The Breakdown of Parity Conservation in the $\pi$-$\mu$-$\epsilon$ Decay and a Test of the Two Component Neutrino Theory†

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[Received April 9, 1957]

ABSTRACT

6149 complete $\pi$-$\mu$-$\epsilon$ events have been measured in nuclear emulsion. 3021 events were in ordinary Ilford G5 emulsion and 3128 were in G6 emulsion containing twice the usual amount of gelatin relative to halide. The angular correlation between the initial directions of motion of $\mu$-meson and electron, after possible edge effects have been eliminated, is of the form $1 - (0.149 \pm 0.033) \cos \theta$ in ordinary emulsion and $1 - (0.190 \pm 0.033) \cos \theta$ in diluted emulsion. This demonstrates conclusively the breakdown of parity conservation and charge conjugation in both steps of the $\pi$-$\mu$-$\epsilon$ process. It is shown that, with the aid of other data, the partial quenching of the $\mu$-meson's initial spin orientation may be eliminated from these data and the true correlation of the fundamental process obtained: it is $1 - (0.233 \pm 0.060) \cos \theta$. This asymmetry is consistent with the prediction of the two-component neutrino theory with

$$\left(\sum \left| f \right|^2 \right)^{-1} (f^*_\alpha f_\beta^* + f_\alpha^* f^*_\beta) = 0.85 \pm 0.18.$$
A specific objective of our emulsion experiment was to determine the coefficient \( \alpha \) which measures the asymmetry in order to compare it with the predictions of particular models for the \( \pi-\mu-e \) decay. The only detailed model so far proposed is that based on the 2-component neutrino theory of Salam (1957), Lee and Yang (1957), Landau (1967), and Touschek (1957). This model predicts an extreme value of \( \alpha = -\frac{1}{2} \) if electrons of all energies are detected without discrimination. It also suggests that the asymmetry in the electron emission should be strongly dependent upon the energy of the electron. Now while from the experiment of Garwin et al. one may infer a distribution of the form \( 1 + A \cos \theta \) with considerable accuracy the asymmetry coefficient so determined cannot be immediately related to the desired \( \alpha \) of the fundamental process. This is because: firstly, the angular resolution so far achieved is poor and one must know how the electron detection sensitivity varies as a function of position across the face of the detector before a proper folding-in of this resolution function can be achieved; secondly, the sensitivity of the detector varies with the electron energy in a manner which is not easy to determine with certainty; thirdly, the degree of polarization of the \( \mu \)-meson beam in terms of that for the fundamental process is not known. These difficulties are overcome by using emulsions: the angular resolution is very precise; all electrons are detected without regard to energy; since the complete \( \pi-\mu-e \) chain is observed it is sure that the polarization of the \( \mu \)-meson is as high as is allowed by the fundamental process. It appears at first sight, however, that these advantages are useless because the asymmetry observed for \( \mu \)-mesons stopping in nuclear emulsion is appreciably lower than that observed for other substances such as carbon (see Garwin et al. 1957). It is therefore apparent that some 'quenching' or loss of memory of the initial \( \mu \)-meson spin orientation takes place in emulsion and so it seems that measurements in emulsion, despite the advantages listed above, cannot yield the true asymmetry coefficient \( \alpha \) of the fundamental process. We shall show, however, that this is not so, and that the emulsion data may be analysed to give the desired true asymmetry coefficient if we make use also of the relative apparent asymmetry coefficients observed at Columbia for carbon and nuclear emulsion. We do not make use of the Columbia asymmetry measurements directly in obtaining our result but rather use them to deduce the degree of quenching taking place in nuclear emulsion.

The aim of the present experiment is therefore two-fold: to establish clearly the \( \pi-\mu-e \) asymmetry in nuclear emulsions; and to obtain a value for the true asymmetry coefficient \( \alpha \) integrated over the whole electron spectrum.

