Measurement of the Regeneration Phase in Carbon from 4 to 10 GeV/c

W. C. Carithers,† T. Modis,‡ D. R. Nygren,§ T. P. Pun,‖ E. L. Schwartz,¶ and H. Sticker**
Columbia University, New York, New York 10027
and
J. H. Christenson
New York University, New York, New York 10003
(Received 20 January 1975)

A regeneration experiment exploring \( K_s - K_L \) interference in the decay modes \( K_{s,L} \rightarrow \pi^+\pi^- \) and \( K_{s,L} \rightarrow \pi^+\nu \) \((l=\mu \text{ or } e)\) has been performed at the Brookhaven National Laboratory alternating-gradient synchrotron. The regeneration phases in carbon obtained from the time-dependent charge asymmetry of the \( K_{s} \) and \( K_{l} \) modes are in good agreement and yield a combined result \( \phi L = \text{arg} \{f(0) - f(0)\} = -40.9^\circ \pm 2.6^\circ \) at the average \( K^0 \) momentum of 7.5 GeV/c.

Since the discovery of \( CP \) nonconservation,\(^1\) the phenomenon of coherent \( K_s \) regeneration from an initially pure \( K_L \) beam has been exploited in a number of experiments\(^8\) designed to measure the phase \( \phi_+ \) of the \( CP \)-nonconservation parameter,\(^3\)

\[
\eta_+ = \langle \pi^+\pi^- | T | K_L \rangle / \langle \pi^+\pi^- | T | K_s \rangle = |\eta_+| \exp(i\phi_+).
\]

The principal limitation in this type of experiment arises from the uncertainty of the regeneration phase, \( \phi_+ \), which enters directly in any interference of the \( K_L \) and regenerated \( K_s \) amplitudes.

After traversing a block of matter (the regenerator), a pure \( | K_L \rangle \) beam is transformed into a coherent mixture \( | \rangle = a | K_L \rangle + b | K_S \rangle \). The regeneration amplitude, defined at the exit face of the regenerator for the undeflected beam, is

\[
\rho = b/a = |\rho| \exp(i\phi_+),
\]

\[
\frac{\rho N L \{ f(0) - f(0) \}}{P_K} = \frac{1 - \exp \left( (\pm m_0 - \Gamma /2)L M_K / P_K \right)}{1 - \exp \left( (\pm m_0 - \Gamma /2)L M_K / P_K \right)},
\]

where \( N \) is the atomic density, \( L \) is the length of the regenerator, \( P_K \) is the \( K \) momentum, and \( f(0) \) is the \( K_0 \) (\( K^0 \)) nucleus forward scattering amplitude. In the determination of \( \phi_+ \), the poorest known part is \( \phi_+ = \text{arg} \{f(0) - f(0)\} \).

Several methods have been employed for the determination of \( \phi_+ \).\(^4\) The method followed here utilizes the time-dependent charge asymmetry in the decay modes \( K_s \rightarrow \pi^+l^-\nu_l \), where \( l \) is either a muon or electron. The charge asymmetry after a regenerator is defined by \( \delta(\tau) = \Gamma_\pm(\tau) - \Gamma_\mp(\tau) / \left( \Gamma_\pm(\tau) + \Gamma_\mp(\tau) \right) \), where \( \Gamma_\pm (\Gamma_\mp) \) refers to positive (negative) leptonic decay rate, and is given to sufficient accuracy for the present discussion by

\[
\delta(\tau) = 2K \left( \cos(\Delta m \tau + \phi_+) + \alpha \cos(\Delta m \tau + \phi - \phi') \right) \pm \delta_L.
\]
Here $\chi = [1 - |x|^2] / (1 - |x|^2)$, where $x$ is the ratio of possible $\Delta S = - \Delta Q$ amplitudes to $\Delta S = + \Delta Q$ amplitudes, $\tau$ is proper time measured from the exit face of the regenerator, and $\delta_L$ is the observed $CP$-nonconserving $K_L^0$ charge asymmetry. The first term enclosed by the curly brackets of Eq. (3) arises from the coherently produced (transmission regeneration) events; $\alpha$ and $\phi'$ are introduced to account for incoherently produced events caused by diffractive and/or inelastic scattering within the regenerator. At short proper time, the observed asymmetry can be of the order of 0.1, whereas the asymptotic value given by $\delta_L$ is known to be $\sim (3.4 \pm 0.2) \times 10^{-3}$.

The apparatus (Fig. 1) is essentially the same as that used to search for $K_L^0 - \mu^+ \mu^-$, except for the replacement of the vacuum decay region by a regenerator assembly and helium decay region. The bulk of the data was taken with a carbon regenerator 81.28 cm long, density 1.72 g/cm$^3$, and with chemical impurities measured to be less than 0.1%. The veto counter, in contact with the exit face, was 0.32 cm thick and dc coupled to dead-time-free electronics. The event trigger required the traversal of the spectrometer by exactly two tracks, and that no veto count was present. The magnetic field was reversed regularly to allow the cancelation of possible apparatus asymmetry.

