pi^+)}{\text{Br}(D^+ \to K^0 \pi^+)} \frac{|S|^2 \tau(D^0)}{|T|^2 \tau(D^+)} = \frac{3.0}{2.3},$$

which gives

$$|S|^2/|T|^2 \approx \frac{3}{2.3} \tau(D^+) \approx 2 \sim 5.\quad (2.39a)$$

$$|S|/2|T| \approx 3.5 \sim 5.5. \quad (2.39b)$$

We do not have the overwhelming dominance of a single amplitude as in the strange particle decays [1.50, 2.16, 2.30].

From table 2.7 we can see that contributions from the two amplitudes $S$ and $T$ can adequately fit the presently known charm decay data, tables 2.2, 2.3. Much more data and theoretical understanding are needed before a clear picture emerges for the nonleptonic weak decays.

2.4. Concluding remarks

Traditionally the nonleptonic decays of strange particles are analyzed in terms of the symmetry properties of the weak interaction Hamiltonian $H_w$ and the particles involved. Except for the outstanding difficulty with the $p$-wave hyperon decays the $\Delta I = \frac{1}{2}$ rule, the $[8]$ dominance of the $H_w$ works quite well. In the charmed particle decays, however, there has been no clear indication which is the dominant SU(3), SU(4) transformation properties of $H_w$. The esthetically preferred representation $[9^*]$ for the charm changing $H_{w,Ac=-1}$ definitely does not dominate to the same extent as $[8]$ does for $H_{w,Ac=0,As=1}$. Another form of representation e.g. [15] for $H_{w,Ac=-1}$ may be as important.

During the study of charmed decay in the past few years, an alternate approach has developed, i.e. analyzing charmed decays according to quark diagrams. Various leading-order QCD calculations for the quark diagrams appeared prior to the experimental results. Nature certainly had some surprises in stock! Assuming the experimental results do not change as error bar decreases, the leading-order QCD preferred W-emission graphs fail to explain many charmed decays. The W-exchange or W-annihilation diagrams, which were argued to be negligible from the perturbative QCD point of view, are needed. This has been explained by considering one gluon emission in the W-exchange diagram, or more than two gluon emissions in the W-annihilation diagram.

Using this leading-order QCD quark diagram approach back to the $K$ system, it has been estimated that the so-called “Penguin” diagram is important. It has the interesting implication (see discussions later in section 3) that the $\Delta I = \frac{3}{2}$ CP violation part of $K_L \to 2\pi$ can be large $1/600 < |e'/|e| < 1/80$.

Besides providing a framework for perturbative QCD calculation, the quark diagrams can also be used in a more phenomenological way to fit the data. It is interesting to note that for the mixing-matrix nonsuppressed decays the W-exchange diagram contributes only to the neutral particles, $D^0$, $K^0$ decays.
The dominance by a factor of two or three of the W-exchange diagram over the W-emission diagram can easily fit the D° decays into two pseudoscalars. The dominance of the W-exchange diagram over the W-emission diagrams also gives the interesting result of \( \tau(D^0) < \tau(D^+) \), since the W-exchange diagram does not contribute to the D° decay. It is currently supported by experimental results, table 2.2. The importance of the W-exchange diagram offers an alternate reason for \( \epsilon_l = \epsilon' \) rule to the "Penguin" diagram. The W-exchange diagram alone gives \( \epsilon' = 0 \). Therefore the magnitude of \( |\epsilon'/\epsilon| \) can test the relative importance of the W-exchange and the "Penguin" diagrams in the \( K^0 \) system.

The corresponding diagram for charged particles (D°, F°) decays to the W-emission diagram in the D° decay is the W-annihilation diagram. For D° decays, it is mixing matrix suppressed. Its dominance will give fewer numbers of strange particles in the inclusive decay of D°. This seems to be supported by data, table 2.3. The W-annihilation diagram for F° decay is mixing matrix nonsuppressed. If its dominance over W-emission is about the same magnitude as the W-exchange diagram in the D° decay, one would expect \( \tau(F^0) \approx \tau(D^0) \), and there will be fewer numbers of strange particles in the decay than expected from the W-emission diagram.

Besides the determination of the relative importance of the quark diagram amplitudes or the symmetry properties of the weak decay Hamiltonian, heavier flavored-particle decays can also provide information about other mixing matrix elements, just like hyperon decays determined the value of \( V_{us} \).

Here we have given a very qualitative discussion. Much more data and ideas are needed to make further progress in our understanding of the difficult and fascinating subject of nonleptonic decays.

3. \( CP \) violation

3.1. The neutral particle–antiparticle mixing and its \( CP \) violation effects

Here we shall discuss the neutral particle–antiparticle mixing and \( CP \) violation effects of the heavy-quark systems \( D_0D^0, B_s\bar{B}_s^0, B^0\bar{B}^0_s \) and \( T^0\bar{T}^0 \). We denote them with a general notation \( P^0\bar{P}^0 \). The \( K_s, K_L \)-like states are \( p^0, p^0 \), respectively. The formalism of \( CP \) violation due to \( K^0 \rightarrow \bar{K}^0 \) transition as introduced in section 1 can be easily used here. The main differences are heavier masses and shorter lifetimes, and many more final states available in the decay.

In the case of the \( K^0 \) system, having the \( K^0 \) mass so close to the three-pion masses, nature has provided us with a wonderful state to study the \( CP \) violation properties. Due to this phase space limitation, \( K_L \), which mainly decays into \( 3\pi \), has a much longer lifetime than \( K_s \) and can travel a few meters before they decay away. This is the case of maximal mixing, i.e. regardless of what the combinations of \( K^0, \bar{K}^0 \) to begin with, in \( 10^{-10} \) sec mainly \( K_L \) will remain, which is an almost equal mixture of \( K^0 \) and \( \bar{K}^0 \). The mass difference \( \Delta m = m_K - m_s \), the \( CP \) violation parameters \( \epsilon, \epsilon' \) can be studied from the \( K_L, K_s \) decays separately, and from interference effects in the common \( \pi^+\pi^- \) final states through the regeneration of \( K_s \) in a \( K_L \) beam, or from charge asymmetry of \( \ell^+\ell^- \) in the decay of \( K_L \rightarrow \pi^+\ell^-\nu, [1.28, 1.29, 3.1] \). It is remarkable and yet frustrating that so far the only \( CP \) violation observed is still the originally observed system of \( K_s, K_L \) in 1964.

The study of \( CP \) violation effects in the heavy quark system will become much harder. Due to the high masses, the lifetimes of both \( p^0, p^0 \) are much shorter, e.g. the charmed particle lifetime is observed of the order of magnitude \( \tau_c \approx 10^{-13} \) sec, and the lifetimes \( \tau_b, \tau_t \) for the b and the t particles are expected to be even smaller, \( \tau_b \approx 10^{-14} \) sec, \( \tau_t < 10^{-18} \) sec, as estimated in section 1.5.4. In the many decay channels available, there is no phase space limitation for either \( p^0 \) or \( p^0 \) as for \( K_L \). The lifetimes for both \( p^0 \) are
expected to be comparable. Therefore the $CP$ violation property of the heavy quark system must be studied via time integrated results [3.2].

3.1.1. Formalism for the mass and width difference $\delta m$, $\delta \Gamma$; parameters of mixing $r$, $\bar{r}$; of $CP$ violation $\eta$; asymmetry $a$

First let us summarize the formalism, which is exactly analogous to that for the $K^0\bar{K}^0$ system, eqs. (1.20–1.29). Due to the transition between $P^0 \leftrightarrow \bar{P}^0$, the original $P^0$, $\bar{P}^0$ states are no longer physical states. The physical states are obtained after the diagonalization of the effective Hamiltonian

$$
H = \begin{pmatrix}
M_0 - i\Gamma/2 & M_{12} - i\Gamma_{12}/2 \\
M_{12}^* - i\Gamma_{12}^*/2 & M_0 - i\Gamma/2
\end{pmatrix}
$$

$$
\Rightarrow \begin{pmatrix}
M_0 - i\Gamma/2 + (1/2)(\delta m - i\delta\Gamma/2) & 0 \\
0 & M_0 - i\Gamma/2 - (1/2)(\delta m + i\delta\Gamma/2)
\end{pmatrix},
$$

(3.1a)

where

$$(1/2)(\delta m - i\delta\Gamma/2) = [(M_{12} - i\Gamma_{12}/2)(M_{12}^* - i\Gamma_{12}^*/2)]^{1/2}
$$

(3.1b)

and the eigenstates are

$$|P^0_\pm\rangle = [2(1 + |\tilde{\epsilon}_p|^2)]^{-1/2} [(1 + \tilde{\epsilon}_p)|P^0\rangle \pm (1 - \tilde{\epsilon}_p)|\bar{P}^0\rangle],
$$

(3.1c)

$$
\frac{1 - \tilde{\epsilon}_p}{1 + \tilde{\epsilon}_p} \frac{M_{12}^* - i\Gamma_{12}^*/2}{M_0 - i\Gamma/2} = \frac{(1/2)(\delta m - i\delta\Gamma/2)}{M_0 - i\Gamma/2 - (1/2)(\delta m + i\delta\Gamma/2)}.
$$

(3.1d)

As we discussed in section 1.3 the phase-convention independent physical measure of $CP$ violation is the deviation from unity of

$$
\eta_P = \left| \frac{1 - \tilde{\epsilon}_p}{1 + \tilde{\epsilon}_p} \right| = \left| \frac{M_{12}^* - i\Gamma_{12}/2}{M_{12} - i\Gamma_{12}/2} \right|^{1/2}
$$

(3.2)

It is related to other quantities by

$$
\frac{1 - \eta_P}{1 + \eta_P} = \frac{2 \Re \tilde{\epsilon}_p}{1 + \eta_P} = \frac{2 \Im (M_{12}^0 \Gamma_{12}/2)}{1 + \eta_P} = 
$$

$$
\frac{\Im (M_{12}^0 \Gamma_{12}/2)}{1 + \eta_P} = \left[ |\Gamma_{12}|^2 + |M_{12}|^2 + (\delta m)^2 + (\delta\Gamma)^2/4 \right].
$$

(3.3)

Now the time evolution for the physical states $p^0_\pm$, is very simple

$$
|p^0_\pm(t)\rangle = \exp(im_\pm t - \Gamma_\pm t/2)|p^0_\pm(0)\rangle.
$$

(3.4)

The time evolution for the states $|\phi(t = 0)\rangle = |p^0\rangle$, and $|\bar{\phi}(t = 0)\rangle = |\bar{p}^0\rangle$ are respectively,

$$
|\phi(t)\rangle = a_\rho(t)|p^0\rangle + a_{\bar{\rho}}(t)|\bar{p}^0\rangle,
$$

(3.5a)

$$
|\bar{\phi}(t)\rangle = \bar{a}_\rho(t)|p^0\rangle + \bar{a}_{\bar{\rho}}(t)|\bar{p}^0\rangle,
$$

(3.5b)
Ling-Lie Chau, Quark mixing in weak interactions

where

\[ a_\rho(t) = \left[ \exp(\imath m_\theta t - \Gamma_\theta t/2) - \exp(\imath m_\rho t + \Gamma_\rho t/2) \right], \]

\[ a_\bar{\rho}(t) = \left[ (1 - \tilde{e}_\rho)/(1 + \tilde{e}_\rho) \right] \left[ -\exp(\imath m_\theta t - \Gamma_\theta t/2) + \exp(\imath m_\rho t - \Gamma_\rho t/2) \right]. \]

Clearly the ratio of the time integrated probability of \( |\phi(t = 0)\rangle = |p^0\rangle \) state goes into \( |p^0\rangle \) and \( |\bar{p}^0\rangle \) is

\[ r \equiv \frac{\langle \bar{p}^0 \rightarrow \bar{p}^0 \rangle}{\langle p^0 \rightarrow \bar{p}^0 \rangle} = \frac{\int_0^\infty |a_\rho(t)|^2 \, dt}{\int_0^\infty |a_\bar{\rho}(t)|^2 \, dt} = \eta^2 \Delta, \]

(3.6a)

and

\[ \bar{r} \equiv \frac{\langle \bar{p}^0 \rightarrow \bar{p}^0 \rangle}{\langle p^0 \rightarrow \bar{p}^0 \rangle} = \frac{\int_0^\infty |\bar{a}_\rho(t)|^2 \, dt}{\int_0^\infty |\bar{a}_\bar{\rho}(t)|^2 \, dt} = \eta^{-2} \Delta, \]

(3.6b)

where

\[ \Delta \equiv \frac{(\delta m/\Gamma)^2 + [(1/2)\delta \Gamma/\Gamma]^2}{2 + (\delta m/\Gamma)^2 - [(1/2)\delta \Gamma/\Gamma]^2}, \]

(3.6c)

\[ \delta m = m_+ - m_-, \quad \delta \Gamma = \Gamma_+ - \Gamma_-, \quad \Gamma = (\Gamma_+ + \Gamma_-)/2. \]

There are two physical situations when the mixing is maximal. First, \( |\delta \Gamma/2| = \Gamma \), i.e. either \( \Gamma_+ \gg \Gamma_- \) or \( \Gamma_- \gg \Gamma_+ \), like that of the \( K^0 \) system. In this situation either \( p^0 \) or \( \bar{p}^0 \) to begin with will quickly end in the \( p^0_\theta \) or \( p^0_\rho \) state respectively which has approximately equal mixture of \( p^0 \) and \( \bar{p}^0 \), therefore maximal mixing. Second, \( \delta m \gg \Gamma \) so that before decaying the system oscillates very quickly between \( p^0 \) and \( \bar{p}^0 \) and appears as an equal mixture of \( p^0 \) and \( \bar{p}^0 \). We shall see that this happens in the \( B^0_\theta \bar{B}^0_\rho \) system. In both situations of maximal mixing

\[ r \cdot \bar{r} \approx 1. \]

(3.7)

For small \( CP \) violation \( \eta \ll 1 \)

\[ r = \bar{r}. \]

(3.8)

For both maximal mixing and small \( CP \) violation

\[ r = \bar{r} \approx 1. \]

(3.9)

This mixing of \( p^0 \) and \( \bar{p}^0 \) has the marvelous apparent effect that \( |p^0\rangle \) can have decay products that belong distinctively to \( |\bar{p}^0\rangle \).

For example \( D^0 \) and \( \bar{D}^0 \) can only have the following semileptonic inclusive decays respectively
\[ D^0 \to \ell^+ \nu_\ell X^- , \quad (3.10a) \]
\[ \bar{D}^0 \to \ell^- \bar{\nu}_\ell X^+ . \quad (3.10b) \]

But as time evolves \( D^0, \bar{D}^0 \) both become a mixed state of \( D^0 \) and \( \bar{D}^0 \), and we have the apparent
\[ \text{""}D^0 \to \ell^- \bar{\nu}_\ell X^- \text{""} , \quad (3.11a) \]
\[ \text{""}\bar{D}^0 \to \ell^+ \nu_\ell X^- \text{""} , \quad (3.11b) \]

where " " underline the time integrated probability. Actually the ratio of ""\( D^0 \to \ell^- \bar{\nu}_\ell X^- \)"" to ""\( D^0 \to \ell^+ \nu_\ell X^- \)"" is given by the same \( r, \tilde{r} \) as in eqs. (3.6a) and (3.6b), respectively,
\[ r = \frac{\text{""}D^0 \to \ell^- \bar{\nu}_\ell X^- \text{""}}{\text{""}D^0 \to \ell^+ \nu_\ell X^- \text{""}} , \quad (3.12a) \]
\[ \tilde{r} = \frac{\text{""}\bar{D}^0 \to \ell^+ \nu_\ell X^- \text{""}}{\text{""}\bar{D}^0 \to \ell^- \bar{\nu}_\ell X^- \text{""}} . \quad (3.12b) \]

This \( D^0, \bar{D}^0 \) mixing will give the same sign dilepton production both in \( \bar{\nu} \) and \( \nu \) reactions, contrary to the usual opposite sign dilepton productions; and \( \mu^- \mu^- \mu^- \mu^+, \mu^- \mu^+ \mu^+ \mu^- \), \( \mu^+ \mu^+ \mu^+ \mu^- \), \( \mu^+ \mu^- \mu^- \mu^- \) production in \( \mu^- n (\mu^- n) \) scattering [3.3]. In \( e^+ e^- \) and nucleon nucleon scattering there will be same sign dilepton production.