\section*{\textbf{§ 2. Experimental Method}}

The Columbia workers observed a weaker asymmetry in nuclear emulsion than in carbon and certain hydrocarbons. They also found
that this weaker asymmetry did not depend on the time, after the stopping of the \( \mu \)-meson in the emulsion, at which they observed the electron. This last observation suggests that some \( \mu \)-mesons stopped in emulsion are 'quenched' very quickly and lose their initial polarization while others retain their spin direction for at least some microseconds. It is natural to associate the first class of mesons with those that stop in one component of the emulsion, probably the halide, and the second class with those stopping in the other component, the gelatin. This argument suggested to us that it may be possible to observe a stronger asymmetry if the content of gelatin in the emulsion were increased relative to halide. Such an enhanced asymmetry is obviously highly desirable if emulsions are to be used for similar studies of non-conservation of parity for rare particles such as the K-mesons. We therefore carried out the experiment using both ordinary Ilford G5 emulsions and identical emulsions in which the proportion of gelatin to halide is increased by a factor two (twice-diluted or \( \times 2 \) emulsion). The two sets of emulsions were prepared at the same time from the same batches of halide and gelatin and they contained the same additives—stabilizers and so on. In order to achieve a good grain density on the minimum-ionizing electron tracks and also to minimize distortion the emulsions were not stripped but were exposed on their glass backing. The ordinary G5 emulsions showed a density for fast electrons of 30 blobs per 100 \( \mu \text{m} \); the density for the \( \times 2 \) emulsions was 27 blobs per 100 \( \mu \text{m} \).

The emulsions were exposed to a flux of about \( 10^4 \pi^+ \) mesons per cm\(^2\) at the Liverpool University synchrocyclotron. The incident pion energy was about 35 mev. The emulsions were contained in a steel box with a lining of mu-metal. The magnetic field at the position of the emulsions was measured with a flipcoil to be 0.15 gauss. This magnetic field is so weak that its effect in depolarizing the \( \mu \)-mesons before they decay is completely negligible (the precession is by 1.6° in one mean life). It is sufficient, however, to effect a very large depolarization of any triplet 'muonium' atoms (\( \mu^+-e^- \)) which may be formed (a precession of about 180° in one mean life). It is nevertheless likely that such atoms would be depolarized very quickly by internal magnetic fields in so complicated a structure as nuclear emulsion.

Before processing, the total thickness, emulsion plus glass backing, was measured at several points on each plate using a dial gauge. These total thickness measurements were repeated at the same points on the processed emulsions and the thickness of the processed emulsion itself was measured at the same points under the microscope immediately afterwards. By combining these measurements we know the original thickness of the unprocessed emulsion at several points on each plate and so know the shrinkage factors accurately for the processed emulsions. The original thickness of the ordinary G5 emulsion was about 610 \( \mu \text{m} \); that of the \( \times 2 \) emulsion was about 480 \( \mu \text{m} \) (although both nominal thicknesses were 600 \( \mu \text{m} \)).
The emulsions were scanned using air objectives under low magnification and $\pi - \mu$ vertices were sought. The apparent ending of each $\mu$-meson was examined under the same low magnification and if it appeared to end in the body of the emulsion the whole event was examined under high magnification using oil objectives. It is felt that the $\mu$-mesons ending more than about 30 pm from either surface (in the unprocessed emulsion) would be detected as ending in the emulsion in the low power examination. We consider only complete $\pi - \mu - e$ events lying within a single emulsion and so we are sure of the initial direction of motion of the $\mu$-meson whose ultimate decay we also observe.

The $\mu$-mesons frequently suffer strong scattering but because they are very non-relativistic their spins are not rotated appreciably by these (purely electric) deviations. The angle $\theta$ which we want to measure is therefore that between the initial directions of the $\mu$-meson and decay electron tracks.

The readings taken for each event were: the angles (read to 1°) of the $\mu$-meson and electron tracks projected in the plane of the emulsion; and depth measurements (read to 1 pm) for the $\pi - \mu$ vertex, the point along the $\mu$-meson track whose projection on the plane of the emulsion was 50 pm from the $\pi - \mu$ vertex, the $\mu - e$ vertex and the point 50 pm in projected distance along the electron track; in addition depth measurements were made for the top and bottom surfaces of the emulsion at each $\mu - e$ vertex so that the shrinkage factor was continuously monitored and we knew the closest approach of each $\mu - e$ vertex to an emulsion surface. Sometimes smaller projected distances than 50 pm had to be used for the $\mu$-meson track because of scattering. Smaller projected distances were also used for a few very steep electron tracks as a precaution against the effects of distortion.

From these measurements and from the known original unprocessed thicknesses $\cos \theta$ was computed. The computation was programmed for EDSAC, the electronic digital computer at Cambridge University. Because of our knowledge of the original emulsion thicknesses the determination of $\cos \theta$ is rather accurate and introduces no appreciable error into our final analysis of the asymmetry.

Measurements were made on 3021 complete $\pi - \mu - e$ events in ordinary G5 emulsion and on 3128 complete events in the twice diluted emulsion.