The analysis of the semileptonic decays is substantially more complicated than that of the $\pi^+ \pi^-$ mode because of the missing kinematic information for the neutrino. It is impossible to separate event by event the coherently produced events from the incoherently produced events, and the calculation of the $K$ momentum (and hence proper time) contains an inherent twofold ambiguity. Space limitations preclude a complete discussion of the analysis here, and an appropriate presentation will be published elsewhere.

Constraints on event reconstruction were described by the following quantities: (1) the observed "kink" angle of each trajectory in a plane parallel to the magnetic field; (2) the closest distance of approach (CDA) of the trajectories in the decay region; (3) the variable $\Delta \nu = |P_\mu|_c.m. - P_\pi$, i.e., the magnitude of the neutrino momentum as determined from the invariant mass of the pion-lepton system minus the observed transverse momentum of the pion-lepton system; (4) the visible longitudinal momentum $P_L' = (P_\mu + P_\pi)_z$; and (5) the invariant mass computed under various decay-mode hypotheses. For each event, a $\chi^2$ sum for the two trajectory kinks and CDA was computed.

The following requirements were applied to the $K_{}\Lambda$ decay candidates: (1) $\chi^2 < 10$, designed to reject erroneously matched track segments, $\pi^- \mu^-$ decays within the spectrometer, and other backgrounds; (2) $M_{\pi\pi} < 460$ MeV/$c^2$, intended to reject feedthrough of the relatively copious $K_S^0 - \pi^+ \pi^-$ decays; (3) $M_{\pi\pi}$ outside the interval 1105–

![FIG. 1. Views of the apparatus. Chambers A, B, and D are multiwire proportional chambers. After passing through a hole in the wall directly behind the trigger counters, the neutral beam was absorbed in a lead and uranium insert.](image-url)
1125 MeV/c², designed to reject feedthrough of Λ → pπ decays; (4) \( -2 < \Delta \nu < 118 \text{ MeV/c} \), designed to discriminate against the incoherent events; (5) \( P'_\mu \leq 10 \text{ GeV/c} \); (6) \( 1 \leq P_\mu \leq 8 \text{ GeV/c} \); (7) \( P'_\mu > 2 \text{ GeV/c} \); (8) various fiducial-volume, spectrometer, and muon-hodoscope boundary cuts to ensure proper registration of the event; (9) in-bending trajectories, i.e., both trajectories are bent towards the beam center line; and (10) exactly one muon track, defined by appropriate counts in both horizontal and vertical hodoscope arrays along the extrapolated trajectory. The number of events which satisfy the \( K'_{\mu 3} \) criteria is \( 3.5 \times 10^6 \), with an estimated background less than \( 1\% \).

The \( K'_{e 3} \) decays are less prone to background contamination and have been subjected to less restrictive requirements: (1) \( \chi^2 \leq 16 \); (2) \( \Delta \nu > -2 \text{ MeV/c} \); (3) \( P'_\mu < 11 \text{ GeV/c} \); (4) \( 1 < P_\mu < 7 \text{ GeV/c} \); (5) \( 1 < P'_\mu < 8 \text{ GeV/c} \); (6) appropriate fiducial, spectrometer, and Cherenkov acceptance cuts; and (7) in-bending trajectories. The number of events passing the \( K'_{e 3} \) criteria is \( 7 \times 10^6 \) with negligible background.

The twofold ambiguity in the proper-time computation has been dealt with by defining an apparent proper time \( \tau' = M_k (z - z_{\text{vert}}) / P'_k \), where \( z \) is the location of the decay vertex given by the CDA calculation. The relationship of \( \tau \) and \( \tau' \) was then obtained by use of a Monte Carlo-derived transformation matrix \( A: \delta \tau' = \sum \delta \tau \) transformed by \( A \). The requisite accuracy of the Monte Carlo simulation was carefully checked by high-statistics comparisons of a large number of variable distributions for both \( K^0 \rightarrow \pi^0 \pi^0 \) and \( K^0 \rightarrow \pi l \nu \) modes. As the regeneration phase and amplitude are both expected to vary slowly with momentum, the data have been analyzed separately in 1-GeV/\( c \) \( P'_k \) intervals. For clarity of presentation here, however, the data have been summed over momentum and are shown in Fig. 2.