For the associated production of \( p^0 \bar{p}^0 \), a nice quantity to measure the mixing is the sum of same sign dilepton production
\[ r_+ = \frac{(p^0 \bar{p}^0) + (\bar{p}^0 p^0)}{(\bar{p}^0 p^0) + (p^0 \bar{p}^0) + (p^0 p^0) + (\bar{p}^0 \bar{p}^0)} = \frac{r + \tilde{r}}{r + \tilde{r} + 1 + r \tilde{r}} = \frac{(1 + \eta^{-4}) \Delta}{(1 + \Delta)(1 + \eta^{-2} \Delta)} . \quad (3.13a) \]

where
\[ (p^0 \bar{p}^0) = (\text{""}p^0 \to p^{0\prime}\text{""}) \cdot (\text{""}p^0 \to p^{0\prime}\text{""}) \], and
\[ (\bar{p}^0 p^0) = (\text{""}\bar{p}^0 \to \bar{p}^{0\prime}\text{""}) \cdot (\text{""}p^0 \to \bar{p}^{0\prime}\text{""}) . \]

Note that for maximal mixing
\[ r_+ = \frac{1}{2} . \quad (3.13b) \]

The nice quantity to measure \( CP \) violation is the fractional difference of same sign dilepton production
\[ a(p) = \frac{(\bar{p}^0 p^0) - (p^0 \bar{p}^0)}{(\bar{p}^0 p^0) + (p^0 \bar{p}^0) + (p^0 p^0) + (\bar{p}^0 \bar{p}^0)} = \frac{r - \tilde{r}}{r + \tilde{r}} = \frac{-4 \text{Re} \bar{e}_p (1 + |\bar{\xi}_p|^2)}{(1 + |\bar{\xi}_p|^2)^2 + 4(\text{Re} \bar{\xi}_p)^2} = \frac{\eta^4 - 1}{\eta^4 + 1} . \quad (3.14) \]

Of course the measurability of \( a(p) \) depends on the existence of appreciable mixing, i.e. \( r, \tilde{r} \) not too small.

For the mixing and \( CP \) violation effects from other initial coherent states of \( p^0 \bar{p}^0 \), see refs. [1.48, 3.5].

3.1.2. Calculation of the particle–antiparticle mixing and its \( CP \) violation effects in the \( B^0, \bar{B}^0, B^0_d, B^0_{d\bar{d}} \) and \( T^0, T^0_d \) systems

Now we shall proceed to estimate the mixing and the \( CP \) violation in the heavy quark system. We
need to calculate the width $\Gamma$, the dispersive part of $p^0 \leftrightarrow \bar{p}^0$ transition, i.e. $M_{12}$, as well as the absorptive part $\Gamma_{12}$, eqs. (1.20b, c). We shall assume that all the $p^0 \leftrightarrow \bar{p}^0$ transition, the dispersive part $M_{12}$ as well as the absorptive part $\Gamma_{12}$ are from the box graphs as shown in fig. 1.4 and the total widths are given by the W-emission graph as shown in figs. 1.1, 1.2.

The transition mass matrix element is

$$M_{12}(p^0 \leftrightarrow \bar{p}^0) = -\left[G_F^2 B_q f_p m_p M_w^2/(12\pi^2)\right] \sum_{i,j} A_{ij} \alpha_i \alpha_j.$$  \hspace{1cm} \text{(3.15a)}

Here $\alpha_i = V_{iQ}^* V_{iq}$ for the B-like system, or $\alpha_i = V_{Qi} V_{q'i}$ for the D, T-like system. $m_r$ is the neutral heavy meson mass, approximately given by $m_r = m_Q + m_q$, where the subscript Q denotes the heavy quark. The factor $f_p$ would be the leptonic decay constant if the vacuum insertion saturates the intermediate states. It is known in the $K^0$ system that the value for $B$ is very model dependent. Symbolically, our $B_q f_p$ here denotes a quantity which has all uncertainties combined. We use

$$B_D f_D = (0.19 \text{ GeV})^2, \quad B_u f_u = (0.33 \text{ GeV})^2, \quad \text{and} \quad B_T f_T = (1.9 \text{ GeV})^2 \hspace{1cm} \text{(3.15b)}$$

as estimates. The summation in eq. (3.15a) is over $u, c, t$ channels for $Q$ being d-like; or over $d, s, b$ channels for $Q$ being u-like. The amplitude $A_{ij}$ is essentially the same expression of eq. (1.77) in section 1, as long as the external quark masses can be neglected. This is generally valid for the K, D, B systems. But for the case of the T meson of mass near $M_w$, we have to replace it by the following exact formula \cite{3.4}

$$A_{ij} = 2 \int_0^1 dx_j \alpha \left\{ -x_i x_j - \frac{1}{2}(x_i + x_j) x_0 \alpha - \frac{a}{2}(1 + \frac{1}{4} x_i x_j) \alpha (1 - \alpha) - \frac{1}{4} x_0 x_i x_j x_0 (1 + 2\alpha) \right\} \ln \left| \frac{d(x_i x_j) d(1, 1)}{d(1, x_j) d(x_i, 1)} \right|$$

$$+ (1 + \frac{1}{4} x_i x_j) \alpha \left[ x_j \ln \left| \frac{d(x_i x_j)}{d(1, x_j)} \right| + \ln \left| \frac{d(1, 1)}{d(x_i, 1)} \right| + (i \leftrightarrow j) \right],$$

d$x_i, x_j \equiv x_i (1 - \alpha) + x_0 \alpha - x_0 \alpha (1 - \alpha), \hspace{1cm} \text{(3.16)}$

with $x_i = m_i^2/M_w^2; \ x_0 = m_0^2/M_w^2$. We take $m_u = m_d = 0.3 \text{ GeV}, \ m_s = 0.5 \text{ GeV}, \ m_b = 4.9 \text{ GeV}$. For our illustrative purpose, we assume $m_t = 30 \text{ GeV}$.

The absorptive amplitude $\Gamma_{12}$ arises from cutting the box-diagrams as shown in figs. 3.1a, b. The exact expression of $\Gamma_{12}$ is obtained by replacing the logarithms in eq. (3.16) by $\pi$ and integrate over the domain of negative argument within the absolute symbols:

Fig. 3.1. Diagrams for calculating the absorptive parts of $p^0 \leftrightarrow \bar{p}^0$ transition; (a) gives the W-emission contribution; (b) gives the W-exchange contribution.
\begin{align}
\Gamma_{12} &= \sum_{i,j} \frac{G^2 F_{\pi}^2 m_{\pi} M_{\pi} \lambda_i \lambda_j}{8 \pi (1 - \frac{x_i}{x_j}) (1 + \frac{x_i}{x_j})} \left[ 1 - \frac{2(x_i + x_j)}{x_\Omega} \right]^{1/2} \\
&\quad \cdot \left\{ (1 + \frac{1}{2} x_i x_j) \left[ x_\Omega - \frac{1}{3} (x_i + x_j) - \frac{2}{3} (x_i - x_j)^2 / x_\Omega \right] + \frac{8}{3} x_i x_j + \frac{8}{3} x_i x_\Omega (x_i x_j) + \frac{8}{3} x_i x_j x_\Omega \right\}. 
\end{align}
\tag{3.17}

When \( m_\Omega, m_\Omega \ll M_\pi \), we can safely neglect W-propagator effects. From eq. (3.17) we obtain

\begin{align}
\Gamma_{12} &= -\sum_{i,j} \frac{G^2 F_{\pi}^2 m_{\pi} \lambda_i \lambda_j}{(16 + 8) \pi} \left[ 1 - \frac{2(m_i^2 + m_j^2)}{(m_\Omega \pm m_i)^2} + \frac{(m_i^2 - m_j^2)^2}{(m_\Omega \pm m_j)^2} \right]^{1/2} \\
&\quad \cdot \left[ (m_i^2 + m_j^2) (\frac{1}{2} \pm \frac{1}{2}) - (m_\Omega - m_i)^2 \left( \frac{1}{2} \pm \frac{1}{2} \right) - (m_i^2 - m_j^2)^2 / (m_\Omega \pm m_j)^2 \right]. 
\end{align}
\tag{3.18}

The choice of negative sign gives the contribution of the W-emission graph, fig. 3.1a. The choice of positive sign in the above expression corresponds to the W-exchange graph, fig. 3.1b, which is suppressed by helicity flipping for small \( m_i, m_j \). The indices \( i, j \) run through all kinematically allowed channels.

From these calculated \( M_{12}, \Gamma_{12} \), and substituting them into eq. (3.18), we obtain the mass and width differences \( \delta m, \delta \Gamma \) for the \( B_0, B_d^0, T^0 \) systems.

We estimate the total width of heavy meson in the approximation of free quark decay via the W-emission diagram, i.e. \( Q \to q \pi W \to q \alpha \beta \), and \( Q \to q \pi W \to q \ell \nu \ell \)

\begin{align}
\Gamma &= [(3 G^2 m_\pi^5 / (192 \pi^3))] \sum_i |V_{q_i q}| \left[ \sum_{\alpha \beta} |V_{q_i q}|^2 I(m_i / m_\pi, m_\alpha / m_\pi, m_\beta / m_\pi) \right] + \sum_{\ell} 4 I(m_i / m_\pi, m_\ell / m_\pi, 0), 
\end{align}
\tag{3.19}

with

\begin{align}
I(a, b, c) &= 12 \int_{(b+c)^2}^{(1-a)^2} (dw/w) (1 + a^2 - w) (w - b^2 - c^2) [\lambda(1, a^2, w) \lambda(w, b^2, c^2)]^{1/2}, 
\end{align}
\tag{3.20}

and

\begin{align}
\lambda(x, y, z) &= x^2 + y^2 + z^2 - 2xy - 2xz - 2yz.
\end{align}

The summation is over all available quark \( q_i, \alpha, \beta \) states and all available lepton \( \ell \nu \ell \) states. To ensure the appropriate kinematic boundaries of the phase space in the total width calculation, we use physical meson masses for the quarks: \( m_b = 5.2 \text{ GeV} \) for the \( B_d \) meson, or \( m_s = 5.0 \text{ GeV} \) for the \( B_s \) meson, \( m_c = 1.86 \text{ GeV} \), \( m_s = 0.5 \text{ GeV} \), \( m_u = m_d = 0.3 \text{ GeV} \). We also assume \( m_t = 30.3 \text{ GeV} \) for the \( T_u \), and \( m_t = 31.5 \text{ GeV} \) for the \( T_c \) mesons.

Now substituting \( \delta m, \delta \Gamma \) into eqs. (3.6a, 3.6b) and (3.14), we obtain the mixing parameters \( r, \tilde{r} \), and the \( CP \)-violation asymmetry \( a(p) \). The results are obtained in ref. [1.48] and shown in figs. 3.2–3.7. \textit{Note that in calculating the \( CP \)-violation parameter \( \epsilon_p, \eta_p \), the exact formula eqs. (3.1d, 3.2) ought to be used, otherwise spurious sharp peaking-up will appear in the \( CP \)-violating parameters \( \eta_p \) and \( a(p) \) in the \( B^0 B^0, T^0 T^0 \) system as \( \delta \approx 90^\circ \) as happened in some of the previous calculations [3.8].}

In fig. 3.2, \( \Gamma(D), \delta \Gamma(D), \delta m(D) \) are given for the K–M phase \( \delta \) varying in the region allowed by the \( K^0 \overline{K}^0 \) fit in section 1. The reason that \( \Gamma(D) \gg |\delta \Gamma(D)|, |\delta m(D)| \) is because \( \Gamma(D) \) is given by the
mixing-matrix non-suppressed diagram, yet \( \delta \Gamma(D), \delta m(D) \) are given by the box diagrams, which are necessarily mixing-matrix suppressed. Since \( \Gamma(D) \gg |\delta \Gamma(D)|, |\delta m(D)| \) in eq. (3.6c) therefore \( \Delta \ll 1 \), so there is very little mixing. In physical terms, this is the situation that the lifetime is so much shorter than the oscillation period as well as the lifetime difference that there is no time for mixing. In fig. 3.3 the mixing parameters \( r(D) \) and \( CP \) violation parameter \( a(D) \) are given. The mixing \( r(D) < 10^{-4} \) is very small, so that even the fractional \( CP \) violation asymmetry \( a(D) \) can be \( 10^{-2} \), it will be hard to be observed.

In fig. 3.4 \( \Gamma, \delta \Gamma, \delta m \) are given for \( B_a \) and \( B_c \). Since \( \Gamma(B) \)'s are given by the mixing-matrix suppressed diagrams, \( \delta m(B), \delta \Gamma(B) \) can be comparable to \( \Gamma(B) \). We see that using \( B_{B} f_{B}^{2} \approx (0.32 \text{ GeV})^{2} \), \( |\delta m(B_{a})| > |\Gamma(B_{a})| \) for \( \delta \approx 180^\circ \); and \( \delta m(B_{c}) > |\Gamma(B_{c})| \) for all regions of \( \delta \). In fig. 3.5, \( r(B_{a}), r(B_{c}) \), are shown. In the region \( \delta m > \Gamma \) there is large \( B^{0}\bar{B}^{0} \) mixing. The \( B_{a} \) has large mixing for \( \delta \approx 180^\circ \). Though the mixing is very sensitive to the value of \( B_{B} f_{B}^{2} \) used, the possibility of large \( B^{0}\bar{B}^{0} \) mixings should have very interesting implications for high energy \( e^{+}e^{-}, \) \( e^{+}e^{-}, \text{ ep, and pp} \) reactions. In the same graph, the \( CP \)-violation parameters \( a(B_{a}), a(B_{c}) \) are also given, which are unfortunately rather small when \( r(B_{a}), r(B_{c}) \) are appreciable. In figs. 3.6, 3.7 the mixing and \( CP \)-violation parameters \( r \) and \( a(T) \) for \( T_c, T_u \) are given. They are all extremely small due to the reason that \( \Gamma(T) \gg |\delta m(T)|, |\delta \Gamma(T)|, \) since the \( \Gamma(T) \)'s are given by mixing-matrix nonsuppressed diagram, yet \( \delta m(T) \) and \( \delta \Gamma(T) \) are given by the mixing-matrix suppressed box diagram with GIM mechanism [3.9].

It is rather disappointing that the \( CP \)-violation effects discussed here are so small. This means that, except serving to confirm the K-M model, the future non-observation of \( CP \) violation from \( p^{0} \leftrightarrow \bar{p}^{0} \) transition in the \( D^{0}\bar{D}^{0}, B^{0}\bar{B}^{0}, T^{0}\bar{T}^{0} \) system cannot be used to determine the poorly known \( \delta \). It seems
that to further constrain $\delta$ we need to rely on the other $CP$-violation effects, which shall be discussed in the next section, 3.2.