§ 3. Analysis of the Results

These events should not be analysed directly because one may suspect a bias in favour of backward-moving electrons for those $\mu$-mesons which stop very close to either emulsion surface. This bias has two origins: firstly, in many cases the decay electron can be seen even under the low power used for scanning and, if the electron is emitted into the body of the emulsion, this may draw attention to events in which the $\mu$-meson stops so close to one surface that it might have been rejected as having left the emulsion had the decay electron been emitted, unnoticed, through
the surface of the emulsion; secondly, when the \( \mu \)-meson is identified as stopping in the emulsion, but close to a surface, there is a greater chance of failing to find the electron, under oil, if it is emitted out of the surface than if emitted (generally backwards) into the body of the emulsion. Because of this possibility of bias we analysed the results in two groups according to the distance of the \( \mu-e \) vertex from the nearer surface. The first group, called the 'centre group', is those events for which the \( \mu-e \) vertex is further than 50 \( \mu \)m from either surface of the (unprocessed) emulsion. The second group, the 'surface group', is the remainder. The figure 50 \( \mu \)m was chosen because it was felt to be a generous upper limit to the distance from a surface at which a stopping \( \mu \)-meson might be mistakenly rejected as having left the emulsion in the low power scan to find the events. A warning sign that more detailed analysis is needed would be that this surface group of events show an anomalous apparent preference for backward emission of the electron.

In the ordinary G5 emulsion the centre group consisted of 2789 complete events, while 3 \( \mu \)-mesons which ended in the same region of the emulsion did not give decay electrons which could be identified with certainty. The corresponding figures for the twice diluted emulsions were 2731 and 11. These numbers of missed electrons are so small as to have a completely negligible effect on the reliability of the results.

The best way to analyse the results was considered carefully. The method chosen was to assume that the distribution was of the form \( 1+A \cos \theta \) and to compute the mean value of \( \cos \theta \), which is equal to \( A/3 \). It may be shown that, when the number \( N \) of events is large and when \( A \) is small, the answer given by this method approximates very closely to the maximum likelihood estimate and that the standard deviation in \( A \) so deduced is \( \sqrt{3/N} \).

The results of this analysis are presented in table 1.

<table>
<thead>
<tr>
<th>Emulsion</th>
<th>Group</th>
<th>Events</th>
<th>( A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>G5</td>
<td>centre</td>
<td>2789</td>
<td>(-0.149 \pm 0.033)</td>
</tr>
<tr>
<td>G5</td>
<td>surface</td>
<td>232</td>
<td>(-0.198 \pm 0.11)</td>
</tr>
<tr>
<td>\times 2</td>
<td>centre</td>
<td>2731</td>
<td>(-0.190 \pm 0.033)</td>
</tr>
<tr>
<td>\times 2</td>
<td>surface</td>
<td>397</td>
<td>(-0.090 \pm 0.087)</td>
</tr>
</tbody>
</table>

We see at once that the asymmetry is established with virtual certainty. The second observation is that the events of the surface groups do not show a stronger backwards asymmetry than the bulk of the events in the corresponding centre groups. This, together with the generous nature
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of the 50 $\mu$m thickness of the surface zones† and the fact that we miss only 1% of the electrons from the centre zone† makes us feel confident that the values of $A$ displayed in table 1 for the centre groups are not biased. These $A$ values for the centre groups we shall use later, rejecting all events in the 50 $\mu$m surface groups.

We also note that the asymmetry observed for the twice diluted emulsions is a little stronger, though scarcely significantly so, than that for the ordinary G5 emulsion. This is at any rate consistent with the idea

Angular distribution of the $\pi-\mu-e$ decays observed in ordinary G5 emulsion (top graph) and in twice-diluted emulsion, containing only half as much halide relative to gelatin (bottom graph). The errors shown are the square roots of the numbers of events represented by the points and the full lines correspond to the asymmetry coefficients displayed in table 1 for distributions of the form $1 + A \cos \theta$, where $\theta$ is the angle between the initial directions of motion of the $\mu$-meson and electron. The events shown here are only those of the centre groups for which the $\mu-e$ vertex is at least 50 $\mu$m from either surface of the unprocessed emulsion.

† Even when the analysis was repeated for surface zones of only 25 $\mu$m no significantly stronger backward asymmetry emerged.