The understanding and treatment of the inelastically and diffractively produced events have been facilitated by a comparison of the easily separated coherent and incoherent \( K^0 \rightarrow \pi^+ \pi^- \) events. Unlike previous work, the present trigger accepts the latter with relatively high efficiency. The detailed analysis, lengthy but straightforward, shows that with judiciously cho-

\[ \varphi' = (-41.6^\circ \pm 2.6^\circ) - 65^\circ \left( \frac{\Delta m - 0.540}{\Delta m} \right) - 30^\circ \left( \frac{\Gamma_5 - 1.124}{1.124} \right). \]
TABLE I. Results for the regeneration phase referred to values $\Delta m=0.540$ and $\Gamma_S=1.124$. $(P_K)$ is the true mean $K^+$ momentum for the indicated $P_K'$ interval. Momentum units are GeV/c; $\varphi$ units are rad.

<table>
<thead>
<tr>
<th>$P_K'$</th>
<th>$\langle P_K' \rangle$</th>
<th>$K_{e3}$</th>
<th>$\varphi_f$</th>
<th>$\langle P_K \rangle$</th>
<th>$K_{p3}$</th>
<th>$\varphi_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3–4</td>
<td>4.5</td>
<td>-0.64±0.20</td>
<td>5.6</td>
<td>-0.66±0.17</td>
<td>5.6</td>
<td>-0.66±0.17</td>
</tr>
<tr>
<td>4–5</td>
<td>5.6</td>
<td>-0.96±0.13</td>
<td>6.7</td>
<td>-1.00±0.18</td>
<td>6.7</td>
<td>-1.00±0.18</td>
</tr>
<tr>
<td>5–6</td>
<td>6.6</td>
<td>-0.82±0.12</td>
<td>7.7</td>
<td>-0.64±0.16</td>
<td>7.7</td>
<td>-0.64±0.16</td>
</tr>
<tr>
<td>6–7</td>
<td>7.6</td>
<td>-0.74±0.12</td>
<td>8.7</td>
<td>-0.63±0.18</td>
<td>8.7</td>
<td>-0.63±0.18</td>
</tr>
<tr>
<td>7–8</td>
<td>8.5</td>
<td>-0.56±0.17</td>
<td>10.0</td>
<td>-0.79±0.19</td>
<td>10.0</td>
<td>-0.79±0.19</td>
</tr>
<tr>
<td>8–9</td>
<td>9.5</td>
<td>-0.88±0.18</td>
<td>11.2</td>
<td>-0.66±0.42</td>
<td>11.2</td>
<td>-0.66±0.42</td>
</tr>
<tr>
<td>9–10</td>
<td>10.4</td>
<td>-0.26±0.20</td>
<td>11.2</td>
<td>-0.66±0.42</td>
<td>11.2</td>
<td>-0.66±0.42</td>
</tr>
<tr>
<td>10–11</td>
<td>11.2</td>
<td>-0.66±0.42</td>
<td>11.2</td>
<td>-0.66±0.42</td>
<td>11.2</td>
<td>-0.66±0.42</td>
</tr>
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Subsequent to the completion of the analysis, new results for $\Delta m$ have become available which significantly affect the average $\Delta m$ value and uncertainty. For reasons discussed in the following Letter, we prefer to employ the value $\Delta m/h = (0.5348±0.0021) \times 10^{-11}$ sec$^{-1}$, leading to the result $\varphi_f = -40.9^\circ ± 2.6^\circ$.

This result may be compared with an optical-model calculation using input values of Ref. (0) and $\varphi_f(0)$ obtained by dispersion-relation calculations. The results of Lusignoli et al. yield $\varphi_f = -41^\circ ± 7^\circ$ whereas those of Carter yield $\varphi_f = 50^\circ ± 7^\circ$. The errors quoted are estimates of the input errors and do not reflect possible inadequacies of the optical model in this context.

Nevertheless, the experimental results for $\varphi_f$, given in Table I can be directly utilized in the analysis of interference in $K^+\pi^- \rightarrow \pi^+\pi^-$ decays, the subject of the following Letter.

We wish to acknowledge the outstanding efforts of Bill Sippach, Yin Au, and many other members of the Nevis Laboratory staff for their contributions to the success of the experiment. It is a pleasure to thank Dr. David Berley and S. Ozaki for the generous support provided by Brookhaven National Laboratory. Finally, we wish to acknowledge the numerous contributions of Professor Jack Steinberger in the conception and development of the experiment.

*Research supported by the National Science Foundation.
‡Present address: CERN, Geneva, Switzerland.
§Present address: Lawrence Berkeley Laboratory, Berkeley, Calif. 94720.
∥Present address: Stanford Linear Accelerator Center, Stanford, Calif. 94305.
¶Present address: Brain Research Laboratory, New York Medical College, New York, N.Y.
**Present address: Rockefeller University, New York, N.Y. 10021.
9The value of $\Delta m$ as given by V. Chaloupka, Phys. Lett. 50B, 1 (1974); the value of $\Gamma_S$ as given in the following Letter; the value of $\delta_2$ as given in Ref. 5.