In figs. 3.8, 3.9 the lifetimes for D, B, T are given. It is interesting to note that in this calculation the b lifetime, though its decay is mixing suppressed, is always shorter than the c lifetime, and a b lifetime longer than $10^{-13}$ sec is unlikely in this model calculation.
In section 3.5 we shall compare mixing and \( CP \)-violation effects from other sources and the results are summarized in table 3.1.

3.2. \( CP \) violation in partial decay rates

Besides contributing \( CP \) violation effects in the mass matrix, the complexity in the mixing matrix can also give rise to \( CP \) violation effects in the partial decay rates due to interference between the weak interaction amplitudes and the strong interaction amplitudes. It was discussed in generality quite some
time ago by the authors of ref. [3.2] that, though CPT predicts equal total decay rate for particle and antiparticle, the partial decay rates of particle and antiparticle into CP conjugated final particles can be different if CP is not invariant. The complexity of $V_{ij}$ in the six-quark model of K–M provides an explicit mechanism to such difference in partial decay rates [3.10–3.14]. The quark-diagram scheme discussed in section 2.2 provides an easy way to sort out the decay channels where particle and antiparticle decay rates can be different [3.12]. We shall see that the partial decay rates for particle and antiparticle are allowed to be different for many channels in the K–M scheme of CF violation. Such differences are not allowed in the super-weak theory for CP violation [3.15].

3.2.1. Exclusive semileptonic decays

From CPT the inclusive semileptonic decays of a particle and its antiparticle must be the same, i.e.

$$\Gamma(D^0 \rightarrow \ell^+ \nu\pi^-) = \Gamma(D^0 \rightarrow \ell^- \bar{\nu}\pi^+)$$

(3.21)

The K–M model predicts that to the leading order there are no CP violation effects in all exclusive semileptonic decays. This is due to the fact that for semileptonic decays, inclusive or exclusive, there is only the W-emission diagram in leading order, so no source of interference, which is needed for CP violation effects from the amplitudes as we shall demonstrate next. This prediction is in agreement with the super weak interaction for CP violation. CP violation for other sources, like Higgs and right-handed intermediate bosons, can provide CP-violation effects in exclusive semileptonic decays, see discussion in section 3.5.

3.2.2. Strange meson decay

The two-pion decay amplitudes of K*, K° are given in tables 2.1a,b. The corresponding decay amplitudes of K°, K° are given by the same equations, except replacing $V_{ij}$ by $V_{ij}^*$. Typically, the decay amplitudes for particles and antiparticle are of the form

$$A(K^0 \rightarrow \pi^+ \pi^-) = V_{us}^* V_{ud} A_1 + V_{cs}^* V_{cd} A_2,$$

(3.22a)

$$\bar{A}(\bar{K}^0 \rightarrow \pi^- \pi^+) = V_{us}^* V_{ud}^* A_1 + V_{cs}^* V_{cd} A_2,$$

(3.22b)

where $A_1 = \alpha + \epsilon + \epsilon + 2\gamma$, $A_2 = \epsilon + 2\gamma$. The partial decay rate difference between the two is given by

$$\Delta_s = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2}$$

$$= -\frac{4 \text{Im}(V_{us}^* V_{ud} V_{cs}^* V_{cd}^*) \text{Im}(A_1 A_2^*)}{|A|^2 + |\bar{A}|^2} = -\frac{4(s_5 s_6 c_1 c_3) \text{Im}(A_1 A_2^*)}{(|A|^2 + |\bar{A}|^2) s_5^2},$$

(3.23)

where $\Gamma$ is the particular partial decay width, and Br the corresponding branching ratio. $\Gamma$ and Br are quantities for the corresponding antiparticle. Since the total decay rates between particle and antiparticle are the same, the $\Gamma$ in eq. (3.23) can also be the branching ratio Br. In eq. (3.23) we divide the denominator by $s_5^2$ because both $|A|^2$ and $|\bar{A}|^2$ have a factor of $s_5^2$. $\Delta_s$ is proportional to $s_5^2$. $\Delta_s$ is zero if $A_1$ and $A_2$ have
Ling-Lie Chau. Quark mixing in weak interactions

the same phase. Unfortunately we do not have reliable ways to calculate $A_1$ and $A_2$. The present scheme, however, provides the information about the channels where particle-antiparticle decay rates can be different. Here we see that the decay rate $K^0 \rightarrow \pi^+ \pi^-$, $\bar{K}^0 \rightarrow \pi^+ \pi^-$ can be different.

Note that

$$
\Gamma(K^+ \rightarrow \pi^+ \pi^0) = \Gamma(K^- \rightarrow \pi^- \pi^0).
$$

(3.24)

since $\pi^+ \pi^0(\pi^- \pi^0)$ has only $I = 2$ state, so there is only one phase for the strong interaction amplitude $S$, i.e. $\text{Im}(A_1 A_2^*) = 0$, our quark-diagram scheme gives the same result, see table 2.1a.

Using table 2.1 gives also the decay amplitudes for the three-pion final states. Now, other amplitudes $(a)$, $(e)$, $(\varphi)$, besides $(\alpha)$ and $(\delta)$, will also contribute to the $K^+ \rightarrow 3\pi$ decays. Therefore $K^+ \rightarrow \pi^+ \pi^+ \pi^-$ can have different decay rates, so can $K^+ \rightarrow \pi^+ \pi^0 \pi^0$. This difference in the decay rates of $K^+ \rightarrow (3\pi)^+$ has been calculated within the K–M scheme of CP violation to relate to the CP violation in the $K_1 \rightarrow 2\pi$, using current algebra techniques [3.16].

$$
\Delta_{K \rightarrow 3\pi} = \frac{\text{Br}(K^+ \rightarrow \pi^+ \pi^+ \pi^-) - \text{Br}(K^- \rightarrow \pi^- \pi^- \pi^+)}{\text{Br}(K^+ \rightarrow \pi^+ \pi^+ \pi^-) + \text{Br}(K^- \rightarrow \pi^- \pi^- \pi^+)}
$$

(3.25)

where $\epsilon'$ is the CP violation in the $\Delta I = \frac{1}{2}$ transition as given in eq. (1.48). We see that $\Delta_{K \rightarrow 3\pi}$ is unfortunately very small.

3.2.3. Strange baryon decays

It is worthwhile to emphasize that based on the same quark diagram analysis, $\Lambda$ and $\bar{\Lambda}$ (similarly $\Sigma$, $\bar{\Sigma}$) can have different particle-antiparticle decay rates [3.12]. From figs. 3.10, 3.11

$$
A(\Lambda \rightarrow \pi^- p) = V_{us}V_{ud}A_1' + V_{cs}V_{cd}A_2',
$$

(3.26a)

$$
\bar{A}(\bar{\Lambda} \rightarrow \pi^+ \bar{p}) = V_{us}^*V_{ud}A_1' + V_{cs}^*V_{cd}A_2'.
$$

(3.26b)

where $A_1' = \alpha' + \beta' + \epsilon' + \epsilon''$, the sum of all four graphs in fig. 3.10, and $A_2' = \epsilon'$ the "Penguin" diagram. Again we obtain the difference in partial decay rates,

$$
\Delta_4 = -\frac{\text{Br}(\Lambda \rightarrow \pi^- p) - \text{Br}(\bar{\Lambda} \rightarrow \pi^+ \bar{p})}{\text{Br}(\Lambda \rightarrow \pi^- p) + \text{Br}(\bar{\Lambda} \rightarrow \pi^+ \bar{p})} = -\frac{4(s_2s_3s_8c_1c_3)}{|A_1'|^2 + |A_1''|^2} \frac{\text{Im}(A_1'A_2'^*)}{|A_1'|^2 + |A_2'|^2}
$$

(3.27)

exactly the same form as in eq. (3.23) for strange meson decays. The factor $s_2s_3s_8 \sim 10^{-3} - 10^{-4}$, from the analysis in section 1, depending on the t-quark mass. This is the same factor as appeared in $\epsilon$. However we expect $\text{Im}(A_1'A_2'^*)/(|A_1'|^2 + |A_2'|^2) \sim \text{Im}(a_1a_2d_3d_2)/|a_1|^2$, which is suppressed by $\Delta I = \frac{1}{2}$ dominance. Though this difference is small, it is so simple and it is worthwhile to embark on a systematic search for such differences in partial decay rates.

3.2.4. Charmed meson decay

Using table 2.5 for formulae of $D^0$, $D^+$, $F^+$ decaying into pseudoscalar mesons, and the same corresponding formulae for $D^0$, $D^+$, $F$ with $V_{ij}$ replaced by $V_{ij}^*$, we can discuss which decay channel
can have different partial decay rates for particle and antiparticle. Notice immediately that in the mixing-matrix nonsuppressed and doubly-suppressed channels the partial decay rates are the same for the particle and antiparticle. For the mixing-matrix singly suppressed channels, typically the decay amplitudes for the particle and antiparticle are of the following form, e.g.,

\[
A(D^+ \rightarrow \bar{K}^0 K^+) = V_{us} V_{cs}^* A_1 + V_{ud} V_{cd}^* A_2 ,
\]

\[
\tilde{A}(D^- \rightarrow K^0 \bar{K}^-) = V_{us}^* V_{cs} A_1 + V_{ud}^* V_{cd} A_2 ,
\]

where \( A_1 = a + c \); \( A_2 = d + e \). For different decays, \( A_1, A_2 \) represent the corresponding combination of amplitudes \((a), (b), (c), (d), (e)\) as given in tables 2.5, 2.6. Again the partial rate differences are exactly in the form of eq. (3.23). Therefore we would expect that the percentage difference in partial decay rates will be of order of \( \varepsilon \).

From table 2.5 of charmed decays we can find out which partial decay rates can be different between particle and antiparticle. All, but \( D^* \rightarrow \pi^\pm \pi^0 \), will have different particle and antiparticle decay rates. To give explicit examples, here we shall list the exclusive and inclusive decays of \( D^*, F^*, D^0, \bar{D}^0 \) which can
have different partial decays among particle and antiparticle

\[ D^z \rightarrow \bar{K}^0 K^+, \eta^0 \pi^+, K^+ \bar{K} X_{1=0}, \eta^0 \pi^+ X_{1=0}, K^+ X_{1=0}, \eta^0 X_{1=0}, \text{etc.} \]  \hspace{2cm} (3.29a)

\[ F^z \rightarrow K^0 \pi^+, K^- \pi^0, K^- \eta^0, K^0 \pi^+ X_{1=0}, K^+ \bar{K} X_{1=0}, K^- \eta^0 X_{1=0}, \pi^- X_{1=0}, K^+ X_{1=0}; \]  \hspace{2cm} (3.29b)

\[ \frac{D^0}{D^0} \rightarrow K^- K^+, \pi^+ \pi^-, \pi^0 \pi^0, \eta^0 \eta^0, \pi^0 \eta^0, \text{and their inclusive states}. \]  \hspace{2cm} (3.29c)

Here \( s \) denotes strangeness. \( X \) denotes the inclusive states. The equality

\[ \Gamma(D^+ \rightarrow \pi^0 \pi^+) = \Gamma(D^- \rightarrow \pi^0 \pi^-), \]  \hspace{2cm} (3.29d)

because of the same reason as \( \Gamma(K^+ \rightarrow \pi^+ \pi^0) = \Gamma(K^- \rightarrow \pi^- \pi^0), \) eq. (3.24).

Here we see a rich variety of channels where one can search for \( CP \)-violation effects. Needless to say high experimental sensitivity, in the range of \( \varepsilon \), is needed in such searches.

3.2.5. The \( B \) meson decay

The \( B_u, B_d, B_s \rightarrow \text{double charm } c\bar{c} \text{ particle final states} \): From the quark diagrams like those in fig. 2.4, the mixing matrix and amplitude dependences of \( B_u \rightarrow D^0 D^-, B_d \rightarrow D^+ D^-, B_s \rightarrow F^+ D^- \) can be easily worked out and are listed as follows:

\[ A(B_u \rightarrow D^0 D^-) = V_{ub} V_{cd}^*(a + \delta + \epsilon) - V_{ud} V_{ud}^*(a + \epsilon), \]  \hspace{2cm} (3.30a)

\[ A(B_u \rightarrow D^+ D^-) = V_{ub} V_{cd}^*(a + \delta + \epsilon) + V_{ub} V_{ud}^*(\epsilon), \]  \hspace{2cm} (3.30b)

\[ A(B_s \rightarrow F^+ D^-) = V_{cd} V_{cd}^*(a + \delta + \epsilon) + V_{ub} V_{ud}^*(\epsilon). \]  \hspace{2cm} (3.30c)

The decay amplitudes for the antiparticle states are the same except replacing \( V_{ij} \) by \( V_{ij}^\dagger \). Therefore all these decay amplitudes have the general form of, e.g.,

\[ A(B_u \rightarrow D^0 D^-) = V_{ub} V_{ud}^* A_1 + V_{eb} V_{cd}^* A_2 \]  \hspace{2cm} (3.31a)

\[ \bar{A}(B_u \rightarrow \bar{D}^0 \bar{D}^-) = V_{ub} V_{ud}^* A_1 + V_{eb} V_{cd}^* A_2. \]  \hspace{2cm} (3.31b)

The difference of partial decay rate in \( CP \) conjugated decays are of the form

\[ \Delta_{B_{u-c}} = \frac{\text{Br} - \bar{\text{Br}}}{\text{Br} + \bar{\text{Br}}} = -\frac{4 \text{Im}(V_{ub} V_{ud}^* V_{cb}^* V_{cd}) \text{Im}(A_1 A_2^*)}{|A|^2 + |\bar{A}|^2} = -\frac{4 s_s s_c c_c c_1 \text{Im}(A_1 A_2^*)}{|A|^2 + |\bar{A}|^2} s_1^2 \]  \hspace{2cm} (3.32a)

where \( |A|^2 + |\bar{A}|^2 \) is proportional to \( s_1^2 \). Note then that except for eqs. (3.30a), the difference in partial decay rates are from the loop diagrams, the so-called “Penguin” diagrams. The point here is that even though the factor of the angles in the numerator of \( \Delta_{B_{u-c}} \) is the same as those of strange and charmed decays, eq. (3.23), the denominator of \( \Delta_{B_{u-c}} \), i.e. the rate is rather small too. Therefore \( \Delta_{B_{u-c}} \) can be appreciable. This can be easily seen if we rewrite eq. (3.32a) in the form
\[ \Delta_{B \to \ell e} = \frac{2(s_2/s_3)s_6c_1c_2c_3 \text{Im}(A_1A_2^*)}{\frac{1}{2}[|A|^2 + |\bar{A}|^2](s_1)^{-2}(s_3)^{-2}} \] (3.32b)

where

\[ \frac{1}{2}[|A|^2 + |\bar{A}|^2](s_1)^{-2}(s_3)^{-2} = [c_1A_1|^2 + |c_2[c_1c_2 + (s_2/s_3)c_3 e^{i\delta}]A_2|^2 \]
\[ - 2c_1c_2[c_1c_2 + (s_2/s_3)c_3c_8] \cdot \text{Re}(A_1A_2^*) - 2c_1c_2(s_2/s_3)c_3s_8 \cdot \text{Im}(A_1A_2^*) \]

we see that as \((s_2/s_3) \gg 1,\)

\[ \Delta_{B \to \ell e} \propto s_6c_1c_2c_3, \] (3.33)

which can be appreciable. The difficulty lies in estimating the hadronic interference \(\text{Im}(A_1A_2^*)\). Some of these effects have been calculated using various models [3.10–14, 3.17]. Fig. 3.12 shows a calculation by J. Bernabéu and C. Jarlskog [3.13, 3.18] for the decay \(B_u \rightarrow D^{*+}D^-\), which has the same amplitude form as given by eqs. (3.32, 3.33). In their calculation they let \(s_8, s_2/s_3\) be independent parameters. However, from \(CP\) violation analysis discussed in section 1, \(s_8\) and \(s_2/s_3\) are correlated. The circles on the curves in fig. 3.12 are the allowed values of \(s_8\) vs \(s_2/s_3\) from the box-graph fit to \(CP\) violation in the \(K^0\bar{K}^0\) system [1.48]. We see that the fractional differences in partial decay rates are typically a few to twenty percent. Other decays in this category that can have different particle–antiparticle partial decay rates are, e.g.,