‡ In a very few cases what appeared to be a decay electron was displaced by a few microns from the end of the $\mu$-meson track. Scattering of the electron through a large angle very close to its origin is most unlikely and it is conceivable that we are here witnessing the diffusion of a neutral muonium atom prior to its decay. We are informed by Professor G. Bernardini that a similar phenomenon has been noticed by workers in Rome.
that the quenching of the $\mu$-meson spin orientation takes place chiefly in the halide. In the figure we display the angular distributions themselves, grouping the events in 0.2 intervals of $\cos \theta$. The distributions are well-behaved and quite consistent with the assumed form of $1 + A \cos \theta$.

For comparison the results of other workers using emulsions (see § 1) (in all cases ordinary G5) are gathered in table 2.

<table>
<thead>
<tr>
<th>Authors</th>
<th>Events</th>
<th>$A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bhowmik et al.</td>
<td>1582</td>
<td>$-0.08 \pm 0.05$</td>
</tr>
<tr>
<td>Biswas et al.</td>
<td>2003</td>
<td>$-0.095 \pm 0.044$</td>
</tr>
<tr>
<td>Castagnoli et al.</td>
<td>1028</td>
<td>$-0.222 \pm 0.067$</td>
</tr>
<tr>
<td>Friedman and Telegdi</td>
<td>1300</td>
<td>$-0.12 \pm 0.05$</td>
</tr>
</tbody>
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§ 4. THE TRUE ASYMMETRY COEFFICIENT

Our first objective, the demonstration of a clear asymmetry, is reached and we now consider how we may infer the true asymmetry coefficient of the fundamental $\pi-\mu-e$ distribution: $1 + \alpha \cos \theta$. This we are obviously unable to do without calling on some other data from which we may determine the fraction $\gamma$ of the $\mu$-mesons which are quenched in the emulsion and lose their polarization before decaying.

As we remarked earlier the Columbia experiments may not be interpreted directly for a variety of reasons; they may be used, however, to give a quite accurate indication of the relative initial quenching in carbon and G5 nuclear emulsions. If, furthermore, we may assume that there is little quenching in carbon we may use the Columbia data to determine the quenching $\gamma$ in emulsion, which, again following the Columbia data, we take to be rapid compared with the lifetime of the $\mu$-meson. We would then have the true asymmetry coefficient $\alpha = A/(1 - \gamma)$ where $A$ is our apparent coefficient of table 1. The assumption that quenching is weak in carbon is likely to be a good one because just the same apparent asymmetry is observed for a variety of substances at Columbia and also at Liverpool (Cassels et al. 1967) including carbon and copper which certainly differ widely in their structure. It would be very surprising if the quenching were appreciable but accurately the same in such different substances. It is also known (Cassels et al. 1967) that with very good accuracy no loss of memory of the $\mu$-mesons' spin orientation in carbon takes place over periods as long as 4 microseconds. Since carbon is a pure substance the most reasonable conclusion is that the quenching in it is small. We make this assumption.

The Columbia workers measure the emission of electrons in a fixed counter as the $\mu$-mesons process in a magnetic field. This emission shows a succession of maxima and minima as the magnetic field is changed
because of the asymmetric emission of the electron relative to the 
$\mu$-meson's spin orientation. Call the observed peak-to-valley ratio for 
this emission $R_c$ when the $\mu$-mesons are stopped in carbon and $R_e$ when 
they are stopped in nuclear emulsion. One may now easily show that, 
irrespective of the bad geometry, lengthy time gate and unknown energy 
response of the detecting equipment, and of the unknown degree of 
polarization of the $\mu$-meson beam,

$$\gamma = 1 - \frac{R_e - 1}{R_e + 1} \frac{R_c - 1}{R_c + 1}.$$ 

We use the results $R_e = 1 \cdot 86 \pm 0.07$ and $R_c = 1 \cdot 40 \pm 0.07$ to deduce 
$\gamma_{0s} = 0.45 \pm 0.11$.

Before considering this result we turn to the problem of determining 
$\gamma_{xy}$, the quenching in the twice diluted emulsion of the second half of 
the experiment. This we may do, continuing our hypothesis that the 
quenching takes place chiefly in the halide. We do not need to assume, 
however, that all $\mu$-mesons which stop in the halide are quenched. We 
find

$$\gamma_{xy} = \frac{1 - 0.86(1 - p)}{2 - 0.86(2 - p)}.$$ 

In the expression 0.86 is the proportion by weight of halide in ordinary 
G5 emulsion and $p$ is the mass stopping power of halide relative to 
gelatin. $p$ may be deduced from the standard datum that the number of 
particles ending their tracks in halide and gelatin in ordinary G5 emulsion 
stand in stopping ratio 7:3. This gives $p = 0.38$ so $\gamma_{xy}/\gamma_{0s} = 0.77$ and 
$\gamma_{xy} = 0.35 \pm 0.10$. The final result is, however, extremely insensitive 
to this assumed stopping ratio and changes in it over a factor of two 
produce changes in $\gamma$ small compared with errors from other sources. 
We have increased the error in the above value of $\gamma_{xy}$ to allow for this 
uncertainty in the stopping ratio.