![Graph showing percentage partial decay-rate difference \(\Delta_{B \to \ell e}\) between \(B_u \rightarrow D^{*+}D^-\) and \(B_u^* \rightarrow \bar{D}^{*+}D^-\) as a function of \(s_2/s_3\) from [3.13]. The circles are the points relating \(\delta\) to \(s_2/s_3\) given by the box-graph fit to the \(\Delta m. \tau\) in the \(K^0\bar{K}^0\) system. [1.48].](image-url)
\[
\begin{align*}
B_u^- &\rightarrow D^0 D^+, D^- D^0 X_{s=0}, D^- X_{c=-1}, \quad (3.34a) \\
B_d^- &\rightarrow D^- D^-, D^- D^- X_{s=0}, D^- X_{c=-1}, D^- X^+_{c=-1}, \quad (3.34b) \\
B_s^- &\rightarrow F^+ D^+, F^- D^+ X_{s=0}, F^- X^+_{c=1}, D^+ X^+_{s=1, c=-1}. \quad (3.34c)
\end{align*}
\]

The \( B_u^-, B_d^-, B_s^- \rightarrow \) ordinary (no charm) particle final states: We first list the decay amplitudes of the \( B_u^-, B_d^-, B_s^- \) to two ordinary pseudomeson (no charm particle in the final states),

\[
\begin{align*}
A(B_u^- \rightarrow \pi^- \pi^0) &= \frac{1}{\sqrt{2}} V_{ub} V_{ud}(\alpha + \beta + \epsilon + \delta), \quad (3.35a) \\
A(B_d^- \rightarrow \pi^- \pi^-) &= V_{ub} V_{ud}(\alpha + \epsilon + \delta) + V_{cb} V_{cd}(\epsilon + \delta), \quad (3.35b) \\
A(B_s^- \rightarrow \pi^- K^+) &= V_{ub} V_{ud}(\alpha) + V_{cb} V_{cd}(\epsilon). \quad (3.35c)
\end{align*}
\]

We see that the interference can only come from the loop diagrams (\( \epsilon \)) and (\( \delta \)). The partial decay rates are the same for \( B^+ \rightarrow \pi^+ \pi^0 \) just as eqs. (3.24, 3.29d) for \( K^+, D^+ \rightarrow \pi^+ \pi^0 \), but different for

\[
\begin{align*}
B_d^0 &\rightarrow \pi^+ \pi^-, \pi^- \pi^0 X_{s=0}, \pi^+ X_{s=0}, \pi^- X^+_0, \pi^- X^+_{s=0}, \quad (3.36a) \\
B_s^0 &\rightarrow \pi^- K^+, \pi^- K^* X_{s=0}, K^* X_{s=0}, \pi^- X^+_{s=0}, \pi^+ X^+_{s=0}. \quad (3.36b)
\end{align*}
\]

The difference of partial decay rate in the \( CP \) conjugated decays are exactly of the form given by eqs. (3.32a,b)

The \( B_u^-, B_d^-, B_s^- \) mesons \( \rightarrow \) single-charm particle final states: These are the dominant modes. However just like in the charm decay, there is no partial decay rate difference in the mixing-matrix nonsuppressed particle and antiparticle decays.

Using the same method we can easily sort out channels of \( B_c \) decays, and flavored particle decays where the partial decay rates for particle and antiparticle are different. Hopefully this has opened a new fruitful area for the search of \( CP \)-violation effects and their origins.

3.3. The estimate of \( \epsilon' \)

It was proposed in ref. [2.5], that the \( K \) decay is dominated by the “Penguin” diagram. In that scenario of explaining \( \Delta I = \frac{1}{2} \) dominance with the \( K\!-\!M \) phase convention, \( a_0 \) is complex and has a rather small imaginary part, but \( a_2 \) is real. This phase difference between \( a_0 \) and \( a_2 \) gives \( \epsilon' \neq 0 \). This was the point made by Gilman and Wise [3.19]. Putting in the experimental values \( |a_2/a_0| = 1/20, \delta_2 - \delta_0 = \pi/4 \) [3.20], eq. (1.48) becomes

\[
\epsilon' = -i(\sqrt{2}/2)(1/20)t_0 \exp(i\pi/4) = -\frac{i}{20\sqrt{2}}(\text{Im} a_0/\text{Re} a_0) \exp(i\pi/4). \quad (3.37)
\]
The question now is what the value of $\text{Im } a_0/\text{Re } a_0$ is. Using the formulation given in section 2, the amplitude $a_0$ has the form

$$a_0 = V_{us}V^*_{ud}A_1 + V_{cs}V^*_{cd}A_2,$$

(3.38)

and

$$\frac{\text{Im } a_0}{\text{Re } a_0} = \frac{s_1s_2s_3s_8c_2A_2}{s_1c_3A_1 - s_1c_2(c_1c_2c_3 - s_2s_3c_8)A_2},$$

(3.39)

where $A_2$ can have contributions from the “Penguin” diagrams ($\epsilon$) and ($\phi$) only, but $A_1$ can have contributions from the W exchange, the W emission diagrams, ($\epsilon$) and ($\phi$), ($\delta$) respectively, see table 2.1.

Assuming dominance from “Penguin” diagrams alone, elaborate calculations have been made including various considerations of perturbative QCD effects [3.19, 3.21]. The value of $\text{Im } a_0/\text{Re } a_0$, thus $\epsilon'/\epsilon$, can vary a great deal

$$1.2 \times 10^{-4} < |\text{Im } a_0/\text{Re } a_0| < 8 \times 10^{-4},$$

(3.40)

or

$$1/600 < |\epsilon'/\epsilon| < 1/80,$$

(3.41)

depending on the approximation scheme, and the QCD scale $0.1 \text{ GeV} < \Lambda < 0.7 \text{ GeV}$. The smaller values of $\Lambda$ tend to give smaller values of $\epsilon'/\epsilon$. Combining this calculation of $\epsilon'$ with that of $\epsilon$ in the K–M model, as given in section 1, it was noted by Hagelin [3.22] that $\epsilon'$ and $\epsilon$ are parallel as $s_8 > 0$, and antiparallel as $s_8 < 0$, therefore the third quadrant solutions of $\delta$ given in section 1 can be distinguished from those of $180^\circ > \delta > 0^\circ$ by the measurement of the sign of $|\epsilon'/\epsilon|$.

Actually we ought to be cautious about considering the “Penguin” diagram alone. The exchange diagram may be as important. If it dominates, as indicated from the charm nonleptonic decay (see section 2.2.2), $A_1$ in eq. (3.39) can be even larger, thus the value of $\text{Im } a_0/\text{Re } a_0$ can be smaller [3.23].

Currently the experimental bounds on $|\epsilon'/\epsilon|$ is, $|\epsilon'/\epsilon| \leq 1/50.$

(3.42)

Experiments [3.25–3.26] with sensitivity of measuring $|\epsilon'/\epsilon|$ down to 0.1% level are being carried out. The observation of $|\epsilon'/\epsilon|$ at that level certainly can be viewed as a triumph of the K–M scheme over that of super-weak theory. A small value of $|\epsilon'/\epsilon| \leq 10^{-3}$ can still be accommodated by the K–M scheme, however it can put even more severe limits on the complex Higgs coupling as the origin of CP violation (see discussion later in section 3.5).

3.4. The estimate of the neutron electric dipole moment $d_n$

There are three form factors for the neutron [1.28]

$$\langle n|J_{\mu}^\text{em}(0)|n\rangle = \bar{u}(p') [F_1(q^2) \gamma_\mu + iF_2(q^2) \sigma_\mu \nu q^\nu + F_3(q^2) \gamma_2 \sigma_\mu \nu q^\nu] u(p),$$

(3.43)
where \( F_1(0) = 0 \) the charge form factor, \( F_2(0) = \mu_n \) the magnetic moment and \( F_3(0) = d_n \) the electric dipole moment. Again the complexity in \( V_{ij} \) can give \( d_n \) of the neutron. The relevance of the \( CP \) violation effect of the K–M model to the neutron electric dipole moment was first discussed in refs. [3.27]. It was noted by Shabelin [3.28] that the electric dipole moment from a single quark is zero, see fig. 3.13a where a photon is meant to be attached in all possible ways. Graphs including gluon correction [3.29], or graphs of inter-quark interaction fig. 3.13b have to be considered [3.30]. The estimated value of \( d_n \) varies a great deal:

\[
10^{-35} \text{ e cm} < d_n < 10^{-30} \text{ e cm}, \tag{3.44}
\]

which is way below the experimental limit [3.31],

\[
d_n < (2.3 \pm 2.3) \times 10^{-25} \text{ e cm}. \tag{3.45}
\]

The smallness of \( d_n \) is one salient feature of the K–M model due to the GIM cancellation mechanism. Recent analysis including the “Penguin” diagram, fig. 3.13c have given values of \( d_n \) close to the larger values of eq. (3.44), [3.32].

\( CP \) violations of other sources like complex Higgs coupling, left–right symmetric gauge theories, tend to give values of \( d_n \) close to the experimental bound of eq. (3.45), (for more discussions on comparisons with other \( CP \) violation mechanisms and references, see the next section, 3.5), therefore an improvement on the measurement of \( d_n \) will be very helpful in clarifying the origin of \( CP \) violation.

3.5. \( CP \) violation from other sources

So far we have taken the approach assuming that the K–M mixing with the phase \( \delta \neq 0 \) is the sole source for the \( CP \) violation. It is quite successful: the parameters adequately fitted the \( \Delta m, \epsilon \) of the \( K^0 \bar{K}^0 \) system, and the fitted parameters give consistent descriptions of other experimental measurements, including \( \epsilon' \) and \( d_n \). There are other possible sources of \( CP \) violation which are of current interest. I shall give a brief discussion.

\( CP \) violation from super-weak interaction: So far all the experimental information is also in agreement with \( CP \) violation from super-weak interaction. The super-weak interaction [1.28, 3.15] was designed to explain the \( CP \) violation in \( K_L \rightarrow 2\pi \) through a super-weak interaction with strength \( G_{SW} \)

\[
\frac{G_{SW}M_N^2}{4\pi} = |\eta_{\pm}| \left( \frac{G_F M_N^2}{4\pi} \right)^2 \approx 10^{-3} \left( \frac{G_F M_N^2}{4\pi} \right)^2 \approx 10^{-9} \cdot \frac{G_F M_N^2}{4\pi}, \tag{3.46}
\]

which contributes only to the \( \Delta S = 2, K^0 \leftrightarrow \bar{K}^0 \) transitions. It has no effect in \( \Delta S = 1 \) reactions. The
observation of $\epsilon'$ and many other interesting $CP$ violation effects like partial rate differences in $K^- \to 3\pi$ and hyperon nonleptonic decays will eliminate the possibility.

$CP$ violation from complex Higgs: In the electroweak unified theories, there are basically two origins of $CP$ violation from the Higgs fields. One is from the complexity in the coupling constants, e.g. Yukawa coupling $y_u$ and $\lambda \phi^4$ with complex $\lambda$. This has been called hard $CP$ violation in the literature. The other is from the complexity in the vacuum expectation values of the Higgs fields, i.e. $CP$ symmetry is broken spontaneously. This is also called soft $CP$ violation.

The K–M phase can be generated either way. (For more discussions of generating the K–M phase spontaneously through the quark mass matrix, see section 4.) It is known that in order to generate a nontrivial phase in the K–M matrix, at least two doublets of complex Higgs fields with complex vacuum expectation values are needed [3.33, 3.34]. Thus after serving the purpose of generating $W^+$, $Z^0$ masses, there are still two charge zero Higgs $H^0_{1,2}$ and two charged $H^\pm$ left over. Their coupling to the fermions are complex and therefore $CP$ violating. Since $H^0_{1,2}$ can give tree-level $K^0 \leftrightarrow \bar{K}^0$ mixing, the masses of $H^0$ are bounded to be $m_{H^0} \geq 100 \text{ GeV}$, which in turn gives very small $K^0 \to \mu\bar{\mu}$, $2\pi$, and has very little physical effects. For the charged Higgs bosons $H^\pm$, current experimental limits [3.35] set $m_{H^\pm} > 10 \text{ GeV}$. In this case the contribution from $H^\pm$ to $\epsilon$, $\epsilon'$, $\Delta m$ of the $K^0$'s are negligible. So in this scenario, physical effects of the Higgs are negligible.

Another scenario of spontaneously breaking $CP$ invariance is that the K–M matrix has the phase $\delta = 0$, and $CP$ violation comes solely from the complexities of the Higgs fields. Such a model was constructed by Weinberg [3.34], with the characteristics that there is no flavor changing neutral coupling [3.36], i.e. no tree level contribution to the $K^0 \leftrightarrow \bar{K}^0$ transition. To be specific, we shall only discuss the Weinberg model. (Other models with $CP$ violation from complex Higgs fields have not been very thoroughly investigated because of the many arbitrariness involved in the models.) In this model the $CP$ violation $\epsilon$ from $K^0 \leftrightarrow \bar{K}^0$ transition is from the box diagrams of figs. 3.14a, b, and $\text{Im} \ a_0/\text{Re} \ a_0$ is from the Higgs "Penguin" diagram of fig. 3.14c. In a recent analysis [3.37], it is found that

$$|\text{Im} \ a_0/\text{Re} \ a_0| \gg |M_{12}^H/2M_{13}^H|, \text{ or } |t_0| \gg |t_M/2|,$$

which exceeds the current experimental limit eqs. (1.63–1.67). This is a very interesting result. However we must be cautious in concluding now that $CP$ violations purely from complex Higgs are ruled out. There are uncertainties which typically plague such calculations [3.38, 1.33]. A better measurement of $|\epsilon'/\epsilon|$ will definitely be helpful in deciding the fate of this mechanism.

The mixing and $CP$ violation for the $D^0\bar{D}^0$, $B^0\bar{B}^0$, $T^0\bar{T}^0$ systems in this model has also been investigated in the literature [3.38–3.40]. Because of the many parameters involved in the theory it is hard to reach definite conclusions. Results from the latest investigation [3.40, 1.42] indicate that both the mixing and $CP$ violation from $p^0 \leftrightarrow \bar{p}^0$ transitions are very small except in the $T^0\bar{T}^0$ systems, which is an interesting contrast to the results from the K–M model, see table 3.1.
Another characteristic of $CP$ violation from complex Higgs coupling is that it gives $d_n$ bigger than the K–M mechanism [3.41]

$$10^{-27} \text{e cm} < d_n < 10^{-25} \text{e cm},$$

(3.48)

which is very close to the experimental bounds, eq. (3.45).

Since the charged Higgs bosons mediate like the $W^\pm$, we expect that they will also contribute to the difference of partial decay rates for particle and antiparticle. However there have been no detailed calculations. The $CP$ violation from complex Higgs coupling also tend to give bigger $CP$ violation in rare decays [3.42]. From the interference of the $W^\pm$ and $H^\pm$ exchange, there are $CP$ violation effects in exclusive semileptonic decays [3.43]. For example the model can give the time reversal invariance violation in the $\mu$ polarization from $K_{\mu3}$. The recent experimental bound on such time reversal invariance violation [3.44] actually provides limitations on the parameters in this model.

The K–M model predicts zero $CP$ violation in all semileptonic decays, as we mentioned in subsection 3.2.1.

$CP$ violation from strong interactions: Then there is the $CP$ violation from strong interaction due to the instanton solutions [3.45]. The bound on $d_n$ gives very severe limits on the effective $CP$ violation.

<table>
<thead>
<tr>
<th>Models</th>
<th>$e$</th>
<th>$e'/e$</th>
<th>$r$</th>
<th>$a(p)$</th>
<th>$\Delta_\rho$</th>
<th>$d_n$(e cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>superweak fitted</td>
<td>0</td>
<td>0</td>
<td>nonzero only in the K$^0\bar{K}^0$ system</td>
<td>nonzero only in the K$^0\bar{K}^0$ system</td>
<td>zero</td>
<td>0</td>
</tr>
<tr>
<td>the K–M model fitted</td>
<td>1/600–1/80</td>
<td></td>
<td>fitted in the K$^0\bar{K}^0$ system; mixing can be large in the $B^+\bar{B}^-\pi^0$ systems and in the $B^0\bar{B}^0\pi^0$ systems for $\delta &gt; 177^\circ$; small for the $D^0\bar{D}^0$ and $T^0\bar{T}^0$ systems</td>
<td>appreciable ($10^{-2}$) only in the $D^0\bar{D}^0$ system for $\delta = 110^\circ$</td>
<td>zero for all semileptonic decays; nonzero for many hadronic channels</td>
<td>$&lt;10^{-30}$</td>
</tr>
<tr>
<td>complex Higgs (a la Weinberg)</td>
<td>&gt;1/50</td>
<td>close to</td>
<td>appreciable only for the $T^0\bar{T}^0$ system; small for the $D^0\bar{D}^0$, $B^0\bar{B}^0$ systems</td>
<td>appreciable only for the $T^0\bar{T}^0$ system; small for the $D^0\bar{D}^0$, $B^0\bar{B}^0$ systems</td>
<td>nonzero for many channels including exclusive semileptonic decays</td>
<td>$10^{-25}$, close to experimental bounds</td>
</tr>
<tr>
<td>no flavor-changing neutral coupling</td>
<td>&gt;1/50</td>
<td>exceed</td>
<td>appreciable only for the $T^0\bar{T}^0$ system; small for the $D^0\bar{D}^0$, $B^0\bar{B}^0$ systems</td>
<td>appreciable only for the $T^0\bar{T}^0$ system; small for the $D^0\bar{D}^0$, $B^0\bar{B}^0$ systems</td>
<td>nonzero for many channels including exclusive semileptonic decays</td>
<td>$10^{-25}$, close to experimental bounds</td>
</tr>
</tbody>
</table>

$\tau$ = parameter for neutral particle–antiparticle mixing, eq. (3.12).

$a(p)$ = asymmetry between $p^+/p^-$ and $p^0/\bar{p}^0$ due to $p^3/\bar{p}^3$ mixing, eq. (3.14).

$\Delta_\rho$ = partial decay rate difference between particle and its antiparticle decaying into their corresponding $CP$ conjugated final states, see eqs. (3.23, 25, 27, 32).
phase from the instanton effect [3.46]. It is still a hotly pursued topic—why the phase is so small [3.47]. The important feature from the CP violation points of view is that the severe limitation from the bound on \( d_\alpha \) renders totally negligible strong CP-violation effects in the \( K^0 \) system. The readers are referred to [3.46] for detailed calculations.

**Left–right symmetric models**: An alternative to the left-handed model is the left–right symmetric models [3.48] which provides more of a difference in point of view than difference in interpreting the low energy phenomenology. In its left-handed sector, the K–M scheme works essentially the same way. Due to the additional right-handed coupling, there is more flexibility in interpreting the low energy phenomenology. In one version of the left–right symmetric model, when the mass matrix is Hermitian, the strong CP violation is zero for lowest order diagrams [3.49]. Due to the existence both of left and right couplings, there are CP-violation effects in exclusive semileptonic decays [3.50].

**3.6. Concluding remarks**

We summarize the discussion of comparison in table 3.1. We see that the present status is that the K–M mechanism can adequately fit all the known phenomena of mixing and CP violation in the \( K^0\bar{K}^0 \) system, \( \Delta m, \epsilon \), and gives \( d_\alpha \) within experimental bounds. Further, with all uncertainties estimated so far the model cannot produce values beyond \( |\epsilon'|/\epsilon | \leq 1/80, d_\alpha \leq 10^{-30} \text{e cm} \). It is therefore crucial to have measurements on \( \epsilon' \) and \( d_\alpha \) to see whether other CP-violation mechanisms are needed. The prediction of extremely small CP-violation effects from \( p^0 \leftrightarrow \bar{p}^0 \) transition for all the \( D^0\bar{D}^0, B^0\bar{B}^0, T^0\bar{T}^0 \) systems is disappointing, however it can serve as a test of the model. The effects of the possible large \( B^0\bar{B}^0 \) mixing are very interesting to look for. Observations of the many CP-violation effects (e.g. the partial rate difference \( \Delta A \) for \( A, \bar{A} \); \( \Delta B \) for \( B, \bar{B} \), which can be significant if \( \delta = 90^\circ \)) in the partial decay rate difference between particle and antiparticle predicted by the K–M mechanism will be very interesting [3.51].

**4. Quark mixing matrix and quark masses**

**4.1. Introduction and formulation**

Ever since the observation that the quark states in weak interaction currents are mixed states of the physical quark states, attempts have been made to understand the dynamical origins of the mixing angles [4.1]. Ever since the observation that the Cabibbo angle \( \theta_c \) is very close to the mass ratios,

\[
\theta_c \approx m_u/m_K \approx m_d/m_s,
\]

many discussions have evolved around expressing the mixing matrix in terms of the quark masses [4.2]. Recently there has been an increase of activities in such studies [4.3].

First, we demonstrate how the mixing matrix is related to the procedure of quark mass matrix diagonalization. The mass term in the weak Lagrangian can be written as

\[
\mathcal{L}_m = \bar{p}_{iL}M_{ij}(2/3)p_{jR} + \bar{n}_{iL}M_{ij}(-1/3)n_{jR} + \text{H.C.}
\]

(4.2a)

where the subindices \( i, j \) denote quark flavors, \( p_i \) are mixed quark states of charge \( Q = \frac{2}{3} \), \( n_i \) are mixed quark states of charge \( Q = -\frac{1}{3} \); or in matrix notation in the quark flavor space,
\[ L_m = \bar{p}_L M(2/3) p_R + \bar{n}_L M(-1/3) n_R + \text{H.C.} \],

(4.2b)

where

\[ p_R = (1 + \gamma s) p, \quad \bar{p}_L = \bar{p}(1 + \gamma s). \]

The mass matrices \( M(Q) \) are arbitrary complex matrix. To find the physical quark states we diagonalize the mass matrix by a bi-unitary transformation:

\[ U_L(Q) M(Q) U_R^\dagger(Q) = \hat{M}(Q), \quad \text{with} \ U^\dagger U = U U^\dagger = I, \]

(4.3a)

where \( \hat{M} \) is diagonal. To find \( U_R \) and \( U_L \), we observe that multiplying eq. (4.3a) by its Hermitian conjugate on the left and right, one obtains

\[ U_R M' M U_R^\dagger = \hat{M}', \quad U_L M M^\dagger U_L^\dagger = \hat{M}'. \]

(4.3b)

Since \( M' M \) and \( M M^\dagger \) are Hermitian, there are well known procedures to find their eigenvectors and their diagonalizing transformations \( U_R \) and \( U_L \) respectively. We also see that \( U_R \) and \( U_L \) are in general different unless \( M' M = M M'^\dagger \). The physical quark states are now

\[ u_L = U_L(2/3) p_L, \quad u_R = U_R(2/3) p_R, \]

(4.4a)

\[ d_L = U_L(-1/3) n_L, \quad d_R = U_R(-1/3) n_R, \]

(4.4b)

the charged current coupled to W bosons is

\[ J^\nu = \bar{u}_L \gamma^\nu n_L \]

\[ = \tilde{u}_L U_L(2/3) \gamma^\nu U_L^\dagger(-1/3)d_L \]

\[ = \tilde{u}_L \gamma^\nu U_L(2/3) U_L^\dagger(-1/3)d_L. \]

(4.5)

where

\[ U_L(2/3) U_L^\dagger(-1/3) = V. \]

(4.6)

is precisely the quark mixing matrix of eqs. (1.8), (1.10) defined previously in section 1.

4.2. Parameter counting, calculability of quark mixing matrix in terms of quark masses, and some examples

The \( U(Q) \) matrices are determined, up to a diagonal unitary matrix, in terms of \( M(Q) \). The hope is that if, from some general principle in some models, \( M(Q) \) are given in terms of the quark masses, then the quark mixing matrix elements can be expressed in terms of quark masses.

In the following we give the counting procedure that provides the information of how many parameters the mass matrix at most can have so that the mixing matrix can be related to quark masses.
The results are: for example, in the case of \(2 \times 2\) mass matrices, each of the mass matrices can have two magnitude parameters and arbitrary number of phase parameters. All of these phase parameters eventually can be removed from the mixing matrix by defining the phases of the quark fields, as discussed in eqs. (1.11) and leaving the mixing matrix elements as functions of the two magnitude parameters that, in turn, are expressed in terms of quark masses. In the case of \(3 \times 3\) mass matrices, totally there are six invariant constraints in terms of the masses of the three charge \(2/3\) and three charge \(-1/3\) quarks. Therefore, in order to express the mixing matrix in terms of quark masses each of the charge \(-1/3\) and \(2/3\) mass matrices together can have five magnitude parameters and an arbitrary number of phase parameters. Among these phase parameters, at most, one will appear in the mixing matrix, the rest can be absorbed in the redefinition of the quark phases.

For a \(n \times n\) Hermitian matrix \(MM^\dagger\), there are \(n\) real invariants which can be expressed in terms of the eigenvalues that in this case are the quark masses. For a \(2 \times 2\) matrix \(MM^\dagger\), the two invariants are \(m_1^2m_2^2\) and \(m_1^2 + m_2^2\), i.e. the determinant and the trace; for a \(3 \times 3\) matrix, the invariants are \(m_1^2m_2^2m_3^2\), \(m_1^2m_2^2 + m_2^2m_3^2 + m_3^2m_1^2\), \(m_1^2 + m_2^2 + m_3^2\); in general the \(n\) invariants are

\[
\prod_{i=1}^{n} m_i^2, \sum \left( \prod_{i=1}^{n-1} m_i^2 \right); \quad \sum \left( \prod_{i=1}^{n-2} m_i^2 \right); \quad \ldots; \quad \sum m_i^2;
\]

for a \(n \times n\) Hermitian matrix there are \(n^2\) parameters, of which \(n(n+1)/2\) are real magnitude parameters and \(n(n-1)/2\) phase parameters. Among the \(n(n-1)/2\) phase parameter \((n-1)\) phase parameters can be removed by the similarity transformation of diagonal unitary matrix, therefore only the \(n(n-1)/2 - (n-1) = (n-1)(n-2)/2\) phase factors are left as true phase parameters. Thus the total number of arbitrary parameters in \(MM^\dagger\) is \(n(n+1)/2 + (n-1)(n-2)/2 = n^2 - (n-1)\) which is just the original \(n^2\) parameter minus the \((n-1)\) removable phase parameters.

From the general method of diagonalizing a Hermitian matrix [4,4], one can easily show that the complexity in the matrix elements \(U(Q)\) must come from the phase parameters in \(M^\dagger\). In general the \(2 \times (n-1)(n-2)/2\) phase parameters result in \(V = U_L^\dagger(2/3) U_L(-1/3)\), of which only \((n-1)(n-2)/2\) are true quark mixing phases, as we had also counted in section 1. So the most economical way of parametrizing the mass matrix is to let \(M(2/3) M^\dagger(2/3)\) and \(M(-1/3) M^\dagger(-1/3)\) have a total of \((n-1)(n-2)/2\) phase parameters, e.g. for \(n = 2\), no phase parameter is needed; for \(n = 3\) only one phase parameter is needed, either in \(M(2/3) M^\dagger(2/3)\) or in \(M(-1/3) M^\dagger(-1/3)\). Any additional phase parameters more than the minimal in \(\cap (Q)\) can eventually be removed from the mixing matrix \(V\).

Now we see that in order to give mixing angles in terms of quark masses, restrictions must be put on the mass matrix so that each of \(M(Q) M^\dagger(Q)\) have only \(n\) parameters. Of these total \(2n\) parameters, \(n-1(n-2)/2\) phase parameters are sufficient to generate all independent phases in the mixing matrix. [Of course for \((n-1)(n-2)/2 > 2n\), i.e. \(n \geq 7\) phases in the mixing matrix are not all independent.] For example, in the case of \(n = 2\), in order for the mixing matrix to be expressible in terms of masses, each of \(M(Q)\) can have two magnitude parameters, and arbitrary numbers (including zero) of phase parameters, all which can eventually be eliminated by requiring that the \(2 \times 2\) mixing matrix \(V\) be real, as shown in eqs. (1.11); for \(n = 3\), all the mixing matrix elements of three mixing angles and one phase can be given in terms of masses, if there are five magnitude parameters and one or more phase parameters jointly in \(M(2/3)\) and \(M(-1/3)\). In the end, all the phase parameters except for one in the mixing matrix can be removed by the quark field phase redefinition. We see that by this simple counting we can easily tell, given the parametrization of mass matrices, which model can give quark mixing angles in terms of quark masses.
So far there is no good guiding principle for such restrictions. Various possibilities of imposing horizontal (flavor) discrete symmetries or horizontal global gauge invariance among the quarks with the same charge have been tried. For example, in the case of two generations it was observed that a mass matrix of the form [4.5]

\[
M(Q) = \begin{pmatrix} 0 & a \\ a & b \end{pmatrix}, \tag{4.7}
\]
gives

\[
\tan^2 \theta_c = m_d/m_s. \tag{4.8}
\]

Note that using our counting rule \(\tan^2 \theta_c\) can be expressed in terms of masses regardless of whether \(a, b\) are real or complex, since the phases can be removed.