We now use these values of $\gamma$ to correct our observed centre group $A$ 
values of table 1 to find the true asymmetry coefficient $\alpha$. We display 
the results in table 3.

| Table 3. The Observed Asymmetry Coefficients $A$, the Quenching 
Factors $\gamma$, and the Deduced True Asymmetry Coefficients $\alpha$ for 
the Two Types of Emulsion. The Mean Value of $\alpha$ Quoted Allows 
for the Fact that the Error in $\gamma$ is Common to the Two Emulsions |
<table>
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<tbody>
<tr>
<td>Emulsion</td>
<td>$A$</td>
<td>$\gamma$</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>G5</td>
<td>$-0.149 \pm 0.033$</td>
<td>$0.45 \pm 0.11$</td>
<td>$-0.271 \pm 0.080$</td>
</tr>
<tr>
<td>$\times 2$</td>
<td>$-0.190 \pm 0.033$</td>
<td>$0.35 \pm 0.10$</td>
<td>$-0.282 \pm 0.067$</td>
</tr>
<tr>
<td>Mean value $\alpha = -0.283 \pm 0.060$</td>
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</table>
Our best value $\alpha = -0.283 \pm 0.060$ has been worked out allowing for the common error in $\gamma$ (which is more serious than those in the $A$ values).

We now consider the effect of errors in our assumptions. Even if as much as 20% of the $\mu$-mesons are rapidly quenched in the carbon of the Columbia experiment, instead of none as we have assumed, our final result for $\alpha$ would be made more negative by less than the above standard error. Again, even if it is un-plausibly contended that the quenching in nuclear emulsion is complete for the gelatin phase instead of zero as we have assumed then our final value for $\alpha$ becomes more negative just to the limit of its present standard error. The two values for $\alpha$ derived from the two different emulsions are then in much poorer agreement, however, and this in itself argues for the correctness of our more reasonable assumption that the gelatin phase does not quench strongly relative to the halide phase. Finally the result is very insensitive to the assumption that some $\mu$-mesons are quenched immediately in nuclear emulsion and that the rest retain their memory until they decay. Even if we make the diametrically opposite assumption, which is certainly contrary to the evidence of the Columbia experiments, namely that there is no immediate quenching and that all their lowered asymmetry in emulsion is due to a gradual loss of polarization exponentially with time, our result for $\alpha$ merely becomes more negative by 0.02 which is well within the error limits. This fortunate insensitivity is due to a happy choice of the time gate in the Columbia experiment. We see that our result for $\alpha$ is surprisingly insensitive to the assumptions made in deriving it. It is consistent with the extreme value $\alpha = -\frac{1}{2}$ allowed by the two-component neutrino theory and relaxation of any of our assumptions moves the experimental result into better agreement with that extreme theoretical value. Alternatively we may accept the two-component theory and use our result to determine its parameter $\xi$ (Lee and Yang 1957) which measures a relationship between the axial vector and vector couplings,

$$\xi = (|V_{ud}|^2 + |V_{ut}|^2)^{-1} (V_{ud}^* V_{ud}^* + V_{ut}^* V_{ut}^*).$$

We find $\xi = 0.86 \pm 0.18$.

**Acknowledgments**

We thank Professor J. M. Cassels and Professor H. W. B. Skinner of Liverpool University who very kindly allowed us to make the exposure.
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to the $\pi^{+}$-meson beam. We thank Dr. H. E. Daniels for considerable help with the statistical aspects of the analysis. We thank especially Dr. A. J. Oxley who wrote the programme for EDSAC and who was ever-helpful in processing the data; we also thank Dr. M. V. Wilkes who allowed us the use of this machine.

G. B. Chadwick, S. A. Durrani, P. B. Jones and J. W. G. Wignall wish to thank respectively the following bodies for grants: the Shell Petroleum Company; Gonville and Caius College, Cambridge; the D.S.I.R.; the Royal Commissioners of the 1861 Exhibition.

REFERENCES