In the case of \(n = 3\), for example, the following form of mass matrices for both \(Q = -\frac{1}{3}\) and \(\frac{2}{3}\),

\[
M(Q) = \begin{pmatrix} 0 & a & 0 \\ a^* & 0 & b \\ 0 & b^* & c \end{pmatrix}, \quad a, b, c \text{ complex} ; \tag{4.9}
\]

\[
= \begin{pmatrix} 0 & a & 0 \\ a & 0 & b \\ 0 & b & c \end{pmatrix}, \quad a, b, c \text{ complex} ; \tag{4.10}
\]

\[
= \begin{pmatrix} 0 & a e^{i\alpha} & ba/c \\ a e^{i\alpha} & b & 0 \\ ba/c & 0 & c e^{i\alpha} \end{pmatrix}, \quad a, b, c, \alpha \text{ real} ; \tag{4.11}
\]

\[
= \begin{pmatrix} 0 & 0 & c \\ 0 & 0 & b \\ c & b & a \end{pmatrix}, \quad a \text{ real} ; b, c \text{ complex} ; \tag{4.12}
\]

have been used based upon discrete horizontal symmetry [4.5], and for example

\[
M(Q) = \begin{pmatrix} 0 & c & 0 \\ -c & -a & b \\ 0 & -b & a \end{pmatrix}, \quad a \text{ real} ; b, c \text{ complex} \tag{4.13}
\]

based upon horizontal SU(2) gauge symmetry [4.6]. Same form of the matrices is used for both \(Q = -\frac{1}{3}\) and \(Q = \frac{2}{3}\) quarks but the parameters are unrelated. In all these examples there are three magnitude parameters and more than one phase factor for each of \(M(-1/3)\) and \(M(2/3)\), so there are more than the six parameters that can be related to quark masses. Thus only three of the four K–M parameters can be given in terms of masses. Also, all the mass matrices are either symmetric or Hermitian. This is a
necessary result of left right symmetric models. If we let the number of parameters be one less than the
number of invariant constraints, the unknown t quark mass can be related to known quark masses. For
example, the mass matrix [4.6]

$$
M(2/3) = \begin{pmatrix}
0 & A & 0 \\
A & 0 & B \\
0 & B & C
\end{pmatrix},
$$

(4.14a)

$$
M(-1/3) = \begin{pmatrix}
0 & \lambda A & 0 \\
\lambda A & 0 & \varepsilon B \exp(i\delta/2) \\
0 & \varepsilon B \exp(i\delta/2) & \lambda C
\end{pmatrix}
$$

(4.14b)

has all together five magnitude parameters, A, B, C, \(\varepsilon\), \(\lambda\). The phase parameter \(\delta\) is a removable type,
i.e., \(M(-1/3)M^*(-1/3)\) has a phase parameter that can be rotated away by a similar transformation of a
diagonal unitary matrix \(D = \text{diag}(\exp(i\delta), 1, \exp(i\delta/2))\), such that \(DM(-1/3)M^*(-1/3)D^T\) has only real
matrix elements. Then the t quark mass \(m_t\) can be treated as a free parameter to be determined with the
other five parameters by the six constraint equations of the six invariants.

In grand unified models the generation of lepton masses are related to those quarks. For example, in
ref. [4.7], the mass matrix is of the form

$$
M(Q) = \begin{pmatrix}
a_0 \exp(i\phi_0) & b_0 \exp(-i\phi_0) & \frac{1}{\sqrt{3}} b_0 \exp(i\phi_0) \\
a_0 \exp(-i\phi_0) & b_0 \exp(i\phi_0) & \frac{1}{\sqrt{3}} b_0 \exp(i\phi_0) \\
\kappa a_0 \exp(-i\phi_0) & 0 & -\frac{\sqrt{2}}{3} \kappa b_0
\end{pmatrix}
$$

(4.14c)

for \(Q = -1, 2/3\) and \(-1/3\); where \(\phi_{-1} = \phi_{-1/3} = -\phi_{2/3} = \phi\) the number of free parameters are eight: \(a_{2/3}, b_{2/3}, a_{-1/3}, b_{-1/3}, a_{-1}, b_{-1}, \phi, \kappa\), thus they are overly constrained by the nine-mass invariants and again
the t quark mass \(m_t\) can be made into a free parameter to be determined.

4.3. Relating quark mixing matrix to quark masses necessarily leads to flavor changing neutral couplings

In the unified gauge theories, the masses of the quarks are solely generated through their Yukawa
coupling with the Higgs bosons (elementary or dynamically generated), and the spontaneous symmetry
breaking of the Higgs fields, i.e. the Higgs fields gaining nonzero vacuum expectation value [4.8]. It has
been noted that the calculability of mixing matrix in terms of quark masses through the imposition of
symmetry in the quark flavor space is incompatible with the requirement that there is no flavor changing
neutral couplings, i.e. the mixing matrix becomes trivial with mixing angles being 0, \(\pi\), or \(\pi/2\). Thus the
imposition of relating the mixing matrix to quark masses necessarily leads to the existence of flavor
changing neutral coupling. This conclusion was originally reached by [4.9] for a single Higgs coupling,
and has been generalized to multi-Higgs boson coupling [4.10]. Here we should sketch the essential
ingredients of the proof.

The Lagrangian of the Yukawa coupling term of the quark fields in the standard SU(2) \(\times\) U(1)
electroweak unified theory is

$$\mathcal{L}_V = \bar{\psi}_L \Gamma^\alpha ( -1/3)n_R \phi_\alpha + \bar{\psi}_L \Gamma^\alpha (2/3)p_R \chi_\alpha . \quad (4.15a)$$

where

$$\phi_\alpha = \begin{pmatrix} \phi_\alpha^0 \\ \phi_\alpha^+ \end{pmatrix}, \quad \chi_\alpha = \begin{pmatrix} \chi_\alpha^0 \\ \chi_\alpha^+ \end{pmatrix}$$

are the Higgs doublets, and

$$\psi_{l,l} = (p, n)_{l,l}.$$ The $\Gamma^\alpha (Q)$'s are matrices operating in the flavor space of quarks with charge $Q$. Eq. (4.15a) can also be written as

$$\mathcal{L}_V = \bar{\psi}_L \Gamma^\alpha ( -1/3)n_R \phi^+_\alpha + \bar{\psi}_L \Gamma^\alpha (2/3)p_R \chi^+_\alpha + \bar{\psi}_L \Gamma^\alpha (2/3)p_R \chi^0_\alpha . \quad (4.15b)$$

The mass terms $\mathcal{L}_m$ of the fermions in the Lagrangian are from the spontaneous vacuum symmetry breaking, i.e. the nonzero vacuum expectation values of the Higgs fields,

$$\lambda_\alpha = \langle \phi^0_\alpha \rangle, \quad \mu_\alpha = \langle \chi^0_\alpha \rangle . \quad (4.16)$$

$$\mathcal{L}_m = \lambda_\alpha \bar{n}_L \Gamma^\alpha ( -1/3)n_R + \mu_\alpha \bar{p}_L \Gamma^\alpha (2/3)p_R$$

$$= \bar{n}_L M(-1/3)n_R + \bar{p}_L M(2/3)p_R . \quad (4.17)$$

where the mass matrices are

$$M(-1/3) = \lambda_\alpha \Gamma^\alpha ( -1/3), \quad M(2/3) = \mu_\alpha \Gamma^\alpha (2/3). \quad (4.18)$$

After diagonalization of the mass matrix we obtain the physical quark states as given in eqs. (4.4a) and (4.4b). The requirement that there is no flavor changing neutral coupling among physical quarks means that the transformations that diagonalize the mass matrices must also diagonalize the neutral Higgs couplings, i.e.

$$U_L (Q) \Gamma_\alpha (Q) U_R (Q) = \hat{\Gamma}_\alpha (Q) , \quad (4.19)$$

where $\hat{\Gamma}_\alpha (Q)$'s are diagonal.

As we discussed previously, in order to express the mixing matrix elements in terms of masses, there must be certain constraints put on the mass matrix. In order not to be spoiled by high order effect of radiative corrections, it is best to impose such constraints through symmetry transformations of the Lagrangian. For example, consider symmetry transformations $\{K^\alpha_{l,r}\}$,

$$n_L \rightarrow K^\alpha_L ( -1/3)n_L , \quad n_R \rightarrow K^\alpha_R ( -1/3)n_R , \quad \phi_\alpha \rightarrow \mathcal{D}^\alpha_{ab} \phi_\beta . \quad (4.20a)$$

$$p_L \rightarrow K^\alpha_L (2/3)p_L , \quad p_R \rightarrow K^\alpha_R (2/3)p_R , \quad \chi_\alpha \rightarrow \mathcal{D}^\alpha_{ab} \chi_\beta . \quad (4.20b)$$

Note that these are horizontal transformations, quarks in the same isospin multiplet should transform the same way, thus
\[ K_L^*(2/3) = K_L^*(1/3) = K_L^* . \] (4.20c)

The physical quark states transform under \( \{ K_{L,R}^* \} \), like
\[
\begin{align*}
d_L &\rightarrow S_L^*(—1/3) d_L , \\
u_L &\rightarrow S_L^*(2/3) u_L ,
\end{align*}
\] (4.21)

where
\[ S_L^*(Q) = U_L(Q) K_L^* U_L^*(Q) . \] (4.22)

Since the mixing matrix \( V = U_L(2/3) U_L^*(—1/3) \), eq. (4.6), relates the transformations in the \( Q = \frac{2}{3} \) quarks to \( Q = —\frac{1}{3} \) quarks, one easily obtains
\[ V^* S_L^*(2/3) V = S_L^*(—1/3) . \] (4.23)

The invariance of the Lagrangian
\[ \mathcal{L}_Y \xrightarrow{\{ K^* \}} \mathcal{L}_Y , \] (4.24)

implies
\[ K_L^* \Gamma_\alpha(Q) K_R^* = D_{ab}^{* \alpha} \Gamma_b(Q) , \] (4.25)

which can be rewritten as
\[
K_L^* U_L^* U_L \Gamma_\alpha(Q) U_R^*(Q) U_R(Q) K_R^* = D_{ab}^{* \alpha} U_L^* U_L \Gamma_b(Q) U_R^*(Q) U_R(Q)
\]

and after re-arranging and using eq. (4.19), we obtain
\[ U_L^* K_L^* U_L^* \hat{\Gamma}_\alpha(Q) U_R^*(Q) K_R^* U_R^* = D_{ab}^{* \alpha} \hat{\Gamma}_b(Q) , \] (4.26)

which is, from eq. (4.22),
\[ S_L^{\alpha \tau}(Q) \hat{\Gamma}_\alpha(Q) S_R^*(Q) = D_{ab}^{* \alpha} \hat{\Gamma}_b(Q) . \] (4.27)

Multiplying eq. (4.27) by its Hermitian conjugate, it follows that
\[ S_L^*(Q) \hat{\Gamma}_\alpha(Q) \hat{\Gamma}_\alpha(Q)^* S_L^{\alpha \tau}(Q) = D_{ab}^{* \alpha} D_{\alpha \gamma}^{* \alpha} \hat{\Gamma}_b(Q) \hat{\Gamma}_a(Q) . \] (4.28)

In the special case of one Higgs doublet
\[ D_{ab} = e^{i\delta_{ab}} , \] (4.29)
eq (4.28) reduces to
\[ S_t^t(Q) \hat{\Gamma}(Q) \hat{\Gamma}^*(Q) S_t^t(Q) = \hat{\Gamma}(Q) \hat{\Gamma}^*(Q) . \] (4.30)

This implies that the unitary matrix \( S_t^t(Q) \) must be diagonal. Now let's look at eq. (4.23). Since the left-hand side of eq. (4.23) is diagonal the column vector \( v_i \) of \( V = (v_1, v_2, \ldots v_n) \) must be eigenvectors of \( S_t^t(2/3) \). But now \( S_t^t(-1/3) \) is also diagonal, its eigenvectors must be of the form, up to a phase factor,

\[
\begin{pmatrix}
1 \\
0 \\
0 \\
. \\
. \\
0
\end{pmatrix}, \quad
\begin{pmatrix}
0 \\
1 \\
0 \\
. \\
. \\
0
\end{pmatrix}, \quad
\begin{pmatrix}
0 \\
0 \\
1 \\
. \\
. \\
0
\end{pmatrix}, \quad \text{etc.} \quad (4.31)
\]

which is unique if there is no degeneracy in the eigenvalues of \( S_t^t(2/3) \). These are the physical quark states in some order. We have thus shown that the mixing matrix \( V \) has exactly one nonzero entry in each column and each row. Such a matrix is called monomial. It is a trivial mixing matrix.

In the case of more than one Higgs field, from eq. (4.28), we can only show that \( S_t^t(Q) \) are monomial, rather than diagonal. To prove again that \( V \) is monomial from eq. (4.23) is more involved. The readers are referred to [4.10].

Therefore all those models with horizontal symmetry have flavor changing neutral coupling either through Higgs bosons or horizontal gauge fields. These couplings must be suppressed to the level consistent with experimental data. In the formulation of the K–M model, the flavor changing neutral couplings of the \( Z^0 \) are suppressed equally for all flavors. Therefore the observation of abnormally high \( b \)-flavor changing neutral coupling will have the interesting implication that either the \( t \) quark is not there, or there are flavor changing Higgs or horizontal gauge couplings. As we have emphasized in section 5, it is important to establish the suppression level of the \( b \)-changing neutral coupling.

4.4. Concluding remarks

This duplication of quarks and leptons is one of the most fascinating phenomena. The \( e-\mu \) puzzle turns out to be only a small part of a bigger one. Why this duplication? Will it ever end? The impressive number of papers on the subject shows the intense interest on the subject. Some of these models have some phenomenological success, and some are already ruled out. There is no lack in providing mechanisms relating mixing matrix, or even \( t \) quark mass, to the “known” quark masses. However there is a lack of compelling fundamental reasons or guiding principles for imposing those horizontal symmetries or for certain ways of generating masses. It probably will be difficult to understand quark mixing before we understand the quark “family problem”. Does the answer lie in going to larger groups [4.11]? Maybe it is important to study those theories naturally having three families of quark fields with specified spontaneous-symmetry-breaking mechanism built in. It is most interesting to see how nature eventually reveals her secrets.

5. Alternative to the three left-handed doublet model

The discoveries of the charm and the \( b \) particles in the past few years have certainly made a brilliant chapter in the development of particle physics. They demonstrated the fruitful results of the interplay
between experimental observations and elegant physical reasonings. The particle world with three left-handed quark doublets and three left-handed lepton doublets would appear rather self sufficient, simple and elegant, though may be boring to some physicists. The electroweak unified theory for these quarks and leptons are anomaly free and thus renormalizable [5.1]. The recently discovered heavy lepton $\tau^+$ have all the decay properties expected of them from such a theory [5.2]. However the expected $t$ quark is, so far, still out of sight up to the highest energy of PETRA and PEP, $\sqrt{s} = 37$ GeV [1.16]. This makes many theorists look for alternative models, if indeed the $t$ quark is not there.

5.1. Alternative models

Basically there are three types of alternatives offered in the literature:

Alternative (1). The $b$ is a left-handed singlet. It is the Cabibbo World enlarged, compared to eq. (1.7),

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L, b_L; u_R, d_R, c_R, s_R, b_R. \tag{5.1}$$

However in order not to spoil the GIM mechanism, i.e., $d'd' + s's' = ss + d\bar{d}$, a new quantum number is assigned to $b_L$ [5.3]. We underline this point by putting a wavy line under the $b_L$. Now the $b$ cannot decay into the other quarks. But experimentally we know that the $b$ decays. So new leptons $\ell$ and new gauge bosons $W$ with this new quantum number must be introduced in the theory, and the $b$ can only decay semileptonically into such new lepton

$$b \to qW \to q\ell\bar{\ell}, \tag{5.2}$$

where $q$ denotes the light quarks, and $\ell$ is an ordinary lepton. Note that these models from a theoretical point of view have two undesirable theoretical features: a right-handed doublet lepton pair must be included in the theory to make it anomaly free [5.4] the flavor changing coupling is not suppressed in a "natural" way [3.36].

Some models in this alternative have the bizarre situation of having a gauge boson $W$ which couples to antiquark $u$ and $\tau^+$:

$$b \to \bar{u} W \to \bar{u}\bar{u}\tau^+, \tag{5.3a}$$

or

$$B \to \bar{p}\tau^+. \tag{5.3b}$$

This alternative has already been ruled out by experiments from CESR and PETRA. The $b$ particles are observed to decay more than 85% pure hadronically [1.64, 5.5].

Alternative (2.a): This is the same as alternative (1) but the neutral flavor conservation is not enforced for the $b$ flavor. Therefore $b$ can decay into other quarks through mixing [5.6]. Now we have,

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L, b_L; u_R, d_R, c_R, s_R, b_R. \tag{5.4}$$
\[ d' = V_{ud}d + V_{us}s + V_{ub}b, \quad s' = V_{cd}d + V_{cs}s + V_{cb}b. \]

This is eq. (1.1) enlarged.

Taking the known value of \( V_{ud} = 0.973 \), \( V_{us} = 0.23 \) from section 1, and using the orthogonality, unitarity, and the constraint of no strangeness changing neutral current

\[
|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1,
\]
\[
V_{ud}V_{cd} + V_{us}V_{cs} + V_{ub}V_{cb} = 0, \quad (5.5)
\]
\[
V_{ud}V_{us} + V_{cd}V_{cs} = 0,
\]

one can actually find solutions for \( V_{ij} \). The important point here is that now there are \( b\bar{s}, b\bar{d} \) neutral flavor changing couplings, which produce sizeable \( Br(b \rightarrow s\ell^+\ell^-) \) or \( Br(b \rightarrow d\ell^+\ell^-) \). It was estimated \( Br(b \rightarrow s\ell^+\ell^-) \approx 2\% \) [5.6]. Another important consequence is that the sizeable \( b \rightarrow s\nu\bar{\nu} \) decay will cause large enough missing energy to be tested. Both these features have now been analysed for the \( b \) decays at CESR [1.64, 5.5], and by Mark J and PETRA [5.7]. The predicted amount of \( Br(B \rightarrow \ell^+\ell^-X) \) and missing energy in the decay both exceed experimental observation. The experimental information about the \( b \) decay will be summarized at the end of this section.

This alternative suffers from the same undesirable feature as alternative (1): neutral-strange-changing suppression is not "natural". A new right-hand lepton doublet is needed to make the theory anomaly free.

**Alternative (2.b):** To make the neutral flavor strangeness changing suppression natural, one can put \( b \) with \( u \) and \( d' \) to form a triplet but then another new charge \(-1/3\) quark \( f \) is needed in order to form with \( c \) and \( s' \) another triplet. Now we have

\[
\begin{pmatrix}
  u \\
  d'' \\
  b_1^r
\end{pmatrix}_L, \quad \begin{pmatrix}
  c \\
  s'' \\
  b_2^r
\end{pmatrix}_L.
\]

(5.6)

where \( d'', s'', b_1^r, b_2^r \) can all be mixtures of \( d, s, b, f \). The only requirement is orthogonality, unitarity and no neutral strangeness changing neutral coupling. Such arrangements are rather common in unified models: \( SU(3) \times U(1) \) [5.8], \( SU(3)_c \times SU(3)_R \) [5.9], and E_6 for the grand unification models [5.10]. Of course for the cancellation of anomaly the leptons have to form triplets too.

Insofar as the decay properties are concerned, this alternative has the same two outstanding characteristics as alternative (2.a): sizeable \( Br(b \rightarrow q\ell^+\ell^-) \) and \( Br(b \rightarrow q\nu\bar{\nu}) \).

There have been many studies on the \( B \) decay properties, especially the \( \ell^+\ell^- \) in the final product and their phenomenological implications [5.11]. In a remarkable analysis by Kane and Peskin [5.12], it is shown for both alternatives (2.a) and (2.b) in which the \( b \) quark decays via the normal \( W^\pm \), \( Z^0 \) gauge boson, the inclusive neutral dilepton decay with inclusive charge dilepton decay ratio

\[
\frac{\Gamma(B \rightarrow X\ell^+\ell^-)/\Gamma(B \rightarrow X\ell^+\nu)}{\Gamma(B \rightarrow X\ell^+\ell^-) > 0.12}. \quad (5.7)
\]

However the experimental observation is
Br(B → ℓ⁺ ℓ⁻) ≤ 0.74% ,

or equivalently

\[ \Gamma(B → Xℓ⁺ ℓ⁻)/\Gamma(B → Xℓ⁺ ν) < 0.08 , \]

not consistent with eq. (5.7), [1.64, 5.5, 5.7].

**Alternative (3):** The b forms a right-handed doublet with the right-handed charm quark:

\[ \begin{pmatrix} u^r \\ d^r \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, b^r, u^r, d^r, s^r, \begin{pmatrix} c \\ b \end{pmatrix} \]

where

\[ d' = \cos \theta_c d + \sin \theta_c s , \]
\[ s' = -\sin \theta_c d + \cos \theta_c s . \]

This is a phenomenological model designed to be consistent with the present experimental observation of the B decay [5.13]. Since the b is now put in a right-handed doublet, there is no mixing of b with s and d. It avoids the existence of bS or bd neutral currents. By construction, it couples only to the charm c. However its full coupling strength to the c can be easily changed by mixing the b with another charge \( -1/3 \) quark, i.e. in eq. (5.9a)

\[ \begin{pmatrix} c \\ b \end{pmatrix} \rightarrow \begin{pmatrix} c \\ b' \end{pmatrix}, f_R , \]

where \( b' = b \cos \theta_b + f \sin \theta_b \). Therefore it cannot be tested based upon its coupling strength, like the b lifetime or its rare-mode decay rates. How can we experimentally test this model? In principle the lepton distribution from the semilepton distribution of the b is different if the coupling is \( (V - A) \) or \( (V + A) \). Unfortunately the difference is rather small. It needs much more accurate data than presently available to make the distinction. See fig. 5.1 from ref. [5.14]. The crucial test against the model is the positive identification of b → u decay modes. Like other alternatives, the model is not anomaly free, we

![Fig. 5.1. Single-lepton spectra from B decays (a) at \( \sqrt{s} = 5.27 \) GeV for left-handed and right-handed models compared with electron spectra. (b) at \( \sqrt{s} = 5.27 \) GeV for bL singlet model, from refs. [5.14, 1.64].](image-url)
must increase quark or lepton doublets: (1) increase two right-handed heavy neutral leptons forming two right-handed doublets with e_R, μ_R or τ_R; or two completely new heavy right-handed doublets; (2) or increase two left-hand doublets of heavy quarks (m_q > 20 GeV); (3) one left-handed doublet of quarks plus one right-handed doublet of leptons; (4) other exotic states of quarks and leptons so that the theory is anomaly free. Esthetically these are not a very elegant way out. We might as well have a world with a very heavy, unobservable t quark.

5.2. Experimental status of the b-particle decay

Let’s now sum up the experimental situation:

(1) The semilepton leptonic branching ratios of the B decay is about 8 ~ 14%, see table 5.1 [1.62, 1.64, 5.5]. This rules out alternative (1) type of models.

Table 5.1
Branching ratios for semileptonic decay of the B-meson

<table>
<thead>
<tr>
<th>Experiments</th>
<th>Br(B→eνX)</th>
<th>Br(B→μνX)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CESR: CLEO</td>
<td>13.6 ± 2.1 ± 1.7%</td>
<td>10.0 ± 1.3 ± 2.1%</td>
</tr>
<tr>
<td>CUSB</td>
<td>13.6 ± 2.5 ± 3.0%</td>
<td>-</td>
</tr>
<tr>
<td>PETRA: Mark J</td>
<td>-</td>
<td>8 ± 2.7 ± 2%</td>
</tr>
</tbody>
</table>

(2) The single lepton energy spectrum is consistent with B→Xℓ⁻̄ν where m_ν ≈ 1.8 GeV, and give a limit [1.64, 5.5]

\[ \text{Br}(b→u)/\text{Br}(b→c) < 50\% . \]  \hspace{1cm} (5.10)

(3) The average number of kaons is 2.0 ± 0.3 kaons per nonleptonic B decay, and 0.8 ± 0.07 kaons per semileptonic B decay [1.64, 5.5]. This is consistent with b→c.

(4) The inclusive opposite-charge dilepton decay of the B is bounded [5.5, 5.7]

\[ \text{Br}(B→X\ell^+\ell^-) < 0.74\% . \]  \hspace{1cm} (5.11a)

Fig. 5.2. Single-lepton spectra from b decay at rest in the standard model with |b→u|/|b→c| = 0.3, 0.7, 1.0, ∞, from ref. [5.14].
or equivalently

\[
\text{Br}(B \rightarrow X \ell^+\ell^-)/\text{Br}(B \rightarrow X \ell^-\bar{\nu}) < 0.08 .
\] (5.11b)

This is not in agreement with alternatives (2.a), (2.b) type of models.

(5) The charged (assumed to be pions) energy fraction of the B decay is about 60%, see table 5.2 [5.5]. This is not consistent with those models in alternative (1), producing a baryon in the B decay, or those models producing two neutrinos in the B decay, due to b-flavor changing neutral coupling like alternative (2.a), (2.b).

(b) The charged energy fraction is measured as function of semileptonic branching fraction. This seems to rule out B decay via a charged Higgs boson emission. The Higgs boson coupling to fermions is proportional to the fermion masses, it couples predominantly to \( \tau \bar{\nu} \) or \( c\bar{s} \):

\[
b \rightarrow uH^- \rightarrow u\tau^-\bar{\nu} \text{ or } uc\bar{s} .
\] (5.12)

This gives a lepton yield different from experimental results, table 5.1, and give less semileptonic branching fraction.

5.3. Concluding remarks

It is a remarkable achievement by the experimentalists that, except those models in which the b quark does not decay via the normal \( W^\pm \), \( Z^0 \) gauge bosons, all the models without the t quark dreamed up by the theorists have been ruled out. This, of course, still does not mean that t quark exists. It may be just our lack of imagination.

The question to what degree the neutral b-flavor changing is suppressed is still extremely important to know. Since if our theory is composed of only left-handed doublets, the generalized GIM mechanism will suppress very much the inclusive opposite charge dilepton \( \ell^+\ell^- \) decays of the B, just like for \( K^0 \rightarrow \mu^+\mu^- \) and \( K \rightarrow \pi e^+e^- \), [3.42]. Some estimates show [5.15]

\[
\text{Br}(B^0 \rightarrow \tau^+\tau^-) \approx 10^{-8} ,
\] (5.13)

\[
\text{Br}(B \rightarrow X_\ell\ell^+\ell^-) \approx 10^{-5} .
\] (5.14)

As we mentioned in section 3, if horizontal gauge formulation is introduced, flavor changing neutral coupling can be present. They are mediated by neutral Higgs bosons or horizontal gauge bosons, and can give much larger branching ratios \( \text{Br}(b \rightarrow \ell^+\ell^-X) \) than those given by the K–M model, eqs. (5.13,
Therefore the observation of neutral b-flavor changing coupling at a level above that predicted by the K–M model has the interesting implication that either the t quark is absent, or there are neutral flavor changing Higgs fields or horizontal gauge bosons. All these possibilities are extremely interesting.

6. Summary and outlook

The three left-handed doublet quark model with its mixing matrix of three angles and one phase is so far consistent with existing relevant experimental information. Though still crude, a consistent range of values of the quark mixing matrix elements has emerged. An interesting feature resulting from these analyses is the generation-gap phenomenon: i.e. quarks of different generations do mix but the mixing gets weaker as the generation gap increases. Physically this gives the cascade pattern of decays, e.g. \( t \rightarrow b \rightarrow c \rightarrow s \). The analysis of \( V_{td} \) can be refined as better theoretical understanding and experimental information about the decays of the charmed and b-flavored particles improve. Most important, of course, is the finding of the t quark.

While we do have the approximate range of values of the quark mixing matrix elements, our knowledge about the values of the angles \( \theta_2, \theta_3 \), and the phase \( \delta \) (except for \( \cos \theta_1 = 0.9737, \sin \theta_2 < 0.5, \sin \theta_3 < 0.5 \)) are still rather poor. Better determination of \( \delta \) will require the future observation of new \( CP \) violation phenomena in the heavy quark system. Unfortunately the K–M model seems to predict very little \( CP \) violation from the neutral particle–antiparticle transition. We need to rely upon the observation of \( CP \)-violation effects in the partial decay rate difference between particle and antiparticle into their \( CP \)-conjugated final states, e.g., \( \Lambda \rightarrow \pi^- p \) compared to \( \bar{\Lambda} \rightarrow \pi^- \bar{p} \); \( B^+ \rightarrow \pi^- \pi^0 \), the difference of which can be significant if \( \delta \approx 90^\circ \).

The distinct features of \( CP \) violation from the K–M model is the absence of first-order weak \( CP \)-violation effects in semileptonic decays, the extreme small value of the neutron electric dipole moment \( d_n < 10^{-36} \) e cm, and a maximal \( |e'/e| < 1/80 \), and possible large \( B^{0}/\bar{B}^{0} \) mixing. Better experimental information on these phenomena will help to distinguish the model from other sources of \( CP \) violation.

The analysis of the quark mixing matrix is intertwined with the study of nonleptonic decays of particles. Furthermore the \( CP \)-violation effects can manifest in the difference of nonlepton partial decay rates between particle and antiparticle decaying into their \( CP \)-conjugate final states. There has been some progress in our understanding of the difficult subject of nonleptonic decay. Unlike the overwhelming \( \Delta I = 1/2 \) or \([8]\) dominance in the weak decay of strange particles, there is no clear dominance of one single group representation in the weak decay of the charmed particles. The new development has been the use of the quark-diagram approach. Though quite successful for semileptonic decays leading-order QCD calculations of these nonleptonic-decay diagrams had predicted the single dominance of the W-emission diagram. It met with great difficulty when compared with the present initial data of D decays. Other perturbative diagrams have been introduced to accommodate the data. If a more phenomenological approach is taken, inclusion of either the W-exchange or the W-annihilation diagrams in addition to the W-emission diagram seems to be adequate in fitting the present data. Much work, both theoretical and experimental, is needed. It will be probably some time before we truly understand the dynamics of nonleptonic decays. The better determination of the quark-mixing matrix will certainly be helpful in the study of the weak decay dynamics.

One of the searching questions is why the quark states entering into the weak interaction are certain mixtures of the physical quark state. Much work has been done attempting to understand the origin and the amount of quark mixing from the symmetry of the quark mass matrix. It is a beautiful idea.
However there has been a lack of fundamental guiding principle in imposing such symmetries. Other searching questions are why the quark generation repeats; and if the t quark exists, how many more generations of quark doublets there really are. The generation-gap phenomenon of the quark mixing matrix prevents us from detecting the existence of high quark doublets from the deviation of unitarity of the present quark mixing matrix. It was the similar situation as the leakage to the third generation was so small that the possible existence of the third quark doublet was not suspected until a good physical reason was found by Kobayashi and Maskawa. Do we have compelling reasons now to expect more generations of quark doublets? It is interesting to see if the reasoning that more heavy quark doublets than three might be needed in order to explain the matter–antimatter asymmetry in our universe can be substantiated in the future. One cannot but have a keen sense of curiosity and excitement waiting to see what the next secret in nature will be unveiled.

Acknowledgments

This review is a result of the many collaborations with W.Y. Keung, R.E. Schrock, M.D. Tran, S.B. Treiman and F. Wilczek. Discussions with them and many other colleagues have helped enormously in clarifying and improving the presentation, special thanks are due to M.K. Gaillard, G.L. Kane, L.B. Okun, S. Pakvasa, A. Sanda, M. Schmidt, S.C.C. Ting, T.L. Trueman, S.-H. Tye and C.N. Yang, and the many friends who responded to my letter asking them to bring me up to date on their contributions in the field. However, the author is solely responsible for any mistakes in the review. My sincere thanks also goes to M. Jacob for inviting me to write this review. It has been a most educational experience. Finally, I cannot express enough my appreciation to Mrs. Isabell Harrity for her artistry, and unfailing support and patience in typing the manuscript.

I would like to thank Mr. F.L. Olness for careful proof reading of the references and eliminating many errors.

References

for earlier work on the subject, see M. Gell-Mann and M. Lévy, Nuovo Cim. 16 (1960) 705.
in e\'e\' reaction see G. Goldhaber et al., Phys. Rev. Lett. 37 (1976) 255;
[1.10] In theories with interactions between different doublets due to certain horizontal gauge coupling, such redefinition of the phases of quark fields can result in the complexity of other couplings in the theory. That in turn can give CP violation effects. See discussions in section 4.
[1.11] For the earlier discussion on the phenomenological implications from the K-M model, see
Ling-Lie Chau, Quark mixing in weak interactions


[1.14] The observation of the b particles:


[1.16] For the limit of $m_\beta > 18$ GeV, see

[1.17] For the discussions of parametrization of mixing matrix in higher dimensions, see

- A. Salam and J.C. Ward, Phys. Lett. 13 (1964) 168;


[1.20] For a review, see


[1.22] For the discovery of the $W^\pm$ boson with mass predicted by the standard SU(2) × U(1) electroweak theory, see UAI collaboration, CERN, G. Arnison et al., Phys. Lett. 122B (1983) 103;
- For a review of other experimental tests of the standard SU(2) × U(1) electroweak theory, see talk by


[1.26] Recently new, improved data on hyperon semileptonic decays have become available, see
- CERN WA 2 Collaboration, M. Bourquin et al., Measurement of $\Sigma \to \Lambda e\nu$, $\Lambda \to p e\nu$, $\Xi \to \Lambda e\nu$, $\Xi \to \Xi e\nu$ and $\Sigma \to \Lambda e\nu$ Branching Ratios and Form Factors, Contributed paper to Int’l. Conf. on High Energy Physics, Lisbon, Portugal, 9–15 July 1981;
- The difficulty related to the fit of $\Delta s = 0$, $\Sigma^0 \to \Lambda e^+\nu$ first noted in ref. [1.24] (because of this difficulty the authors of ref. [1.24] did two fits, one for $\Delta s = 0$, the other one including $\Delta s = 0$) stands even more sharply as the error bar shrinks. For more recent analysis, see
- Another discrepancy with the model has just recently been observed, i.e. $g_1/g_2 = +0.44 \pm 0.03$ from electron asymmetry measurement of $\Sigma \to ne^+\bar{\nu}_e$. The theory predicts a negative $g_1/g_2$ for the reaction, see
- It is important to confirm this result from other independent experiments. A word of caution must be added to the interpretation of the data. Strictly speaking Cabibbo theory works only in the SU(3) symmetric world. One might be able to attribute this disagreement to the breaking of SU(3) symmetry. The recent fit by
- A. Bohn, Fits of Hyperon Data, U. of Texas at Austin preprint 1982, these works do not answer the fundamental question of the source of the breaking, demonstrated the SU(3) breaking of mass equality can accommodate the data.


[1.30] The phase-convention independent analysis has been recently emphasized in
- E. Ma, W.A. Simmons and S.F. Tuan, Phys. Rev. D20 (1979) 2888;
For original analysis see

L. Wolfenstein, in: Theory and Phenomenology in Particle Physics, ed. A. Zichichi (Academic Press, 1969);
The formula for $e'$ in the phase convention $\Im a_0 \neq 0$ had been done before. However the expression for $e'$ given in these papers is not obviously vanishing as the phases of $a_0$ and $a_2$ become the same.

See also ref. [1.28], eq. (5.173). I am especially grateful to W.-Y. Keung and A. Sanda for discussions on this subject.


For a critical discussion on using the box graph alone as the estimate for the $K^0\bar{K}^0$ transition, see


For a recent review on $\mathcal{CP}$ violation see

For further discussions see

Recently $M_{12}$ with external momentum non-neglected has been given by H.Y. Chen, Phys. Rev. D26 (1982) 143. The effects of the external momenta for $K^0\bar{K}^0$, $D^0\bar{D}^0$, $B^0\bar{B}^0$ systems are small, and for the $T^0\bar{T}^0$ system can be important depending on the $t$ quark mass.

For discussions on these uncertainties of the box-graph calculation, see
D.F. Greenberg, Nuovo Cim. 56 (1968) 597;
P.D. Burnett and D.G. Sutherland, The Contribution of the Two Pion Intermediate State of the $K_\pi K_\pi$ Mass Difference, Glasgow Univ. preprint-78-0788 (1978). This work emphasizes that there are significant modifications to the real part of $\mathcal{M}$ from the box-graph calculation; in other words, the constraints on the $K-M$ mixing from the box-graph calculation may be modified.

For the original bag calculation of the $B$, see
For a recent discussion on bag calculation see
P. Colic, B. Guberina, D. Tadic and J. Trapetic, $K_\pi-K_\pi$ Mass Difference and Quark Models, Max-Planck Institute, etc., preprint, MPI-PAE/Phb, 1982. For some parameters of the bag model $B$ can be negative. However even for $B$ negative there are still solutions in the allowed ranges of $s_2$, $s_3$, though very limited, as noted by the authors of ref. [1.48].

For the original bag calculation of the $B$, see
For a recent discussion on bag calculation see
P. Colic, B. Guberina, D. Tadic and J. Trapetic, $K_\pi-K_\pi$ Mass Difference and Quark Models, Max-Planck Institute, etc., preprint, MPI-PAE/Phb, 1982. For some parameters of the bag model $B$ can be negative. However even for $B$ negative there are still solutions in the allowed ranges of $s_2$, $s_3$, though very limited, as noted by the authors of ref. [1.48].


For general framework of perturbative QCD corrections, see
A.I. Vainshtein, V.I. Zakharov, V.A. Novikov and M.A. Shifman, Phys. Lett. 60B (1975) 71;

For the predictive power of such calculations for nonleptonic decays, for an earlier review see ref. [1.50], also see section 2.2. It has not been too successful in fitting the data.

Ling-Lie Chau, Quark mixing in weak interactions


They used the technique introduced by K.G. Wilson, Phys. Rev. 179 (1959) 1499.

For another explanation of the $\Delta J=\frac{1}{2}$ rule in $K\rightarrow\pi\pi$, see G. Nardulli, G. Preparata and D. Rotondo, Phys. Rev. D27 (1983) 557.


[2.6] Higher order effects were considered and were found not to affect the conclusions of ref. [2.5].

M. Wise and E. Witten, Phys. Rev. D20 (1979) 1216. However, whether the Penguin diagram can explain the full $\Delta J=\frac{1}{2}$ rule is somewhat uncertain.

In an estimate by C.T. Hill and G.G. Ross, Phys. Lett. 94B (1980) 234, the Penguin diagram can only account for 10% of the K and A decays.

[2.7] The development of the complete six-quark diagram for meson decays was first reported in L.-L. Chau Wang, Flavor Mixing and Charm Decay, Proc. 1980 Guangzhou (Canton) Conf. on Theoretical Particle Physics, Jan. 5-14, 1980, p. 1218. The importance of the W-exchange diagram for the D° decay was also discussed.


[2.11] For a recent review on charm lifetimes see: 
K. Reibel, Talk at 8th Int'l. Workshop on Weak Interactions and Neutrino, Sept. 5-11, 1982, Javea, Spain.


[2.13] Three remarks about the tables for the quark-diagram amplitudes:

(1) In these quark diagram amplitudes, for the Penguin diagrams the contribution from the b quark for charmed particle decays (and the contributions from the t quark for strange particle decays) are not explicitly written down since they can be combined with contributions from the d and the s (from the u and the c) contributions.

(2) If one needs to take into consideration the differences due to different quark intermediate states, the Penguin amplitudes $\epsilon$ with different mixing matrix can be treated differently.

(3) The convention for the quark mixing matrix element at a $q_dW$ vertex is such that it is $V_\mu$ if the outgoing quark $q_\mu$ is a u-like quark; is $V_\nu$ if the outgoing quark $q_\nu$ is a d-like quark.


[2.16] For comments on the SU(4) dominance, 


[2.19] G. Kane, SLAC Rept. No. SLAC-PUB-2326, 1979 (unpublished); 

K. Ishikawa, UCLA preprint UCLA/TEP/11 (1979); 
M. Glück, Phys. Lett. 88B (1979) 145; 

The results of the calculation are very much dependent on the value of $\alpha$, used, as discussed in the last reference.


[2.34] For tests on W-exchange and W-annihilation diagrams in some other decay modes, see


[2.37] For detailed discussion on the comparison of the predictions from \( M = 1/2 \) and \([8]\) dominance, for strange particle and hyperon decays see Chapter 6.3 of ref. [1.28], and a review,


[2.39] For recent leading-order QCD analysis of hyperon decay see


H.C. Lee and K.B. Winterbon, Canadian Journal of Physics 50 (1982) 405. The simultaneous understanding of the s-wave as well as the

p-wave is still the main difficulty. A possible remedy for this difficulty has been recently pointed out by


[3.1] For reviews on the \( CP \) violation in the K system, see


See also the proposal, Aachen, The relevance of the W-exchange diagram in K decays and its implication on J.S.
The original estimate of K~ experiment at CERN, PS F.J. Gilman and M.B. Wise, Phys. Lett. In ref. [3.13]
The authors in ref. [3.10] considered the interference effects in b decay between a W-emission diagram and a “Penguin” diagram with q{ bar}q production via a gluon emission.
In ref. [3.13] the “Penguin” diagrams are not considered, therefore interferences in some channels, like eqs. (3.30b,c) (3.35b,c) are missed.
The original estimate of e'/e was too big, see F.J. Gilman and M.B. Wise, Phys. Rev. D20 (1979) 2392,
and other calculations, M.B. Wise and E. Witten, Phys. Rev. D20 (1979) 1216;
This technique was originally used by L.-F. Li and L. Wolfenstein, Phys. Rev. D21 (1980) 178, to calculate $\eta_{--\alpha} = (\langle \pi^- \pi^- \pi^0 \rangle)^2 (\langle \pi^- \pi^- \pi^0 \rangle / H_{W}(K^0))$. They obtained $|\eta_{--\alpha} - \epsilon| = (2/3)|\eta_{--\alpha} - \eta_{0K^0}|$. The uncertainty involved in the calculation was discussed in the paper.
[3.17] Ling-Lie Chau, Quark mixing in weak interactions
[3.21] The original estimate of $\epsilon'/\epsilon$ was too big, see F.J. Gilman and M.B. Wise, Phys. Rev. D20 (1979) 2392,
and other calculations, M.B. Wise and E. Witten, Phys. Rev. D20 (1979) 1216;
Princeton, BNL experiment, M. Banner et al., Phys. Rev. 28 (1972) 1597 and see K. Kleinknecht [3.1].
[3.25] Univ. Chicago, Stanford, Saclay, FNAL experiment #617, A Study of Direct CP Violation in the decay of the Neutral Kaon via a Precision Measurement of $|\eta_{90}|$,$\eta_{--\alpha}|$. R. Berstein et al.;
Yale–Brookhaven, BNL experiment #749, A Measurement of Milliweak CP Violation in $K_L-K_S$ Decays Through the Determination of $\epsilon', R.C. Larsen et al.
[3.26] See also the proposal, D. Dundy et al., CERN/SPSC/81-110/P174, CERN–Dortmund–Pisa–Siegen, Measurement of $|\eta_{90}|$,$\eta_{--\alpha}|$. This proposal plans to have a sensitivitiy of 0.01%.


The author here argues that the analysis in the previous paper might have overestimated $d_2$ in the inter-quark interaction. fig. 3.1b, calculation.
For other contributions to $d_2$ see ref. [1.7].


J.G. Körner and D. McKay, DESY preprint DESY/81/034;
These results differ from a previous calculation by A.A. Anselm et al., see ref. [3.41].

Y. Dupont and T.N. Pham. Dispersion Contribution to $K^0$–$ar{K}^0$ Transition and Higgs–Boson Exchange Model of CP Violation. École Polytechnique preprint. A498.0482, 1982;


[3.40] Ref. [1.42]. This calculation shows the uncertainties involved. Using a different set of parameters with the limitations from $\mu$-polarization from $K_\pi$, the author reaches very different conclusions from ref. [3.37]: the neutral particle–antiparticle mixing can be big only in $T^0$ and $CP$ asymmetry is not at few percent level.

The reader will find the interesting evolution of the estimates of $d_2$ however it stays about $(10^{-23} - 10^{-24})$ e cm.

[3.42] For discussions on rare decays, see ref. [3.27] and


H. Georgi. Hadr. J. 1 (1978) 155. See also
Ling-Lie Chau, Quark mixing in weak interactions

For alternative attempts in the context of technicolor gauge theories, see

For a review of basics of L-R symmetry see
G. Senjanovic, Nucl. Phys. B153 (1979) 334 and


N. Cabibbo and L. Maiani, Phys. Lett. 28B (1968) 131;

T.C. Yang, Phys. Rev. D13 (1976) 1322;
A. Zee, Phys. Rev. D13 (1976) 713;
G. Preparate, Phys. Lett. 82B (1979) 398;
E. Derman, Phys. Rev. D19 (1979) 317;
E. Derman and H.S. Tsao, Phys. Rev. D20 (1979) 1207;
D. Wyler, Phys. Rev. D19 (1979) 330, 3369;
H. Hayashi, M.J. Hayashi and A. Murayama, École Polytechnique Report A397 (1980);

F. Wilczek and A. Zee, Phys. Lett. 70B (1977) 418;
H. Fritzsch, Phys. Lett. 70B (1977) 436;
See Chapter II of J. Heading, Matrix Theory for Physicists (Longmans, Green and Co. Ltd.).

S. Pakvasa and H. Sugawara, Phys. Lett. 73B (1978) 61, 82B (1979) 105;
G. Segré, H. Weldon and J. Weyers, Phys. Lett. 83B (1979) 351;
R.N. Mohapatra and G. Senjanovic, Phys. Lett. 73B (1978) 176;

F. Wilczek and A. Zee, Phys. Rev. Lett. 42 (1979) 421;
M. Yanagida, Phys. Rev. D20 (1979) 2986; Prog. of Th. Phys. 64 (1980) 1103;
C.L. Ong, Phys. Rev. D19 (1979) 2738;


J.S. Bell and R. Jackiw, Nuovo Cimento 60 (1969) 47;

B. Gittelman, Proc. of the XXth Int'l Conf. on High Energy Physics, Madison, Wisconsin, 1980, p. 687;
The search for such heavy leptons using the innovative \((\text{e}^+\text{e}^-)\)-search method in \(\text{e}^+\text{e}^-\) reactions was first proposed by M. Bernardini, D. Bollini, E. Fiorentino, T. Massam, L. Monari, F. Palmonari and A.Z. Zichichi, ADONE proposal INFN/AE-67/3, 1967.


[5.4] The condition for anomaly free is
\[
\Sigma_i (\Sigma_i O_i^Q - \Sigma_i O_i^Q) T_i^Q(1 + T_i^Q) - \Sigma_i (\Sigma_i O_i^Q) T_i^Q(1 + T_i^Q) = 0 \text{ where } j \text{ denotes the multiplet flavor, } T_j^Q \text{ denotes the weak isospin of the } j^{th} \text{ right-handed multiplets, } i \text{ is the } i^{th} \text{ quark in a multiplet and } O_i^Q \text{ is the charge. The superscript } L \text{ symbolizes "left-handed".}
\]
In the case of three left-hand doublets of quarks and leptons, the first term in eq. (5.3) is zero due to \(T_j^Q = 0\) since all right-handed quarks and leptons are singlets; the second term in eq. (5.3) is also zero due to the exact cancellation between the three colored quark doublets and the color neutral lepton doublets.

[5.5] For reviews see


Y. Achiman and B. Stech, Phys. Lett. 77B (1978) 589;

M. Gorn, Phys. Rev. D20 (1979) 2380;


